Chapter

8

Differential Equations

Contents

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One of the principal languages of science is that of differential equations. Interprestingly, the date of birth of differential equations is taken to be **November 11, 1675**, ***when*** **Gottfried Wilthelm Freiherr Leibnitz (1646-1716)** ***first put in black and white the identity*** $\int y\,dy = \frac{1}{2}y^2$ ***thereby introducing both the symbols*** $\int$ ***and dy. Leibnitz was actually interested in the problem of finding a curve whose tangents were prescribed. This led him to discover the*** **'method of separation of variables'** ***in*** **1691.** ***A year later he formulated the*** **'method of solving the homogeneous differential equations of the first order'.** ***He went further in a very short time to the discovery of the*** **'method of solving a linear differential equation of the first-order'.** ***How surprising is it that all these methods came from a single man and that too within 25 years of the birth of differential equations.***

Differential Equations

Many of the practical problems in physics and engineering can be converted into differential equations. The solution of differential equations is, therefore of paramount importance. This chapter deals with some elementary aspects of differential equations. These are addressed through simple application of differential and integral calculus.

There are two important aspects of differential equation, which have just been touched in this chapter. How to formulate a problem as a differential equation is the one, and the other is how to solve it.

8.1 Definition

An equation involving independent variable x, dependent variable y and the differential coefficients $\frac{dy}{dx}, \frac{d^2y}{dx^2}, ...$ is called differential equation.

Examples : (i) $\frac{dy}{dx} = 1 + x + y$ (ii) $\frac{dy}{dx} + xy = \cot x$

(iii) $\left(\frac{d^4y}{dx^4}\right)^3 - 4\frac{dy}{dx} + 4y = 5\cos 3x$ (iv) $x^2\frac{d^2y}{dx^2} + \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = 0$

(1) **Order of a differential equation :** The order of a differential equation is the order of the highest derivative occurring in the differential equation. For example, the order of above differential equations are 1,1,4 and 2 respectively.

Note : ❑ The order of a differential equation is a positive integer. To determine the order of a differential equation, it is not needed to make the equation free from radicals.

(2) **Degree of a differential equation :** The degree of a differential equation is the degree of the highest order derivative, when differential coefficients are made free from radicals and fractions. In other words, the degree of a differential equation is the power of the highest order derivative occurring in differential equation when it is written as a polynomial in differential coefficients.

Note : ❑ The definition of degree does not require variables x, y, t etc. to be free from radicals and fractions. The degree of above differential equations are 1, 1, 3 and 2 respectively.

Example: 1 The order and degree of the differential equation $y = x\frac{dy}{dx} + \sqrt{a^2\left(\frac{dy}{dx}\right)^2 + b^2}$ are

(a) 1, 2 (b) 2, 1 (c) 1, 1 (d) 2, 2

Solution: (a) Clearly, highest order derivative involved is $\frac{dy}{dx}$, having order 1.

Expressing the above differential equation as a polynomial in derivative, we have $\left(y - x\frac{dy}{dx}\right)^2 = a^2\left(\frac{dy}{dx}\right)^2 + b^2$

i.e., $(x^2 - a^2)\left(\frac{dy}{dx}\right)^2 - 2xy\frac{dy}{dx} + y^2 - b^2 = 0$

In this equation, the power of highest order derivative is 2. So its degree is 2.

Example: 2 The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{3}} + x^{\frac{1}{4}} = 0$, are respectively

(a) 2, 3 (b) 3, 3 (c) 2, 6 (d) 2, 4

Solution: (a) The highest order derivative involved is $\frac{d^2y}{dx^2}$ which is the 2nd order derivative. Hence order of the differential equation is 2. Making the above equation free from radical, as far as the derivatives are concerned, we have

$$\left(\frac{d^2y}{dx^2} + x^{\frac{1}{4}}\right)^3 = -\frac{dy}{dx} \quad i.e. \left(\frac{d^2y}{dx^2} + x^{\frac{1}{4}}\right)^3 + \frac{dy}{dx} = 0$$

The exponent of highest order derivative $\frac{d^2y}{dx^2}$ will be 3. Hence degree of the differential equation is 3.

Example: 3 The degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$ is

(a) 1 (b) 2 (c) 3 (d) None of these

Solution: (d) The above equation cannot be written as a polynomial in derivatives due to the term $x^2 \log\left(\frac{d^2y}{dx^2}\right)$.

Hence degree of the differential equation is 'not defined'.

Example: 4 The order of the differential equation whose solution is $x^2 + y^2 + 2gx + 2fy + c = 0$, is

(a) 1 (b) 2 (c) 3 (d) 4

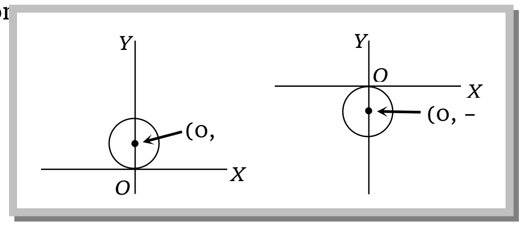
Solution: (c) To eliminate the arbitrary constants *g*, *f* and *c*, we need 3 more equations, that by differentiating the equation 3 times. Hence highest order derivative will be $\frac{d^3y}{dx^3}$. Hence order of the differential equation will be 3.

Example: 5 The order of the differential equation of all circles of radius *r*, having centre on y-axis and passing through the origin is

(a) 1 (b) 2 (c) 3 (d) 4

Solution: (a) The equation of the circle will be $x^2 + (y \pm r)^2 = r^2$. Hence the constant to be eliminated is *r*.

∴ order of differential equation



Example: 6 The order of the differential equation, whose general solution is $y = c_1e^x + c_2e^{2x} + c_3e^{3x} + c_4e^{x+c_5}$, where c_1, c_2, c_3, c_4, c_5 are arbitrary constants is

(a) 5 (b) 4 (c) 3 (d) None of these

Solution: (c) Rewriting the given general solution, we have

$$y = c_1e^x + c_2e^{2x} + c_3e^{3x} + c_4e^x.e^{c_5}$$

$$= (c_1 + c_4.e^{c_5})e^x + c_2e^{2x} + c_3e^{3x} = c_1'e^x + c_2e^{2x} + c_3e^{3x}$$

where $c_1' = c_1 + c_4.e^{c_5}$. So there are 3 arbitrary constant associated with different terms. Hence the order of the differential equation formed, will be 3.

Example: 7 The degree of the differential equation satisfying $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ is

(a) 1 (b) 2 (c) 3 (d) None of these

Solution: (a) To eliminate a the above equation is differentiated once and exponent of $\frac{dy}{dx}$ will be1. Hence degree is 1

Example: 8 The order and degree of $y = 1 + \frac{dy}{dx} + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + ...$ is

(a) 1, 2 (b) 1, 1

(c) Order 1, degree not defined (d) None of these

Solution: (b) The given differential equation can be re-written as $y = e^{\frac{dy}{dx}} \Rightarrow \ln y = \frac{dy}{dx}$

This is a polynomial in derivative. Hence order is 1 and degree 1.

Example: 9 The order and degree of $\frac{d^2y}{dx^2} = \sin\left(\frac{dy}{dx}\right) + x$ is

(a) 2, 1 (b) Order 2, degree not defined

(c) 2, 0 (d) None of these

Solution: (b) As the highest order derivative involved is $\frac{d^2y}{dx^2}$. Hence order is 2.

The given differential equation cannot be written as a polynomial in derivatives, the degree is not defined.

8.2 Formation of Differential Equation

Formulating a differential equation from a given equation representing a family of curves means finding a differential equation whose solution is the given equation. If an equation, representing a family of curves, contains *n* arbitrary constants, then we differentiate the given equation *n* times to obtain *n* more equations. Using all these equations, we eliminate the constants. The equation so obtained is the differential equation of order *n* for the family of given curves.

Consider a family of curves $f(x, y, a_1, a_2 ....., a_n) = 0$ ......(i)

where $a_1, a_2, ...a_n$ are *n* independent parameters.

Equation (i) is known as an *n* parameter family of curves *e.g.* $y = mx$ is a one-parameter family of straight lines. $x^2 + y^2 + ax + by = 0$ is a two parameters family of circles.

If we differentiate equation (i) *n* times *w.r.t.* *x*, we will get *n* more relations between $x, y, a_1, a_2, ...a_n$ and derivatives of *y* *w.r.t.* *x*. By eliminating $a_1, a_2, ..., a_n$ from these *n* relations and equation (i), we get a differential equation.

Clearly order of this differential equation will be n *i.e.* equal to the number of independent parameters in the family of curves.

Algorithm for formation of differential equations

Step (i) **:** Write the given equation involving independent variable x (say), dependent variable y (say) and the arbitrary constants.

Step (ii) **:** Obtain the number of arbitrary constants in step (i). Let there be n arbitrary constants.

Step (iii) **:** Differentiate the relation in step (i) n times with respect to x.

Step (iv) **:** Eliminate arbitrary constants with the help of n equations involving differential coefficients obtained in step (iii) and an equation in step (i). The equation so obtained is the desired differential equation.

Example: 10 Differential equation whose general solution is $y = c_1 x + \frac{c_2}{x}$ for all values of c_1 and c_2 is

(a) $\frac{d^2y}{dx^2} + \frac{x^2}{y} + \frac{dy}{dx} = 0$ (b) $\frac{d^2y}{dx^2} + \frac{y}{x^2} - \frac{dy}{dx} = 0$ (c) $\frac{d^2y}{dx^2} - \frac{1}{2x}\frac{dy}{dx} = 0$ (d) $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} = 0$

Solution: (d) $y = c_1 x + \frac{c_2}{x}$ .....(i)

There are two arbitrary constants. To eliminate these constants, we need to differentiate (i) twice.

Differentiating (i) with respect to x,

$$\frac{dy}{dx} = c_1 - \frac{c_2}{x^2} \qquad .....(ii)$$

Again differentiating with respect to x,

$$\frac{d^2y}{dx^2} = \frac{2c_2}{x^3} \qquad ......(iii)$$

From (iii), $c_2 = \frac{x^3}{2}\frac{d^2y}{dx^2}$ and from (ii), $c_1 = \frac{dy}{dx} + \frac{c_2}{x^2}$;

$\therefore\ c_1 = \frac{dy}{dx} + \frac{x}{2}\frac{d^2y}{dx^2}$

From (i), $y = \left(\frac{dy}{dx} + \frac{x}{2}\cdot\frac{d^2y}{dx^2}\right)x + \frac{x^2}{2}\cdot\frac{d^2y}{dx^2} \Rightarrow y = x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx}$

$\therefore\ \frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} = 0$

Example: 11 $y = \frac{x}{x+1}$ is a solution of the differential equation

(a) $y^2\frac{dy}{dx} = x^2$ (b) $x^2\frac{dy}{dx} = y^2$ (c) $y\frac{dy}{dx} = x$ (d) $x\frac{dy}{dx} = y$

Solution: (b) We have $y = \frac{x}{x+1} \Rightarrow \frac{1}{y} = \frac{x+1}{x} = 1 + \frac{1}{x}$

Differentiating *w.r.t.* x,

$$-\frac{1}{y^2}\frac{dy}{dx} = 0 - \frac{1}{x^2}$$

$\therefore\ x^2\frac{dy}{dx} = y^2$

Example: 12 The differential equation of all parabolas whose axes are parallel to y axis is

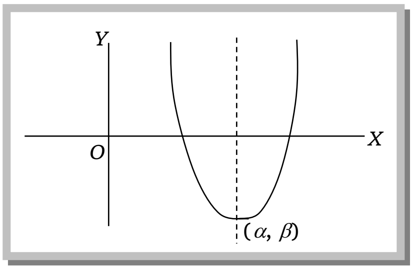
(a) $\frac{d^3y}{dx^3}=0$ (b) $\frac{d^2x}{dy^2}=c$ (c) $\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}=0$ (d) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}=c$

Solution: (a) The equation of a parabola whose axis is parallel to y-axis may be expressed as

$$(x-\alpha)^2=4a(y-\beta) \quad ....(i)$$

There are three arbitrary constants α, β and a.

We need to differentiate (i) 3 times



Differentiating (i) *w.r.t.* x, $2(x-\alpha)=4a\frac{dy}{dx}$

Again differentiating *w.r.t.* x,

$$2=4a\frac{d^2y}{dx^2} \Rightarrow \frac{d^2y}{dx^2}=\frac{1}{2a}$$

Differentiating *w.r.t.* x,

$$\frac{d^3y}{dx^3}=0$$

Example: 13 The differential equation of family of curves whose tangent form an angle of $\pi/4$ with the hyperbola $xy=c^2$ is

(a) $\frac{dy}{dx}=\frac{x^2+c^2}{x^2-c^2}$ (b) $\frac{dy}{dx}=\frac{x^2-c^2}{x^2+c^2}$ (c) $\frac{dy}{dx}=-\frac{c^2}{x^2}$ (d) None of these

Solution: (b) The slope of the tangent to the family of curves is $m_1=\frac{dy}{dx}$

Equation of the hyperbola is $xy=c^2 \Rightarrow y=\frac{c^2}{x}$

$$\therefore \frac{dy}{dx}=-\frac{c^2}{x^2}$$

$\therefore$ Slope of tangent to $xy=c^2$ is $m_2=-\frac{c^2}{x^2}$

Now $\tan\frac{\pi}{4}=\frac{m_1-m_2}{1+m_1m_2} \Rightarrow 1=\frac{\frac{dy}{dx}+\frac{c^2}{x^2}}{1-\frac{c^2}{x^2}\frac{dy}{dx}} \Rightarrow \frac{dy}{dx}\left(1+\frac{c^2}{x^2}\right)=\left(1-\frac{c^2}{x^2}\right)$

$$\therefore \frac{dy}{dx}=\frac{x^2-c^2}{x^2+c^2}$$

8.3 Variable Separable type Differential Equation

(1) **Solution of differential equations :** If we have a differential equation of order 'n' then by solving a differential equation we mean to get a family of curves with n parameters whose differential equation is the given differential equation. Solution or integral of a differential equation is a relation between the variables, not involving the differential coefficients such that this relation and the derivatives obtained from it satisfy the given differential equation. The solution of a differential equation is also called its primitive.

For example $y=e^x$ is a solution of the differential equation $\frac{dy}{dx}=y$.

(i) **General solution :** The solution which contains as many as arbitrary constants as the order of the differential equation is called the general solution of the differential equation. For example, $y = A\cos x + B\sin x$ is the general solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$. But $y = A\cos x$ is not the general solution as it contains one arbitrary constant.

(ii) **Particular solution** : Solution obtained by giving particular values to the arbitrary constants in the general solution of a differential equation is called a particular solution. For example, $y = 3\cos x + 2\sin x$ is a particular solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$

(2) **Differential equations of first order and first degree :** A differential equation of first order and first degree involves *x*, *y* and $\frac{dy}{dx}$. So it can be put in any one of the following forms: $\frac{dy}{dx} = f(x,y)$ or $f\left(x, y, \frac{dy}{dx}\right) = 0$ or $f(x,y)dx + g(x,y)dy = 0$ where $f(x,y)$ and $g(x,y)$ are obviously the functions of *x* and y.

(3) **Geometrical interpretation of the differential equations of first order and first degree :** The general form of a first order and first degree differential equation is $f\left(x, y, \frac{dy}{dx}\right) = 0$ .....(i)

We know that the direction of the tangent of a curve in Cartesian rectangular coordinates at any point is given by $\frac{dy}{dx}$, so the equation in (i) can be known as an equation which establishes the relationship between the coordinates of a point and the slope of the tangent *i.e.*, $\frac{dy}{dx}$ to the integral curve at that point. Solving the differential equation given by (i) means finding those curves for which the direction of tangent at each point coincides with the direction of the field. All the curves represented by the general solution when taken together will give the locus of the differential equation. Since there is one arbitrary constant in the general solution of the equation of first order, the locus of the equation can be said to be made up of single infinity of curves.

(4) **Solution of first order and first degree differential equations :** A first order and first degree differential equation can be written as

$$f(x, y)dx + g(x, y)dy = 0$$

$$\text{or } \frac{dy}{dx} = \frac{f(x,y)}{g(x,y)'} \text{ or } \frac{dy}{dx} = \phi(x,y)$$

Where $f(x,y)$ and $g(x,y)$ are obviously the functions of *x* and *y*. It is not always possible to solve this type of equations. The solution of this type of differential equations is possible only when it falls under the category of some standard forms.

(5) **Equations in variable separable form :** If the differential equation of the form

$$f_1(x)dx = f_2(y)dy \quad .....(i)$$

where f_1 and f_2 being functions of x and y only. Then we say that the variables are separable in the differential equation.

Thus, integrating both sides of (i), we get its solution as $\int f_1(x)dx = \int f_2(y)dy + C$,

where c is an arbitrary constant.

There is no need of introducing arbitrary constants to both sides as they can be combined together to give just one.

(i) **Differential equations of the type** $\frac{dy}{dx} = f(x)$

To solve this type of differential equations we integrate both sides to obtain the general solution as discussed following : $\frac{dy}{dx} = f(x) \Leftrightarrow dy = f(x)dx$

Integrating both sides, we obtain, $\int dy = \int f(x)dx + C$ or $y = \int f(x)dx + C$.

(ii) **Differential equations of the type** $\frac{dy}{dx} = f(y)$

To solve this type of differential equations we integrate both sides to obtain the general solution as discussed following :

$$\frac{dy}{dx} = f(y) \Rightarrow \frac{dx}{dy} = \frac{1}{f(y)} \Rightarrow dx = \frac{1}{f(y)}dy$$

Integrating both sides, we obtain, $\int dx = \int \frac{1}{f(y)}dy + C$ or $x = \int \frac{1}{f(y)}dy + C$.

(6) **Equations reducible to variable separable form**

(i) Differential equations of the form $\frac{dy}{dx} = f(ax + by + c)$ can be reduced to variable separable form by the substitution $ax + by + c = Z$

$$\therefore \quad a + b\frac{dy}{dx} = \frac{dZ}{dx}$$

$$\therefore \quad \left(\frac{dZ}{dx} - a\right)\frac{1}{b} = f(Z) \Rightarrow \frac{dZ}{dx} = a + bf(Z).$$

This is variable separable form.

