Ratio of two quantities is the fraction that one

quantity is relative to the other. Ratio of **a** to **b** is  $\frac{a}{b}$  or a : b. **a**, the first term is called **antecedent** and the second term **b** is called **consequent**. It should be noted that both **a** and **b** are of the same units.

#### Important Note:

If any ratio  $\frac{a}{b}$  is given such that  $\frac{a}{b} > 1$  or a > b, and there is an integer x such that x is +ve, then

$$\frac{a+x}{b+x} < \frac{a}{b}$$
 and  $\frac{a-x}{b-x} > \frac{a}{b}$ .

If any ratio  $\frac{a}{b}$  is given such that  $\frac{a}{b} < 1$  or a < b, and there is an integer x such that x is +ve, then

$$\frac{a+x}{b+x} > \frac{a}{b}$$
 and  $\frac{a-x}{b-x} < \frac{a}{b}$ .

# Proportion

A proportion is a statement of equality that exists between two ratios.

**Example:** 1 : 4 :: 2 : 8 is a proportion consisting of four terms. The first and the last terms are called extremes, and the second and the third terms are called the means.

# **Direct Proportion**

Two variables are directly proportional if their corresponding values have constant ratio, if one quantity is multiplied or divided by the same number, the ratio of the variables remains unchanged.

Suppose 'x' & 'y' are two variable under Direct proportion,

then their ratio remains constant. i.e  $\frac{x}{v}$  = constant.

Hence, for two iterations in which the values of x are  $x_1$ and  $x_2$ , and the corresponding values of y are  $y_1 \& y_2$ 

Then,  $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ .

By componendo and dividendo property

If 
$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$
, then  $\frac{x_1 + y_1}{x_1 - y_1} = \frac{x_2 + y_2}{x_2 - y_2}$ 

### Example :

If one cup of oil can be obtained by crushing 50 groundnuts, then how many cups of oil can be obtained by crushing 150 groundnuts?

#### Solution :

	No. of Groundnut	No. of cups
Case – I	50	1

Case – II 150 y Since no. of Groundnuts is more, then obviously

we'll have more no. of cups of oil. So it is a clear case of Direct proportion.

$$\frac{50}{1} = \frac{150}{y}$$
$$\therefore y = 3$$

 $\therefore$  No. of cups obtained = 3.

# **Inverse Proportion**

Two variables are inversely proportional, if they are so related that an increase in one variable results in corresponding decrease in the other, and a decrease in one variable results in a corresponding increase in the other. In this case product of the two variable remains same. Hence, for the case or iteration for two values of x to be  $x_1$  and  $x_2$  and the corresponding values of y to be  $y_1$  and  $y_2$ ,

then,  $x_1y_1 = x_2y_2$ 

Example :

If one man can build a shed in 12 days, then how long will 4 men take to build the shed?

Solution :

	No. of men	No. of days
Case – I	1	12
Case – II	4	y

Since no. of men are increasing, hence work will be divided and it will take lesser no. of days. This is a case of inverse proportion. Hence, the product will remain constant.

So, it will take 3 days for 4 men to build the shed.

# Partnership

When two or more persons invest their money in a joint business, they are called partners and their association is known as partnership.

# **Types of Partnership**

# 1. Simple partnership

If the money of the partners are invested for equal interval of time, the partnership is called **simple**.

# Example :

A, B and C invested ₹1,000, ₹600 and ₹400 respectively to start a business. The profit is ₹200 which is to be divided among A, B and C in the ratio of their capital invested. How much share does each get?

2

# Solution :

Ratios of investment of A, B and C

A's share of profit = 
$$\frac{5}{10}$$
 × 200 = ₹100

B's share of profit = 
$$\frac{3}{10}$$
 × 200 = ₹60

C's share of profit = 
$$\frac{2}{10}$$
 × 200 = ₹40

# 2. Compound partnership

If the money of the partners are invested for different intervals of time, the partnership is called **compound.** 

# Example :

A, B and C enter into a partnership with an amount of ₹10,000 each. After 4 months, A invests an additional ₹2,000. Three months later, B invests ₹4,000, and C at the same time withdraws ₹2,000. The profit at the end of the year was₹2,17,000. What are their respective shares if C is allowed ₹2,000 as salary per month from profits at the end?

# Solution :

Ratio in which profits are to be shared

Salary of C = 2000 × 12 = ₹24,000

A's share = 
$$\frac{193000 \times 68}{193}$$
 = ₹68,000  
B's share = ₹70,000  
C's share = ₹55.000



# Important Note:

In the problems related to partnership, we always divide the profits in the proportion of the capital invested by the partners unless some special conditions are given.

# Alligation

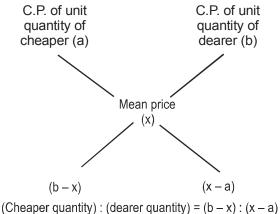
The word alligation means 'linking' and states that when different quantities of same or different ingredients of different value are mixed together to produce a mixture of a mean value, the ratio of their quantities are inversely proportional to the differences in their value from the mean value.

This rule is also applicable in case of their cost prices.

# **Rule of Alligation**

Quantity of cheaper	CP of dearer – Mean Price	
Quantity of dearer	Mean price – CP of cheaper	
It can also be represented as:		

It can also be represented as:



# Removal by equal amount

Consider a vessel containing only ingredient 'x'. From this 'a' units of ingredient is taken out and replaced by an equal amount of ingredient 'y'. This process is repeated 'n' times, then after 'n' operations,

$$\frac{\text{Amount of x left}}{\text{Amount of x originally present}} = \left(\frac{x-a}{x}\right)^n$$

# Example :

A container contained 100 kg of milk. From this container 10 kg of milk was taken out and replaced by water. This process was further repeated three times. How much milk is now contained by the container?

# 5.2

#### Solution :

Amount of liquid left after 'n' operations, when the container originally contains 'a' units of liquid from which 'b' units is taken out each time, is

$$a\left(\frac{a-b}{a}\right)^{n} \text{ units} = 100\left(\frac{100-10}{100}\right)^{4} \text{ kg}$$
$$= 100 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} = 65.61 \text{ kg}$$

#### Average

An average or an arithmetic mean is the sum of all observations divided by the total number of observations.

