SAMPLE PAPER-04 (solved)

PHYSICS (Theory)

Class - XI

Time allowed: 3 hours Maximum Marks: 70

General Instructions:

- a) All the questions are compulsory.
- b) There are **26** questions in total.
- c) Questions 1 to 5 are very short answer type questions and carry **one** mark each.
- d) Questions 6 to 10 carry two marks each.
- e) Questions 11 to 22 carry three marks each.
- f) Questions 23 is value based questions carry four marks.
- g) Questions **24** to **26** carry **five** marks each.
- h) There is no overall choice. However, an internal choice has been provided in one question of two marks, one question of three marks and all three questions in five marks each. You have to attempt only one of the choices in such questions.
- i) Use of calculators is **not** permitted. However, you may use log tables if necessary.
- j) You may use the following values of physical constants wherever necessary:

$$c = 3x10^{8} m / s$$

$$h = 6.63x10^{-34} Js$$

$$e = 1.6x10^{-19} C$$

$$\mu_{o} = 4\pi x10^{-7} TmA^{-1}$$

$$\frac{1}{4\pi\varepsilon_{0}} = 9x10^{9} Nm^{2} C^{-2}$$

$$m_{e} = 9.1x10^{-31} kg$$

- 1. Why a man standing near the top of an old wooden step-ladder feels unstable?
- 2. What happens to the potential energy of the spring when it is compressed or stretched?
- 3. Find the angular velocity of the hour hand of clock.
- 4. Write down the value of Stefan's constant in SI units if its value in CGS system is $5.67 \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ K}^{-4}$.
- 5. Give the unit and dimension of moment of inertia?
- 6. What will be the magnitude and the direction of acceleration of the stone when a stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed and if the stone makes 14 revolutions in 25 s?
- 7. Explain if angular momentum and rotational kinetic energy can be conserved in a system whose moment of inertia is decreased.

- 8. At what temperature will the average velocity of oxygen molecules be sufficient so as to escape from the earth? [Given: Escape velocity from the earth is 11.0 km/sec and the mass of one molecule of oxygen is $5.34 \times 10^{-26} \text{ kg}$].
- 9. Write the expression for C_v and C_p of a gas in terms of gas constant R and constant γ where $\gamma = \frac{C_p}{C}$
- 10. On what factors do the critical velocity of the liquid depends?
- 11. Find how high will it rise and how much time will it take to return to its point of projection if a body is thrown up with a velocity of 748.4ms⁻¹?
- 12. How should one kg of water at 5°C be so divided that one part of it when converted into ice at 0°C would by this change of state provide a quantity of heat that would be sufficient to vaporize the other part?
- 13. A perfect Carnot engine utilizes an ideal gas. The source temperature is 500 K and sink temperature is 375 K. If the engine takes 600 kcal per cycle from the source, then calculate
 - a. The efficiency of the engine
 - b. Work done per cycle
 - c. Heat rejected to the sink per cycle
- 14. What is the frequency of a second pendulum in an elevator moving up with an acceleration of $\frac{g}{2}$?

Or

If a rocket moving at a speed of 200 m/s towards a stationary target emitting a wave of frequency 1000 Hz and some of the sound reaching the target gets reflected back to the rocket as an echo, then calculate

- a) The frequency of the sound detected by the target
- b) The frequency of the echo detected by the rocket
- 15. Calculate the minimum energy required to launch a 250 kg satellite from earth's surface at an altitude of 2R when r is the radius of the earth and is equal to 6400 km.
- 16. An enclosure of volume four litres contains a mixture of 8 g of oxygen 14 g of nitrogen and 22 g of carbon dioxide. If the temperature of the mixture is 27° C, find the pressure of the mixture of gases. Given R = 8.315 J k⁻¹ mol⁻¹
- 17. Justify the statement: "On the earth satellite, the ratio of its velocity at apogee to its velocity at perigee is equal to the inverse ratio of its distance from apogee and perigee".