(ii) **Differential equation of the form**

$$\frac{dy}{dx} = \frac{ax + by + c}{Ax + By + C}, \text{ where } \frac{a}{A} = \frac{b}{B} = K \text{ (say)}$$

$$\therefore \frac{dy}{dx} = \frac{K(Ax + By) + C}{Ax + By + C}$$

Put $Ax + By = Z$

$$\therefore \quad A + B\frac{dy}{dx} = \frac{dZ}{dx}, \quad \therefore \left[\frac{dZ}{dx} - A\right]\frac{1}{B} = \frac{KZ + C}{Z + C} \Rightarrow \frac{dZ}{dx} = A + B\frac{KZ + C}{Z + C}$$

This is variable separable form and can be solved.

Example: 14 The solution of the differential equation $(1+x^2)\frac{dy}{dx}=x(1+y^2)$ is **[AISSE 1983]**

(a) $2\tan^{-1}y=\log(1+x^2)+c$ (b) $\tan^{-1}y=\log(1+x^2)+c$

(c) $2\tan^{-1}y+\log(1+x^2)+c=0$ (d) None of these

Solution: (a) Separating the variables, we can re-write the given differential equation as

$$\frac{xdx}{1+x^2}=\frac{dy}{1+y^2} \Rightarrow \int \frac{2x\,dx}{1+x^2}=2\int \frac{dy}{1+y^2} \Rightarrow 2\tan^{-1}y=\log_e(1+x^2)+c$$

Example: 15 The solution of the differential equation $\frac{dy}{dx}=x^2+\sin 3x$ is **[DSSE 1981]**

(a) $y=\frac{x^3}{3}+\frac{\cos 3x}{3}+c$ (b) $y=\frac{x^3}{3}-\frac{\cos 3x}{3}+c$ (c) $y=\frac{x^3}{3}+\sin 3x+c$ (d) None of these

Solution: (b) We have $dy=(x^2+\sin 3x)dx \Rightarrow \int dy=\int (x^2+\sin 3x)dx \Rightarrow y=\frac{x^3}{3}-\frac{\cos 3x}{3}+c$

Example: 16 The solution of $\frac{dy}{dx}=\frac{1}{y^2+\sin y}$ is

(a) $x=\frac{y^3}{3}-\cos y+c$ (b) $y + \cos y = x + c$ (c) $x=\frac{y^3}{3}+\cos y+c$ (d) None of these

Solution: (a) Given equation may be re-written as $dx=(y^2+\sin y)dy$

Integrating, $\int dx=\int (y^2+\sin y)\,dy$

$\therefore\ x=\frac{y^3}{3}-\cos y+c$

Example: 17 The solution of the differential equation $\frac{dy}{dx}=(4x+y+1)^2$ is

(a) $4x - y + 1 = 2\tan(2x - 2c)$ (b) $4x - y - 1 = 2\tan(2x - 2c)$

(c) $4x + y + 1 = 2\tan(2x + 2c)$ (d) None of these

Solution: (c) Let $4x + y + 1 = z \Rightarrow 4+\frac{dy}{dx}=\frac{dz}{dx} \Rightarrow \frac{dy}{dx}=\frac{dz}{dx}-4$

$\therefore\ \frac{dy}{dx}=(4x+y+1)^2$

$$\Rightarrow \frac{dz}{dx}-4=z^2 \Rightarrow \frac{dz}{dx}=z^2+4 \Rightarrow \frac{dz}{z^2+4}=dx \Rightarrow \frac{1}{2}\tan^{-1}\frac{z}{2}=x+c \Rightarrow \tan^{-1}\left(\frac{4x+y+1}{2}\right)=2x+2c$$

$\therefore\ 4x + y + 1 = 2\tan(2x + 2c)$

Example: 18 Solution of the differential equation $\frac{dy}{dx}=\frac{x+y+7}{2x+2y+3}$ is

(a) $6(x + y) + 11\log(3x + 3y + 10) = 9x + c$ (b) $6(x + y) - 11\log(3x + 3y + 10) = 9x + c$

(c) $6(x+y)-11\log\left(x+y+\frac{10}{3}\right)=9x+c$ (d) None of these

Solution: (b, c) Given equation may be re-written as $\frac{dy}{dx}=\frac{x+y+7}{2(x+y)+3}$

Let $x + y = z$

$$\Rightarrow 1+\frac{dy}{dx}=\frac{dz}{dx} \Rightarrow \frac{dy}{dx}=\frac{dz}{dx}-1$$

$$\therefore \frac{dz}{dx}-1=\frac{z+7}{2z+3}$$

$$\Rightarrow \frac{dz}{dx}=1+\frac{z+7}{2z+3}=\frac{3z+10}{2z+3} \Rightarrow \frac{2z+3}{3z+10}dz=dx \Rightarrow \frac{\frac{2}{3}(3z+10)-\frac{11}{3}}{3z+10}dz=dx \Rightarrow \int \frac{2}{3}dz-\frac{11}{9}\int \frac{3dz}{3z+10}=\int dx$$

$$\Rightarrow \frac{2}{3}z-\frac{11}{9}\log(3z+10)=x+c_1 \Rightarrow 6z-11\log(3z+10)=9x+9c_1$$

$$\therefore 6(x+y)-11\log(3x+3y+10)=9x+c \qquad [9c_1=c]$$

$$\Rightarrow 6(x+y)-11\log 3\left(x+y+\frac{10}{3}\right)=9x+c \Rightarrow 6(x+y)-11\log\left(x+y+\frac{10}{3}\right)=9x+(c+11\log 3)$$

$$\therefore 6(x+y)-11\log\left(x+y+\frac{10}{3}\right)=9x+k \qquad (k=c+11\log 3)$$

8.4 Homogeneous Differential Equation

(1) **Homogeneous differential equation :** A function $f(x, y)$ is called a homogeneous function of degree n if $f(\lambda x, \lambda y)=\lambda^n f(x,y)$.

For example, $f(x,y)=x^2-y^2+3xy$ is a homogeneous function of degree 2, because $f(\lambda x,\lambda y)=\lambda^2x^2-\lambda^2y^2+3\lambda x.\lambda y=\lambda^2 f(x,y)$. A homogeneous function $f(x, y)$ of degree n can always be written as $f(x,y)=x^n f\left(\frac{y}{x}\right)$ or $f(x,y)=y^n f\left(\frac{x}{y}\right)$. If a first-order first degree differential equation is expressible in the form $\frac{dy}{dx}=\frac{f(x,y)}{g(x,y)}$ where $f(x, y)$ and $g(x, y)$ are homogeneous functions of the same degree, then it is called a homogeneous differential equation. Such type of equations can be reduced to variable separable form by the substitution $y=vx$. The given differential equation can be written as $\frac{dy}{dx}=\frac{x^n f(y/x)}{x^n g(y/x)}=\frac{f(y/x)}{g(y/x)}=F\left(\frac{y}{x}\right)$. If $y=vx$, then $\frac{dy}{dx}=v+x\frac{dv}{dx}$. Substituting the value of $\frac{dy}{dx}=F\left(\frac{y}{x}\right)$, we get $v+x\frac{dv}{dx}=F(v) \Rightarrow \frac{dv}{F(v)-v}=\frac{dx}{x}$. On integration, $\int \frac{1}{F(v)-v}dv=\int \frac{dx}{x}+C$ where C is an arbitrary constant of integration. After integration, v will be replaced by $\frac{y}{x}$ in complete solution.

(2) **Algorithm for solving homogeneous differential equation**

Step (i) **:** Put the differential equation in the form $\frac{dy}{dx}=\frac{\phi(x,y)}{\psi(x,y)}$

Step (ii) **:** Put $y = vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$ in the equation in step (i) and cancel out x from the right hand side. The equation reduces to the form $v+x\frac{dv}{dx}=F(v)$.

Step (iii) **:** Shift v on RHS and separate the variables v and x

Step (iv) **:** Integrate both sides to obtain the solution in terms of v and x.

Step (v) **:** Replace v by $\frac{y}{x}$ in the solution obtained in step (iv) to obtain the solution in terms of x and y.

(3) **Equation reducible to homogeneous form**

A first order, first degree differential equation of the form

$$\frac{dy}{dx} = \frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}, \text{ where } \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad .....(i)$$

This is non-homogeneous.

It can be reduced to homogeneous form by certain substitutions. Put $x = X + h, y = Y + k$

Where h and k are constants, which are to be determined.

$$\therefore \frac{dy}{dx} = \frac{dy}{dY} \cdot \frac{dY}{dX} \cdot \frac{dX}{dx} = \frac{dY}{dX}$$

Substituting these values in (i), we have $\frac{dY}{dX} = \frac{(a_1X + b_1Y) + a_1h + b_1k + c_1}{(a_2X + b_2Y) + a_2h + b_2k + c_2}$ .....(ii)

Now h, k will be chosen such that $\left.\begin{matrix} a_1h + b_1k + c_1 = 0 \\ a_2h + b_2k + c_2 = 0 \end{matrix}\right]$

.....(iii)

i.e. $\frac{h}{b_1c_2 - b_2c_1} = \frac{k}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$ .....(iv)

For these values of h and k the equation (ii) reduces to $\frac{dY}{dX} = \frac{a_1X + b_1Y}{a_2X + b_2Y}$ which is a homogeneous differential equation and can be solved by the substitution $Y = vX$. Replacing X and Y in the solution so obtained by $x - h$ and $y - k$ respectively, we can obtain the required solution in terms of x and y.

Example: 19 The solution of the differential equation $x\frac{dy}{dx} = y(\log y - \log x + 1)$ is **[IIT 1986]**

(a) $y = xe^{cx}$ (b) $y + xe^{cx} = 0$ (c) $y + e^x = 0$ (d) None of these

Solution: (a) Given equation may be expressed as $\frac{dy}{dx} = \frac{y}{x}\left[\log\left(\frac{y}{x}\right) + 1\right]$ ....(i)

Let $\frac{y}{x} = v \Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ From (i), $v + x\frac{dv}{dx} = v(\log v + 1) \Rightarrow x\frac{dv}{dx} = v\log v \Rightarrow \frac{dv}{v\log v} = \frac{dx}{x} \Rightarrow \int \frac{1}{\log v} d(\log v) = \int \frac{dx}{x}$

$\therefore \log(\log v) = \log x + \log c \Rightarrow \log(\log v) = \log(cx) \Rightarrow \log v = cx \Rightarrow v = e^{cx} \Rightarrow \frac{y}{x} = e^{cx}$, $\therefore y = xe^{cx}$

Example: 20 The solution of differential equation $yy' = x\left(\frac{y^2}{x^2} + \frac{\phi(y^2/x^2)}{\phi'(y^2/x^2)}\right)$ is

(a) $\phi(y^2/x^2) = cx^2$ (b) $x^2\phi(y^2/x^2) = c^2y^2$ (c) $x^2\phi(y^2/x^2) = c$ (d) $\phi(y^2/x^2) = \frac{cy}{x}$

Solution: (a) Given equation may be re-written as $\frac{y}{x}\cdot\frac{dy}{dx}=\left(\frac{y}{x}\right)^2+\frac{\phi((y/x)^2)}{\phi'((y/x)^2)}$ .....(i)

Let $y = vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$ and $\frac{y}{x}=v$

$\therefore$ From (i), $v\left(v+x\frac{dv}{dx}\right)=v^2+\frac{\phi(v^2)}{\phi'(v^2)} \Rightarrow vx\frac{dv}{dx}=\frac{\phi(v^2)}{\phi'(v^2)} \Rightarrow \frac{\phi'(v^2)(2v\,dv)}{\phi(v^2)}=2\frac{dx}{x}$

Integrating, $\ln(\phi(v^2))=2\ln x+\ln c \Rightarrow \phi(v^2)=cx^2$

$\therefore\ \phi(y^2/x^2)=cx^2$

Example: 21 The solution of $\frac{dy}{dx}=\frac{y^3+2x^2y}{x^3+2xy^2}$ is

(a) $(x^2-y^2)^3=Bx^2y^2$ (b) $(x^2+y^2)^3=Bx^2y^2$ (c) $(x^2-y^2)^3=x^2y^2$ (d) None of these

Solution: (a) Given equation is homogeneous. Let $y = vx$ $\therefore \frac{dy}{dx}=v+x\frac{dv}{dx}$

$\Rightarrow \frac{y^3+2x^2y}{x^3+2xy^2}=v+x\frac{dv}{dx} \Rightarrow \frac{(y/x)^3+2(y/x)}{1+2(y/x)^2}=v+x\frac{dv}{dx} \Rightarrow \frac{v^3+2v}{1+2v^2}=v+x\frac{dv}{dx} \Rightarrow x\frac{dv}{dx}=v\left\{\frac{v^2+2}{1+2v^2}-1\right\}=v\left\{\frac{1-v^2}{1+2v^2}\right\}$

$\Rightarrow \frac{1+2v^2}{v(1-v^2)}dv=\frac{dx}{x} \Rightarrow \frac{1+2v^2}{v(1-v)(1+v)}dv=\frac{dx}{x} \Rightarrow \left(\frac{A}{v}+\frac{B}{1-v}+\frac{D}{1+v}\right)dv=\frac{dx}{x}$

where $A(1-v)(1+v)+Bv(1+v)+Dv(1-v)=1+2v^2$

Putting $v = 0$, $A = 1$

$v = 1$, $B=\frac{3}{2}$

$v = -1$, $D=-\frac{3}{2}$

$\therefore \left(\frac{1}{v}+\frac{3}{2}\frac{1}{1-v}-\frac{3}{2}\frac{1}{1+v}\right)dv=\frac{dx}{x}$

Integrating both side, we get $\ln v+\frac{3}{2}\frac{\ln(1-v)}{-1}-\frac{3}{2}\ln(1+v)=\ln x+\ln c \Rightarrow \ln v-\frac{3}{2}\ln(1-v)-\frac{3}{2}\ln(1+v)=\ln cx$

$\Rightarrow v/\{(1-v)1+v\}^{3/2}=cx \Rightarrow \left(\frac{v}{cx}\right)^2=(1-v^2)^3 \Rightarrow \left(\frac{y}{cx^2}\right)^2=\left(1-\frac{y^2}{x^2}\right)^3 \Rightarrow (x^2-y^2)^3=\frac{x^2y^2}{c^2}$

$\therefore (x^2-y^2)^3=Bx^2y^2$, $\left(\because \frac{1}{c^2}=B\right)$

Example: 22 The solution of $\frac{dy}{dx}=\frac{x-3y+2}{3x-y+6}$ is

(a) $y^2+6(x+2)y+(x+2)^2=c$ (b) $y^2-6(x+2)y+(x+2)^2=c$

(c) $y^2-6(y+2)x+x^2=c$ (d) None of these

Solution: (b) Given equation is non-homogeneous

Let $x = X + h$, $y = Y + k$

$\Rightarrow \frac{dy}{dx}=\frac{dY}{dX}$

$\therefore \frac{dY}{dX}=\frac{(X+h)-3(Y+k)+2}{3(X+h)-(Y+k)+6}=\frac{X-3Y+(h-3k+2)}{3X-Y+(3h-k+6)}$

Let us select h and k so that $h - 3k + 2 = 0$ and $3h - k + 6 = 0$

Solving, $k = 0$, $h = -2$ $\therefore X = x - h = x + 2$, $Y=y-k=y$

$\therefore \frac{dY}{dX}=\frac{X-3Y}{3X-Y}$, which is homogeneous

Now, let $Y = vX$

$\Rightarrow \frac{dY}{dX} = v + X\frac{dv}{dX} \Rightarrow \frac{X-3Y}{3X-Y} = v + X\frac{dv}{dX} \Rightarrow \frac{1-3(Y/X)}{3-(Y/X)} = v + X\frac{dv}{dX} \Rightarrow \frac{1-3v}{3-v} = v + X\frac{dv}{dX}$

$\Rightarrow X\frac{dv}{dX} = \frac{1-3v}{3-v} - v = \frac{v^2-6v+1}{3-v} \Rightarrow \frac{(3-v)dv}{v^2-6v+1} = \frac{dX}{X} \Rightarrow \frac{2v-6}{v^2-6v+1}dv = -2\frac{dX}{X}$

Integrating, $\ln(v^2-6v+1) = -2\ln X + \ln c \Rightarrow \ln(v^2-6v+1) + \ln X^2 = \ln c \Rightarrow X^2(v^2-6v+1) = c \Rightarrow$ $Y^2 - 6XY + X^2 = c$

$\therefore\ y^2 - 6(x+2)y + (x+2)^2 = c$

8.5 Exact Differential Equation

(1) **Exact differential equation :** If M and N are functions of x and y, the equation $Mdx + Ndy = 0$ is called exact when there exists a function $f(x, y)$ of x and y such that

$d[f(x, y)] = Mdx + Ndy$ *i.e.*, $\frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = Mdx + Ndy$

where $\frac{\partial f}{\partial x} =$ Partial derivative of $f(x, y)$ with respect to x (keeping y constant)

$\frac{\partial f}{\partial y} =$ Partial derivative of $f(x, y)$ with respect to y (treating x as constant)

Note : ❑ An exact differential equation can always be derived from its general solution directly by differentiation without any subsequent multiplication, elimination etc.

(2) **Theorem :** The necessary and sufficient condition for the differential equation $Mdx + Ndy = 0$ to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ *i.e.*, partial derivative of $M(x, y)$ *w.r.t.* y = Partial derivative of $N(x, y)$ *w.r.t.* x

(3) **Integrating factor :** If an equation of the form $Mdx + Ndy = 0$ is not exact, it can always be made exact by multiplying by some function of x and y. Such a multiplier is called an integrating factor.

(4) **Working rule for solving an exact differential equation :**

Step (i) **:** Compare the given equation with $Mdx + Ndy = 0$ and find out M and N. Then find out $\frac{\partial M}{\partial y}$ and $\frac{\partial N}{\partial x}$. If $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, the given equation is exact.

Step (ii) **:** Integrate M with respect to x treating y as a constant.