Average = 
$$\frac{\text{Sum of all observations}}{\text{Total number of observations}}$$

#### Example :

If the ages of five citizens in a club are 40, 45, 55, 60 and 40 years respectively, then

Average age = 
$$\frac{40 + 45 + 55 + 60 + 40}{5}$$
  
=  $\frac{240}{5}$  = 48 years.

But, suppose in the same club, 2 citizens are 40year-old, 3 of them are 45-year-old, 4 are 50-yearold and 5 are 55-year-old, then their average is

$$\frac{\{(2 \times 40) + (3 \times 45) + (4 \times 50) + (5 \times 55)\}}{(2 + 3 + 4 + 5)}$$
$$= \frac{690}{14} \text{ years.}$$

Here,  $\left(\frac{2}{14}\right)$ ,  $\left(\frac{3}{14}\right)$ ,  $\left(\frac{4}{14}\right)$  and  $\left(\frac{5}{14}\right)$  are called the weights of each category of citizens.

Weights are the fraction of the numbers in that category with respect to the total citizens in that club. This average is also called the weighted average of the club.

# Solved Examples

#### 1. Express the following in ratios.

- (i) A and B are two students who got 175 and 225 marks respectively. Find the ratio of their marks.
- (ii) X got 105 marks out of 150, and Y got 175 marks out of 200. Express the ratio of marks they got to their respective total marks in ratio.

Solution :

(i) 
$$\frac{175}{225} = \frac{7}{9} = 7:9$$
 (ii)  $\frac{105}{150}:\frac{175}{200} = 4:5$ 

2. If 5 kg of wheat flour is mixed with 500 g of sugar extract, what is the ratio of sugar extract to the rest of the mixture, after adding 1.5 kg of water? Solution :

We first need to convert all figures into one single unit.

Wheat flour = 5 kg; Water = 1.5 kg

Sugar extract = 500 g = 0.5 kg

Total weight of the mixture = 7 kg

Ratio of sugar extract to the rest of mixture

$$=\frac{0.5}{6.5}=1:13$$

3. Ratio of a : b is same as that of b : c, all three are positive integers. If a = 10 and c = 40, find b.

#### Solution :

a : b = b : c (Same proportion)  $b \times b = a \times c$ 

 $b = \sqrt{400} = \pm 20$ . As it is +ve, so, b = 20.

4. What must be subtracted from the numerator and

denominator of fraction  $\frac{6}{7}$  to give a fraction equal

to 
$$\frac{10}{21}$$
?

Solution :

Let the number to be subtracted be x.

$$\frac{6-x}{7-x} = \frac{16}{21};$$
  
On solving, we get x = 2.8

5. Pompo and Rompo are two countries engaged in a war. Pompo possesses 8 tanks and Rompo possesses 11 tanks. They get external support from neighbouring countries, of two tanks each. Which of the two countries is supposed to have a relatively greater increase in strength?

#### Solution :

Pompo's strength : Rompo's strength (initial) = 8 : 11. Pompo's strength : Rompo's strength (after aid received) = 10 : 13.

 $\frac{10}{13} > \frac{8}{11}$ 

Therefore, Pompo's strength increased in a greater proportion as compared to Rompo's strength.

6. Divide ₹1,000 between A and B in the ratio 7 : 3. Solution :

7: 3 implies 
$$\frac{7}{10}$$
 and  $\frac{3}{10}$  of ₹1,000.  
Therefore, A gets ₹700 and B gets ₹300

7. ₹1,150 is to be divided between A, B and C such that the ratio of share of A to that of B is equal to 3 : 2 and the share of B to share of C is equal to 3 : 4. Find their individual share?

Solution :

Here A : B = 3 : 2 = 
$$\frac{3}{2}$$
 and B : C = 3 : 4 =  $\frac{3}{4}$ 

[We take the LCM of the values of the common term {B over here}

e.g. LCM of 2 and 3 is 6

So A : B = 3 : 2 = 9 : <u>6</u> and B : C = 3 : 4 = <u>6</u> : 8 Therefore, A : B : C = 9 : 6 : 8

1150 can be divided between them as follows,

A's share = 
$$\frac{1150 \times 9}{23}$$
 = ₹450  
B's share =  $\frac{1150 \times 6}{23}$  = ₹300;  
C's share =  $\frac{1150 \times 8}{23}$  = ₹400

8. Which is the greatest among the following fraction?

$$\frac{13}{11}$$
,  $\frac{15}{13}$ ,  $\frac{11}{9}$  and  $\frac{12}{10}$ 

#### Solution :

Difference in the numerator and denominator is 2 in each case. Also, the fractions are all more than

- 1. Therefore, the greatest fraction is  $\frac{11}{9}$ .
- **9.** Which of the following is the smallest among following fraction?
  - $\frac{14}{25}$ ,  $\frac{57}{100}$ ,  $\frac{49}{86}$ ,  $\frac{3}{5}$ Solution :

#### Solution :

One way of solving such a problem is to convert each into decimals and then compare.

Another way is to either equalize the numerator or denominator of every two fractions that are being compared.

Suppose we have  $\frac{a}{b}$  and  $\frac{c}{d}$  to be compared.

If a = c, then the fraction with the higher value of denominator is smaller.

If b = d, then the fraction with the lower value of numerator is smaller.

Comparing 
$$\frac{14}{25}$$
 with  $\frac{57}{100}$ ,  $\frac{14}{25} = \frac{56}{100} < \frac{57}{100}$ .  
So,  $\frac{57}{100}$  is eliminated.

Now comparing 
$$\frac{14}{25}$$
 with  $\frac{49}{86}$ ,  $\frac{14}{25} = \frac{98}{175}$  and  $\frac{49}{86} = \frac{98}{172}$ . So,  $\frac{49}{86}$  is eliminated. Comparing  $\frac{14}{25}$  with  $\frac{3}{5}$ ,  $\frac{3}{5} = \frac{15}{25} > \frac{14}{25}$ .  
So,  $\frac{14}{25}$  is the smallest.