- 18. Calculate the acceleration due to gravity at the surface of Mars if its diameter is 6760 km and mass one tenth that of the earth by assuming that the diameter of earth is 12742 km and acceleration due to gravity on the earth is 9.8 m/s^2 .
- 19. A bird is sitting on the floor of a closed glass cage and the cage is in the hand of a girl. Will the girl experience any change in the weight of the cage when the bird (i) starts flying in the cage with a constant velocity (ii) flies upwards with acceleration (iii) flies downwards with acceleration?
- 20. Calculate the total number of air molecule in a room of capacity 25.0 m³ at a temperature of 270 C and 1 atm pressure.
- 21. Under what conditions would your weight become zero?
- 22. What is the angle at which the two nuclei fly apart if a nucleus at rest is all of a sudden splits into two small nuclei?
- 23. Vinita went to her grandfather's village for vacation. She saw a bullock cart got struck in wet mud and the driver was not able to push it out by himself. Vinita ran to his help and together they pushed it out, but the iron rim of the wheel came out. They tried to put it on the wheel but it was smaller than diameter of wheel. Suddenly she collected some wood and set them on fire and heated the rim and it slipped on the wheel.
 - a) What nature is shown by Vinita?
 - b) Name the property of solid used here?
 - c) To what temperature had Vinita heated the ring so as to fit the rim of the wheel if the diameter of the rim and ring were 6.243 m and 6.231 m respectively at 27° C? [Coefficient of linear expansion of iron = 1.20×10^{-5} K⁻¹
- 24. Calculate the percentage increase in the length of a wire of diameter 2.5 mm stretched by a force of 100 kg f if Young's modulus Y for the wire = $12.5 \times 10^{11} \text{ dyne cm}^{-2}$

Or

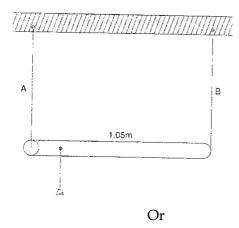
Water stands at a depth 'H' in a tank whose side walls are vertical. If a hole is made on one of the walls at a depth 'h' below the water surface, then find

- i. At what distance from the foot of the wall does the emerging stream of water strike the floor?
- ii. What value of h this range is maximum?
- 25. A cylindrical piece of work of density of base area 'A' and high 'h' floats in a liquid of density ' ρ '. If the cork is depressed slightly and then released, then show that the cork oscillates up and down simple harmonically with a period $T = 2\pi \sqrt{h\rho/\rho_1 g}$.

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Give the characteristic of stationery waves.

- 26. A rod of length 1.05 m having mass is supported at its ends by two wires of steel A and aluminium B of equal length having cross-sectional areas 1.0 mm² and 2.0 mm² respectively. At what point along the rod should a mass m be suspended in order to produce
 - a) Equal stresses
 - b) Equal strains in both steel and aluminium wires.



Air is streaming past a horizontal air plane wing such that its speed is 120 ms⁻¹ over the upper surface and 90 ms⁻¹ at the lower surface. If the density of air is 1.3 kg m⁻³, then

- a) Find the difference in pressure between the top and bottom of the wing.
- b) Calculate the gross lift of the wing if wing is 10 m long and has an average width of 2 m.

SAMPLE PAPER-04 (solved)

PHYSICS (Theory)

Class - XI

ANSWERS

1. Why a man standing near the top of an old wooden step-ladder feels unstable?

Ans:

The point of contact of the ladder with the ground is the point about which the ladder can rotate. When the man is at the top of the ladder, the lever arm of force is large.

2. What happens to the potential energy of the spring when it is compressed or stretched?

Ans:

When a spring is compressed or stretched, the potential energy of the spring increases in both the cases due to work done by us in compressing as well as stretching.

3. Find the angular velocity of the hour hand of clock.

Ans:

$$\Omega = 2 \pi / 12 \text{ hour} = 2 \pi / 12 \text{ x} 3600 \text{ rad/sec}$$

4. Write down the value of Stefan's constant in SI units if its value in CGS system is 5.67×10^{-5} erg s⁻¹ cm⁻² K⁻⁴.

Ans:

$$\sigma$$
 = 5.67 x 10⁻⁸ Js⁻¹ m⁻² K⁻⁴.