Step (iii) **:** Integrate N with respect to y treating x as constant and omit those terms which have been already obtained by integrating M.

Step (iv) **:** On adding the terms obtained in steps (ii) and (iii) and equating to an arbitrary constant, we get the required solution.

In other words, solution of an exact differential equation is $\underset{\substack{\text{Regarding } y \\ \text{as constant}}}{\int} Mdx + \underset{\substack{\text{Only those terms} \\ \text{not containing } x}}{\int} Ndy = c$

(5) **Solution by inspection :** If we can write the differential equation in the form $f(f_1(x,y))d(f_1(x,y)) + \phi(f_2(x,y))d(f_2(x,y)) + ... = 0$, then each term can be easily integrated separately. For this the following results must be memorized.

(i) $d(x + y) = dx + dy$ (ii) $d(xy) = xdy + ydx$

(iii) $d\left(\frac{x}{y}\right)=\frac{ydx-xdy}{y^2}$

(iv) $d\left(\frac{y}{x}\right)=\frac{xdy-ydx}{x^2}$

(v) $d\left(\frac{x^2}{y}\right)=\frac{2xydx-x^2dy}{y^2}$

(vi) $d\left(\frac{y^2}{x}\right)=\frac{2xydy-y^2dx}{x^2}$

(vii) $d\left(\frac{x^2}{y^2}\right)=\frac{2xy^2dx-2x^2ydy}{y^4}$

(viii) $d\left(\frac{y^2}{x^2}\right)=\frac{2x^2ydy-2xy^2dx}{x^4}$

(ix) $d\left(\tan^{-1}\frac{x}{y}\right)=\frac{ydx-xdy}{x^2+y^2}$

(x) $d\left(\tan^{-1}\frac{y}{x}\right)=\frac{xdy-ydx}{x^2+y^2}$

(xi) $d[\ln(xy)]=\frac{xdy+ydx}{xy}$

(xii) $d\left(\ln\left(\frac{x}{y}\right)\right)=\frac{ydx-xdy}{xy}$

(xiii) $d\left[\frac{1}{2}\ln(x^2+y^2)\right]=\frac{xdx+ydy}{x^2+y^2}$

(xiv) $d\left[\ln\left(\frac{y}{x}\right)\right]=\frac{xdy-ydx}{xy}$

(xv) $d\left(-\frac{1}{xy}\right)=\frac{xdy+ydx}{x^2y^2}$

(xvi) $d\left(\frac{e^x}{y}\right)=\frac{ye^xdx-e^xdy}{y^2}$

(xvii) $d\left(\frac{e^y}{x}\right)=\frac{xe^ydy-e^ydx}{x^2}$

(xviii) $d(x^my^n)=x^{m-1}y^{n-1}(mydx+nxdy)$

(xix) $d\left(\sqrt{x^2+y^2}\right)=\frac{xdx+y\,dy}{\sqrt{x^2+y^2}}$

(xx) $d\left(\frac{1}{2}\log\frac{x+y}{x-y}\right)=\frac{x\,dy-y\,dx}{x^2-y^2}$

(xxi) $\frac{d[f(x,y)]^{1-n}}{1-n}=\frac{f'(x,y)}{(f(x,y))^n}$

Example: 23 The general solution of the differential equation $(x+y)dx+xdy=0$ is **[MP PET 1994, 95]**

(a) $x^2+y^2=c$ (b) $2x^2-y^2=c$ (c) $x^2+2xy=c$ (d) $y^2+2xy=c$

Solution: (c) We have $xdx+(ydx+xdy)=0 \Rightarrow xdx+d(xy)=0$

Integrating, $\frac{x^2}{2}+xy=\frac{c}{2}$

$\therefore\ x^2+2xy=c$

Example: 24 Solution of $y(2xy+e^x)dx=e^xdy$ is

(a) $yx^2+e^x=cy$ (b) $xy^2+e^x=cx$ (c) $xy^2+e^{-x}=c$ (d) None of these

Solution: (a) Re-writing the given equation,

$2xy^2dx+ye^xdx=e^xdy \Rightarrow 2x\,dx+\frac{ye^xdx-e^xdy}{y^2}=0 \Rightarrow d(x^2)+d\left(\frac{e^x}{y}\right)=0$

Integrating, $x^2+\frac{e^x}{y}=c$

$\therefore\ yx^2+e^x=cy$

Example: 25 Solution of $(x^2-4xy-2y^2)dx+(y^2-4xy-2x^2)dy=0$ is

(a) $x^3+y^3-6xy(x+y)=c$ (b) $x^3+y^3+6xy(x-y)=c$

(c) $x^3+y^3+6xy(x+y)=c$ (d) $x^3+y^3-6xy(x-y)=c$

Solution: (a) Comparing given equation with $Mdx + Ndy = 0$,

We get, $M = x^2 - 4xy - 2y^2$, $N = y^2 - 4xy - 2x^2$

$\frac{\partial M}{\partial y} = -4x - 4y$

$\frac{\partial N}{\partial x} = -4y - 4x$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

So the given differential equation is exact.

Integrating *m w.r.t. x,* treating *y* as constant,

$\int M dx = \int (x^2 - 4xy - 2y^2) dx = \frac{x^3}{3} - 2x^2 y - 2y^2 x$

Integrating *N w.r.t. y,* treating *x* as constant,

$\int N dy = \int (y^2 - 4xy - 2x^2) dy = \frac{y^3}{3} - 2xy^2 - 2x^2 y = \frac{y^3}{3}$; (omitting $-2xy^2 - 2x^2 y$ which already occur in $\int M dx$)

$\therefore$ Solution of the given equation is $\frac{x^3}{3} - 2x^2 y - 2xy^2 + \frac{y^3}{3} = \lambda \Rightarrow x^3 + y^3 - 6xy(x+y) = 3\lambda$

$\therefore x^3 + y^3 - 6xy(x+y) = c$ $(3\lambda = c)$

8.6 Linear Differential Equation

(1) **Linear and non-linear differential equations** : A differential equation is a linear differential equation if it is expressible in the form $P_o \frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}} + ... + P_{n-1} \frac{dy}{dx} + P_n y = Q$ where $P_0, P_1, P_2, ..., P_{n-1}, P_n$ and Q are either constants or functions of independent variable x.

Thus, if a differential equation when expressed in the form of a polynomial involves the derivatives and dependent variable in the first power and there are no product of these, and also the coefficient of the various terms are either constants or functions of the independent variable, then it is said to be linear differential equation. Otherwise, it is a non linear differential equation.

It follows from the above definition that a differential equation will be non-linear differential equation if (i) its degree is more than one (ii) any of the differential coefficient has exponent more than one. (iii) exponent of the dependent variable is more than one. (iv) products containing dependent variable and its differential coefficients are present.

(2) **Linear differential equation of first order :** The general form of *a* linear differential equation of first order is

$$\frac{dy}{dx} + Py = Q \quad .....(i)$$

Where *P* and *Q* are functions of *x* (or constants)

For example, $\frac{dy}{dx} + xy = x^3$, $x\frac{dy}{dx} + 2y = x^3$, $\frac{dy}{dx} + 2y = \sin x$ etc. are linear differential equations. This type of differential equations are solved when they are multiplied by a factor, which is called integrating factor, because by multiplication of this factor the left hand side of the differential equation (i) becomes exact differential of some function.

Multiplying both sides of (i) by $e^{\int Pdx}$, we get $e^{\int Pdx}\left(\frac{dy}{dx}+Py\right)=Q\,e^{\int Pdx} \Rightarrow \frac{d}{dx}\left\{y\,e^{\int Pdx}\right\}=Q\,e^{\int Pdx}$

On integrating both sides *w. r. t. x*, we get ; $y\,e^{\int Pdx}=\int Q\,e^{\int Pdx}dx+C$

........(ii)

which is the required solution, where C is the constant of integration. $e^{\int Pdx}$ is called the integrating factor. The solution (ii) in short may also be written as $y.(I.F.)=\int Q.(I.F.)dx+C$

(3) **Algorithm for solving a linear differential equation :**

Step (i) **:** Write the differential equation in the form $\frac{dy}{dx}+Py=Q$ and obtain P and Q.

Step (ii) **:** Find integrating factor (I.F.) given by $I.F.=e^{\int Pdx}$.

Step (iii) **:** Multiply both sides of equation in step (i) by I.F.

Step (iv) **:** Integrate both sides of the equation obtained in step (iii) *w. r. t.* x to obtain $y(I.F.)=\int Q(I.F.)dx+C$

This gives the required solution.

(4) **Linear differential equations of the form** $\frac{dx}{dy}+Rx=S$. Sometimes a linear differential equation can be put in the form $\frac{dx}{dy}+Rx=S$ where R and S are functions of y or constants. Note that y is independent variable and x is a dependent variable.

(5) **Algorithm for solving linear differential equations of the form** $\frac{dx}{dy}+Rx=S$

Step (i) **:** Write the differential equation in the form $\frac{dx}{dy}+Rx=S$ and obtain R and S.

Step (ii) **:** Find I.F. by using $I.F.=e^{\int R\,dy}$

Step (iii) **:** Multiply both sides of the differential equation in step (i) by *I.F.*

Step (iv) **:** Integrate both sides of the equation obtained in step (iii) *w. r. t. y* to obtain the solution given by

$x(I.F.)=\int S\,(I.F.)dy+C$ where C is the constant of integration.

(6) **Equations reducible to linear form (Bernoulli's differential equation) :** The differential equation of type $\frac{dy}{dx}+Py=Qy^n$

......(i)

Where P and Q are constants or functions of x alone and n is a constant other than zero or unity, can be reduced to the linear form by dividing by y^n and then putting $y^{-n+1}=v$, as explained below.

Dividing both sides of (i) by y^n, we get $y^{-n}\frac{dy}{dx}+Py^{-n+1}=Q$

Putting $y^{-n+1}=v$ so that $(-n+1)y^{-n}\frac{dy}{dx}=\frac{dv}{dx}$, we get $\frac{1}{-n+1}\frac{dv}{dx}+Pv=Q \Rightarrow \frac{dv}{dx}+(1-n)Pv=(1-n)Q$ which is a linear differential equation.

Remark : If $n=1$, then we find that the variables in equation (i) are separable and it can be easily integrated by the method discussed in variable separable from.

(7) **Differential equation of the form :** $\frac{dy}{dx}+P\phi(y)=Q\psi(y)$

where P and Q are functions of x alone or constants.

Dividing by $\psi(y)$, we get $\frac{1}{\psi(y)}\frac{dy}{dx}+\frac{\phi(y)}{\psi(y)}P=Q$

Now put $\frac{\phi(y)}{\psi(y)}=v$, so that $\frac{d}{dx}\left\{\frac{\phi(y)}{\psi(y)}\right\}=\frac{dv}{dx}$ or $\frac{dv}{dx}=k\cdot\frac{1}{\psi(y)}\frac{dy}{dx}$, where k is constant

We get $\frac{dv}{dx}+kPv=kQ$

Which is linear differential equation.

Example: 26 Which of the following is a linear differential equation

(a) $\left(\frac{d^2y}{dx^2}\right)^2+x^2\left(\frac{dy}{dx}\right)^2=0$ (b) $y=\frac{dy}{dx}+\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ (c) $\frac{dy}{dx}+\frac{y}{x}=\log x$ (d) $y\frac{dy}{dx}-4=x$

Solution: (c) (a), (b), (d) do not fulfill the criteria of a linear differential equation but (c) do.

$\frac{dy}{dx}+\frac{y}{x}=\log x$ is a linear differential equation.

Example: 27 Find the integral factor of equation $(x^2+1)\frac{dy}{dx}+2xy=x^2-1$ **[UPSEAT 2002]**

(a) x^2+1 (b) $\frac{2x}{x^2+1}$ (c) $\frac{x^2-1}{x^2+1}$ (d) None of these

Solution: (a) Given equation may be written as $\frac{dy}{dx}+\frac{2x}{x^2+1}y=\frac{x^2-1}{x^2+1}$

Comparing with $\frac{dy}{dx}+Py=Q$,

$P=\frac{2x}{x^2+1}$

I.F. $=e^{\int Pdx}=e^{\int\frac{2xdx}{1+x^2}}=e^{\ln(1+x^2)}=1+x^2$

Example: 28 The solution of $\frac{dy}{dx}+2y\tan x=\sin x$ is

(a) $y\sec^3 x=\sec^2 x+c$ (b) $y\sec^2 x=\sec x+c$ (c) $y\sin x=\tan x+c$ (d) None of these

Solution: (b) Comparing with $\frac{dy}{dx}+Py=Q$,

$P=2\tan x$, $Q=\sin x$

$\text{I.F.}=e^{\int 2\tan x dx}=e^{2\ln\sec x}=e^{\ln\sec^2 x}=\sec^2 x$

Multiplying given equation by I.F. and integrating, $y\sec^2 x=\int\sin x.\sec^2 x dx=\int\sec x\tan x dx$

$\therefore\ y\sec^2 x=\sec x+c$

Example: 29 The solution of the differential equation $(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx}=0$ is **[AIEEE 2003]**

(a) $(x-2)=ke^{\tan^{-1}y}$ (b) $2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$

(c) $xe^{\tan^{-1}y}=\tan^{-1}y+k$ (d) $xe^{2\tan^{-1}y}=e^{\tan^{-1}y}+k$

Solution: (b) We have $(x-e^{\tan^{-1}y})\frac{dy}{dx}=-(1+y^2)\Rightarrow\frac{dx}{dy}=-\left(\frac{x-e^{\tan^{-1}y}}{1+y^2}\right)\Rightarrow\frac{dx}{dy}+\frac{1}{1+y^2}x=\frac{e^{\tan^{-1}y}}{1+y^2}$ ....(i)

This is a linear differential equation of the form $\frac{dx}{dy}+R(y).x=S(y)$

$R=\frac{1}{1+y^2}$, $S=\frac{e^{\tan^{-1}y}}{1+y^2}$

Integrating factor $=e^{\int Rdy}=e^{\int\frac{dy}{1+y^2}}=e^{\tan^{-1}y}$

Multiplying (i) by I.F. and integrating, $xe^{\tan^{-1}y}=\int\frac{e^{\tan^{-1}y}}{1+y^2}\cdot e^{\tan^{-1}y}dy=\int\frac{(e^{\tan^{-1}y})^2dy}{1+y^2}=\frac{(e^{\tan^{-1}y})^2}{2}+\frac{k}{2}$

$\therefore\ 2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$

Example: 30 Solution of $\frac{dy}{dx}-y\tan x=-y^2\sec x$ is

(a) $y\sec x=\tan x+c$ (b) $\frac{\sec x}{y}=\tan x+c$ (c) $y\cos x=\tan x+c$ (d) None of these

Solution: (b) Re-writing the given equation, $y^{-2}\frac{dy}{dx}-y^{-1}\tan x=-\sec x$

Let $y^{-1}=v\Rightarrow -y^{-2}\frac{dy}{dx}=\frac{dv}{dx}$

$\therefore\ \frac{dv}{dx}+\tan x.v=\sec x$ ....(i)

$\text{I.F.}=e^{\int\tan x}=e^{\ln\sec x}=\sec x$

Multiplying (i) by $\sec x$ and integrating, $v\sec x=\int\sec^2 x dx=\tan x+c$

$\therefore\ \frac{\sec x}{y}=\tan x+c$

Example: 31 The solution of $\frac{dz}{dx}+\frac{z}{x}\log z=\frac{z}{x^2}(\log z)^2$ is

(a) $\left(\frac{1}{\log z}\right)x=2-x^2c$ (b) $\left(\frac{1}{\log z}\right)x=2+x^2c$ (c) $\left(\frac{1}{\log z}\right)x=x^2c$ (d) $\left(\frac{1}{\log z}\right)x=\frac{1}{2}+cx^2$

Solution: (d) Dividing the given equation by $z(\log z)^2$, $\frac{1}{z(\log z)^2}\frac{dz}{dx}+\frac{1}{x}\frac{1}{\log z}=\frac{1}{x^2}$

Let $\frac{1}{\log z}=t \Rightarrow -\frac{1}{(\log z)^2}\cdot\frac{1}{z}\frac{dz}{dx}=\frac{dt}{dx}$

$\therefore -\frac{dt}{dx}+\frac{t}{x}=\frac{1}{x^2}$

$\Rightarrow \frac{dt}{dx}-\frac{t}{x}=-\frac{1}{x^2}$ ....(i)

$\text{I.F.}=e^{\int -\frac{dx}{x}}=e^{-\ln x}=e^{\ln\frac{1}{x}}=\frac{1}{x}$

Multiplying (i) by $\frac{1}{x}$ and integrating, $\frac{t}{x}=\int -\frac{1}{x^3}dx=\frac{1}{2x^2}+c \Rightarrow \frac{1}{x\log z}=\frac{1}{2x^2}+c$

$\therefore \left(\frac{1}{\ln z}\right)x=\left(\frac{1}{2}\right)+cx^2$

8.7 Application of Differential Equation

Differential equation is applied in various practical fields of life. It is used to define various physical laws and quantities. It is widely used in physics, chemistry, engineering etc.

Some important fields of application are ;

(i) Rate of change (ii) Geometrical problems etc.

Differential equation is used for finding the family of curves for which some conditions involving the derivatives are given.