**10.** A, B and C are to be given monthly allowance in year 1996 in the ratio 5 : 3 : 1. If C's share of such allowance is ₹1,000, how much did A get for the year ended in December 31, 1996.

Solution :

C's share = ₹1,000

A's share : C's share = 5 : 1

Therefore, A's share (monthly) = ₹5,000

A's share for whole year = 5000 × 12 = ₹60,000

**11.** Ram, Sham and Pran share profits in the ratio 12 : 1 : 5. If Pran's share is ₹12,500, what is the profit of the firm?

Solution :

Pran's share =  $12500 = \frac{5}{18}$  of total profit. ∴ Total profit =  $\frac{12500 \times 18}{5} = ₹45,000$ 

12. ₹1,59,000 is to be divided in the ratio 5 : 3 : 3 among A, B and C such that a certain sum of ₹5,000 from the total is divided in the ratio 2 : 3 between A and B before such division is made. What are the shares of A and B?

#### Solution :

₹5,000 is to be divided between A and B in ratio 2 : 3.

A's share = ₹2,000, and B's share = ₹3,000.

Amount left is ₹1,54,000 and to be divided in the ratio 5 : 3 : 3.

A's share = 
$$\frac{5}{11} \times 1,54,000 = ₹70,000$$
  
B's share =  $\frac{3}{11} \times 1,54,000 = ₹42,000$   
C's share =  $\frac{3}{11} \times 1,54,000 = ₹42,000$ 

A's total share = ₹72,000 and B's total share = ₹45,000.

**13.** Speed of car 'A' is twice that of truck 'B'. Both start from a point A to point Y. If 'B' took 3 hr more than 'A', what is the time taken by 'B'?

#### Solution :

Since distance is constant, speed and time are inversely proportional.

Therefore, if ratio of speeds of 'A' and 'B' = 2 : 1, ratio of time taken by 'A' and 'B' = 1 : 2.

Let's assume the time taken by 'A' be X and by 'B' be 2X.

$$2X - X = 3$$

 $\Rightarrow$  X = 3

 $\therefore$  Time taken by B = 6 hr.

**14.** Divide 156 in four parts in continuous proportion such that the ratio of sum of first and third part to the sum of second and fourth part is 1 : 5.

#### Solution :

Continuous proportion means that ratio is in a geometric progression, i.e. a ratio which is like a : ad :  $ad^2$  :  $ad^3$  ... is a continuous proportion.

If 
$$(a + ad^2)$$
 :  $(ad + ad^3) = 1 : 5$   
⇒  $a(1 + d^2)$  :  $ad(1 + d^2) = 1 : 5$   
⇒  $1 : d = 1 : 5$   
⇒  $d = 5$ .Ratio  
⇒  $a : 5a : 25a : 125a$   
⇒  $a + 5a + 25a + 125a = 156$   
⇒  $156a = 156$   
⇒  $a = 1$ 

Therefore, four parts are 1, 5, 25 and 125.

**15.** Two solutions contain alcohol and water in the ratio 3 : 2 and 5 : 3. If the two solutions are mixed in equal quantities, what is the ratio of alcohol to water in the resulting solution?

Solution :

Required ratio of alcohol to water 
$$=\frac{\left(\frac{3}{5}+\frac{5}{8}\right)}{\left(\frac{2}{5}+\frac{3}{8}\right)}=\frac{49}{31}$$

**16.** B's income is  $\frac{3}{2}$  of A's income and 75% of C's income. What is C's income as a percentage of A's income?

Solution :

B's income = 
$$\frac{3}{2}$$
 of A's income.  
C's income =  $\frac{4}{3}$  of B's income  
=  $\frac{4}{3} \times \frac{3}{2}$  of A's income = 2(A's income).  
∴ Required percentage = 200%.

**17.** In an election there are only 2 candidates, A and B. A got 59% and won by a majority of 144 votes. What is the total number of votes?

#### Solution :

A got 59%. Therefore, B got 41%. Difference = 59 – 41 = 18% 18% = 144 votes

Therefore, total votes =  $\frac{144}{18} \times 100 = 800$  votes.

18. Out of an inheritance of ₹80,000, Anil invested ₹26,000 in shares, 30% of total in land and deposited the rest in 10% fixed deposit for 2 years. What is the gain from the fixed deposit after 2 years?

# Solution :

Amount in fixed deposit

= 80000 - 26000 - 
$$\frac{30}{100}$$
 × 80000 = ₹30,000

Amount after 2 years =  $30000 \times \left(1 + \frac{10}{100}\right)^2$ 

=₹36,300

Interest = 36,300 – 30,000 = ₹6,300

**19.** A man has certain number of potatoes out of which 13% were bad, 75% of the rest were sold and he was left with 261. How many potatoes did he have initially?

#### Solution :

Let's assume that he had P potatoes initially.

Then Number of fresh potatoes = 100 - 13= 87% and

Number of unsold potatoes = 25% of 87% of initial number potatoes

$$\begin{bmatrix} \frac{87}{100} \times \frac{25}{100} \end{bmatrix} \times P = 261$$
$$\Rightarrow P = 1200$$

**20.** A man lost 12.5% of his money. After spending 70% of the remainder, he has ₹210 left with him. How much did he have at the beginning?

#### Solution :

Let 'x' be the actual amount of money.

Now, 12.5% = 
$$\frac{1}{8}$$

So, money left with him, after losing =  $\frac{7}{8}$  of x and

Money left after spending = 30% of  $\frac{7}{8}$  of x

$$\Rightarrow \frac{30}{100} \times \frac{7}{8} \times x = 210 \Rightarrow \frac{3 \times 7}{80} \times x = 210$$
$$\Rightarrow \frac{21}{80} \times x = 210 \Rightarrow x = ₹800$$

1. If A : B = 3 : 7 and the sum of A and B is 45. Find the value of B. (a) 28 (b) 33.5 (a) 2:9 (c) 31.5 (d) 36 (c) 4 : 5 2. A fraction bears the same ratio to  $\frac{3}{7}$  as  $\frac{1}{27}$  to The first number is  $\frac{1}{35}$ . The fraction is (b)  $\frac{1}{3}$ (a)  $\frac{4}{9}$ (c) 55 (c)  $\frac{3}{5}$ (d) 3. Mean proportion between 8 and 72 is (a) ₹2,790 (a) 24 (b) 40 (c) ₹3,300 (c) 16 (d) 32 4. Fourth proportional to 3, 15 and 27 is (a) 4 : 3 : 2 (a) 39 (b) 45 (c) 3:4:2 (c) 81 (d) 135 5. Third proportional to 20 and 30 is (a) 40 (b) 45 first part is (c) 60 (d) 50 6. A, B and C join a partnership contributing (c) 28 ₹2,000, ₹1,500 and ₹1,250 respectively. What is A's share in a total profit of ₹3,610? (a) ₹1,500 (b) ₹1,520 (c) ₹1,870 (d) ₹2,290 7. A starts a business with ₹4,000. B joins him after 3 months with ₹8,000. C puts a sum of ₹12,000 in the business for 2 months only. At the end of the year the business gave a profit of ₹5,200. Find the share of B.