5. Give the unit and dimension of moment of inertia?

- a) Unit kg m²
- b) Dimension [M¹L²T¹]
- 6. What will be the magnitude and the direction of acceleration of the stone when a stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed and if the stone makes 14 revolutions in 25 s?

$$r = 80 \text{ cm} = 0.8 \text{ m}$$

$$v = \frac{14}{25} \text{ rev / s}$$

$$\omega = 2\pi v = 2 \times \frac{22}{7} \times \frac{14}{25} \text{ rad /s}$$

$$=\frac{88}{25}$$
 rad s⁻¹

The centripetal acceleration

$$a = \omega^2 r = \left(\frac{88}{25}\right)^2 \times 0.80$$
$$= 9.90 \text{ ms}^{-2}$$

The direction of centripetal acceleration is along the string directed towards the centre of circular path.

7. Explain if angular momentum and rotational kinetic energy can be conserved in a system whose moment of inertia is decreased.

Ans:

$$L = I\omega = constant.$$

K.E of rotation K = $\frac{1}{2}$ I ω^2

$$K = \frac{1}{2I}I\omega^2 = \frac{L^2}{2I}$$

$$K \infty \frac{1}{I}$$

When moment of inertia (I) decreases, K.E of rotation (K) increases. Thus K.E. of rotation is not conserved.

8. At what temperature will the average velocity of oxygen molecules be sufficient so as to escape from the earth? [Given: Escape velocity from the earth is 11.0 km/sec and the mass of one molecule of oxygen is $5.34 \times 10^{-26} \text{ kg}$].

$$\frac{1}{2}mv^{2} = \frac{3}{2}kT$$

$$T = \frac{mv^{2}}{3k}$$

$$m = 5.34 \times 10^{-26} \text{ kg}$$

$$v = 11.0 \text{ km/s} = 11 \times 10^{3} \text{ ms}^{-1}$$

$$k = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

$$T = \frac{5.34 \times 10^{-26} \times \left(11 \times 10^{3}\right)^{2}}{3 \times 1.38 \times 10^{-23}}$$

$$= 1.56 \times 10^{5} \text{K}$$
Or

A spring of force constant k has a mass M suspended from it. If the spring is cut into the two halves and the same mass is attached to one of the pieces. What will be the frequencies of oscillation of the mass?

Ans:

When the spring is cut into two equal halves, the force constant of each part will be doubled.

The original frequency
$$v = \frac{1}{2\pi} \sqrt{\frac{k}{M}}$$

$$v' = \frac{1}{2\pi} \sqrt{\frac{2k}{M}} = \sqrt{2v}$$

9. Write the expression for C_{ν} and C_{p} of a gas in terms of gas constant R and constant γ where

$$\gamma = \frac{C_p}{C_v}$$

Ans:

$$C_p$$
- C_v = R ----- (i)

$$\frac{C_p}{C_v} = \gamma - - - - (ii)$$

From equation (ii) $C_p = \gamma C_v$ and substituting this value in (i)

$$\gamma C_{v} - C_{v} = R \Rightarrow C_{v} = \frac{R}{(\gamma - 1)}$$

$$C_{v} = \gamma C_{p} = \frac{\gamma R}{(\gamma - 1)}$$

10. On what factors do the critical velocity of the liquid depends?

Ans:

Critical velocity (v_0) of the liquid are:

- a) Directly proportional to the coefficient of viscosity of the liquid
- b) Inversely proportional to the density of the liquid $V_c \approx \frac{1}{\rho}$
- c) Inversely proportional to the diameter of the tube through which it flows $V_c \propto \frac{1}{D}$

11. Find how high will it rise and how much time will it take to return to its point of projection if a body is thrown up with a velocity of 748.4ms⁻¹?

