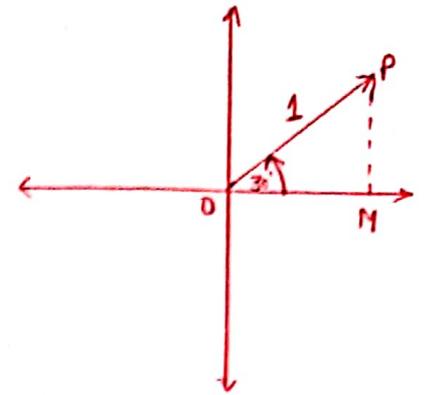


MISCELLANEOUS EXERCISE - VECTOR ALGEBRA.

6.

QNo1 Write down a unit vector in XY-plane, making an angle of 30° with +ve direction of x-axis.

Sol. Let \vec{OP} be required unit vector in XOY plane, so that $\angle XOP = 30^\circ$.
Let M be the foot of $\perp r$ from P on $X'Ox$.



$$\frac{OM}{|OP|} = \cos 30^\circ$$

$$\Rightarrow OM = |OP| \cos 30^\circ = 1 \cos 30^\circ = \frac{\sqrt{3}}{2} \quad [\because |OP| = 1]$$

$$\text{Also } \frac{MP}{|OP|} = \sin 30^\circ \Rightarrow MP = 1 \sin 30^\circ = \frac{1}{2}$$

\therefore Required Unit Vector $\vec{OP} = \frac{\sqrt{3}}{2} \hat{i} + \frac{1}{2} \hat{j}$.

QNo2. Find the scalar component and magnitude of the vector joining the points P(x_1, y_1, z_1) and Q(x_2, y_2, z_2)

Sol. Here $\vec{OP} =$ P.V. of P = $x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k}$

and $\vec{OQ} =$ P.V. of Q = $x_2 \hat{i} + y_2 \hat{j} + z_2 \hat{k}$

$$\begin{aligned} \text{Now } \vec{PQ} &= \vec{OQ} - \vec{OP} = (x_2 \hat{i} + y_2 \hat{j} + z_2 \hat{k}) - (x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k}) \\ &= (x_2 - x_1) \hat{i} + (y_2 - y_1) \hat{j} + (z_2 - z_1) \hat{k} \end{aligned}$$

\therefore Scalar components of Vector \vec{PQ} are $x_2 - x_1, y_2 - y_1, z_2 - z_1$

and Magnitude $|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

QNo.3 A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure.

Sol. Let $|OA| = 4 \text{ km}$ and $|AB| = 3 \text{ km}$ as shown in fig.

$$\text{then } \angle OAB = 90^\circ - 30^\circ = 60^\circ$$

Girl's displacement from initial point O
 $= \vec{OB} = x\hat{i} + y\hat{j}$ (say)

$$\begin{aligned} |x| &= OM = |OA| - |MA| \\ &= 4 - (|AB| \cos 60^\circ) \\ &= 4 - 3 \times \frac{1}{2} = 4 - \frac{3}{2} = \frac{5}{2} \end{aligned}$$

$$\therefore x = -\frac{5}{2}$$

$$\text{and } y = MP = |AB| \sin 60^\circ = 3 \times \frac{\sqrt{3}}{2}$$

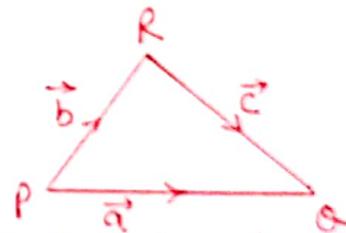
$$\therefore \vec{OB} = x\hat{i} + y\hat{j} = -\frac{5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

Distance of B from initial point $|OB| = |\vec{OB}|$

$$= \left| -\frac{5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j} \right| = \sqrt{\left(-\frac{5}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2} = \sqrt{\frac{25}{4} + \frac{27}{4}} = \sqrt{\frac{52}{4}} = \sqrt{13} \text{ km.}$$

QNo 4. If $\vec{a} = \vec{b} + \vec{c}$, then it is true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$.
 Justify your answer.

Sol When $\vec{a} = \vec{b} + \vec{c}$ then
 $\vec{a}, \vec{b}, \vec{c}$ form a triangle
 as shown.