(a) ₹1,500	(b) ₹2,000
(c) ₹2,600	(d) ₹4,000

Exercise

8. If ratio of shares of X and Y is 2 : 7. what is the ratio of X's share to the difference between Y's and X's shares?

(a) 2:7	(b) 4 : 12
(c) 2:5	(d) 2:9

9. ₹3,960 is divided among A, B and C such that half of A's part, one-third of B's part and one-sixth of C's part are equal. Then B's part is

60

(c) ₹1,720 (d) ₹1,540

- 10. In the above question, what is the ratio of A's share to the difference of B and C's share? (b) 2:5 (d) 2:3
- 11. Two numbers are in the ratio 5:3. If 9 is subtracted from both of them, they will be in the ratio 23 : 12.

(a) 45 (b)	65
------------	----

- (d) None of these
- 12. If ratio of A's money to B's is 4 : 5 and B's money to C's is 2 : 3, and A has ₹800, then the total money of A, B, and C is
  - (b) ₹3,000
  - (d) ₹3,620
- 13. If 4x = 3y = 2z, then x : y : z is
  - (b) 2:3:4
  - (d) 3:4:6
- 14. If 68 is divided into two parts such that 7th part of the first is equal to 10th part of the second. The

(b) 22

- (d) 32
- 15. ₹6,800 is to be divided among A, B and C such that A gets two-thirds of what B gets, and B gets one-fourth of what C gets. Find B's share?

(a) ₹1,200	(b) ₹800
(c) ₹1,000	(d) ₹1,600

16. ₹9,700 has been divided among X, Y and Z such that if their shares are reduced by ₹30, ₹20 and ₹50, the balance is in the ratio 3 : 4 : 5. What is Y's share?

(a) ₹3,180	(b) ₹3,220
(c) ₹3,253.33	(d) ₹3,200

17. The sum of ₹530 is divided among A, B and C such that A gets ₹70 more than B, and B gets ₹80 more than C. The ratio of A and C is

(a) 25 : 18	(b) 18 : 10
(c) 5:9	(d) 5:2

18. In a ratio equal to 4:9, the antecedent is 36. The consequent is

(c) 72 (d) None of these

 An amount is distributed amongst A, B and C such that A gets half that of B and B gets twice that of C. What is the ratio of B to that of the sum of A and B.

(a) 2:5 (b	) 2:3
------------	-------

(c) 3:2 (d) 4	4:3
---------------	-----

20. A bag contains ₹600 in the form of one-rupee,
50-paisa and 25-paisa coins in the ratio 3 : 4 :
12. The number of 25-paisa coins is

(a`	) 600	(b) 900
(a	) 000	(D) 900

- (c) 1,200 (d) 1,376
- 21. In a mixture of 180 L, the ratio of milk and water is2 : 1. If the ratio of milk and water is to be 1 : 2, the water to be added is (in L)

(a) 80 L	(b) 90 L

- (c) 120 L (d) 180 L
- 22. In a mixture of 100 L, the ratio of milk and water is 3 : 1. If 200 L of water is added in the mixture, what is the new ratio of milk and water?
  - (a) 1:3 (b) 3:1
  - (c) 2:5 (d) None of these
- 23. Out of a class of 500, boys and girls are equal. In new batch, one-fifth of the girls left the class and 25 boys joined in. What is the ratio of boys and girls now?

(a) 3:2	(b) 12 : 7

(c) 11 : 8	(d) 9:8
------------	---------

24. The population of a country increased by one-tenth and then decreased by one-tenth in two successive years. If the original population was 15,000, what would be the population after 2 years?

(a) 14,550	(b) 15,000
(c) 14,850	(d) 16,270

25. The ages of a man and his son are in the ratio 7 : 2. After 15 years, they would be in the ratio 2 : 1. The father's age, when the son was born, was
(a) 25 years
(b) 25 years

(a) 25 years	(b) 30 years
(a) $2E$ vector	

- (c) 35 years (d) 42 years
- 26. Four years ago, father's age was six times that of his son. 12 years from now, father's age will be twice that of the son. The present ratio of ages of father and son is

(a) 6 : 1	(b) 7 : 1
-----------	-----------

(c) 8:2 (d) 7:2

- 27. If  $\frac{a}{3} = \frac{b}{4} = \frac{c}{7}$ , then  $\frac{(a+b+c)}{c}$  is (a) 7 (b) 2 (c)  $\frac{1}{2}$  (d)  $\frac{1}{7}$
- 28. Two whole numbers whose sum is 84 cannot be in the ratio

(a) 9:3	(b) 3:5
(c) 19:2	(d) 5:7

29. If 0.35 of a number is equal to 0.07 of another number, the ratio of the number is

(a) 1:2	(b) 2 : 1
(c) 1:5	(d) 1:4

30. Two alloys containing copper and iron in the ratios 5:8 and 5:3 are melted in equal quantities. The ratio of iron to copper in the resulting alloy is

(a) 103 : 105	(b) 15 : 24
(c) 3:8	(d) 105 : 103

31. 80 L of a mixture of milk and water is in the ratio 5: 3. If 16 L of this mixture is replaced by 16 L of milk, ratio of milk and water becomes