Equation of the tangent at a point (x, y) to the curve $y = f(x)$ is given by $Y-y=\frac{dy}{dx}(X-x)$ ....(i)

and equation of normal at (x, y) is $Y-y=-\frac{1}{\left(\frac{dy}{dx}\right)}(X-x)$ ....(ii)

The tangent meets X-axis at $\left(x-\frac{y}{\left(\frac{dy}{dx}\right)},0\right)$ and Y-axis at $\left(0,y-x\frac{dy}{dx}\right)$

The normal meets X-axis at $\left(x+y\frac{dy}{dx},0\right)$ and Y-axis at $\left(0,y+\frac{x}{\left(\frac{dy}{dx}\right)}\right)$

Example: 32 A particle moves in a straight line with a velocity given by $\frac{dx}{dt}=(x+1)$ (x is the distance described). The time taken by a particle to transverse a distance of 99 *metres*

(a) $\log_{10} e$ (b) $2\log_e 10$ (c) $\log_{10} e$ (d) $\frac{1}{2}\log_{10} e$

Solution: (b) We have $\frac{dx}{x+1}=dt$

Integrating, $\int_0^{99} \frac{dx}{x+1} = \int_0^t dt \Rightarrow [\ln(x+1)]_0^{99} = t$

$\therefore\ t = \ln 100 = \log_e (10)^2 = 2\log_e 10$

Example: 33 The slope of the tangent at (x, y) to a curve passing through $\left(1, \frac{\pi}{4}\right)$ as given by $\frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$, then the equation of the curve is **[Kurukshetra CEE 2002]**

(a) $y = \tan^{-1}\left[\log\left(\frac{e}{x}\right)\right]$ (b) $y = x\tan^{-1}\left[\log\left(\frac{x}{e}\right)\right]$ (c) $y = x\tan^{-1}\left[\log\left(\frac{e}{x}\right)\right]$ (d) None of these

Solution: (c) We have $\frac{dy}{dx} = \frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx} \Rightarrow v + x\frac{dv}{dx} = v - \cos^2 v \Rightarrow x\frac{dv}{dx} = -\cos^2 v \Rightarrow \sec^2 v\,dv = -\frac{dx}{x} \Rightarrow \tan v = -\ln x + c$

$\Rightarrow \tan(y/x) = -\ln x + c$

For $x = 1$, $y = \pi/4$

$\Rightarrow \tan\pi/4 = -\ln 1 + c \Rightarrow 1 = 0 + c$

$\therefore c = 1$

$\therefore \tan(y/x) = 1 - \ln x$

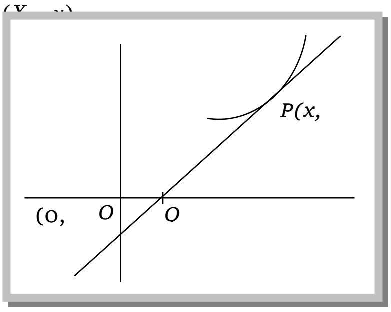
$\Rightarrow y/x = \tan^{-1}(1 - \ln x) = \tan^{-1}(\ln e - \ln x) = \tan^{-1}\left[\ln\left(\frac{e}{x}\right)\right]$

$\therefore\ y = x\tan^{-1}\left[\ln\left(\frac{e}{x}\right)\right]$

Example: 34 The equation of the curve which is such that the portion of the axis of x cut off between the origin and tangent at any point is proportional to the ordinate of that point (b is constant of proportionality)

(a) $y = \frac{x}{(a - b\log x)}$ (b) $\log x = by^2 + a$ (c) $x^2 = y(a - b\log y)$ (d) None of these

Solution: (d) Tangent at $P(x, y)$ to the curve $y = f(x)$ may be expressed as $Y - y = \frac{dy}{dx}$



$\therefore\ Q = \left(x - y\frac{dx}{dy}, 0\right)$

As per question, $OQ \propto y$

$\Rightarrow x - y\frac{dx}{dy} \propto y \Rightarrow x - y\frac{dx}{dy} = by \Rightarrow \frac{x}{y} - \frac{dx}{dy} = b$

$\therefore\ \frac{dx}{dy} = \frac{x}{y} - b$

Let $\frac{x}{y} = v \Rightarrow x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy} \Rightarrow \frac{x}{y} - b = v + y\frac{dv}{dy} \Rightarrow v - b = v + y\frac{dv}{dy} \Rightarrow -b = y\frac{dv}{dy} \Rightarrow -b\frac{dy}{y} = dv$

Integrating, $\int dv = -b\int\frac{dy}{y} \Rightarrow v = -b\ln y + a \Rightarrow \frac{x}{y} = a - b\ln y$ (a, an arbitrary constant)

$\therefore\ x = y(a - b\ln y)$

8.8 Miscellaneous Differential Equation

(1) **A special type of second order differential equation :** $\frac{d^2y}{dx^2} = f(x)$ ....(i)

Equation (i) may be re-written as $\frac{d}{dx}\left(\frac{dy}{dx}\right) = f(x) \Rightarrow d\left(\frac{dy}{dx}\right) = f(x)dx$

Integrating, $\frac{dy}{dx} = \int f(x)dx + c_1$ *i.e.* $\frac{dy}{dx} = F(x) + c_1$ ....(ii)

Where $F(x) = \int f(x)dx + c_1 dx$

From (ii), $dy = f(x)dx + c_1 dx$

Integrating, $y = \int F(x)dx + c_1 x + c_2$

$\therefore\ y = H(x) + c_1 x + c_2$

where $\boxed{H(x) = \int F(x)dx}$ c_1 and c_2 are arbitrary constants.

(2) **Particular solution type problems :** To solve such a problem, we proceed according to the type of the problem (*i.e.* variable-separable, linear, exact, homogeneous etc.) and then we apply the given conditions to find the particular values of the arbitrary constants.

Example: 35 The solution of the equation $x^2 \frac{d^2y}{dx^2} = \ln x$ when $x = 1$, $y = 0$ and $\frac{dy}{dx} = -1$ is **[Orissa JEE 2003]**

(a) $\frac{1}{2}(\ln x)^2 + \ln x$ (b) $\frac{1}{2}(\ln x)^2 - \ln x$ (c) $-\frac{1}{2}(\ln x)^2 + \ln x$ (d) $-\frac{1}{2}(\ln x)^2 - \ln x$

Solution: (d) We have $\frac{d^2y}{dx^2} = \frac{\ln x}{x^2} \Rightarrow d\left(\frac{dy}{dx}\right) = \frac{\ln x}{x^2}dx$

Integrating, $\frac{dy}{dx} = \int \ln x\, d\left(-\frac{1}{x}\right) = -\frac{\ln x}{x} + \int \frac{1}{x^2}dx = -\frac{\ln x}{x} - \frac{1}{x} + c \Rightarrow \frac{dy}{dx} = -\frac{1+\ln x}{x} + c$

When $x = 1$, $\frac{dy}{dx} = -1$

$\therefore -1 = -1 + c \Rightarrow c = 0$

$\therefore\ \frac{dy}{dx} = -\frac{1+\ln x}{x} \Rightarrow dy = -\frac{1+\ln x}{x}dx \Rightarrow -\int dy = +\int \frac{dx}{x} + \int \ln x . \frac{1}{x}dx \Rightarrow -y = \ln x + \frac{1}{2}(\ln x)^2 + \lambda$

$y = 0$ when $x = 1$

$\therefore\ 0 = 0 + 0^2 + \lambda \Rightarrow \lambda = 0 \Rightarrow -y = \ln x + \frac{1}{2}(\ln x)^2$

$\therefore\ y = -\frac{1}{2}(\ln x)^2 - \ln x$

Example: 36 A continuously differentiable function $\phi(x)$ in $(0, \pi)$ satisfying $y' = 1 + y^2$, $y(0) = 0 = y(\pi)$ is

(a) $\tan x$ (b) $x(x - \pi)$ (c) $(x-\pi)(1-e^x)$ (d) Not possible

Solution: (d) For $\phi(x) = y$, $y' = 1 + y^2 \Rightarrow \frac{dy}{dx} = 1 + y^2 \Rightarrow \int \frac{dy}{1+y^2} = \int dx \Rightarrow \tan^{-1} y = x + c$

$\therefore\ y = \tan(x + c)$

i.e., $\phi(x) = \tan(x + c)$

As $y(0) = 0$, $0 = \tan c$ and $y(\pi) = 0 \Rightarrow 0 = \tan(\pi + c) = \tan c$

$\therefore\ c = 0$

$\therefore\ \phi(x) = y = \tan x$.

But $\tan x$ is not continuous in $(0, \pi)$

Since $\tan \frac{\pi}{2}$ is not defined.

Hence there exists not a function satisfying the given condition.

Assignment

Order and Degree of Differential Equation

Basic Level

1. The order of the differential equation of a family of curves represented by an equation containing four arbitrary constants, will be

(a) 2 (b) 4 (c) 6 (d) None of these

2. The order and degree of the differential equation $\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is **[DCE 2002]**

(a) 4, 2 (b) 1, 2 (c) 2, 2 (d) $2, \frac{1}{2}$

3. The order and degree of the differential equation $\left(\frac{d^2s}{dt^2}\right)^2 + 3\left(\frac{ds}{dt}\right)^3 + 4 = 0$ are

(a) 2, 2 (b) 2, 3 (c) 3, 2 (d) None of these

4. The order and degree of differential equation $\frac{d^4y}{dx^4} - 4\frac{d^3y}{dx^3} + 8\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 4y = 0$ are respectively

(a) 4, 1 (b) 1, 4 (c) 1, 1 (d) None of these

5. The order and the degree of the differential equation $\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0$ are **[MP PET 1993]**

(a) 1 and 1/2 (b) 2 and 1 (c) 1 and 1 (d) 1 and 2

6. The order and the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 - xy = 0$ are respectively **[MP PET 2003]**

(a) 2 and 4 (b) 3 and 2 (c) 4 and 5 (d) 2 and 3

7. $\frac{d^3y}{dx^3} + 2\left[1 + \frac{d^2y}{dx^2}\right] = 1$ has degree and order as **[UPSEAT 2003]**

(a) 1, 3 (b) 2, 3 (c) 3, 2 (d) 3, 1

8. The order and the degree of the differential equation $x\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + y^2 = 0$ are respectively **[Karnataka CET 2001]**

(a) 2 and 2 (b) 1 and 1 (c) 2 and 1 (d) 1 and 2

9. The order of the differential equation whose general solution is given by $y = c_1e^{2x+c_2} + c_3e^x + c_4\sin(x + c_5)$ is **[AMU 2000]**

(a) 5 (b) 4 (c) 3 (d) 2

10. The order and the degree of the differential equation representing the family of curves $y^2 = 2k(x + \sqrt{k})$ (where k is a positive parameter) are respectively **[MP PET 2002]**

(a) 1 and 2 (b) 2 and 4 (c) 1 and 4 (d) 1 and 3

11. The degree of differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + 6y = 0$ is **[Kerala (Engg.) 2002]**

(a) 1 (b) 3 (c) 2 (d) 5

12. The differential equation of first order and first degree is

(a) $x\left(\frac{dy}{dx}\right)^2 - x + a = 0$ (b) $\frac{d^2y}{dx^2} + xy = 0$ (c) $dy + dx = 0$ (d) None of these

Advance Level

13. Order and degree of differential equation $\frac{d^2y}{dx^2} = \left\{ y + \left(\frac{dy}{dx} \right)^2 \right\}^{1/4}$ are **[MP PET 1996]**

(a) 4 and 2 (b) 1 and 2 (c) 1 and 4 (d) 2 and 4

14. The degree of the differential equation $(\sqrt{1+x^2} + \sqrt{1+y^2}) = A(x\sqrt{1+y^2} - y\sqrt{1+x^2})$ is

(a) 2 (b) 3 (c) 4 (d) None of these

15. The differential equation $\left(\frac{d^2y}{dx^2} \right)^2 - \left(\frac{dy}{dx} \right)^{1/2} = y^3$ has the degree **[Roorkee 1999]**

(a) 1/2 (b) 2 (c) 3 (d) 4

16. The degree and order of the differential equation of the family of all parabolas whose axis is x-axis are respectively **[AIEEE 2003]**

(a) 2, 1 (b) 1, 2 (c) 3, 2 (d) 2, 3

17. Degree of the given differential equation $\left(\frac{d^2y}{dx^2} \right)^3 = \left(1 + \frac{dy}{dx} \right)^{1/2}$, is **[MP PET 1997]**

(a) 2 (b) 3 (c) $\frac{1}{2}$ (d) 6

18. The differential equation $x\left(\frac{d^2y}{dx^2} \right)^3 + \left(\frac{dy}{dx} \right)^4 + y = x^2$ is of

(a) Degree 3 and order 2 (b) Degree 1 and order 1 (c) Degree 4 and order 3 (d) Degree 4 and order 4

19. The second order differential equation is **[MP PET 2000]**

(a) $y'^2 + x = y^2$ (b) $y'y'' + y = \sin x$ (c) $y''' + y'' + y = 0$ (d) $y' = y$

20. The order and degree of the differential equation $\left(1 + 3\frac{dy}{dx} \right)^{\frac{2}{3}} = 4\frac{d^3y}{dx^3}$ are **[AIEEE 2002]**

(a) $1, \frac{2}{3}$ (b) 3, 1 (c) 3, 3 (d) 1, 2

21. The order of the differential equation whose solution is $y = a\cos x + b\sin x + ce^{-x}$ is

(a) 3 (b) 2 (c) 1 (d) None of these

22. The differential equation of all circles of radius a is of order

(a) 2 (b) 3 (c) 4 (d) None of these

23. The differential equation of all circles in the first quadrant which touch the coordinate axes is of order

(a) 1 (b) 2 (c) 3 (d) None of these

24. If m and n are the order and degree of the differential equation $\left(\frac{d^2y}{dx^2} \right)^5 + 4\frac{\left(\frac{d^2y}{dx^2} \right)^3}{\left(\frac{d^3y}{dx^3} \right)} + \frac{d^3y}{dx^3} = x^2 - 1$, then **[Karnataka CET 1999]**

(a) $m = 3$ and $n = 5$ (b) $m = 3$ and $n = 1$ (c) $m = 3$ and $n = 3$ (d) $m = 3$ and $n = 2$

25. The order and degree of the differential equation $\rho = \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}}{d^2y / dx^2}$ are respectively **[MP PET 2001; UPSEAT 2002]**

(a) 2, 2 (b) 2, 3 (c) 2, 1 (d) None of these

26. Order of the differential equation of the family of all concentric circles centred at (h, k) is **[EAMCET 2002]**

(a) 1 (b) 2 (c) 3 (d) 4

27. Let a and b be respectively the degree and order of the differential equation of the family of circles touching the lines $y^2 - x^2 = 0$ and lying in the first and second quadrant then

(a) $a = 1, b = 2$ (b) $a = 1, b = 1$ (c) $a = 2, b = 1$ (d) $a = 2, b = 2$

28. The order and degree of differential equation of all tangent lines to the parabola $x^2 = 4y$ is

(a) 1, 2 (b) 2, 2 (c) 3, 1 (d) 4, 1

29. The order and degree of differential equation $xy\frac{dy}{dx} = \left(\frac{1+y^2}{1+x^2}\right)(1+x+x^2)$ are

(a) 1, 1 (b) , 2 (c) 2, 1 (d) 2, 2

30. The differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + \sin y + x^2 = 0$ is of the following type

(a) Linear (b) Homogeneous (c) Order two (d) Degree one

31. The order and degree of differential equation $(xy^2 + x)dx + (y - x^2y)dy = 0$ are

(a) 1, 2 (b) 2, 1 (c) 1, 1 (d) 2, 2

32. Family $y = Ax + A^3$ of curve represented by the differential equation of degree **[MP PET 1999]**

(a) Three (b) Two (c) One (d) None of these

33. Which of the following differential equations has the same order and degree **[Kurukshetra CEE 1998]**

(a) $\frac{d^2y}{dx^4} + 8\left(\frac{dy}{dx}\right)^6 + 5y = e^x$ (b) $5\left(\frac{d^3y}{dx^3}\right)^4 + 8\left(1 + \frac{dy}{dx}\right)^2 + 5y = x^8$

(c) $\left[1 + \left(\frac{dy}{dx}\right)^3\right]^{2/3} = 4\frac{d^3y}{dx^3}$ (d) $y = x^2\frac{dy}{dx} + \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

34. The order of the differential equation whose general solution is given by $y = (C_1 + C_2)\cos(x + C_3) - C_4e^{x+C_5}$ where $C_{1,} C_2, C_3, C_4, C_5$ are arbitrary constants, is **[IIT 1998]**

(a) 5 (b) 4 (c) 3 (d) 2

35. The degree of the differential equation $3\frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2}$ is **[MP PET 1994, 95]**

(a) 1 (b) 2 (c) 3 (d) 6

36. The order of the differential equation $y\left(\frac{dy}{dx}\right) = x \Big/ \left[\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3\right]$ is **[MP PET 1994]**

(a) 1 (b) 2 (c) 3 (d) 4

37. The order and degree of the differential equation $\left[4 + \left(\frac{dy}{dx}\right)^2\right]^{2/3} = \frac{d^2y}{dx^2}$ are

(a) 2, 2 (b) 3, 3 (c) 2, 3 (d) 3, 2

38. The degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{2/3} + 4 - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0$ is

(a) 1 (b) 2 (c) 3 (d) None of these

Formation of Differential equation

Basic Level

39. $y = 4\sin 3x$ is a solution of the differential equation **[AI CBSE 1986]**

(a) $\frac{dy}{dx} + 8y = 0$ (b) $\frac{dy}{dx} - 8y = 0$ (c) $\frac{d^2y}{dx^2} + 9y = 0$ (d) $\frac{d^2y}{dx^2} - 9y = 0$

40. The differential equation of all straight lines passing through the origin is **[DCE 2002; Kerala (Engg.)2002]**

(a) $y = \sqrt{x \frac{dy}{dx}}$ (b) $\frac{dy}{dx} = y + x$ (c) $\frac{dy}{dx} = \frac{y}{x}$ (d) None of these

41. The differential equation obtained on eliminating A and B from the equation $y = A\cos\omega t + B\sin\omega t$ is **[Karnataka CET 2000]**

(a) $y'' = -\omega^2 y$ (b) $y'' + y = 0$ (c) $y'' + y' = 0$ (d) $y'' - \omega^2 y = 0$

42. The elimination of the arbitrary constants A, B and C from $y = A + Bx + Ce^{-x}$ leads to the differential equation **[AMU 1999]**

(a) $y''' - y' = 0$ (b) $y''' - y'' + y' = 0$ (c) $y''' + y'' = 0$ (d) $y''' + y'' - y' = 0$

43. A differential equation associated to the primitive $y = a + be^{5x} + ce^{-7x}$ is

(a) $y_3 + 2y_2 + y_1 = 0$ (b) $4y_3 + 5y_2 - 20y_1 = 0$ (c) $y_3 + 2y_2 - 35y_1 = 0$ (d) None of these