Now since the sum of two sides of a triangle is always greater than the third side

$$\therefore |PR| + |RQ| > |PQ|$$

$$\text{i.e. } |\vec{b}| + |\vec{c}| > |\vec{a}|$$

$$\text{or } |\vec{a}| < |\vec{b}| + |\vec{c}|$$

Thus in general $\vec{a} = \vec{b} + \vec{c}$ does not imply $|\vec{a}| = |\vec{b}| + |\vec{c}|$

QNo 5. find value of x for which $x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.

Sol ATQ $|x(\hat{i} + \hat{j} + \hat{k})| = 1$

$$\text{i.e. } |x\hat{i} + x\hat{j} + x\hat{k}| = 1$$

$$\Rightarrow \sqrt{x^2 + x^2 + x^2} = 1$$

$$\sqrt{3x^2} = 1 \quad \text{or} \quad 3x^2 = 1$$

$$\Rightarrow x^2 = \frac{1}{3} \Rightarrow x = \pm \frac{1}{\sqrt{3}}$$

Q No. 6. find a vector of magnitude 5 units and parallel to the resultant of the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Sol.

$$\begin{aligned} \text{Resultant of vectors } \vec{a} \text{ and } \vec{b} &= \vec{a} + \vec{b} \\ &= (2\hat{i} + 3\hat{j} - \hat{k}) + (\hat{i} - 2\hat{j} + \hat{k}) = 3\hat{i} + \hat{j} + 0\hat{k} \end{aligned}$$

Let \vec{c} be the required vector parallel to $\vec{a} + \vec{b}$ and having magnitude 5.

$$\begin{aligned} \text{Then } \vec{c} &= \lambda(\vec{a} + \vec{b}), \lambda \text{ is any real No.} \\ &= \lambda(3\hat{i} + \hat{j} + 0\hat{k}) = 3\lambda\hat{i} + \lambda\hat{j} + 0\hat{k} \end{aligned}$$

$$\text{and } |\vec{c}| = 5$$

$$\Rightarrow |3\lambda\hat{i} + \lambda\hat{j} + 0\hat{k}| = 5$$

$$\Rightarrow \sqrt{(3\lambda)^2 + (\lambda)^2 + (0)^2} = 5$$

$$\Rightarrow \sqrt{9\lambda^2 + \lambda^2} = 5$$

$$\Rightarrow \sqrt{10\lambda^2} = 5$$

$$\Rightarrow 10\lambda^2 = 25 \Rightarrow \lambda^2 = \frac{25}{10} = \frac{5}{2}$$

$$\Rightarrow \lambda = \pm \sqrt{\frac{5}{2}}$$

Hence the required vector is

$$\text{either } \sqrt{\frac{5}{2}}(3\hat{i} + \hat{j}) \text{ or } -\sqrt{\frac{5}{2}}(3\hat{i} + \hat{j})$$

Q No 7. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a unit vector parallel to $2\vec{a} - \vec{b} + 3\vec{c}$.

Sol.

$$\begin{aligned} \text{Let } \vec{v} &= 2\vec{a} - \vec{b} + 3\vec{c} \\ &= 2(\hat{i} + \hat{j} + \hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k}) \\ &= 3\hat{i} - 3\hat{j} + 2\hat{k} \end{aligned}$$

$$\text{Now } |\vec{v}| = |3\hat{i} - 3\hat{j} + 2\hat{k}| = \sqrt{(3)^2 + (-3)^2 + (2)^2} = \sqrt{9+9+4} = \sqrt{22}$$

$$\therefore \text{Unit Vector parallel to } \vec{v} = \pm \frac{\vec{v}}{|\vec{v}|} = \pm \frac{1}{\sqrt{22}}(3\hat{i} - 3\hat{j} + 2\hat{k})$$

Q No 8. Show that the points $A(1, -3, -8)$, $B(5, 0, -2)$ and $C(11, 3, 7)$ are collinear and find the ratio in which B divides AC.

Sol. A, B and C will be collinear if B divides AC in ratio $k:1$ for unique value of k .