(a) 2 : 1	(b) 6:3
(c) 7:3	(d) 8:3

32. The ratio of copper and zinc in a brass piece is 13:7. How much copper will be there in 500 kg of such alloy?

(a) 300 kg	(b) 325 kg
------------	------------

(c) 175 kg	(d)	150 kg
------------	-----	--------

33. A, B and C join a partnership. A invested ₹16,000

for 6 months, B invested ₹12,000 for  $\frac{2}{3}$  year and C invested ₹1,000 for 12 months. Their profit sharing ratio is

(a) 8 : 7 : 10	(b) 10 : 8 : 7
(c) 6 : 8 : 12	(d) None of these

34. A sum of money is divided among A, B and C such that for each rupee that A gets, B gets 65 paise and C gets 35 paise. If C's share is ₹560, the sum is

(a) ₹2,400	(b) ₹2,800
(u) (Z, +00	(0) (2,000

- (c) ₹3,200 (d) ₹3,600
- 35. A ratio of two numbers is 2 : 3. What is the ratio of their squares, if the two numbers have a difference of 12?

(a) 2:3	(b) 12 : 24
(c) 9:16	(d) 4 : 9

36. The cost of providing a product are labour, material and overheads in the ratio 5 : 7 : 3. If the profit is calculated as 20% on costs, the ratio of material costs to profit is

(c) 3:5 (d) 11:9
------------------

37. ₹11,250 is divided among A, B and C so that A receives half as much as B and C together receive;
 B receives one-fourth of what A and C receive together. The share of A is more than that of B by

))	₹3,000
	))

- (c) ₹1,550 (d) ₹3,200
- 38. The ratio of boys and girls in a class of 72 is 7 : 5. How many more girls should be admitted to make equal number of boys and girls?

(a) 9	(b) 12
(c) 220	(d) 240

39. The biggest of three positive numbers that are in the ratio 2 : 3 : 5 and the sum of their squares is 608, is

(a) 8	(b) 12
(c) 16	(d) 20

40. In a mixture of 28 L, the ratio of milk and water is 2 : 5. If 2 L of milk is added and 5 L of water is removed, the new ratio is

(a) 9:20	(b) 1:4
(c) 3:2	(d) 2:3

41. The incomes of A and B are in the ratio 3 : 2 and their expenses are in the ratio 5 : 3. If each one saves ₹3,000, then B's income is

(a) ₹12,000	(b) ₹6,000
(c) ₹9,000	(d) ₹15,000

42. A, B and C invested in a partnership with a ratio of 8 : 7 : 5. A withdraws half of her money after 5 months. If the profit was ₹26,500 for the year, then B's share is

(a) ₹9,800	(b) ₹10,200
(,,	()

43. The prices of a bicycle and a rickshaw are in the ratio 5 : 4. If the bicycle costs ₹800 more than the rickshaw, the price of the rickshaw is

(a) ₹800	(b) ₹3,200
(c) ₹4.000	(d) ₹2,400

44. If A : B :: 5 : 2, the value of 8A + 9B : 8A + 2B is

(a) 26 : 16	(b) 61 : 26
(c) 29:22	(d) 22 : 29

45. If 1 L water is added to 5 L of a 20% solution of sugar in water, what is the strength of the solution now?

(a) 16.67%	(b) 10%
(c) 8.33%	(d) 12.67%

46. The price of oil is increased by 25%. If the expenditure is not allowed to increase, the ratio of reduction in consumption to regular consumption is

(a) 1:3	(b) 1:4
(c) 1 : 5	(d) 1:6

- 47. Three year ago the average age of Abhishek and Aishwarya was 25 year. Now they have a one year old child. The average age of the family at present is:
  - (a) 25 years (b) 17 years
  - (c) 19 years (d) 28 years
- 48. In an ODI cricket match the run rate was only 4.2 in the first 20 overs. What should be the run rate in the remaining 30 overs to reach the target of 312 runs?

(a) 7.6	(b) 7.2
(c) 8	(d) 7.5

- 49. A family consists of six members A, B, C, D, E and F. The average age of A and B is 60 years, that of C and D is 38 years and that of E and F is 10 years. What is the average age of the family?
  - (a) 38 years (b) 42 years
  - (c) 32 years (d) 36 years
- 50. A shopkeeper has made a sale of ₹6,135, ₹7,227, ₹6,755, ₹7,430 and ₹6,462 for 5 consecutive days. How much sale must he have to make on the sixth day in order to manage an average sales of ₹6600 over the given six days.
  - (a) ₹4991 (b) ₹6054
  - (c) ₹5591 (d) None of these
- 51. The average of 50 numbers is zero. How many of them, at the most, can be greater than zero?

(a) 49	(b) 25
(c) 0	(d) 24

52. The average weight of 10 persons increases by 2 kg when a new person comes in place of one of them of weight 56 kg. The weight of the new person is

(a) 66 kg	(b) 76 kg
(c) 72 kg	(d) 36 kg

#### 5.8

- 53. The average monthly income of Ram and Rahim is ₹4,500. The average monthly income of Rahim and Salim is ₹5,500 and the average monthly income of Ram and Salim is ₹5,100. The monthly income of Ram is
  - (a) ₹4,100 (b) ₹4,000
  - (c) ₹4,500 (d) ₹5,100
- 54. A library has an average of 720 visitors on Sundays and 450 on each of other days of the week. The average number of visitors per day in a month of 30 days beginning with Sunday is:
  - (a) 500 (b) 495
  - (c) 590 (d) 490

- 55. In a cricket match the run rate was only 4.5 in the first 24 overs. What must be the run scored in the remaining 30 overs to reach the overall run rate of 6?
  - (a) 182 (b) 218
  - (c) 212 (d) 192
- 56. A student's marks were wrongly entered as 83 instead of 38. Due to that the average marks for the class got increased by 0.3. The number of students in the class is:

(a) 100	(b) 90
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(c) 150 (d) 200