Ans

Initial velocity
$$u = 78.4 \text{ ms}^{-1}$$

The body reaches to the maximum height 'h

The velocity at maximum height v = 0

From the equation $v^2 - u^2 = 2gh$

$$v^2 = (78.4)^2 = 2(-9.8) \times h$$

 $h = \frac{78.4 \times 78.4}{2 \times 9.8} \text{m}$

$$= 313.6 \text{ m}$$

Using the equation v = u + gt

$$0 = 78.4 + (-9.8) t$$

$$t = \frac{78.4}{9.8} s$$

$$=8s$$

Total time taken to return to the point of projection = Time taken in ascent + time taken in descent.

$$= 2 \times 8 = 16 \text{ s}$$

12. How should one kg of water at 5°C be so divided that one part of it when converted into ice at 0°C would by this change of state provide a quantity of heat that would be sufficient to vaporize the other part?

Ans:

Initially 1000g of water is at 5°C

Let m gram of it be cooled to ice at 0°C

Heat released due to this = $(m \times 1 \times 5) + (m \times 80)$

$$= 5m + 80 m = 85 m cal.$$

The heat required by (1000 - m)g of water at 5°C to become steam at 100°C

$$= [(1000 - m) (100 - 5) + (1000 - m) 540]$$
$$= (1000 - m) (95 + 540) cal$$

$$= (1000 - m) (635) cal$$

$$85 \text{ m} = (1000 - \text{m}) (635)$$

$$720 \text{ m} = 635 \text{ x} 1000$$

$$m = 881.9 g$$

Hence 881.9 g water by turning into at 0°C will supply heat to evaporate 118.1 g of water.

- 13. A perfect Carnot engine utilizes an ideal gas. The source temperature is 500 K and sink temperature is 375 K. If the engine takes 600 kcal per cycle from the source, then calculate
 - a. The efficiency of the engine
 - b. Work done per cycle
 - c. Heat rejected to the sink per cycle

Ans:

$$T_1 = 500 \text{ K}$$

$$T_2 = 375 \text{ K}$$

 Q_1 = heat absorbed per cycle = 600 Kcal

i. Using the relation

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{500 - 375}{375} = \frac{125}{500} = 0.25$$

$$\eta = 0.25 \times 100 = 25\%$$

ii. Let W = work done per cycle

$$\eta = \frac{W}{Q_1}$$

$$W = \eta Q_1 = 0.25 \times 600 \text{ Kcal} = 150 \text{ Kcal}$$

$$= 150 \times 10^3 \times 4.2 \text{ J} = 6.3 \times 10^5 \text{ J}$$

iii. Let Q_2 = heat rejected to the sink

$$W = Q_1 - Q_2$$

$$Q_2 = Q_1 - W = 600 - 150 = 450 \text{ Kcal}$$

14. What is the frequency of a second pendulum in an elevator moving up with an acceleration of $\frac{g}{2}$?

For second pendulum, frequency $v = \frac{1}{2}s^{-1}$

When elevator is moving upwards with acceleration a, the effective acceleration due to

gravity is
$$g_1 = g + a = g + \frac{g}{2} = \frac{3g}{2}$$

$$v = \frac{1}{2\pi} \sqrt{\frac{g}{l}}$$

$$\frac{v_1^2}{v^2} = \frac{g_1}{g} = \frac{\frac{3g}{2}}{g} = \frac{3}{2}$$

$$\frac{v_1}{v} = \sqrt{\frac{3}{2}} = 1.225$$

$$v_1 = 1.225 \text{ v}$$

$$= 1.225 \text{ x} \frac{1}{2} = 0.612 \text{ s}^{-1}$$
Or

If a rocket moving at a speed of 200 m/s towards a stationary target emitting a wave of frequency 1000 Hz and some of the sound reaching the target gets reflected back to the rocket as an echo, then calculate

- c) The frequency of the sound detected by the target
- d) The frequency of the echo detected by the rocket Ans:

a)
$$v = v_0 \left(1 - \frac{v_s}{v} \right)^{-1}$$

= 1000 x (1-200/300) = 2540 Hz
b) $v' = v \left(\frac{v + v_0}{v} \right)$
= 2540 x (200 + 330/330) = 4080 Hz

15. Calculate the minimum energy required to launch a 250 kg satellite from earth's surface at an altitude of 2R when r is the radius of the earth and is equal to 6400 km.

