

8.5 Geometrical optics

Lenses and mirrors^a

<p>lens</p>	<p>mirror</p>																
<table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th colspan="2" style="text-align: center;">sign convention</th> </tr> <tr> <th style="text-align: center;">+</th> <th style="text-align: center;">-</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;"><i>r</i></td><td style="text-align: center;">centred to right</td></tr> <tr> <td style="text-align: center;"><i>u</i></td><td style="text-align: center;">real object</td></tr> <tr> <td style="text-align: center;"><i>v</i></td><td style="text-align: center;">real image</td></tr> <tr> <td style="text-align: center;"><i>f</i></td><td style="text-align: center;">converging lens/ concave mirror</td></tr> <tr> <td style="text-align: center;"><i>M_T</i></td><td style="text-align: center;">erect image</td></tr> <tr> <td></td><td style="text-align: center;">inverted image</td></tr> </tbody> </table>		sign convention		+	-	<i>r</i>	centred to right	<i>u</i>	real object	<i>v</i>	real image	<i>f</i>	converging lens/ concave mirror	<i>M_T</i>	erect image		inverted image
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Fermat's principle ^b	$L = \int \eta \, dl \quad \text{is stationary} \quad (8.63)$																
Gauss's lens formula	$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad (8.64)$																
Newton's lens formula	$x_1 x_2 = f^2 \quad (8.65)$																
Lensmaker's formula	$\frac{1}{u} + \frac{1}{v} = (\eta - 1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \quad (8.66)$																
Mirror formula ^c	$\frac{1}{u} + \frac{1}{v} = -\frac{2}{R} = \frac{1}{f} \quad (8.67)$																
Dioptre number	$D = \frac{1}{f} \quad \text{m}^{-1} \quad (8.68)$																
Focal ratio ^d	$n = \frac{f}{d} \quad (8.69)$																
Transverse linear magnification	$M_T = -\frac{v}{u} \quad (8.70)$																
Longitudinal linear magnification	$M_L = -M_T^2 \quad (8.71)$																
L optical path length η refractive index dl ray path element u object distance v image distance f focal length $x_1 = v - f$ $x_2 = u - f$																	
r_i radii of curvature of lens surfaces R mirror radius of curvature D dioptre number (f in metres) n focal ratio d lens or mirror diameter																	
M_T transverse magnification M_L longitudinal magnification																	

^aFormulas assume “Gaussian optics,” i.e., all lenses are thin and all angles small. Light enters from the left.

^bA stationary optical path length has, to first order, a length identical to that of adjacent paths.

^cThe mirror is concave if $R < 0$, convex if $R > 0$.

^dOr “f-number,” written $f/2$ if $n=2$ etc.

Prisms (dispersing)

Transmission angle	$\sin \theta_t = (\eta^2 - \sin^2 \theta_i)^{1/2} \sin \alpha$ $- \sin \theta_i \cos \alpha$	θ_i angle of incidence θ_t angle of transmission α apex angle η refractive index
Deviation	$\delta = \theta_i + \theta_t - \alpha$	δ angle of deviation
Minimum deviation condition	$\sin \theta_i = \sin \theta_t = \eta \sin \frac{\alpha}{2}$	δ_m minimum deviation
Refractive index	$\eta = \frac{\sin[(\delta_m + \alpha)/2]}{\sin(\alpha/2)}$	D dispersion
Angular dispersion ^a	$D = \frac{d\delta}{d\lambda} = \frac{2 \sin(\alpha/2)}{\cos[(\delta_m + \alpha)/2]} \frac{d\eta}{d\lambda}$	λ wavelength

^aAt minimum deviation.

Optical fibres

Acceptance angle	$\sin \theta_m = \frac{1}{\eta_0} (\eta_f^2 - \eta_c^2)^{1/2}$	θ_m maximum angle of incidence η_0 exterior refractive index η_f fibre refractive index η_c cladding refractive index
Numerical aperture	$N = \eta_0 \sin \theta_m$	N numerical aperture
Multimode dispersion ^a	$\frac{\Delta t}{L} = \frac{\eta_f}{c} \left(\frac{\eta_f}{\eta_c} - 1 \right)$	Δt temporal dispersion L fibre length c speed of light

^aOf a pulse with a given wavelength, caused by the range of incident angles up to θ_m . Sometimes called "intermodal dispersion" or "modal dispersion."