	Answer Key									
<b>1.</b> (c)	<b>2.</b> (d)	<b>3.</b> (a)	<b>4.</b> (d)	<b>5.</b> (b)	<b>6.</b> (b)	<b>7.</b> (c)	<b>8.</b> (c)	<b>9.</b> (a)	<b>10.</b> (d)	
<b>11.</b> (c)	<b>12.</b> (c)	<b>13.</b> (d)	<b>14.</b> (c)	<b>15.</b> (a)	<b>16.</b> (b)	<b>17.</b> (d)	<b>18.</b> (a)	<b>19.</b> (b)	<b>20.</b> (b)	
<b>21.</b> (d)	<b>22.</b> (a)	<b>23.</b> (c)	<b>24.</b> (c)	<b>25.</b> (a)	<b>26.</b> (d)	<b>27.</b> (b)	<b>28.</b> (b)	<b>29.</b> (c)	<b>30.</b> (a)	
<b>31.</b> (c)	<b>32.</b> (b)	<b>33.</b> (d)	<b>34.</b> (c)	<b>35.</b> (d)	<b>36.</b> (b)	<b>37.</b> (a)	<b>38.</b> (b)	<b>39.</b> (d)	<b>40.</b> (d)	
<b>41.</b> (a)	<b>42.</b> (c)	<b>43.</b> (b)	<b>44.</b> (c)	<b>45.</b> (a)	<b>46.</b> (c)	<b>47.</b> (c)	<b>48.</b> (a)	<b>49.</b> (d)	<b>50.</b> (c)	
<b>51.</b> (a)	<b>52.</b> (b)	<b>53.</b> (a)	<b>54.</b> (b)	<b>55.</b> (d)	<b>56.</b> (c)					

1. c  $\frac{A}{B} = \frac{3}{7}$ . Let A = 3x, B = 7x.  $\Rightarrow 3x + 7x = 45, x = \frac{45}{10} = 4.5;$ ∴ B = 7x = 31.5 2. d Let the fraction be  $\frac{x}{y}$ ;  $\frac{x/y}{3/7} = \frac{1/27}{1/35} \Rightarrow \frac{7x}{3y} = \frac{35}{27}$  $\frac{x}{v} = \frac{35}{27} \times \frac{3}{7} = \frac{5}{9}$ 3. a If a, b, c are in continuous proportion. The mean proportion is b. Therefore,  $b^2 = ac$ ,  $b^2 = 8 \times 72$ ,  $b = \sqrt{576} = 24$ . 4. d If a, b, c, d are in proportion,  $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{3}{15} = \frac{27}{d}$  $\Rightarrow$  d = 135. 5. b Third proportional c =  $\frac{b^2}{2} = \frac{30 \times 30}{20} = 45$ . 6. b A's share = P $\left(\frac{x}{x+y+z}\right)$  = 3610 ×  $\frac{2000}{4750}$  =₹1,520, where P = Profit and x, y and z are respective shares of A, B and C. 7. c Ratio of their profits = Ratio of their investments = 4000 × 12 : 8000 × 9 : 12000 × 2 48 : 72 : 24 2:3:1 Bs' share =  $\frac{3}{6}$  × 5200 = ₹2,600. 8. c  $\frac{x}{v} = \frac{2}{7}$ . Let x's share = 2a, y's share = 7a.  $\frac{x's \text{ share}}{y's \text{ share} - x's \text{ share}} = \frac{2a}{5a} = \frac{2}{5} \ .$ 9. a Let  $\frac{A}{2} = \frac{B}{2} = \frac{C}{6} = k$  $\Rightarrow$  A : B : C = 2 : 3 : 6 B's part =  $\frac{2}{11} \times 3960 = ₹1080$ 10. d A's share =  $\frac{A}{A + \frac{3}{2}A + 3A} \times P = \frac{2}{11}P$ . B's share =  $\frac{B}{\frac{2}{2}B + B + 2B} \times P = \frac{3}{11}P$ . C's share =  $P - \frac{(2+3)}{11}P = \frac{6}{11}P$ .  $\frac{A's}{B's-C's} = \frac{\frac{2}{11}}{\frac{6}{11} - \frac{3}{11}} = \frac{2}{3}.$ 

**Explanations** 

11. c Let the number be 5x and 3x. Then,  $\frac{5x-9}{3x-9} = \frac{23}{12}$  $\Rightarrow 60x - 108 = 69x - 207$  $\Rightarrow$  9x = 99  $\Rightarrow$  x = 11 The first number is  $11 \times 5 = 55$ . 12. c B's money =  $\frac{5}{4} \times 800 = ₹1000$ . C's money =  $\frac{3}{2}$  × 1000 =₹1500. Therefore, A's + B's + C's money = ₹3,300. 13. d 4x = 3y = 2zDivide the three terms by their LCM. LCM of 4, 3, 2 = 12;  $\therefore x : y : z = 3 : 4 : 6$ 14. c Let the parts be x, (68 - x).  $\frac{x}{7} = \frac{1}{10}(68 - x) = \frac{68}{10} - \frac{x}{10}$  $\frac{x}{7} + \frac{x}{10} = \frac{68}{10} \Longrightarrow \frac{17x}{70} = \frac{68}{10}$ , x = 28. 15. a A =  $\frac{2}{3}$  B, B =  $\frac{1}{4}$  C  $\frac{A}{B} = \frac{2}{3}, \quad \frac{B}{C} = \frac{1}{4} = \frac{3}{12}$  $A \cdot B \cdot C = 2 \cdot 3 \cdot 12$ B's share =  $\frac{3}{17} \times 6800 = ₹1,200$ 16. b X's share  $\Rightarrow$  3x + 30 Y's share  $\Rightarrow$  4x + 20 Z's share  $\Rightarrow$  5x + 50 Sum is 9700. 12x + 100 = 9700, 12x = 9600, x = 800.Y's share = 4x + 20 = 3200 + 20 = ₹3.220. 17. d Let B get ₹x, A get ₹x + 70, C get ₹x – 80. x + x + 70 + x - 80 = 530 $\Rightarrow$  3x - 10 = 530, 3x = 540 ⇒ x = 180 A's share  $180 + 70 \quad 250 \quad 5$  $\overline{C's \text{ share}} = \overline{180 - 80} = \overline{100} = \overline{2}$ . 18. a Ratio =  $\frac{4}{9}$ . Let antecedent = 4x = 36, x = 9. Consequent =  $9x = 9 \times 9 = 81$ .