Ans:

The total energy of a satellite of a mass m in a circular orbit of radius r is

$$\frac{1}{2}$$
 mv² – GMm/r

Where r is measured from the centre of the earth, then the total mechanical energy in the orbit,

$$E = g \frac{mM}{2r} - G \frac{Mm}{r} = -G \frac{mM}{2r}$$
$$r = 2R + R = 2R$$
$$mM$$

$$E = -G \frac{mM}{6R}$$

The potential energy on the surface of the earth = $-G \frac{mM}{R}$

Minimum energy required
$$=\frac{1}{6} G \frac{mM}{R} - \left(-G \frac{Mm}{R}\right)$$

 $=\frac{5}{6} G \frac{mM}{R} = \frac{5}{6} mgR$
 $=\frac{5}{6} \times 250 \times 9.8 \times 6.4 \times 10^6 J = 1.3 \times 10^{10} J$

16. An enclosure of volume four litres contains a mixture of 8 g of oxygen 14 g of nitrogen and 22 g of carbon dioxide. If the temperature of the mixture is 27° C, find the pressure of the mixture of gases. Given R = 8.315 J k⁻¹ mol⁻¹

Ans:

Temperature
$$T = 300 \text{ K}$$

$$V = 4 \text{ litres} = 4 \times 10^{-3} \text{ m}^3$$

The pressure exerted by a gas is given by

$$p = \frac{nRT}{V} = \frac{mass}{molecular \text{ weight}} \times \frac{RT}{v}$$

Pressure exerted by oxygen
$$p_1 = \frac{8}{32} \frac{RT}{V} = \frac{1}{4} \frac{RT}{V}$$

Pressure exerted by nitrogen
$$p_2 = \frac{14}{28} \frac{RT}{V} = \frac{1}{2} \frac{RT}{V}$$

Pressure exerted by carbon dioxide $p_3 = \frac{24}{44} \frac{RT}{V} = \frac{1}{2} \frac{RT}{V}$

From Dalton's law of partial pressure, the total pressure exerted by the mixture is given by

$$P = P_1 + P_2 + P_3$$

$$P = \frac{1}{4} \frac{RT}{V} + \frac{1}{2} \frac{RT}{V} + \frac{1}{2} \frac{RT}{V}$$

$$P = \frac{5}{4} \frac{RT}{V} = \frac{5}{4} x \frac{8.315 \times 300}{4 \times 10^{-3}}$$

$$P = 7.79 \times 10^5 \text{ N m}^{-2}$$

17. Justify the statement: "On the earth satellite, the ratio of its velocity at apogee to its velocity at perigee is equal to the inverse ratio of its distance from apogee and perigee".

Ans:

Let mass of satellite = m, distance of apogee from the earth = r_a

Distance of perigee from earth = r_p ,

Velocity of satellite at apogee = v_a

Velocity of satellite at perigee = v_p

Now angular momentum of satellite at apogee = mv_ar_a

Angular momentum of the satellite at perigee = mv_pr_p

According to the law of conservation of angular momentum $mv_a r_a = mv_p r_p$

$$\frac{v_a}{v_p} = \frac{r_p}{r_a}$$

18. Calculate the acceleration due to gravity at the surface of Mars if its diameter is 6760 km and mass one tenth that of the earth by assuming that the diameter of earth is 12742 km and acceleration due to gravity on the earth is 9.8 m/s^2 .