44. The differential equation of the family of curves represented by the equation $(x-a)^2 + y^2 = a^2$ is

(a) $2xy\frac{dy}{dx} + x^2 = y^2$ (b) $2xy\frac{dy}{dx} + x^2 + y^2 = 0$ (c) $xy\frac{dy}{dx} + x^2 = y^2$ (d) None of these

45. The differential equation of the family of curves $y = a\cos(x+b)$ is **[MP PET 1993]**

(a) $\frac{d^2y}{dx^2} - y = 0$ (b) $\frac{d^2y}{dx^2} + y = 0$ (c) $\frac{d^2y}{dx^2} + 2y = 0$ (d) None of these

46. The differential equation of all parabolas that have origin as vertex and y-axis as axis of symmetry is

(a) $xy' = 2y$ (b) $2xy' = y$ (c) $yy' = 2x$ (d) $y'' + y = 2x$

47. The differential equation of the family of curves represented by the equation $x^2 + y^2 = a^2$ is

(a) $x + y\frac{dy}{dx} = 0$ (b) $y\frac{dy}{dx} = x$ (c) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ (d) None of these

48. The differential equation whose general solution is $y = A\sin x + B\cos x$, is **[CEE 1993; Kerala (Engg.) 2002]**

(a) $\frac{d^2y}{dx^2} + y = 0$ (b) $\frac{d^2y}{dx^2} - y = 0$ (c) $\frac{dy}{dx} + y = 0$ (d) None of these

49. The differential equation of the line $y = mx + c$ is (where c is arbitrary constant)

(a) $\frac{dy}{dx} = m$ (b) $\frac{dy}{dx} + m = 0$ (c) $\frac{dy}{dx} = 0$ (d) None of these

50. The differential equation of the family of curves represented by the equation $x^2 y = a$, is

(a) $\frac{dy}{dx} + \frac{2y}{x} = 0$ (b) $\frac{dy}{dx} + \frac{2x}{y} = 0$ (c) $\frac{dy}{dx} - \frac{2y}{x} = 0$ (d) $\frac{dy}{dx} - \frac{2x}{y} = 0$

51. The differential equation corresponding to primitive $y = e^{cx}$ is

Or

The elimination of the arbitrary constant m from the equation $y = e^{mx}$ gives the differential equation **[MP PET 1995, 2000]**

(a) $\frac{dy}{dx} = \left(\frac{y}{x}\right)\log x$ (b) $\frac{dy}{dx} = \left(\frac{x}{y}\right)\log y$ (c) $\frac{dy}{dx} = \left(\frac{y}{x}\right)\log y$ (d) $\frac{dy}{dx} = \left(\frac{x}{y}\right)\log x$

52. The differential equation of all straight lines passing through the point (1, –1) is **[MP PET 1994]**

(a) $y = (x+1)\frac{dy}{dx} + 1$ (b) $y = (x+1)\frac{dy}{dx} - 1$ (c) $y = (x-1)\frac{dy}{dx} + 1$ (d) $y = (x-1)\frac{dy}{dx} - 1$

53. The differential equation found by the elimination of the arbitrary constant K from the equation $y = (x+K)e^{-x}$ is

(a) $\frac{dy}{dx} - y = e^{-x}$ (b) $\frac{dy}{dx} - ye^{x} = 1$ (c) $\frac{dy}{dx} + ye^{x} = 1$ (d) $\frac{dy}{dx} + y = e^{-x}$

54. Differential equation whose solution is $y = cx + c - c^3$, is **[MP PET 1997]**

(a) $\frac{dy}{dx} = c$ (b) $y = x\frac{dy}{dx} + \frac{dy}{dx} - \left(\frac{dy}{dx}\right)^3$ (c) $\frac{dy}{dx} = c - 3c^2$ (d) None of these

55. Differential equation of $y = \sec(\tan^{-1} x)$ is **[UPSEAT 2002]**

(a) $(1+x^2)\frac{dy}{dx} = y + x$ (b) $(1+x^2)\frac{dy}{dx} = y - x$ (c) $(1+x^2)\frac{dy}{dx} = xy$ (d) $(1+x^2)\frac{dy}{dx} = \frac{x}{y}$

56. The differential equation of the family of curves $v = \frac{A}{r} + B$, where A and B are arbitrary constants, is

(a) $\frac{d^2v}{dr^2} + \frac{1}{r}\frac{dv}{dr} = 0$ (b) $\frac{d^2v}{dr^2} - \frac{2}{r}\frac{dv}{dr} = 0$ (c) $\frac{d^2v}{dr^2} + \frac{2}{r}\frac{dv}{dr} = 0$ (d) None of these

57. The differential equation of the famlily of parabolas with focus at the origin and the x-axis as axis is **[EAMCET 2003]**

(a) $y\left(\frac{dy}{dx}\right)^2 + 4x\frac{dy}{dx} = 4y$ (b) $-y\left(\frac{dy}{dx}\right)^2 = 2x\frac{dy}{dx} - y$ (c) $y\left(\frac{dy}{dx}\right)^2 + y = 2xy\frac{dy}{dx}$ (d) $y\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx} + y = 0$

58. The differential equation of all the lines in the xy-plane is

(a) $\frac{dy}{dx} - x = 0$ (b) $\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$ (c) $\frac{d^2y}{dx^2} = 0$ (d) $\frac{d^2y}{dx^2} + x = 0$

59. $y = ae^{mx} + be^{-mx}$ satisfies which of the following differential equations **[Karnataka CET 2002]**

(a) $\frac{dy}{dx} - my = 0$ (b) $\frac{dy}{dx} + my = 0$ (c) $\frac{d^2y}{dx^2} + m^2y = 0$ (d) $\frac{d^2y}{dx^2} - m^2y = 0$

60. The differential equation whose solution is $y = c_1 \cos ax + c_2 \sin ax$ is (where c_1, c_2 are arbitrary constants) **[MP PET 1996]**

(a) $\frac{d^2y}{dx^2} + y^2 = 0$ (b) $\frac{d^2y}{dx^2} + a^2y = 0$ (c) $\frac{d^2y}{dx^2} + ay^2 = 0$ (d) $\frac{d^2y}{dx^2} - y^2 = 0$

61. If $y = ce^{\sin^{-1} x}$, then corresponding to this the differential equation is

(a) $\frac{dy}{dx} = \frac{y}{\sqrt{1-x^2}}$ (b) $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ (c) $\frac{dy}{dx} = \frac{x}{\sqrt{1-x^2}}$ (d) None of these

Advance Level

62. The differential equation of the family of circles with fixed radius r and with centre on y-axis is

(a) $y^2(1+y_1^2) = r^2y_1^2$ (b) $y^2 = r^2y_1 + y_1^2$ (c) $x^2(1+y_1^2) = r^2y_1^2$ (d) $x^2 = r^2y_1 + y_1^2$

63. The differential equation of all parabolas having their axis of symmetry coinciding with the axis of X is

(a) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ (b) $x\frac{d^2x}{dy^2} + \left(\frac{dx}{dy}\right)^2 = 0$ (c) $y\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ (d) None of these

64. The function $f(\theta) = \frac{d}{d\theta}\int_0^\theta \frac{dx}{1 - \cos\theta \cos x}$ satisfies the differential equation

(a) $\frac{df}{d\theta} + 2f(\theta)\cot\theta = 0$ (b) $\frac{df}{d\theta} - 2f(\theta)\cot\theta = 0$ (c) $\frac{df}{d\theta} + 2f(\theta) = 0$ (d) $\frac{df}{d\theta} - 2f(\theta) = 0$

65. The differential equation of all ellipses centred at the origin is

(a) $y_2 + xy_1^2 - yy_1 = 0$ (b) $xyy_2 + xy_1^2 - yy_1 = 0$ (c) $yy_2 + xy_1^2 - xy_1 = 0$ (d) None of these

66. The differential equation for which $\sin^{-1} x + \sin^{-1} y = c$ is given by **[Karnataka CET 2003]**

(a) $\sqrt{1-x^2}\,dx + \sqrt{1-y^2}\,dy = 0$ (b) $\sqrt{1-x^2}\,dy + \sqrt{1-y^2}\,dx = 0$ (c) $\sqrt{1-x^2}\,dy - \sqrt{1-y^2}\,dx = 0$ (d) $\sqrt{1-x^2}\,dx - \sqrt{1-y^2}\,dy = 0$

67. The differential equation satisfied by the family of curves $y = ax\cos\left(\frac{1}{x} + b\right)$, where a, b are parameters, is **[MP PET 2003]**

(a) $x^2y_2+y=0$ (b) $x^4y_2+y=0$ (c) $xy_2-y=0$ (d) $x^2y_2-y=0$

68. Differential equation of central conics are

(a) $yy_1=x(y_1^2+yy_2)$ (b) $yy_1=(y_1^2+yy_2)$ (c) $y^2=xy_1(y_1^2+yy_2)$ (d) None of these

69. The differential equation for all the straight lines which are at a unit distance from the origin is **[MP PET 1993]**

(a) $\left(y-x\frac{dy}{dx}\right)^2=1-\left(\frac{dy}{dx}\right)^2$ (b) $\left(y+x\frac{dy}{dx}\right)^2=1+\left(\frac{dy}{dx}\right)^2$ (c) $\left(y-x\frac{dy}{dx}\right)^2=1+\left(\frac{dy}{dx}\right)^2$ (d) $\left(y+x\frac{dy}{dx}\right)^2=1-\left(\frac{dy}{dx}\right)^2$

70. Family of curves $y=e^x(A\cos x+B\sin x)$, represents the differential equation **[MP PET 1999]**

(a) $\frac{d^2y}{dx^2}=2\frac{dy}{dx}-y$ (b) $\frac{d^2y}{dx^2}=2\frac{dy}{dx}-2y$ (c) $\frac{d^2y}{dx^2}=\frac{dy}{dx}-2y$ (d) $\frac{d^2y}{dx^2}=2\frac{dy}{dx}+y$

71. The differential equation for the family of curves $x^2+y^2-2ay=0$, where a is an arbitrary constant is **[AIEEE 2004]**

(a) $(x^2+y^2)y'=2xy$ (b) $2(x^2+y^2)y'=xy$ (c) $(x^2-y^2)y'=2xy$ (d) $(x^2-y^2)y'=xy$

72. The differential equation of all circles which passes through the origin and whose centre lies on y-axis, is **[MNR 1986; DCE 2000]**

(a) $(x^2-y^2)\frac{dy}{dx}-2xy=0$ (b) $(x^2-y^2)\frac{dy}{dx}+2xy=0$ (c) $(x^2-y^2)\frac{dy}{dx}-xy=0$ (d) $(x^2-y^2)\frac{dy}{dx}+xy=0$

73. The differential equation of the family of curves $y=Ae^{3x}+Be^{5x}$, where A and B are arbitrary constants, is **[MNR 1988]**

(a) $\frac{d^2y}{dx^2}+8\frac{dy}{dx}+15y=0$ (b) $\frac{d^2y}{dx^2}-8\frac{dy}{dx}+15y=0$ (c) $\frac{d^2y}{dx^2}-\frac{dy}{dx}+y=0$ (d) None of these

74. The differential equation whose solution is given by $ae^x+b\log y=0$ is

(a) $\frac{dy}{dx}=-\frac{ay}{b}e^x$ (b) $\frac{d^2y}{dx^2}+\frac{1}{y}=0$ (c) $\frac{d^2y}{dx^2}+y=0$ (d) None of these

Variable Separable Type Differential Equation

Basic Level

75. The solution of $\frac{dy}{dx}=e^x(\sin x+\cos x)$ is

(a) $y=e^x(\sin x-\cos x)+c$ (b) $y=e^x(\cos x-\sin x)+c$ (c) $y=e^x\sin x+c$ (d) $y=e^x\cos x+c$

76. The solution of the differential equation $\frac{dy}{dx}=(1+x)(1+y^2)$ is

(a) $y=\tan(x^2+x+c)$ (b) $y=\tan(2x^2+x+c)$ (c) $y=\tan(x^2-x+c)$ (d) $y=\tan\left(\frac{x^2}{2}+x+c\right)$

77. The solution of the differential equation $(1+x^2)\frac{dy}{dx}=x$ is

(a) $y=\tan^{-1}x+c$ (b) $y=-\tan^{-1}x+c$ (c) $y=\frac{1}{2}\log_e(1+x^2)+c$ (d) $y=-\frac{1}{2}\log_e(1+x^2)+c$

78. The solution of the differential equation $\frac{dy}{dx}+\frac{1+x^2}{x}=0$ is

(a) $y=-\frac{1}{2}\tan^{-1}x+c$ (b) $y+\log x+\frac{x^2}{2}+c=0$ (c) $y=\frac{1}{2}\tan^{-1}x+c$ (d) $y-\log x-\frac{x^2}{2}=c$

79. The solution of the differential equation $\frac{dy}{dx}=\sec x(\sec x+\tan x)$ is

(a) $y=\sec x+\tan x+c$ (b) $y=\sec x+\cot x+c$ (c) $y=\sec x-\tan x+c$ (d) None of these

80. The solution of the differential equation $y\,dx-x\,dy=0$ is

(a) $x=cy$ (b) $xy=c$ (c) $x=c\log x$ (d) None of these

81. The solution of differential equation $x\frac{dy}{dx}+y=y^2$ is

(a) $y=1+cxy$ (b) $y=\log\{cxy\}$ (c) $y+1=cxy$ (d) $y=c+xy$

82. The solution of differential equation $\frac{dy}{dx}+\sin^2 y=0$ **[MP PET 1994]**

(a) $y+2\cos y=c$ (b) $y-2\sin y=c$ (c) $x=\cot y+c$ (d) $y=\cot x+c$

83. The solution of the equation $\frac{dy}{dx}=e^{x-y}+x^2e^{-y}$ is

(a) $e^y=e^x+\frac{x^3}{3}+c$ (b) $e^y=e^x+2x+c$ (c) $e^y=e^x+x^3+c$ (d) $y=e^x+c$

84. The solution of the differential equation $x\cos y\,dy=(xe^x\log x+e^x)dx$ is **[DSSE 1988]**

(a) $\sin y=\frac{1}{x}e^x+c$ (b) $\sin y+e^x\log x+c=0$ (c) $\sin y=e^x\log x+c$ (d) None of these

85. The solution of the differential equation $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$ is **[SCRA 1986]**

(a) $1+xy+c(y-x)=0$ (b) $x+y=c(1-xy)$ (c) $y-x=c(1+xy)$ (d) $1+xy=c(x+y)$

86. Solution of $\frac{dy}{dx}=\frac{x\log x^2+x}{\sin y+y\cos y}$ is **[EAMCET 2003]**

(a) $y\sin y=x^2\log x+c$ (b) $y\sin y=x^2+c$ (c) $y\sin y=x^2+\log x+c$ (d) $y\sin y=x\log x+c$

87. The solution of the differential equation $3e^x\tan y\,dx+(1-e^x)\sec^2 y\,dy=0$ is **[MP PET 1993; AISSE 1985]**

(a) $\tan y=c(1-e^x)^3$ (b) $(1-e^x)^3\tan y=c$ (c) $\tan y=c(1-e^x)$ (d) $(1-e^x)\tan y=c$

88. The solution of the differential equation $\frac{dy}{dx}=1+x+y+xy$ is **[AISSE 1985; AI CBSE 1990; MP PET 2003]**

(a) $\log(1+y)=x+\frac{x^2}{2}+c$ (b) $(1+y)^2=x+\frac{x^2}{2}+c$ (c) $\log(1+y)=\log(1+x)+c$ (d) None of these

89. If $\frac{dy}{dx}=\frac{xy+y}{xy+x}$, then the solution of the differential equation is **[SCRA 1980]**

(a) $y=xe^x+c$ (b) $y=e^x+c$ (c) $y=Axe^{x-y}$ (d) $y=x+A$

90. The solution of the differential equation $(1+\cos x)dy=(1-\cos x)dx$ is **[AISSE 1984]**

(a) $y=2\tan\frac{x}{2}-x+c$ (b) $y=2\tan x+x+c$ (c) $y=2\tan\frac{x}{2}+x+c$ (d) $y=x-2\tan\frac{x}{2}+c$

91. The solution of the differential equation $x(e^{2y}-1)dy+(x^2-1)e^y dx=0$ is **[AISSE 1990]**

(a) $e^y+e^{-y}=\log x-\frac{x^2}{2}+c$ (b) $e^y-e^{-y}=\log x-\frac{x^2}{2}+c$ (c) $e^y+e^{-y}=\log x+\frac{x^2}{2}+c$ (d) None of these

92. Solution of the equation $(1-x^2)dy+xy\,dx=xy^2dx$ **[DSSE 1989]**

(a) $(y-1)^2(1-x^2)=0$ (b) $(y-1)^2(1-x^2)=c^2y^2$ (c) $(y-1)^2(1+x^2)=c^2y^2$ (d) None of these

93. The equation of the curve that passes through the point (1, 2) and satisfies the differential equation $\frac{dy}{dx}=\frac{-2xy}{(x^2+1)}$ is

(a) $y(x^2+1)=4$ (b) $y(x^2+1)+4=0$ (c) $y(x^2-1)=4$ (d) None of these

94. The solution of $(\text{cosec }x\log y)dy+(x^2y)dx=0$ is **[AISSE 1986]**

(a) $\frac{\log y}{2}+(2-x^2)\cos x+2\sin x=c$ (b) $\frac{(\log y)^2}{2}+(2-x^2)\cos x+2x\sin x=c$

(c) $\frac{(\log y)^2}{2}+(2-x^2)\cos x+2x\sin x=c$ (d) None of these

95. The general solution of the differential equation $\frac{dy}{dx}=\frac{x^2}{y^2}$ is

(a) $x^3 - y^3 = C$ (b) $x^3 + y^3 = C$ (c) $x^2 + y^2 = C$ (d) $x^2 - y^2 = C$

96. The general solution of the differential equation $\frac{dy}{dx} = \cot x \cot y$ is **[AISSE 1983; MP PET 1994]**

(a) $\cos x = c \operatorname{cosec} y$ (b) $\sin x = c \sec y$ (c) $\sin x = c \cos y$ (d) $\cos x = c \sin y$