19. b  $A = \frac{1}{2}B, B = 2C \Rightarrow \frac{A}{B} = \frac{1}{2}, \frac{B}{C} = \frac{2}{1}$ A: B: C = 1:2:1 Shares of A, B and C are x, 2x and x.  $\frac{B}{A+B} = \frac{2x}{x+2x} = \frac{2x}{3x} = \frac{2}{3}$ 20. b Ratio of coins by value  $\frac{3}{4}:\frac{4}{2}:\frac{12}{4}\Rightarrow 3:2:3$ .  $\therefore 3x + 2x + 3x = 600, x = \frac{600}{8} = 75.$ Value of 25-paisa coins = 75 × 3 = ₹225.  $\therefore$  Number of coins = 225 × 4 = 900. 21. d M : W = 2 : 1. Total volume = 180 L. Milk = 120 L, water = 60 L. Let 'x' litres of water is added to make the ratio 1:2.  $\therefore \frac{120}{60+x} = \frac{1}{2},$  $\Rightarrow$  240 = 60 + x ⇒ x = 180 L. 22. a  $\frac{M}{W} = \frac{3}{1}$  in 100 L mixture. milk = 75 L, water = 25 L. After adding 200 L of water, water = 225 L and milk = 75 L. Ratio =  $\frac{75}{225}$  = 1 : 3. 23. c Initially number of boys = 250, number of girls = 250. New batch  $\frac{1}{5}$  of the girls (50 girls) left the class and 25 boys joined in.

Number of girls remaining = 200, and number of boys = 250 + 25 = 275.

 $\therefore \frac{\text{Boys}}{\text{Girls}} = \frac{275}{200} = \frac{11}{8}$ 

24. c Original population is 15,000.

Population after one year

$$= 15000 + \frac{1}{10} \times 15000 = 15000 + 1500 = 16500.$$

Population after two years

$$= 16500 - \frac{1}{10} \times 16500 = 16500 - 1650 = 14850.$$

#### Short cut:

Change in population  $= 10 - 10 - \frac{10 \times 10}{100} = -1\%$ Population after 2 years = 15,000 × 0.99 = 14,850 25. a Let man's age be 7x, son's age = 2x. 4 according to question,  $\frac{7x+15}{2x+15} = \frac{2}{1}$ , 7x + 15 = 4x + 30, x = 5.

> Present age of father = 35 years. Present age of son = 10 years. Father's age when son was born

= 35 - 10 = 25 years.

26. d Let present age of son = x, and present age of father = y.

$$(y-4) = 6(x-4)$$
 ... (i)

$$(y+12) = 2(x+12) \qquad ... (ii)$$
  
From (i) and (ii),  
 $y - 6x + 20 = 0$   
 $y - 2x - 12 = 0$   
 $4x = 32, x = 8$  years,  $y = 28$  years.  
Ratio of ages  $= \frac{28}{8} = \frac{7}{2}$ .

27. b Let 
$$\frac{a}{3} = \frac{b}{4} = \frac{c}{7} = k$$
  
 $a = 3k, b = 4k, c = 7k$   
 $\frac{a+b+c}{c} = \frac{3k+4k+7k}{7k} = \frac{14k}{7k} = 2$ 

It must be divisible by the sum of the ratios. Sum of the ratios are 12, 8, 21 and 12. Since, 8 does not divide 84. Hence (b) is the correct option.

29. c 0.35 of x = 0.07 of y.

$$\therefore \frac{x}{y} = \frac{0.07}{0.35} = \frac{1}{5}$$
  
30. a  $\frac{C_1}{l_1} = \frac{5}{8}, \quad \frac{C_2}{l_2} = \frac{5}{3}$   
 $\frac{Copper}{Iron} = \frac{\frac{5}{13} + \frac{5}{8}}{\frac{8}{13} + \frac{3}{8}} = \frac{40 + 65}{64 + 39} = \frac{105}{103}$ 

Hence, ratio of iron : copper = 103 : 105.

31. c Quantity of milk in 80L mixture  $=\frac{5}{8} \times 80 = 50L$ and in 16L mixture  $=\frac{5}{8} \times 16 = 10L$ Quantity of water in 80L mixture  $=\frac{3}{8} \times 80 = 30L$  and in 16L mixture  $=\frac{3}{8} \times 16 = 6L$ 

: Required ration = 
$$\frac{(50-10)+16}{(30-6)} = \frac{56}{24}$$
, i.e. 7 : 3.

32. b Let copper = 13x, zinc = 7x. 13x + 7x = 500

$$x = \frac{500}{20} = 25 \text{ kg}$$

- 33. d A's effective investment = 16000 × 6 = ₹96,000.
  B's effective investment = 12000 × 8 = ₹96,000.
  C's effective investment = 1000 × 12 = ₹12,000.
  Profit sharing ratio = 96 : 96 : 12 i.e. 8 : 8 : 1.
- 34. c A gets 100 paise, B gets 65 paise and C gets 35 paise.

C's share =  $560 = Sum \times \frac{35}{200}$ 

Sum =₹3,200.

35. d Let the numbers be 2x and 3x.

Ratio of their squares =  $\frac{4x^2}{9x^2} = \frac{4}{9}$ .

**Note:** Ratio of squares is independent of difference of numbers.

36. b L : M : O = 5 : 7 : 3

Let the labour cost be 5x, material cost = 7x, overheads = 3x. Total cost = 15x. Profits = 20% of 15x = 3x

$$\frac{\text{Material cost}}{\text{Profit}} = \frac{7x}{3x} = \frac{7}{3}$$
37. a A =  $\frac{1}{2}$  (B + C)

$$\Rightarrow 2A = B + C \qquad \dots (i)$$

$$\mathsf{B} = \frac{1}{4}(\mathsf{A} + \mathsf{C})$$

4B = A + C ... (ii)

Subtracting (ii) from (i),  $2A - 4B = B - A \Rightarrow 3A = 5B$ 

$$\frac{3}{5}A = B$$
 ... (iii)

C = 2A - B = 2A - 
$$\frac{3}{5}A = \frac{7}{5}A$$
  
∴ A : B : C = 1:  $\frac{3}{5}$ :  $\frac{7}{5}$  = 5 : 3 : 7  
A's share =  $\frac{5}{15} \times 11250 = 750 \times 5$ .  
B's share =  $\frac{3}{15} \times 11250 = 750 \times 3$ .  
A's share - B's share = 750 × 2 = ₹1500.  
38. b  $\frac{B}{G} = \frac{7}{5}$ 

Let number of boys = 7x. Let number of girls = 5x. 7x + 5x = 72, x = 6Number of boys = 42. Number of girls = 30. 12 more girls should be admitted to make the

number equal.