Ans:

$$g = \frac{GM}{R^2}$$

Let g_M and g_e be the acceleration due to gravity at Mars and earth.

$$\frac{gM}{g_e} = \left(\frac{M_M}{M_e}\right) \left(\frac{R_e}{R_M}\right)^2$$

$$= \left(\frac{1}{10}\right) \left(\frac{12742}{6760}\right)^2$$

$$= 0.35$$

$$g_M = 0.35 \times g_e = 0.35 \times 9.8 = 3.48 \text{ ms}^{-2}$$

19. A bird is sitting on the floor of a closed glass cage and the cage is in the hand of a girl. Will the girl experience any change in the weight of the cage when the bird (i) starts flying in the cage with a constant velocity (ii) flies upwards with acceleration (iii) flies downwards with acceleration?

Ans:

In a closed glass cage, air inside is bound with the cage,

- (i) There would be no change in weight of the cage if the bird flies with a constant velocity.
- (ii) The cage becomes heavier, when bird flies upwards with acceleration.
- (iii) The cage appears lighter, when bird flies downwards with acceleration.
- 20. Calculate the total number of air molecule in a room of capacity 25.0 m³ at a temperature of 27^o C and 1 atm pressure.

Ans:

Volume of the room $V = 25.0 \text{ m}^3$, temperature $T = 27^{\circ} \text{ C} = 300 \text{ k}$

Gas equation
$$PV = \mu RTB = T \mu N_A . k$$

Total number of air molecules in the volume of given gas

$$N = \mu.N_A = \frac{PV}{k_B T}$$

$$N = \frac{1.01 \times 10^5 \times 25.0}{(1.38 \times 10^{-23}) \times 300} = 6.1 \times 10^{26}$$

27. Under what conditions would your weight become zero?

Ans:

The weight will become zero under the following conditions:

- (i) During free fall
- (ii) At the centre of the earth
- (iii) In an artificial satellite
- (iv) At a point where gravitational pull of earth to the gravitational pull of the moon.
- 28. What is the angle at which the two nuclei fly apart if a nucleus at rest is all of a sudden splits into two small nuclei?

Ans:

Let M = mass of nucleus at rest

Momentum of the nucleus before disintegration = $M \times 0 = 0$

Let m_1 and m_2 be the mass of the two smaller nuclei and v_1 and v_2 be their velocities

Momentum of the nucleus after disintegration = $m_1 v_1 + m_2 v_2$

According to the law of conservation of linear momentum $m_1 \, v_1$ + $m_2 \, v_2$ = 0

$$m_1 v_1 = - m_2 v_2$$

The negative sign shows that the velocities v_1 and v_2 must be opposite sign; the two products must be emitted in opposite direction. Thus the angle between two nuclei is 180°

- 29. Vinita went to her grandfather's village for vacation. She saw a bullock cart got struck in wet mud and the driver was not able to push it out by himself. Vinita ran to his help and together they pushed it out, but the iron rim of the wheel came out. They tried to put it on the wheel but it was smaller than diameter of wheel. Suddenly she collected some wood and set them on fire and heated the rim and it slipped on the wheel.
 - d) What nature is shown by Vinita?
 - e) Name the property of solid used here?
 - f) To what temperature had Vinita heated the ring so as to fit the rim of the wheel if the diameter of the rim and ring were 6.243 m and 6.231 m respectively at 27° C? [Coefficient of linear expansion of iron = 1.20×10^{-5} K⁻¹

Ans:

- a) She has presence of mind and is helpful in nature.
- b) She has used linear expansion of solids.
- c) Here,

$$L_1 = 6.231 \text{ m}, L_2 = 6.243 \text{ m}, T_1 = 270 \text{C}$$

Using the formula,
$$\alpha = \frac{L_2 - L_1}{L_1(T_2 - T_1)}$$

We get
$$T_2 = \frac{6.243 - 6.231}{6.243 \times 1.2 \times 10^{-5}} + 27 = 187^{\circ}C$$

30. Calculate the percentage increase in the length of a wire of diameter 2.5 mm stretched by a force of 100 kg f if Young's modulus Y for the wire = 12.5×10^{11} dyne cm⁻²