97. The solution of $\frac{dy}{dx} = \frac{1}{x}$ is

(a) $y + \log x + c = 0$ (b) $y = \log x + c$ (c) $y^{\log x} + c = 0$ (d) None of these

98. Solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is **[Karnataka CET 2002]**

(a) $xy = c$ (b) $x + y = c$ (c) $\log x \log y = c$ (d) $x^2 + y^2 = c$

99. The differential equation $\cot y \, dx = x \, dy$ has a solution of the form **[Orissa JEE 2002]**

(a) $y = \cos x$ (b) $x = c \sec y$ (c) $x = \sin y$ (d) $y = \sin x$

100. The solution of differential equation $dy - \sin x \sin y \, dx = 0$ is **[MP PET 1996]**

(a) $e^{\cos x} \tan \frac{y}{2} = C$ (b) $e^{\cos x} \tan y = C$ (c) $\cos x \tan y = C$ (d) $\cos x \sin y = C$

101. The solution of the equation $(2y - 1)\, dx - (2x + 3)\, dy = 0$ **[Kerala (Engg.) 2002]**

(a) $\frac{2x-1}{2y+3} = c$ (b) $\frac{2x+1}{2y-3} = c$ (c) $\frac{2x+3}{2y-1} = c$ (d) $\frac{2x-1}{2y-1} = c$

102. The solution of the differential equation $(x^2 - yx^2)\frac{dy}{dx} + y^2 + xy^2 = 0$ is

(a) $\log\left(\frac{x}{y}\right) = \frac{1}{x} + \frac{1}{y} + c$ (b) $\log\left(\frac{y}{x}\right) = \frac{1}{x} + \frac{1}{y} + c$ (c) $\log(xy) = \frac{1}{x} + \frac{1}{y} + c$ (d) $\log(xy) + \frac{1}{x} + \frac{1}{y} = c$

103. The solution of the differential equation $\frac{dy}{dx} = (ae^{bx} + c \cos mx)$ is

(a) $y = \frac{ae^x}{b} + \frac{c}{m}\sin mx + k$ (b) $y = ae^x + c \sin mx + k$ (c) $y = \frac{ae^{bx}}{b} + \frac{c}{m}\sin mx + k$ (d) None of these

104. The solution of $\frac{dy}{dx} = x \log x$ is **[MP PET 2003]**

(a) $y = x^2 \log x - \frac{x^2}{2} + c$ (b) $y = \frac{x^2}{2}\log x - x^2 + c$ (c) $y = \frac{1}{2}x^2 + \frac{1}{2}x^2 \log x + c$ (d) None of these

105. The solution of the differential equation $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$ is **[AISSE 1983; Karnataka CET 1999; MP PET 2003]**

(a) $\tan x = c \tan y$ (b) $\tan x = c \tan(x + y)$ (c) $\tan x = c \cot y$ (d) $\tan x \sec y = c$

106. The solution of the differential equation $x^2 dy = -2xy \, dx$ is **[SCRA 1990]**

(a) $xy^2 = c$ (b) $x^2 y^2 = c$ (c) $x^2 y = c$ (d) $xy = c$

107. The solution of the differential equation $x \sec y \frac{dy}{dx} = 1$ is

(a) $x \sec y \tan y = c$ (b) $cx = \sec y + \tan y$ (c) $cy = \sec x \tan x$ (d) $cy = \sec x + \tan x$

108. If $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$, then **[MNR 1983]**

(a) $y + \sin^{-1} x = c$ (b) $y^2 + 2\sin^{-1} x = 0$ (c) $x + \sin^{-1} y = 0$ (d) $x^2 + 2\sin^{-1} y = 1$

109. The solution of the differential equation $\sin x \sin y \, dx + \cos x \cos y \, dy = 0$ is

(a) $\sin y = c \cos x$ (b) $\sin x = c \cos y$ (c) $\sin x \cos y = c$ (d) $\sin y \cos x = c$

110. The general solution of the differential equation $y \, dx + (1 + x^2)\tan^{-1} x \, dy = 0$, is **[MP PET 1995]**

(a) $y \tan^{-1} x = c$ (b) $x \tan^{-1} y = c$ (c) $y + \tan^{-1} x = c$ (d) $x + \tan^{-1} y = c$

111. The solution of the differential equation $\frac{dy}{dx}+\frac{1+\cos 2y}{1-\cos 2x}=0$ **[AISSE 1982]**

(a) $\tan y+\cot x=c$ (b) $\tan y.\cot x=c$ (c) $\tan y-\cot x=c$ (d) None of these

112. Solution of the equation $\cos x\cos y\frac{dy}{dx}=-\sin x\sin y$ is **[DSSE 1987]**

(a) $\sin y+\cos x=c$ (b) $\sin y-\cos x=c$ (c) $\sin y.\cos x=c$ (d) $\sin y=c\cos x$

113. The solution of the equation $\frac{dy}{dx}=y(e^x+1)$ is **[AISSE 1986; AI CBSE 1984]**

(a) $y+e^{(e^x+x+c)}=0$ (b) $\log y=e^x+x+c$ (c) $\log y+e^x=x+c$ (d) None of these

114. The general solution of $x^2\frac{dy}{dx}=2$ is **[AISSE 1984]**

(a) $y=c+\frac{2}{x}$ (b) $y=c-\frac{2}{x}$ (c) $y=2cx$ (d) $y=c-\frac{3}{x^3}$

115. The solution of the differential equation $dy=\sec^2 x\,dx$ is

(a) $y=\sec x\tan x+c$ (b) $y=2\sec x+c$ (c) $y=\frac{1}{2}\tan x+c$ (d) None of these

116. The solution of the equation $(1+x^2)\frac{dy}{dx}=1$ is

(a) $y=\log(1+x^2)+c$ (b) $y+\log(1+x^2)+c=0$ (c) $y-\log(1+x)=c$ (d) $y=\tan^{-1}x+c$

117. The solution of the differential equation $\frac{dy}{dx}=e^x+\cos x+x+\tan x$ is

(a) $y=e^x+\sin x+\frac{x^2}{2}+\log\cos x+c$ (b) $y=e^x+\sin x+\frac{x^2}{2}+\log\sec x+c$

(c) $y=e^x-\sin x+\frac{x^2}{2}+\log\cos x+c$ (d) $y=e^x-\sin x+\frac{x^2}{2}+\log\sec x+c$

118. The general solution of the differential equation $e^y\frac{dy}{dx}+(e^y+1)\cot x=0$ is

(a) $(e^y+1)\cos x=K$ (b) $(e^y+1)\operatorname{cosec} x=K$ (c) $(e^y+1)\sin x=K$ (d) None of these

119. Solution of differential equation $\frac{dy}{dx}=\sin x+2x$, is **[MP PET 1997]**

(a) $y=x^2-\cos x+c$ (b) $y=\cos x+x^2+c$ (c) $y=\cos x+2$ (d) $y=\cos x+2+c$

120. Solution of differential equation $\frac{dy}{dx}=2xy$, is **[MP PET 1997]**

(a) $y=ce^{x^2}$ (b) $y^2=2x^2+c$ (c) $y=e^{-x^2}+c$ (d) $y=x^2+c$

121. The general solution of differential equation $(4+5\sin x)\frac{dy}{dx}=\cos x$ is

(a) $y=\frac{1}{5}\log|4+5\sin x|+C$ (b) $y=\frac{1}{5}\log|4+5\cos x|+C$

(c) $y=-\frac{1}{5}\log|4-5\sec x|+C$ (d) None of these

122. The general solution of differential $\frac{dy}{dx}=\log x$ is

(a) $y=x(\log x+1)+C$ (b) $y+x(\log x+1)=C$ (c) $y=x(\log x-1)+C$ (d) None of these

123. For solving $\frac{dy}{dx}=(4x+y+1)$, suitable substitution is **[MP PET 1999]**

(a) $y=vx$ (b) $y=4x+v$ (c) $y=4x$ (d) $y+4x+1=v$

Advance Level

124. The solution of $\frac{dy}{dx}+\sqrt{\left(\frac{1-y^2}{1-x^2}\right)}=0$ is **[DCE 1999]**

(a) $\tan^{-1}x+\cot^{-1}x=C$ (b) $\sin^{-1}x+\sin^{-1}y=C$ (c) $\sec^{-1}x+\text{cosec}^{-1}x=C$ (d) None of these

125. The solution of the differential equation $\sqrt{a+x}\frac{dy}{dx}+xy=0$ is **[MP PET 1998]**

(a) $y=Ae^{\frac{2}{3}(2a-x)\sqrt{x+a}}$ (b) $y=Ae^{-\left(\frac{2}{3}\right)(a-x)\sqrt{x+a}}$ (c) $y=Ae^{\frac{2}{3}(2a+x)\sqrt{x+a}}$ (d) $y=Ae^{-\frac{2}{3}(2a-x)\sqrt{x+a}}$

Where A is an arbitrary constant

126. The solution of the given differential equation $\frac{dy}{dx}+2xy=y$ is **[Roorkee 1995]**

(a) $y=ce^{x-x^2}$ (b) $y=ce^{x^2-x}$ (c) $y=ce^x$ (d) $y=ce^{-x^2}$

127. The general solution of the differential equation $\log\left(\frac{dy}{dx}\right)=x+y$ is **[DSSE 1984; MP PET 1994, 95]**

(a) $e^x+e^y=c$ (b) $e^x+e^{-y}=c$ (c) $e^{-x}+e^y=c$ (d) $e^{-x}+e^{-y}=c$

128. The solution of the differential equation $\cos y\log(\sec x+\tan x)dx=\cos x\log(\sec y+\tan y)dy$ is **[AI CBSE 1990]**

(a) $\sec^2x+\sec^2y=c$ (b) $\sec x+\sec y=c$ (c) $\sec x-\sec y=c$ (d) None of these

129. Solution of $y\,dx-x\,dy=x^2y\,dx$ is **[MP PET 1999]**

(a) $ye^{x^2}=cx^2$ (b) $ye^{-x^2}=cx^2$ (c) $y^2e^{x^2}=cx^2$ (d) $y^2e^{-x^2}=cx^2$

130. If $\frac{dy}{dx}=1+x+y+xy$ and $y(-1)=0$, then function y is **[MP PET 1998]**

(a) $e^{(1-x)^2/2}$ (b) $e^{(1+x)^2/2}-1$ (c) $\log_e(1+x)-1$ (d) $1+x$

131. The differential equation $\frac{dy}{dx}=\frac{4x+6y+5}{3y+2x+4}$, which is not with separated variables, can be transformed into one which is with separated variables, by the substitution

(a) $2x+3y=v$ (b) $4x+6y+5=v$ (c) $2x+3y+4=v$ (d) $3x+2y=v$

132. The solution of the differential equation $y-x\frac{dy}{dx}=a\left(y^2+\frac{dy}{dx}\right)$ is **[AISSE 1989, 90; MP PET 2002]**

(a) $y=c(x+a)(1+ay)$ (b) $y=c(x+a)(1-ay)$ (c) $y=c(x-a)(1+ay)$ (d) None of these

133. The solution of $\frac{dy}{dx}-\frac{1}{xy}+\frac{1}{y}=0$ is

(a) $cx=e^{x+y^2/2}$ (b) $cy=e^{x+y^2/2}$ (c) $cx=e^{y^2+x/2}$ (d) None of these

134. The solution of the equation $x\frac{dy}{dx}=\frac{1-y^2}{\sqrt{1-x^2}}$ is

(a) $x=\sec\{\lambda(1+y)/(1-y)\}$ (b) $x=\sec\{\lambda(1+x)/(1-x)\}$ (c) $x=\lambda\sec\{(1+y)/(1-y)\}$ (d) None of these

135. The solution of the differential equation $xy\frac{dy}{dx}=\frac{(1+y^2)(1+x+x^2)}{(1+x^2)}$ is **[AISSE 1983]**

(a) $\frac{1}{2}\log(1+y^2)=\log x-\tan^{-1}x+c$ (b) $\frac{1}{2}\log(1+y^2)=\log x+\tan^{-1}x+c$

(c) $\log(1+y^2)=\log x-\tan^{-1}x+c$ (d) $\log(1+y^2)=\log x+\tan^{-1}x+c$

136. The solution of the differential equation $(1+x^2)(1+y)dy+(1+x)(1+y^2)dx=0$ is **[DSSE 1986]**

(a) $\tan^{-1}x+\log(1+x^2)+\tan^{-1}y+\log(1+y^2)=c$ (b) $\tan^{-1}x-\frac{1}{2}\log(1+x^2)+\tan^{-1}y-\frac{1}{2}\log(1+y^2)=c$

(c) $\tan^{-1}x+\frac{1}{2}\log(1+x^2)+\tan^{-1}y+\frac{1}{2}\log(1+y^2)=c$ (d) None of these

137. The solution of $\frac{dy}{dx}=\frac{e^x(\sin^2 x+\sin 2x)}{y(2\log y+1)}$ is **[AISSE 1990]**

(a) $y^2(\log y)-e^x\sin^2 x+c=0$ (b) $y^2(\log y)-e^x\cos^2 x+c=0$ (c) $y^2(\log y)+e^x\cos^2 x+c=0$ (d) None of these

138. The solution of the differential equation $(x-y^2x)dx=(y-x^2y)dy$ is **[DSSE 1984]**

(a) $(1-y^2)=c^2(1-x^2)$ (b) $(1+y^2)=c^2(1-x^2)$ (c) $(1+y^2)=c^2(1+x^2)$ (d) None of these

139. The solution of the equation $\frac{dy}{dx}=\frac{y^2-y-2}{x^2+2x-3}$ is

(a) $\frac{1}{3}\log\left|\frac{y-2}{y+1}\right|=\frac{1}{4}\log\left|\frac{x+3}{x-1}\right|+c$ (b) $\frac{1}{3}\log\left|\frac{y+1}{y-2}\right|=\frac{1}{4}\log\left|\frac{x-1}{x+3}\right|+c$

(c) $4\log\left|\frac{y-2}{y+1}\right|=3\log\left|\frac{x-1}{x+3}\right|+c$ (d) None of these

140. The general solution of the differential equation $(\tan^2 x+2\tan x+5)\frac{dy}{dx}=2(1+\tan x)\sec^2 x$ is

(a) $y=\log|\tan^2 x+2\tan x+5|+c$ (b) $y=\log|\tan^2 x-2\tan x+5|+c$

(c) $y=\log|\sec^2 x-2\tan x+5|-c$ (d) None of these

141. The solution of the equation $\sqrt{a+x}\frac{dy}{dx}+x=0$ is **[DSSE 1988]**

(a) $3y+2\sqrt{a+x}.(x-2a)=3c$ (b) $3y+2\sqrt{x+a}.(x+2a)=3c$

(c) $3y+2\sqrt{x-a}.(x+2a)=3c$ (d) None of these

142. The solution of $e^{2x-3y}dx+e^{2y-3x}dy=0$ is

(a) $e^{5x}+e^{5y}=c$ (b) $e^{5x}-e^{5y}=c$ (c) $e^{5x+5y}=c$ (d) None of these

143. The solution of $(x\sqrt{1+y^2}\,dx+(y\sqrt{1+x^2})dy=0$ is

(a) $\sqrt{1+x^2}+\sqrt{1+y^2}=c$ (b) $\sqrt{1+x^2}-\sqrt{1+y^2}=c$ (c) $(1+x^2)^{3/2}+(1+y)^{3/2}=c$ (d) None of these

144. Solution of the equation $(e^x+1)y\,dy=(y+1)e^x\,dx$ is **[AISSE 1988]**

(a) $c(y+1)(e^x+1)+e^y=0$ (b) $c(y+1)(e^x-1)+e^y=0$ (c) $c(y+1)(e^x-1)-e^y=0$ (d) $c(y+1)(e^x+1)=e^y$

145. The solution of the differential equation $(1-x^2)(1-y)dx=xy(1+y)dy$ is

(a) $\log[x(1-y^2)]=\frac{x^2}{2}+\frac{y^2}{2}-2y+c$ (b) $\log[x(1-y^2)]=\frac{x^2}{2}-\frac{y^2}{2}+2y+c$

(c) $\log[x(1+y^2)]=\frac{x^2}{2}+\frac{y^2}{2}-2y+c$ (d) $\log[x(1-y)^2]=\frac{x^2}{2}-\frac{y^2}{2}-2y+c$

146. The solution of $\frac{dy}{dx}=2^{y-x}$ is **[Karnataka CET 2000]**

(a) $2^x+2^y=C$ (b) $2^x-2^y=C$ (c) $\frac{1}{2^x}-\frac{1}{2^y}=C$ (d) $\frac{1}{2^x}+\frac{1}{2^y}=C$

147. The solution of differential equation $y\frac{dy}{dx}=x-1$ satisfying $y(1)=1$ is

(a) $y^2=x^2-2x+2$ (b) $y^2=2x^2-x-1$ (c) $y=x^2-2x+2$ (d) None of these

148. The differential equation $y\frac{dy}{dx}+x=a$ (a is any constant) represents

(a) A set of circles having centre on the y-axis (b) A set of circles, centre on the x-axis

(c) A set of ellipses (d) None of these

149. The general solution of differential equation $\frac{dy}{dx} = \sin^3 x \cos^2 x + xe^x$ is

(a) $y = \frac{1}{5}\cos^5 x + \frac{1}{3}\text{cosec}^3 x + (x+1)e^x + c$ (b) $y = \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + (x-1)e^x + c$

(c) $y = -\frac{1}{5}\cos^5 x - \frac{1}{3}\text{cosec}^3 x - (x-1)e^x - c$ (d) None of these

150. The solution of the differential equation $\frac{dy}{dx} = \frac{x-y+3}{2(x-y)+5}$ is

(a) $2(x-y)+\log(x-y) = x+c$ (b) $2(x-y)-\log(x-y+2) = x+c$

(c) $2(x-y)+\log(x-y+2) = x+c$ (d) None of these

151. Solution of $(x+y-1)dx + (2x+2y-3)dy = 0$ is **[MP PET 1999]**

(a) $y+x+\log(x+y-2) = c$ (b) $y+2x+\log(x+y-2) = c$ (c) $2y+x+\log(x+y-2) = c$ (d) $2y+2x+\log(x+y-2) = c$