Now P.V of A = $\vec{a} = 1\hat{i} - 3\hat{j} - 8\hat{k}$

P.V of B = $\vec{b} = 5\hat{i} + 0\hat{j} - 2\hat{k}$

P.V of C = $\vec{c} = 11\hat{i} + 3\hat{j} + 7\hat{k}$

Now B divides AC in ratio $k:1$

if $\vec{b} = \frac{k\vec{c} + 1\vec{a}}{k+1}$ for unique value of k

ie if $5\hat{i} + 0\hat{j} - 2\hat{k} = \frac{k(11\hat{i} + 3\hat{j} + 7\hat{k}) + 1(1\hat{i} - 3\hat{j} - 8\hat{k})}{k+1}$

ie if $5\hat{i} + 0\hat{j} - 2\hat{k} = \left(\frac{11k+1}{k+1}\right)\hat{i} + \left(\frac{3k-3}{k+1}\right)\hat{j} + \left(\frac{7k-8}{k+1}\right)\hat{k}$

ie if $\frac{11k+1}{k+1} = 5$, $\frac{3k-3}{k+1} = 0$, $\frac{7k-8}{k+1} = -2$

if $11k+1 = 5k+5$, $3k-3 = 0$, $7k-8 = -2k-2$

if $6k = 4$, $3k = 3$, $9k = 6$

if $k = \frac{4}{6}$, $k = \frac{3}{3}$, $k = \frac{6}{9}$

if $k = \frac{2}{3}$, $k = \frac{2}{3}$, $k = \frac{2}{3}$

Hence A, B, C are collinear and B divides AC in ratio $\frac{2}{3}:1$ ie 2:3.

Q No 9 Find the position vector of a point R which divides the line joining the two points P and Q whose position vectors are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ externally in ratio 1:2. Also show that P is the mid point of RQ.

Sol. Since R divides PQ in Ratio 1:2 externally

$$\therefore \text{P.V of R} = \frac{1(\text{P.V of Q}) - 2(\text{P.V of P})}{1-2} \quad [\text{section formula}]$$

$$= \frac{1(\vec{a} - 3\vec{b}) - 2(2\vec{a} + \vec{b})}{1-2}$$

$$= \frac{\vec{a} - 3\vec{b} - 4\vec{a} - 2\vec{b}}{-1} = \frac{-3\vec{a} - 5\vec{b}}{-1} = 3\vec{a} + 5\vec{b}$$

Now P.V of mid point of RQ = $\frac{\text{P.V of R} + \text{P.V of Q}}{2}$

$$= \frac{(3\vec{a} + 5\vec{b}) + (\vec{a} - 3\vec{b})}{2} = \frac{4\vec{a} + 2\vec{b}}{2} = 2\vec{a} + \vec{b}$$

$$= \text{P.V. of P.}$$

\therefore P is the mid point of RQ.

Q No 10 The two adjacent sides of a llgm are $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector || to diagonal. Also find its Area.

Sol. Let ABCD be the llgm with

$$\vec{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$$

$$\vec{AD} = \hat{i} - 2\hat{j} - 3\hat{k}$$

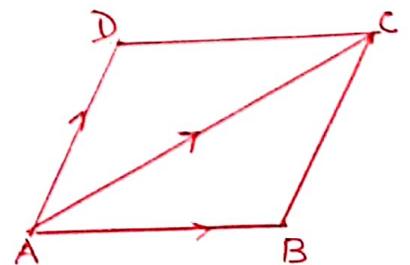
By llgm Law of addition of vectors

$$\vec{AC} = \vec{AB} + \vec{AD}$$

$$= (2\hat{i} - 4\hat{j} + 5\hat{k}) + (\hat{i} - 2\hat{j} - 3\hat{k}) = 3\hat{i} - 6\hat{j} + 2\hat{k}$$

Hence a unit vector parallel to \vec{AC} = $\frac{\vec{AC}}{|\vec{AC}|} = \frac{3\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{(3)^2 + (-6)^2 + (2)^2}}$

$$= \frac{3\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{9 + 36 + 4}} = \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k})$$



$$= \frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}$$

Area of parallelogram ABCD = $|\vec{AB} \times \vec{AD}|$

Now $\vec{AB} \times \vec{AD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 5 \\ 1 & -2 & -3 \end{vmatrix}$

$$= \hat{i}(+12+10) - \hat{j}(-6-5) + \hat{k}(-4+4)$$

$$= 22\hat{i} + 11\hat{j} + 0\hat{k}$$

$$\therefore |\vec{AB} \times \vec{AD}| = \sqrt{(22)^2 + (11)^2 + (0)^2} = \sqrt{11^2 \times 2^2 + 11^2} = \sqrt{11^2(4+1)} = 11\sqrt{5}$$

\therefore Area of Π gram = $11\sqrt{5}$ Square units.

QNo 11 Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Sol. Let a vector \vec{v} be equally inclined to OX, OY and OZ .
Let \vec{v} makes angle α with all the three axes.