- 39. d Let the numbers be 2x, 3x, and 5x respectively. Sum of their squares =  $608 = 4x^2 + 9x^2 + 25x^2$  $38x^2 = 608$ ,  $x^2 = 16$ ,  $x = \pm 4$ . As the numbers are positive, the biggest number is 20.
- 40. d M : W = 2 : 5. Total volume = 28 L.

Milk = 
$$\frac{2}{7} \times 28 = 8$$
 L.  
Water =  $\frac{5}{-} \times 28 = 20$  L

2 L of milk is added and 5 L water is removed.

$$\left(\frac{M}{W}\right)_{new} = \frac{8+2}{20-5} = \frac{10}{15} = \frac{2}{3}$$
 i.e. 2:3

- 41. a Let A's income = ₹3x. B's income = ₹2x. A's expenses = ₹5y. B's expenses = ₹3y. A's savings = 3x - 5y = 3000 ... (i) B's savings = 2x - 3y = 3000 ... (ii) Solving (i) and (ii), x = 6000 B's income = 2x = ₹12,000. 42. c Let A's B's and C's investments be 8x. 7x and 5
- 42. c Let A's, B's and C's investments be 8x, 7x and 5x respectively.A's effective investment = (8x)5 + (4x)7
  - = 40x + 28x = 68x

B's effective investment =  $7x \times 12 = 84x$ . C's effective investment =  $5x \times 12 = 60x$ .

B's share =  $\frac{84}{212}$  × 26500 = ₹10,500.

43. b  $\frac{B}{R} = \frac{5}{4}$ . Let the price be 5x and 4x 5x - 4x = 800, x = 800. Price of rickshaw = 800 × 4 = ₹3,200.

44. c 
$$\frac{A}{B} = \frac{5}{2}$$
, B =  $\frac{2}{5}A$   
 $\frac{8A + 9B}{8A + 2B} = \frac{8A + \frac{18}{5}A}{8A + \frac{4}{5}A} = \frac{58}{44} = \frac{29}{22}$ 

Short cut: Let A = 5. Then B = 2. Therefore, (40 + 18) : (40 + 4) = 58 : 44 = 29 : 22.

#### 5.12

45. a 20% solution of sugar means one-fifth of sugar. In 5 L of solution, 1 L is sugar and 4 L is water. After adding 1 L of water,

Percentage of sugar =  $\frac{1}{6} \times 100 = 16.67\%$ .

46. c Let the original consumption be 1 unit costing₹100. New cost = ₹125.

New consumption =  $\left(\frac{1}{125} \times 100\right) = \frac{4}{5}$  unit.

 $\frac{\text{Reduction in consumption}}{\text{Original consumption}} = \frac{\left(1 - \frac{4}{5}\right)}{1} = \frac{1}{5} \text{, i.e. 1 : 5.}$ 

47. c Sum of present ages of all the three

- :. Average age of the family =  $\frac{57}{3}$  = 19 years.
- 48. a Runs scored in first 20 overs = 20 × 4.2 = 84
  - Runs to be scored to reach the target = 312 84 = 228

$$\therefore$$
 Required run rate =  $\frac{228}{30} = 7.6$ .

- 49. d Sum of ages of
  - A and B =  $60 \times 2 = 120$ C and D =  $38 \times 2 = 76$ E and F =  $10 \times 2 = 20$  $\therefore A + B + C + D + E + F = 120 + 76 + 20 = 216$

Hence, the average age of the family =  $\frac{216}{6}$  = 36 years.

- 50. c Total sales for 5 days
  - =₹(6135 + 7227 + 6755 + 7430 + 6462) = ₹34009.
  - .: Required sales on the sixth day
  - = [(6600 × 6) 34009] = ₹(39600 34009)
  - =₹5591.
- 51. a Average of 50 numbers = 0.
  - $\therefore$  Sum of 50 numbers (0 × 50) = 0.

It is quite possible that atmost 49 of these numbers can be positive and if their sum is 'a' then 50th number will be (-a), making the sum total of 50 numbers zero.

- 52. b Increase in weight due to replacement of the person weighing 56 kg by a new person
  - = 2 × 10 = 20 kg
  - $\therefore$  The weight of the new person = (56 + 20) kg = 76 kg.
- 53. a Let the monthly incomes of Ram, Rahim and Salim are 'x', 'y' and 'z' respectively, then
  - $\begin{array}{ll} x+y = (4500 \times 2) = 9000 & \dots (i) \\ y+z = (5500 \times 2) = 11000 & \dots (ii) \\ x+z = (5100 \times 2) = 10200 & \dots (iii) \\ \mbox{Adding (i), (ii) and (iii), we get; } 2(x+y+z) \\ = 30200 \mbox{ or } x+y+z = 15100 & \dots (iv) \\ \mbox{Subtracting (ii) from (iv), we get } x = 4100. \end{array}$

∴ Ram's monthly income = ₹4,100.

54. b Since the month begins with a Sunday, so there will be five Sundays in the month.

Required average

$$\frac{720 \times 5 + 450 \times 25}{30} = \frac{3600 + 11250}{30} = 495$$

- 55. d Runs made in first 20 overs =  $24 \times 4.5 = 108$ Total run to be scored =  $50 \times 6 = 300$ ∴ Runs needed to be scored = 300 - 108 = 192.
- 56. c Let the number of students in the class = x Increase in the total marks = 0.3x

$$\therefore 0.3x = (83 - 38)$$

$$\Rightarrow x = \frac{45}{0.3} = 150$$

#### Alternative method:

Total number of students =  $\frac{\text{Increase in marks}}{\text{Increase in average}}$ 

$$=\frac{45}{0.3}=150.$$