152. The solution of $\cos(x+y)dy = dx$ is **[DCE 1999]**

(a) $y = \tan\left(\frac{x+y}{2}\right) + c$ (b) $y + \cos^{-1}\left(\frac{y}{x}\right) = c$ (c) $y = x\sec\left(\frac{y}{x}\right) + c$ (d) None of these

153. The solution of $\log\left(\frac{dy}{dx}\right) = ax + by$ is

(a) $\frac{e^{by}}{b} = \frac{e^{ax}}{a} + c$ (b) $\frac{e^{-by}}{-b} = \frac{e^{ax}}{a} + c$ (c) $\frac{e^{-by}}{a} = \frac{e^{ax}}{b} + c$ (d) None of these

154. The solution of the equation $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$ is

(a) $\tan(x+y)+\sec(x+y) = x+c$ (b) $\tan(x+y)-\sec(x+y) = x+c$

(c) $\tan(x+y)+\sec(x+y)-x+c = 0$ (d) None of these

155. The solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 4x - 2y - 2$, $y = 1$ when $x = 1$ is

(a) $2e^{2y+2} = e^{4x} + e^2$ (b) $2e^{2y-2} = e^{4x} + e^4$ (c) $2e^{2y+2} = e^{4x} + e^4$ (d) $3e^{2y+2} = e^{3x} + e^4$

156. The solution of the equation $\frac{dy}{dx} = \frac{3x-4y-2}{3x-4y-3}$ is

(a) $(x-y)^2 + c = \log(3x-4y+1)$ (b) $x-y+c = \log(3x-4y+4)$

(c) $(x-y)^2 + c = \log(3x-4y-3)$ (d) $x-y+c = \log(3x-4y+1)$

157. The solution of $\frac{dy}{dx} = \sin(x+y)+\cos(x+y)$ is

(a) $\log\left[1+\tan\left(\frac{x+y}{2}\right)\right] + c = 0$ (b) $\log\left[1+\tan\left(\frac{x+y}{2}\right)\right] = x+c$ (c) $\log\left[1-\tan\left(\frac{x+y}{2}\right)\right] = x+c$ (d) None of these

158. The solution of the equation $\frac{dy}{dx} = (x+y)^2$ is

(a) $x+y+\tan(x+c) = 0$ (b) $x-y+\tan(x+c) = 0$ (c) $x+y-\tan(x+c) = 0$ (d) None of these

159. The solution of the equation $\frac{dy}{dx} = \cos(x-y)$ is

(a) $y+\cot\left(\frac{x-y}{2}\right) = c$ (b) $x+\cot\left(\frac{x-y}{2}\right) + c = 0$ (c) $x+\tan\left(\frac{x-y}{2}\right) = c$ (d) None of these

160. The solution of the differential equation $(x+y)^2\frac{dy}{dx} = a^2$ is **[AMU 2001]**

(a) $(x+y)^2 = \frac{a^2}{2}x + C$ (b) $(x+y)^2 = a^2x + C$ (c) $(x+y)^2 = 2a^2x + C$ (d) None of these

Homogeneous Equation

Basic Level

161. Solution of differential equation $2xy\frac{dy}{dx}=x^2+3y^2$ is (where p is constant) **[MP PET 1993]**

(a) $x^3+y^2=px^2$ (b) $\frac{x^2}{2}+\frac{y^3}{x}=y^2+p$ (c) $x^2+y^3=px^2$ (d) $x^2+y^2=px^3$

162. The solution of the equation $\frac{dy}{dx}=\frac{x+y}{x-y}$ is **[AI CBSE 1990]**

(a) $c(x^2+y^2)^{1/2}+e^{\tan^{-1}(y/x)}=0$ (b) $c(x^2+y^2)^{1/2}=e^{\tan^{-1}(y/x)}$

(c) $c(x^2+y^2)=e^{\tan^{-1}(y/x)}$ (d) None of these

163. The solution of the differential equation $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$ is

(a) $ay^2=e^{x^2/y^2}$ (b) $ay=e^{x/y}$ (c) $y=e^{x^2}+e^{y^2}+c$ (d) $y=e^{x^2}+y^2+c$

164. The solution of the equation $\frac{dy}{dx}=\frac{x}{2y-x}$ is

(a) $(x-y)(x+2y)^2=c$ (b) $y=x+c$ (c) $y=(2y-x)+c$ (d) $y=\frac{x}{2y-x}+c$

165. The solution of the differential equation $x+y\frac{dy}{dx}=2y$ is

(a) $\log(y-x)=c+\frac{y-x}{x}$ (b) $\log(y-x)=c+\frac{x}{y-x}$ (c) $y-x=c+\log\frac{x}{y-x}$ (d) $y-x=c+\frac{x}{y-x}$

166. The solution of $\frac{dy}{dx}=\left(\frac{y}{x}\right)^{1/3}$ is **[EAMCET 2002]**

(a) $x^{2/3}+y^{2/3}=C$ (b) $x^{1/3}+y^{1/3}=C$ (c) $y^{2/3}-x^{2/3}=C$ (d) $y^{1/3}-x^{1/3}=C$

167. If $y'=\frac{x-y}{x+y}$, then its solution is **[MP PET 2000]**

(a) $y^2+2xy-x^2=C$ (b) $y^2+2xy+x^2=C$ (c) $y^2-2xy-x^2=C$ (d) $y^2-2xy+x^2=C$

168. The solution of the equation $x\frac{dy}{dx}+3y=x$ is

(a) $x^3y+\frac{x^4}{4}+c=0$ (b) $x^3y+\frac{x^4}{4}+c$ (c) $x^3y+\frac{x^4}{4}=0$ (d) None of these

Advance Level

169. The solution of the differential equation $x^2\frac{dy}{dx}=x^2+xy+y^2$ is

(a) $\tan^{-1}\left(\frac{y}{x}\right)=\log x+c$ (b) $\tan^{-1}\left(\frac{y}{x}\right)=-\log x+c$ (c) $\sin^{-1}\left(\frac{y}{x}\right)=\log x+c$ (d) $\tan^{-1}\left(\frac{x}{y}\right)=\log x+c$

170. The general solution of $y^2dx+(x^2-xy+y^2)dy=0$ **[EAMCET 2003]**

(a) $\tan^{-1}\left(\frac{x}{y}\right)+\log y+c=0$ (b) $2\tan^{-1}\left(\frac{x}{y}\right)+\log x+c=0$ (c) $\log(y+\sqrt{x^2+y^2})+\log y+c=0$ (d) $\sinh^{-1}\left(\frac{x}{y}\right)+\log y+=0$

171. The solution of the differential equation $(x^2+y^2)dx=2xy\,dy$ is **[MP PET 2003]**

(a) $x = c(x^2 + y^2)$ (b) $x = c(x^2 - y^2)$ (c) $x + c(x^2 + y^2) = 0$ (d) None of these

172. The solution of the equation $x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$ is **[Roorkee 1982]**

(a) $x\sin\left(\frac{x}{y}\right) + c = 0$ (b) $x\sin y + c = 0$ (c) $x\sin\left(\frac{y}{x}\right) = c$ (d) None of these

173. The solution of the differential equation $x\,dy - y\,dx = (\sqrt{x^2 + y^2})dx$ is

(a) $y - \sqrt{x^2 + y^2} = cx^2$ (b) $y + \sqrt{x^2 + y^2} = cx^2$ (c) $y + \sqrt{x^2 + y^2} + cx^2 = 0$ (d) None of these

174. The solution of the differential equation $(3xy + y^2)dx + (x^2 + xy)dy = 0$ is **[AISSE 1990]**

(a) $x^2(2xy + y^2) = c^2$ (b) $x^2(2xy - y^2) = c^2$ (c) $x^2(y^2 - 2xy) = c^2$ (d) None of these

175. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ is **[DCE 2002]**

(a) $\phi\left(\frac{y}{x}\right) = kx$ (b) $x\phi\left(\frac{y}{x}\right) = k$ (c) $\phi\left(\frac{y}{x}\right) = ky$ (d) $y\phi\left(\frac{y}{x}\right) = k$

176. The solution of $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$ is

(a) $x^2 - y^2 = (x^2 + y^2)c^2$ (b) $x^2 + y^3(x - 2y)^2 = c^2$ (c) $x^2 + y^2(x - 2y)^2 = c^2$ (d) None of these

177. The solution $(x^3 + y^3)dx - 3xy^2dy = 0$ is

(a) $x^3 - 2y^3 = cx$ (b) $x^3 - 2y^2 = cx$ (c) $x^3 + 2y^3 = cx$ (d) None of these

178. Solution of differential equation $\frac{dy}{dx} = \frac{y - x}{y + x}$ is **[MP PET 1997]**

(a) $\log_e(x^2 + y^2) + 2\tan^{-1}\frac{y}{x} + c = 0$ (b) $\frac{y^2}{2} + xy = xy - \frac{x^2}{2} + c$

(c) $\left(1 + \frac{x}{y}\right)y = \left(1 - \frac{x}{y}\right)x + c$ (d) $y = x - 2\log_e y + c$

179. Solution of $(x - y - 1)dx + (4y + x - 1)dy = 0$ is

(a) $\log\{4y^2 + (x - 1)^2\} + \tan^{-1}\{2y/(x - 1)\} = c$ (b) $\log\{4x^2 + (y - 1)^2\} + \tan^{-1}\{2y/(x + 1)\} = c$

(c) $\log\{4y^2 + (x + 1)^2\} + \tan^{-1}\{2y/(x + 1)\} = c$ (d) None of these

180. Solution of $(3y - 7x + 7)dx + (7y - 3x + 3)dy = 0$ is

(a) $(y - x + 1)^2(y + x - 1)^5 = c$ (b) $(y - x + 1)^2(y + x - 1)^3 = c$

(c) $(y + x - 1)^2(y - x + 1)^4 = c$ (d) None of these

181. Solution of $\frac{dy}{dx} = \frac{6x - 2y - 7}{2x + 3y - 6}$ is

(a) $3x^2 - 7xy = c$ (b) $2x - 3y + xy = c$

(c) $3x^2 - 2xy - 7x - \frac{3}{2}y^2 + 6y = c$ (d) None of these

Exact Differentials

Basic Level

182. The solution of $y\,dx - xdy + 3x^2y^2e^{x^3}dx = 0$ is

(a) $\frac{x}{y}+e^{x^3}=C$ (b) $\frac{x}{y}-e^{x^3}=0$ (c) $\frac{-x}{y}+e^{x^3}=0$ (d) None of these

183. The solution of $(x+2y^3)\frac{dy}{dx}=y$ is

(a) $\frac{x}{y}=y^2+c$ (b) $xy=y+c$ (c) $\frac{y}{x}=x+c$ (d) None of these

184. The solution of $(1+xy)y\,dx+(1-xy)x\,dy=0$ is

(a) $\frac{x}{y}+\frac{1}{xy}=k$ (b) $\log\left(\frac{x}{y}\right)=\frac{1}{xy}+k$ (c) $\frac{x}{y}=e^{xy}+k$ (d) $\log\left(\frac{x}{y}\right)=xy+k$

185. The solution of the differential equation $y\,dx+(x+x^2y)dy=0$ is **[AIEEE 2004]**

(a) $\log y=Cx$ (b) $-\frac{1}{xy}+\log y=C$ (c) $\frac{1}{xy}+\log y=C$ (d) $-\frac{1}{xy}=C$

186. The solution of the differential equation $(\sin x+\cos x)dy+(\cos x-\sin x)dx=0$ is

(a) $e^x(\sin x+\cos x)+c=0$ (b) $e^y(\sin x+\cos x)=c$ (c) $e^y(\cos x-\sin x)=c$ (d) $e^x(\sin x+\cos x)=c$

187. Solution of the equation $y\,dx-x\,dy+\log x\,dx=0$ is

(a) $y=cx-(1+\log x)$ (b) $y=cx+(1+\log x)$ (c) $y+cx+(1+\log x)=0$ (d) None of these

188. Solution of the equation $(x+\log y)dy+y\,dx=0$ is

(a) $xy+y\log y=c$ (b) $xy+y\log y-y=c$ (c) $xy+\log y-x=c$ (d) None of these

189. Solution of $(xy\cos xy+\sin xy)dx+x^2\cos xy\,dy=0$ is

(a) $x\sin(xy)=k$ (b) $xy\sin(xy)=k$ (c) $\frac{x}{y}\sin(xy)=k$ (d) $x\sin(xy)+xy\cos xy=k$

190. The solution of $(x-y^3)dx+3xy^2dy=0$ is

(a) $\log x+\frac{x}{y^3}=k$ (b) $\log x+\frac{y^3}{x}=k$ (c) $\log x-\frac{x}{y^3}=k$ (d) $\log xy-y^3=k$

Advance Level

191. The solution of $ye^{-x/y}dx-(xe^{-x/y}+y^3)dy=0$ is

(a) $\frac{y^2}{2}+e^{-x/y}=k$ (b) $\frac{x^2}{2}+e^{-x/y}=k$ (c) $\frac{x^2}{2}+e^{x/y}=k$ (d) $\frac{y^2}{2}+e^{x/y}=k$

192. The solution of the differential equation $x\,dy+y\,dx-\sqrt{1-x^2y^2}\,dx=0$ is

(a) $\sin^{-1}xy=C-x$ (b) $xy=\sin(x+C)$ (c) $\log(1-x^2y^2)=x+C$ (d) $y=x\sin x+C$

193. Solution of the differential equation, $y\,dx-x\,dy+xy^2dx=0$ can be

(a) $2x+x^2y=\lambda y$ (b) $2y+y^2x=\lambda y$ (c) $2y-y^2x=\lambda y$ (d) None of these

194. The solution of the equation $\frac{dy}{dx}=\frac{(1+x)y}{(y-1)x}$ is **[AISSE 1986; AI CBSE 1982]**

(a) $\log(xy)+x+y=c$ (b) $\log\left(\frac{x}{y}\right)+(x-y)=c$ (c) $\log(xy)+x-y=c$ (d) None of these

195. The general solution of the equation $(e^y+1)\cos x\,dx+e^y\sin x\,dy=0$ is **[SCRA 1986]**

(a) $(e^y+1)\cos x = c$ (b) $(e^y-1)\sin x = c$ (c) $(e^y+1)\sin x = c$ (d) None of these

Linear Equation

Basic Level

196. The solution of differential equation $\frac{dy}{dx}+y=e^x$ is **[AI CBSE 1990]**

(a) $y=e^x+ce^{-x}$ (b) $y=e^{-x}+ce^x$ (c) $y=\frac{1}{2}e^x+ce^{-x}$ (d) $y=\frac{1}{2}e^{-x}+ce^x$

197. Which of the following equation is non-linear

(a) $\frac{dy}{dx}+\frac{y}{x}=\log x$ (b) $y\frac{dy}{dx}+4x=0$ (c) $dx+dy=0$ (d) $\frac{dy}{dx}=\cos x$

198. The solution of the differential equation $\frac{dy}{dx}+\frac{3x^2}{1+x^3}y=\frac{\sin^2 x}{1+x^3}$ is

(a) $y(1+x^3)=x+\frac{1}{2}\sin 2x+c$ (b) $y(1+x^3)=cx+\frac{1}{2}\sin 2x$

(c) $y(1+x^3)=cx-\frac{1}{2}\sin 2x$ (d) $y(1+x^3)=\frac{x}{2}-\frac{1}{4}\sin 2x+c$

199. The solution of the differential equation $\frac{dy}{dx}+y\tan x-\sec x=0$ is

(a) $y\tan x=\sec x+c$ (b) $y\sec x=\tan x+c$ (c) $y\sec x=\cot x+c$ (d) None of these

200. Which of the following equation is linear

(a) $\frac{dy}{dx}+xy^2=1$ (b) $x^2\frac{dy}{dx}+y=e^x$ (c) $\frac{dy}{dx}+3y=xy^2$ (d) $x\frac{dy}{dx}+y^2=\sin x$

201. The solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}=x^2$ is

(a) $4xy=x^4+c$ (b) $xy=x^4+c$ (c) $\frac{1}{4}xy=x^4+c$ (d) $xy=4x^4+c$

202. Which of the following equation is non-linear

(a) $\frac{dy}{dx}=\cos x$ (b) $\frac{d^2y}{dx^2}+y=0$ (c) $dx+dy=0$ (d) $x\frac{dy}{dx}+\frac{3}{\frac{dy}{dx}}=y^2$

203. The integrating factor of the differential equation $\frac{dy}{dx}=y\tan x-y^2\sec x$, is **[MP PET 1995]**

(a) $\tan x$ (b) $\sec x$ (c) $-\sec x$ (d) $\cot x$

204. $\frac{dy}{dx}+y=\cos x$ is **[AISSE 1990]**

(a) $y=\frac{1}{2}(\cos x+\sin x)+ce^{-x}$ (b) $y=\frac{1}{2}(\cos x-\sin x)+ce^{-x}$

(c) $y=\cos x+\sin x+ce^{-x}$ (d) None of these

205. The solution of the equation $x\frac{dy}{dx}+3y=x$ is

(a) $x^3y+\frac{x^4}{4}+c=0$ (b) $x^3y=\frac{x^4}{4}+c$ (c) $x^3y+\frac{x^4}{4}=0$ (d) None of these

206. Integrating factor of the differential equation $\frac{dy}{dx}+y\tan x-\sec x=0$ is **[MP PET 2002]**

(a) $e^{\sin x}$ (b) $\frac{1}{\sin x}$ (c) $\frac{1}{\cos x}$ (d) $e^{\cos x}$

207. The solution of the differential equation $x \log x \frac{dy}{dx} + y = 2 \log x$ is

(a) $y = \log x + c$ (b) $y = \log x^2 + c$ (c) $y \log x = (\log x)^2 + c$ (d) $y = x \log x + c$

208. The solution of the differential equation $\frac{dy}{dx} + 2y \cot x = 3x^2 \text{cosec}^2 x$ is

(a) $y \sin^2 x = x^3 + c$ (b) $y \sin x = c$ (c) $y \cos x^2 = c$ (d) $y \sin x^2 = c$