Then unit vector in direction of \vec{v} is

$$\hat{v} = (\cos\alpha)\hat{i} + (\cos\alpha)\hat{j} + (\cos\alpha)\hat{k}$$

$$\Rightarrow 1 = \sqrt{\cos^2\alpha + \cos^2\alpha + \cos^2\alpha}$$

$$\Rightarrow 1 = \sqrt{3\cos^2\alpha}$$

$$\Rightarrow (1)^2 = (\sqrt{3\cos^2\alpha})^2 \Rightarrow 1 = 3\cos^2\alpha \Rightarrow \cos^2\alpha = \frac{1}{3}$$

$$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}} \quad (\text{Taking only +ve value})$$

$$\Rightarrow \therefore \text{Direction cosines of } \vec{v} \text{ are } \left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$$

QNo. 12 Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$; $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.
find a vector \vec{d} which is perpendicular to both \vec{a} and \vec{b}
and $\vec{c} \cdot \vec{d} = 15$.

Sol. The vector which is perpendicular to both \vec{a} and \vec{b} must be parallel to $\vec{a} \times \vec{b}$

Now $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix} = (28+4)\hat{i} - (7-6)\hat{j} + (-2-12)\hat{k}$

$$= 32\hat{i} - \hat{j} - 14\hat{k}$$

Since \vec{a} is \parallel to $\vec{a} \times \vec{b}$

$$\text{Let } \vec{a} = \lambda (\vec{a} \times \vec{b}) = \lambda (32\hat{i} - \hat{j} - 14\hat{k}) = 32\lambda\hat{i} - \lambda\hat{j} - 14\lambda\hat{k}$$

$$\text{Now } \vec{c} \cdot \vec{a} = 15$$

$$\Rightarrow (2\hat{i} - \hat{j} + 4\hat{k}) \cdot (32\lambda\hat{i} - \lambda\hat{j} - 14\lambda\hat{k}) = 15$$

$$\Rightarrow (2)(32\lambda) + (-1)(-\lambda) + (4)(-14\lambda) = 15$$

$$\Rightarrow 64\lambda + \lambda - 56\lambda = 15$$

$$\Rightarrow 9\lambda = 15 \Rightarrow \lambda = \frac{15}{9} = \frac{5}{3}$$

\therefore Required Vector $\vec{a} = \frac{5}{3}(32\hat{i} - \hat{j} - 14\hat{k})$

Q No 13.

The scalar product of the vector: $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ .

Sol.

$$\text{Let } \vec{a} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$$

$$\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\text{Now } \vec{b} + \vec{c} = (2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}$$

$$\therefore \text{Unit Vector along } \vec{b} + \vec{c} = \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + (6)^2 + (-2)^2}} = \vec{u} \text{ (say)}$$

It is given that $\vec{a} \cdot \vec{u} = 1$

$$\text{i.e. } (\hat{i} + \hat{j} + \hat{k}) \cdot \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + (6)^2 + (-2)^2}} = 1$$

$$\text{i.e. } \frac{(2+\lambda)(1) + (6)(1) + (-2)(1)}{\sqrt{4 + \lambda^2 + 4\lambda + 36 + 4}} = 1$$

$$\text{i.e. } \frac{2 + \lambda + 6 - 2}{\sqrt{\lambda^2 + 4\lambda + 44}} = 1$$

$$\text{i.e. } \lambda + 6 = \sqrt{\lambda^2 + 4\lambda + 44}$$

$$\text{i.e. } \lambda^2 + 36 + 12\lambda = \lambda^2 + 4\lambda + 44$$

$$\text{i.e. } 8\lambda = 8 \quad \text{i.e. } \lambda = 1$$

Q No. 14. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitude, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to \vec{a}, \vec{b} and \vec{c} . 13.

Sol. Since \vec{a}, \vec{b} and \vec{c} are mutually perpendicular

$$\therefore \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0 \quad \dots (1)$$

$$\text{Let } \vec{a} + \vec{b} + \vec{c} = \vec{v}$$

Let \vec{v} makes angles α, β and γ with \vec{a}, \vec{b} and \vec{c} resp.