Y = 12.5 x 10¹¹ dyne cm⁻²
= 12.5 x 10¹⁰ Nm⁻².
Diameter D = 2.5 mm = 2.5 x 10⁻³m
F = 100 kg f = 100 x 9.8 N = 980 N

$$\frac{\Delta L}{L} \times 100 = x$$

$$A = \pi r^2 = \pi (1.25 \times 10^{-3})^2 \text{ m}^2.$$

Using the relation,

$$Y = \frac{FL}{A\Lambda L}$$
 we get,

Percentage increase in length = $\frac{\Delta L}{L}$ x 100

$$= \frac{F}{AY} \times 100 = \frac{F}{\pi r^2 Y} \times 100$$

$$= \frac{980}{3.142x(1.25x10^{-3})^2 x12.5x10^{10}} x100$$

$$= 15.96 \times 10^{-2} = 0.16\%$$

Or

Water stands at a depth 'H' in a tank whose side walls are vertical. If a hole is made on one of the walls at a depth 'h' below the water surface, then find

iii. At what distance from the foot of the wall does the emerging stream of water strike the floor?

iv. What value of h this range is maximum?

Ans:

$$v_A = \sqrt{2gh} \qquad (i)$$

$$(H-h) = \frac{1}{2}gt^2 \qquad (ii)$$

The distance R,

$$R = v_A \times t - (iii)$$

From equation (ii)
$$t = \sqrt{\frac{2(H-h)}{g}}$$
 ----- (iv)

Substituting the value of v_A from equation (i) and the value of t from equation (iv) in equation (iii) we get,

$$R = \sqrt{(2gh)}x\sqrt{\{2(H-h)/g\}}$$

$$R = 2\sqrt{h(H-h)}$$

The range R will be at maximum when

$$dR/dh = 0$$

$$22.\frac{1}{2}h^{-1/2}(H-h)^{-1/2}-2h^{1/2}.\frac{1}{2}(H-h)^{1/2}=0$$

$$h=H/2$$

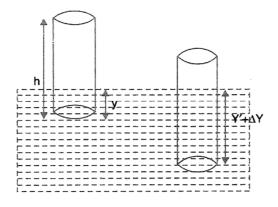
31. A cylindrical piece of work of density of base area 'A' and high 'h' floats in a liquid of density ' ρ '. If the cork is depressed slightly and then released, then show that the cork oscillates up and down simple harmonically with a period $T = 2\pi \sqrt{h\rho/\rho_1 g}$.

Ans:

In equilibrium y height of cylinder is inside the liquid.

Weight of the cylinder = up thrust due to liquid displaced.

$$Ah\rho = Ay\rho_1 g$$



When the cork cylinder is depressed slightly by Δy and released a restoring force equal to additional up thrust acts on it. The restoring force is

$$F = A(y + \Delta y) \rho 1g - Ay \rho_1 g = A \rho_1 g \Delta y$$

Acceleration a = F/m = $A\rho_1g\Delta y$ / $Ah\rho = \rho_1g$ / $h\rho.\Delta y$ and the acceleration is directed in a direction opposite to Δy . As a a = - Δy , the motion of cork cylinder is SHM

$$T = 2\pi \sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

$$T = 2\pi \sqrt{\frac{\Delta y}{a}}$$

$$T = 2\pi \sqrt{\frac{h\rho}{\rho_1 g}}$$
Or

Give the characteristic of stationery waves.

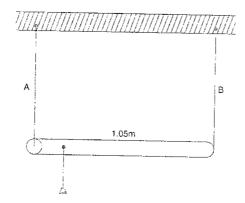
Ans:

- (i) Stationary waves are produced in a bounded medium. A medium whose boundaries are separated from other media by distinct surface is called bounded medium. The boundaries of a bounded medium may be rigid or free.
- (ii) There are certain points in the bounded medium which are always in the state of rest.

