Polynomials

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$F(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$

Based on number of terms

(i) Monomial = 1 term; e g:- ax^2, $5x^3$
(ii) Binomial = 2 terms; e g:- $2x^2 + 3x$
(iii) Trinomial = 3 terms; e g:- $2x^2 + 3x + 5$

Degree of Polynomial

(i) $P(x) = x^1$; degree 1
(ii) $P(x) = x^3 - 5x^2$; degree 3
(iii) $P(x) = x^4 + 4x^2$; degree 4
Highest power of x in polynomial

Based on degree

(i) Zero degree ; $f(x) = ax^\circ$, $a \neq 0$
(ii) Linear ; $f(x) = ax + b$; $a \neq 0$
(iii) Quadratic ; $f(x) = ax^2 + bx + c$; $a \neq 0$
(iv) Cubic ; $f(x) = ax^3 + bx^2 + cx + d$; $a \neq 0$
(v) Biquadratic ; $f(x) = ax^4 + bx^3 + cx^2 + dx + e$; $a \neq 0$

Types of polynomial

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Remainder Theorem
{If $P(x) \div (x - a)$; Rem = $P(a)$}

Value of polynomial
e.g. $f(x) = x^2 - 3x + 4$, value at $x = 2$
$f(2) = 2^2 - 3 \times 2 + 4 = 2$

Factor Theorem
{If $P(x) \div (x - a)$; Rem = $P(a) = 0$}

Zero or root of polynomial
e.g. $f(x) = 2x^2 + x^2 + 7x - 6$, value at $x = 2$
$f(2) = 0$, $x = 2$ is a root or zero

Factors of Polynomials

(i) By taking common factor
eg. $= 2a(x + y) - 3b(x + y)$

(ii) By grouping
eg. $(x^2 + 3x)^2 - 5(x^2 + 3x) - y(x^2 + 3x) + 5y$
$= (x^2 + 3x)(x^2 + 3x - 5) - y(x^2 + 3x - 5)$

(iii) By making perfect square
eg. $4a^2 + 12ab + 9b^2$

(iv) Difference of 2 squares
eg. $x^8 - y^8$
$= (x^4)^2 - (y^4)^2$

(v) By spliting middle term
eg. $x^2 + 6\sqrt{2}x + 10$
$= x^2 + 5\sqrt{2}x + \sqrt{2}x + 10$

Algebraic Identities

(i) $(a + b)^2 = a + 2ab + b^2$
(ii) $(a - b)^2 = a^2 - 2ab + b^2$
(iii) $a^2 - b^2 = (a + b)(a - b)$
(iv) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
(v) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
(vi) $a^3 + b^3 = (a - b)(a^2 + ab + b^2)$
(vii) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
(viii) $(a - b)^3 = a^3 - b^3 - 3ab(a + b)$
(ix) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
Special case : If $a + b + c = 0$, then
$a^3 + b^3 + c^3 = 3abc$

e.g. $\dfrac{(a^2 \forall b^2)^3 \exists (b^2 \forall c^2)^3 \exists (c^2 \forall a^2)^3}{(a \forall b)^3 \exists (b \forall c)^3 \exists (c \forall a)^3}$

Here, $(a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2) = 0$
& $(a - b) + (b - c) + (c - a) = 0$