Thermal Expansion of Solids

Thermal expansion

Definition. Increase in dimensions of a solid on heating, is called thermal expansion of the solid. All dimensions increase in same proportion.

Types

(i) Linear expansion. A rod or wire has length only. When heated, its length increases. Increase in length is called linear expansion.

(ii) Superficial expansion. A sheet has area. When heated, its area increases. Increase in area is called superficial expansion.

(iii) Cubical expansion. A body has volume. When heated, its volume increases. Increase in volume is called cubical expansion.

Coefficient of linear expansion

Introduction. Let a rod of certain material have length L_0 at 0°C. Let the length become L_t at t°C.

Then, linear expansion $= L_t - L_0$.

This linear expansion has been found to depend upon the following factors :

(*i*) *Original length* (L_0), and varies directly with it,

i.e., $$L_t - L_0 \propto L_0.$$

(*ii*) *Rise in temperature* (t), and varies directly with it,

i.e., $$L_t - L_0 \propto t.$$

It also depends upon the material of the rod.

Combining above two relations,

$$L_t - L_0 \propto L_0 t$$

or $$L_t - L_0 = \alpha L_0 t \quad \Rightarrow \quad L_t = L_0 (1 + \alpha t) \quad ...(1)$$

where α is constant of proportionality whose value depends upon the material of the rod.

It is called *coefficient of linear expansion* of the material of the rod.

Definition. In equation (1),

if $$L_0 = 1, t = 1,$$

then, linear expansion, $L_t - L_0 = \alpha$.

Hence, the coefficient of linear expansion of the material of the rod may be defined as the increase in length of a rod of unit length for one degree rise in temperature.

From Eq. (1), $$L_t - L_0 = \alpha L_0 t$$

or $$L_t = L_0 (1 + \alpha t) \quad ...(2)$$

From Eq. (2), L_t can be found, if L_0, α and t are known.

In general, if a rod has length L_1 at t_1°C and L_2 at t_2°C, we may prove that

$$L_2 = L_1 [1 + \alpha(t_2 - t_1)] \quad ...(3)$$

Equation (3) is possible because α has small value and α^2 is negligible.

Coefficient of superficial expansion

Introduction. Let a sheet of certain material have area A_0 at 0°C. Let the area become A_t at °C.

Then, like linear expansion, superficial expansion $(A_t - A_0)$ depends upon original area (A_0) and rise in temperature (t).

It also depends upon the material of the sheet.

Hence, (proceeding as in linear expansion)

$$A_t - A_0 = \beta A_0 t \quad ...(1)$$

where β is constant of proportionality whose value depends upon the material of the sheet. It is called *coefficient of superficial expansion* of the material of the sheet.

Definition. In equation (1),

if $A_0 = 1, t = 1$

then, superficial expansion, $A_t - A_0 = \beta$.

Hence, the coefficient of superficial expansion of the material of the sheet may be defined as the increase in area of a sheet of unit area for one degree rise in temperature.

From Eq. (1), $A_t - A_0 = \beta A_0 t$

or $$A_t = A_0(1 + \beta t) \quad ...(2)$$

From Eq. (2), A_t can be found if A_0, β and t are known.

In general, if a sheet has area A_1 at t_1°C and A_2 at t_2°C, we may prove that,

$$A_2 = A_1[1 + \beta(t_2 - t_1)] \quad ...(3)$$

Equation (3) is possible because β has small value and β^2 is negligible.

Coefficient of cubical expansion

Introduction. Let a body of certain material have volume V_0 at 0°C. Let the volume become V_t at t°C.

Then, like linear and superficial expansion, cubical expansion $(V_t – V_0)$ depends upon original volume (V_0) and rise in temperature (t). It also depends upon the material of the body.

Hence, proceeding as in linear expansion,

$V_t - V_0 = \gamma V_0 t$

where γ is constant of proportionality whose value depends upon the material of the body. It is called coefficient of cubical expansion of the material of the body.

Definition. In Eq. (1),

if $V_0=1$, t=1,

then, cubical expansion, $V_t - V_0 = \gamma$.

Hence, the coefficient of cubical expansion of the material of the body may be defined as the increase in volume of a body of unit volume for one degree rise in temperature.

From Eq. (1), $V_t - V_0 = y V_0 t$

or $V_t = V_0[1 + \gamma(t_2 - t_1)]$ ... (2)

From Eq. (2), Vt can be found if V_0, γ and t are known.

In general, if a body has volume V_1 at t°C and V_2 at t_2°C we may prove that

$V_2 = V_1[1 + \gamma(t_2 - t_1)]$

Equation (3) is possible because γ has Small value and γ^2 is negligible.

Relation between α, β and γ by differential method

(a) Relation between α and β

As,
$$A = L^2$$
$$\Delta A = 2L\Delta L$$
$$\frac{\Delta A}{A} = \frac{2L\Delta L}{L^2} = \frac{2\Delta L}{L}$$

For change in temp. t, $\frac{\Delta A}{At} = 2\frac{\Delta L}{Lt}$ or $\beta = 2\alpha$

(b) Relation between α and γ

As,
$$V = L^3$$
$$\Delta V = 3L^2\Delta L$$
$$\frac{\Delta V}{V} = \frac{3L^2\Delta L}{L^3}$$
$$\frac{\Delta V}{V} = \frac{3\Delta L}{L}$$

For temp. change t, $\frac{\Delta V}{Vt} = \frac{3\Delta L}{Lt}$ or $\gamma = 3\alpha$.

Viva Voce

Question.1. Which part of the bimetallic strips lie outside on heating ?
Answer. The metal which has higher coefficient of linear expansion lies on the outer side.

Question.2. Where bimetallic strips are used ?
Answer. In fire alarm, thermostats, etc.