$$\begin{aligned} \text{Then } \cos \alpha &= \frac{\vec{v} \cdot \vec{a}}{|\vec{v}| |\vec{a}|} = \frac{(\vec{a} + \vec{b} + \vec{c}) \cdot \vec{a}}{|\vec{v}| |\vec{a}|} = \frac{\vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{a} + \vec{c} \cdot \vec{a}}{|\vec{v}| |\vec{a}|} = \frac{|\vec{a}|^2 + 0 + 0}{|\vec{v}| |\vec{a}|} \\ &= \frac{|\vec{a}|^2}{|\vec{v}| |\vec{a}|} \quad [\because \text{of (1)}] \\ &= \frac{|\vec{a}|}{|\vec{v}|} \end{aligned}$$

$$\text{Similarly we get } \cos \beta = \frac{|\vec{b}|}{|\vec{v}|} \quad \text{and} \quad \cos \gamma = \frac{|\vec{c}|}{|\vec{v}|}$$

Now Since it is given that $|\vec{a}| = |\vec{b}| = |\vec{c}|$

$$\therefore \cos \alpha = \cos \beta = \cos \gamma$$

$$\Rightarrow \alpha = \beta = \gamma$$

$\therefore \vec{a} + \vec{b} + \vec{c}$ is equally inclined to \vec{a}, \vec{b} and \vec{c} .

Q No 15. Prove that $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$ if and only if \vec{a} and \vec{b} are perpendicular, given $\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}$

Sol.

$$(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$$

$$\text{iff } \vec{a} \cdot (\vec{a} + \vec{b}) + \vec{b} \cdot (\vec{a} + \vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$$

$$\text{iff } \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} = |\vec{a}|^2 + |\vec{b}|^2$$

$$\text{iff } |\vec{a}|^2 + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{b} + |\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 \quad [\because \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}]$$

$$\text{iff } |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$$

$$\text{iff } \vec{a} \cdot \vec{b} = 0$$

iff \vec{a} and \vec{b} are perpendicular vectors.

Choose the correct answer in questions 18 to 19. 14.

Q.No.16 If θ is the angle between two vectors \vec{a} and \vec{b} , then $\vec{a} \cdot \vec{b} \geq 0$ only when (A) $0 < \theta < \frac{\pi}{2}$ (B) $0 \leq \theta \leq \frac{\pi}{2}$ (C) $0 < \theta < \pi$ (D) $0 \leq \theta \leq \pi$.

Sol. $\vec{a} \cdot \vec{b} \geq 0$ iff $|\vec{a}| |\vec{b}| \cos \theta \geq 0$
ie only if $\cos \theta \geq 0$ ie only if $0 \leq \theta \leq \frac{\pi}{2}$.
 \therefore (B) is the correct option.

Q.No.17 Let \vec{a} and \vec{b} be two unit vectors and θ is the angle between them. Then $\vec{a} + \vec{b}$ is unit vector if

(A) $\theta = \frac{\pi}{4}$ (B) $\theta = \frac{\pi}{3}$ (C) $\theta = \frac{\pi}{2}$ (D) $\theta = \frac{2\pi}{3}$

Sol
 $|\vec{a} + \vec{b}| = 1$
iff $|\vec{a} + \vec{b}|^2 = 1$
iff $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = 1$
iff $|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = 1$ [As in Q.No.15]
iff $1 + 1 + 2\vec{a} \cdot \vec{b} = 1$
iff $1 + 1 + 2 \times 1 \times 1 \cdot \cos \theta = 1$
iff $2 \cos \theta = 1 - 2$
iff $\cos \theta = -\frac{1}{2}$
iff $\theta = \frac{2\pi}{3}$.
 \therefore (D) is correct option.

Q.No.18 The value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{i} \times \hat{j})$ is
(A) 0 (B) -1 (C) 1 (D) 3

Sol.
 $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{i} \times \hat{j})$
 $= \hat{i} \cdot (\hat{i}) + \hat{j} \cdot (-\hat{j}) + \hat{k} \cdot (\hat{k})$
 $= |\hat{i}|^2 - |\hat{j}|^2 + |\hat{k}|^2$
 $= 1^2 - 1^2 + 1^2 = 1$

\therefore (C) is correct option.

Q No 19 If θ is the angle between any two vectors \vec{a} and \vec{b} , then $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$ when θ is equal to

- (A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π .

Sol.

$$|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$$

$$\text{iff } \|\vec{a}\| \|\vec{b}\| |\cos \theta| = \|\vec{a}\| \|\vec{b}\| |\sin \theta|$$

$$\text{iff } (\cos \theta)^2 = (\sin \theta)^2$$

$$\text{iff } \tan^2 \theta = 1$$

$$\text{iff } \theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

\therefore (B) is correct option.

#.

PREPARED BY: Rupinder Kaur, Lect. Maths.
GSSS BHARI (FGS)