 These points are called nodes. If the stationary waves are longitudinal, then the change in pressure and density is maximum at nodes as compared to the other points.
- (iii) There are points between the nodes whose displacement is maximum as compared to other points. These points are called anti-nodes. In the longitudinal stationary waves, there is no-charge in pressure and density of the medium at anti-nodes.
- (iv) The distance between any two successive nodes or antinodes is $\lambda/2$. The distance between a node and the neighbouring anti-node is $\lambda/4$
- (v) All the particles between two successive nodes vibrate in the same phase. They pass simultaneously through their mean positions and also pass simultaneously through their positions of maximum displacement.
- (vi) Stationary waves do not advance in the medium, but remains steady at its place.
- (vii) All the particles except those at nodes, execute simple harmonic motion about their mean positions with the same time period.

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- (viii) In a stationary wave, the medium splits up into a number of segments. Each segment vibrates up and down as a whole.
- 32. A rod of length 1.05 m having mass is supported at its ends by two wires of steel A and aluminium B of equal length having cross-sectional areas 1.0 mm² and 2.0 mm² respectively. At what point along the rod should a mass m be suspended in order to produce
 - c) Equal stresses
 - d) Equal strains in both steel and aluminium wires.



Ans:

For steel wire A $l_1 = 1$; $A_1 = 1 \text{ mm}^2$; $Y_1 = 2 \times 10^{11} \text{ Nm}^{-2}$

For aluminium wire B, l_2 = 1; A_2 =2mm² ; Y_2 = 7 x 10¹⁰ Nm⁻²

a) Let mass m be suspended from the rod at distance x from the end where wire A is connected. Let F_1 and F_2 be the tensions in two wires and there e is equal stress in two wires

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow \frac{F_1}{F_2} = \frac{A_1}{A_2} = \frac{1}{2}$$
 -----(i)

Taking moment of forces about the point of suspension of mass from the rod,

$$F_1x = F_2(1.05 - x)$$

$$2.10-2x = x = 0.70$$
m = 70 cm

(a) Let mass m be suspended from the rod at distance x from the end where wire A is connected. Let F_1 and F_2 be the tension in the wires and there is equal strain in the two wires.

$$\frac{F_1}{A_1 Y_1} = \frac{F_2}{A_2 Y_2} \Rightarrow \frac{F_1}{F_2} = \frac{A_1 Y_1}{A_2 Y_2} = \frac{1}{2} \times \frac{2 \times 10^{11}}{7 \times 10^{10}} = \frac{10}{7}$$

As the rod is stationary $F_1x = F_2(1.05 - x)$

$$10x = 7.35 - 7x$$

$$x = 0.4324 \text{ m} = 43.2 \text{ cm}$$

Or

Air is streaming past a horizontal air plane wing such that its speed is 120 ms⁻¹ over the upper surface and 90 ms⁻¹ at the lower surface. If the density of air is 1.3 kg m⁻³, then

- c) Find the difference in pressure between the top and bottom of the wing.
- d) Calculate the gross lift of the wing if wing is 10 m long and has an average width of 2 m. Ans:

According to Bernoulli's theorem,

$$\frac{P_1}{\rho} + gh_1 + \frac{1}{2}2v_1^2 = \frac{P_2}{\rho} + gh_2 + \frac{1}{2}v_2^2$$

For the horizontal flow h₁=h₂

$$\frac{P_1}{\rho} + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + \frac{1}{2}v_2^2$$

 $v_1 = 90 \text{ ms}^{-1}$; $v_2 = 120 \text{ ms}^{-1}$

$$\rho$$
 = 1.3 kg m⁻³

$$\frac{P_1 - P_2}{\rho} = \frac{1}{2} 1 \left(v_2^2 - v_1^2 \right)$$

$$(P_1 - P_2) = \frac{\rho(v_2^2 - v_2^2)}{2}$$

$$(P_1 - P_2) = \frac{1.3(14400 - 8100)}{2}$$

$$(P_1 - P_2) = \frac{1.3 \times 6300}{2}$$

$$(P_1 - P_2) = 4.095 \times 10^3 \, Nm^{-2}$$

This is the pressure difference between the top and the bottom of the wing.

Gross lift of the wing

= (P_1-P_2) x area of the wing

 $= 4.095 \times 10^3 \times 10 \times 2$

 $= 8.190 \times 10^4 \text{ N}$