209. The solution of $\frac{dy}{dx} + \frac{y}{3} = 1$ is **[EAMCET 2002]**

(a) $y = 3 + Ce^{x/3}$ (b) $y = 3 + Ce^{-x/3}$ (c) $y = C + e^{x/3}$ (d) $y = C + e^{-x/3}$

210. $y + x^2 = \frac{dy}{dx}$ has the solution **[EAMCET 2002]**

(a) $y + x^2 + 2x + 2 = Ce^x$ (b) $y + x + x^2 + 2 = Ce^{2x}$ (c) $y + x + 2x^2 + 2 = Ce^x$ (d) $y^2 + x + x^2 + 2 = Ce^x$

211. Solution of differential equation $x \frac{dy}{dx} = y + x^2$ is **[MP PET 1997]**

(a) $y = \log_e x + \frac{x^2}{2} + a$ (b) $y = \frac{x^3}{3} + \frac{a}{x}$ (c) $y = x^2 + ax$ (d) None of these

212. Which of the following equation is linear

(a) $\sqrt{1-x^2}dx + \sqrt{1-y^2}dy = 0$ (b) $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$

(c) $\frac{1}{x}\frac{d^2y}{dx^2} = e^x$ (d) $(xy^2 + x)dx + (y - x^2y)dy = 0$

213. The solution of the differential equation $x \frac{dy}{dx} + y = x^2 + 3x + 2$ is

(a) $xy = \frac{x^3}{3} + \frac{3}{2}x^2 + 2x + c$ (b) $xy = \frac{x^4}{4} + x^3 + x^2 + c$ (c) $xy = \frac{x^4}{4} + \frac{x^3}{3} + x^2 + c$ (d) $xy = \frac{x^4}{4} + x^3 + x^2 + cx$

214. The integrating factor of the differential equation $x\,dy - y\,dx = xy^2dx$ is

(a) $\frac{1}{x^2}$ (b) $\frac{1}{y^2}$ (c) $\frac{1}{xy}$ (d) $\frac{1}{x^2y^2}$

Advance Level

215. The solution of the equation $\frac{dy}{dx} + y \tan x = x^m \cos x$ is

(a) $(m+1)y = x^{m+1} \cos x + c(m+1)\cos x$ (b) $my = (x^m + c)\cos x$

(c) $y = (x^{m+1} + c)\cos x$ (d) None of these

216. An integrating factor for the differential equation $(1+y^2)dx - (\tan^{-1} y - x)dy = 0$, is **[MP PET 1993]**

(a) $\tan^{-1} y$ (b) $e^{\tan^{-1} y}$ (c) $\frac{1}{1+y^2}$ (d) $\frac{1}{x(1+y^2)}$

217. The equation of the curve passing through the origin and satisfying the equation $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$ is

(a) $3(1+x^2)y = 4x^3$ (b) $3(1-x^2)y = 4x^3$ (c) $3(1+x^2) = x^3$ (d) None of these

218. The solution of the equation $\frac{dy}{dx}=\frac{1}{x+y+1}$ is

(a) $x=ce^y-y-2$ (b) $y=x+ce^y-2$ (c) $x+ce^y-y-2=0$ (d) None of these

219. The solution of the differential equation $\frac{dy}{dx}+y\cot x=2\cos x$ is

(a) $y\sin x+\cos 2x=2c$ (b) $2y\sin x+\cos x=c$ (c) $y\sin x+\cos x=c$ (d) $2y\sin x+\cos 2x=c$

220. The solution of the equation $(x+2y^3)\frac{dy}{dx}-y=0$ is (where A is any arbitrary constant) **[MP PET 1998, 2002]**

(a) $y(1-xy)=Ax$ (b) $y^3-x=Ay$ (c) $x(1-xy)=Ay$ (d) $x(1+xy)=Ay$

221. Solution of the differential equation $y'=y\tan x-2\sin x$, is **[AMU 1999]**

(a) $y=\tan x+2C\cos x$ (b) $y=\tan x+C\cos x$ (c) $y=\tan x-2C\cos x$ (d) None of these

222. The solution of $\frac{dv}{dt}+\frac{k}{m}v=-g$ is

(a) $v=ce^{-\frac{k}{m}t}-\frac{mg}{k}$ (b) $v=c-\frac{mg}{k}e^{-\frac{k}{m}}$ (c) $ve^{-\frac{k}{m}}=c-\frac{mg}{k}$ (d) $ve^{\frac{k}{m}t}=c-\frac{mg}{k}$

223. Integrating factor of differential equation $\cos x\frac{dy}{dx}+y\sin x=1$ is **[MP PET 1996]**

(a) $\cos x$ (b) $\tan x$ (c) $\sec x$ (d) $\sin x$

224. Solution of differential equation $\frac{dy}{dx}+ay=e^{mx}$ is **[MP PET 1996]**

(a) $(a+m)y=e^{mx}+C$ (b) $ye^{ax}=me^{mx}+C$

(c) $y=e^{mx}+Ce^{-ax}$ (d) $(a+m)y=e^{mx}+Ce^{-ax}(a+m)$

225. The solution of $\frac{dy}{dx}+P(x)y=0$ is **[Kerala (Engg.) 2002]**

(a) $y=ce^{\int Pdx}$ (b) $y=ce^{-\int Pdy}$ (c) $y=ce^{-\int Pdx}$ (d) $y=ce^{\int Pdy}$

226. The solution of $\frac{dy}{dx}+y=e^{-x}$, $y(0)=0$ **[Kerala (Enggg.) 2002]**

(a) $y=e^{-x}(x-1)$ (b) $y=xe^{x}$ (c) $y=xe^{-x}+1$ (d) $y=xe^{-x}$

227. Solution of the differential equation $\frac{dy}{dx}+y\sec^2 x=\tan x\sec^2 x$ is **[DCE 2001]**

(a) $y=\tan x-1+ce^{-\tan x}$ (b) $y^2=\tan x-1+ce^{-\tan x}$ (c) $ye^{\tan x}=\tan x-1+c$ (d) $ye^{-\tan x}=\tan x-1+c$

228. An integrating factor of the differential equation $\frac{dy}{dx}+\frac{2xy}{1-x^2}=\frac{x}{\sqrt{1-x^2}}$ is **[AMU 1999]**

(a) $(1+x^2)^{-1}$ (b) $(1-x^2)^{-1}$ (c) $x/(1-x^2)$ (d) $x/\sqrt{1-x^2}$

229. The solution of $\left(\frac{dy}{dx}\right)\cdot(x^2y^3+xy)=1$

(a) $\frac{1}{x}=-y^2+2-ce^{y^2/2}$ (b) $\frac{1}{x}=y^3+2-ce^{-y^2/2}$ (c) $\frac{1}{x}=-y^2+2+ce^{y^2/2}$ (d) None of these

230. An integrating factor of the differential equation $(1-x^2)\frac{dy}{dx}-xy=1$, is **[MP PET 2001]**

(a) $-x$ (b) $-\frac{x}{(1-x^2)}$ (c) $\sqrt{(1-x^2)}$ (d) $\frac{1}{2}\log(1-x^2)$

231. If $y(t)$ is a solution of $(1+t)\frac{dy}{dt}-ty=1$ and $y(0)=-1$, then $y(1)$ is equal to **[IIT Screening 2003]**

(a) $-\frac{1}{2}$ (b) $e+\frac{1}{2}$ (c) $e-\frac{1}{2}$ (d) $\frac{1}{2}$

232. Integrating factor of $\frac{dy}{dx}+\frac{y}{x}=x^3-3$ is **[MP PET 1999]**

(a) x (b) $\log x$ (c) $-x$ (d) e^x

233. Solution of $\cos x\frac{dy}{dx}+y\sin x=1$ is **[MP PET 1999]**

(a) $y\sec x\tan x=C$ (b) $y\sec x=\tan x+C$ (c) $y\tan x=\sec x+C$ (d) $y\tan x=\sec x\tan x+C$

234. If integrating factor of $x(1-x^2)dy+(2x^2y-y-ax^3)dx=0$ is $e^{\int P\,dx}$, then P is equal to **[MP PET 1999]**

(a) $\frac{2x^2-ax^3}{x(1-x^2)}$ (b) $(2x^2-1)$ (c) $\frac{2x^2-1}{ax^3}$ (d) $\frac{(2x^2-1)}{x(1-x^2)}$

235. If $y=f(x)$ passing through (1, 2) satisfies the differential equation $y(1+xy)dx-x\,dy=0$, then

(a) $f(x)=\frac{2x}{2-x^2}$ (b) $f(x)=\frac{x+1}{x^2+1}$ (c) $f(x)=\frac{x-1}{4-x^2}$ (d) $f(x)=\frac{4x}{1-2x^2}$

236. Solution of the equation $x(dy/dx)+2y=x^2\log x$ is

(a) $16yx^2=x^4\log(x^4/e)+c$ (b) $yx^2=\frac{1}{4}x^4\log x-\frac{1}{16}x^4+c$

(c) $16yx^2=4x^4\log x-x^4+c$ (d) None of these

237. Solution of the differential equation $x\cos x\left(\frac{dy}{dx}\right)+y(x\sin x+\cos x)=1$ is

(a) $xy=\sin x+c\cos x$ (b) $xy\sec x=\tan x+c$ (c) $xy+\sin x+c\cos x=0$ (d) None of these

238. The solution of the equation $\frac{dy}{dx}-3y=\sin 2x$ is

(a) $ye^{-3x}=-\frac{1}{13}e^{-3x}(2\cos 2x+3\sin 2x)+c$ (b) $y=-\frac{1}{13}(2\cos 2x+3\sin 2x)+ce^{3x}$

(c) $y=\{-1/\sqrt{13}\}\cos(2x-\tan^{-1}(3/2))+ce^{3x}$ (d) $y=\{-1/\sqrt{13}\}\sin(2x-\tan^{-1}(2/3))+ce^{3x}$

239. Solution of the equation $\frac{dy}{dx}+\frac{1}{x}\tan y=\frac{1}{x^2}\tan y\sin y$ is

(a) $2x=\sin y(1+2cx^2)$ (b) $2x=\sin y(1+cx^2)$ (c) $2x+\sin y(1+cx^2)=0$ (d) None of these

240. Solution of the differential equation $(1+y^2)dx=(\tan^{-1}y-x)dy$ is

(a) $xe^{\tan^{-1}y}=(1-\tan^{-1}y)e^{\tan^{-1}y}+c$ (b) $xe^{\tan^{-1}y}=(\tan^{-1}y-1)e^{\tan^{-1}y}+c$

(c) $x=\tan^{-1}y-1+ce^{-\tan^{-1}y}$ (d) None of these

Application of Differential Equation

Basic Level

241. The equation of the curve which passes through the point (1, 1) and whose slope is given by $\frac{2y}{x}$, is **[Roorkee 1987]**

(a) $y = x^2$ (b) $x^2 - y^2 = 0$ (c) $2x^2 + y^2 = 3$ (d) None of these

242. Equation of curve through point (1, 0) which satisfies the differential equation $(1 + y^2)dx - xy\,dy = 0$, is **[JEE West Bengal 1986]**

(a) $x^2 + y^2 = 1$ (b) $x^2 - y^2 = 1$ (c) $2x^2 + y^2 = 2$ (d) None of these

243. Equation of curve passing through (3, 9) which satisfies the differential equation $\frac{dy}{dx} = x + \frac{1}{x^2}$, is **[JEE West Bengal 1990]**

(a) $6xy = 3x^2 - 6x + 29$ (b) $6xy = 3x^3 - 29x + 6$ (c) $6xy = 3x^3 + 29x - 6$ (d) None of these

244. The equation of family of curves for which the length of the normal is equal to the radius vector is

(a) $y^2 \pm x^2 = k$ (b) $y \pm x = k$ (c) $y^2 = kx$ (d) None of these

245. The equation of a curve passing through $\left(2, \frac{7}{2}\right)$ and having gradient $1 - \frac{1}{x^2}$ at (x, y) is

(a) $y = x^2 + x + 1$ (b) $xy = x^2 + x + 1$ (c) $xy = x + 1$ (d) None of these

246. The equation of the curve through the point (1, 0) and whose slope is $\frac{y-1}{x^2 + x}$ is

(a) $(y-1)(x+1) + 2x = 0$ (b) $2x(y-1) + x + 1 = 0$ (c) $x(y-1)(x+1) + 2 = 0$ (d) None of these

247. The slope of a curve at any point is the reciprocal of twice the ordinate at the point and it passes through the point (4, 3). The equation of the curve is

(a) $x^2 = y + 5$ (b) $y^2 = x - 5$ (c) $y^2 = x + 5$ (d) $x^2 = y - 5$

248. Solution of differential equation $x\,dy - y\,dx = 0$ represents **[MP PET 1996]**

(a) Rectangular hyperbola (b) Straight line passing through origin

(c) Parabola whose vertex is at origin (d) Circle whose centre is at origin

249. The differential equation of the family of circles passing through the fixed points (a, 0) and ($-a$, 0) is

(a) $y_1(y^2 - x^2) + 2xy + a^2 = 0$ (b) $y_1 y^2 + xy + a^2x^2 = 0$

(c) $y_1(y^2 - x^2 + a^2) + 2xy = 0$ (d) None of these

Advance Level

250. If the gradient of the tangent at any point (x, y) of a curve which passes through the point $\left(1, \frac{\pi}{4}\right)$ is $\left\{\frac{y}{x} - \sin^2\left(\frac{y}{x}\right)\right\}$, then the equation of the curve is **[MP PET 1998]**

(a) $y = \cot^{-1}(\log_e x)$ (b) $y = \cot^{-1}\left(\log_e \frac{x}{e}\right)$ (c) $y = x\cot^{-1}(\log_e ex)$ (d) $y = \cot^{-1}\left(\log_e \frac{e}{x}\right)$

251. The differential equation of displacement of all "simple harmonic motions" of given period $\frac{2\pi}{n}$, is

(a) $\frac{d^2x}{dt^2} + nx = 0$ (b) $\frac{d^2x}{dt^2} + n^2x = 0$ (c) $\frac{d^2x}{dt^2} - n^2x = 0$ (d) $\frac{d^2x}{dt^2} + \frac{1}{n^2}x = 0$

252. A curve having the condition that the slope of tangent at some point is two times the slope of the straight line joining the same point to the origin of coordinates is a/an **[Orissa JEE 2003]**

(a) Circle (b) Ellipse (c) Parabola (d) Hyperbola

253. If rate of decrement of N with time is proportional to N, k being proportionality constant, the solution of the differential equation formed is

(a) $N = N_0 + e^{-kt}$ (b) $N = N_0 + e^{kt}$ (c) $N = N_0e^{kt}$ (d) $N = N_0e^{-kt}$

254. The family of curves represented by $\frac{dy}{dx}=\frac{x^2+x+1}{y^2+y+1}$ and the family represented by $\frac{dy}{dx}+\frac{y^2+y+1}{x^2+x+1}=0$

(a) Touch each other (b) Are orthogonal (c) Are one and the same (d) None of these

255. The equation of the curve whose subnormal is constant is

(a) $y=ax+b$ (b) $y^2=2ax+b$ (c) $ay^2-x^2=a$ (d) None of these

256. The curve for which the normal at any point (*x*, *y*) and the line joining origin to that point form an isosceles triangle with the *x*-axis as base is

(a) An ellipse (b) A rectangular hyperbola (c) A circle (d) None of these

257. The solution of $\frac{dy}{dx}=\frac{ax+h}{by+k}$ represents a parabola when

(a) $a=0, b=0$ (b) $a=1, b=2$ (c) $a=0, b\neq 0$ (d) $a=2, b=1$

258. The equation of the curve satisfying the differential equation $y_2(x^2+1)=2xy_1$ passing through the point (0, 1) and having slope of tangent at $x=0$ as 3 is

(a) $y=x^2+3x+2$ (b) $y^2=x^2+3x+1$ (c) $y=x^3+3x+1$ (d) None of these

259. If $\phi(x)=\phi'(x)$ and $\phi(1)=2$, then $\phi(3)$ equals

(a) e^2 (b) $2e^2$ (c) $3e^2$ (d) $2e^3$

260. If *f*(*x*), *g*(*x*) be twice differentiable functions on [0, 2] satisfying $f''(x)=g''(x)$, $f'(1)=2g'(1)=4$ and $f(2)=3, g(2)=9$, then $f(x)-g(x)$ at $x=4$ equals

(a) 0 (b) 10 (c) 8 (d) 2

261. The curve in which the slope of the tangent at any point equals the ratio of the abscissa to the ordinate of the point is

(a) An ellipse (b) A parabola (c) A rectangular hyperbola (d) A circle

262. A particle starts at the origin and moves along the *x*-axis in such a way that its velocity at the point (*x*, 0) is given by the formula $\frac{dx}{dt}=\cos^2\pi x$. Then the particle never reaches the point on **[AMU 2000]**

(a) $x=\frac{1}{4}$ (b) $x=\frac{3}{4}$ (c) $x=\frac{1}{2}$ (d) $x=1$

263. The slope of the tangent at (*x*, *y*) to a curve passing through a point (2, 1) is $\frac{x^2+y^2}{2xy}$ then the equation of the curve is **[MP PET 2002]**

(a) $2(x^2-y^2)=3x$ (b) $2(x^2-y^2)=6y$ (c) $x(x^2-y^2)=6$ (d) $x(x^2+y^2)=10$

264. Integral curve satisfying $y'=\frac{x^2+y^2}{x^2-y^2}, y(1)=2$ has the slope at the point (1, 0) of the curve equal to **[MP PET 2000]**

(a) – 5/3 (b) – 1 (c) 1 (d) 5/3

Miscellaneous Differential Equation

Basic Level

265. The solution of the differential equation $x\frac{d^2y}{dx^2}=1$, given that $y=1$, $\frac{dy}{dx}=0$ when $x=1$, is

(a) $y=x\log x+x+2$ (b) $y=x\log x-x+2$ (c) $y=x\log x+x$ (d) $y=x\log x-x$

266. the solution of the equation $\frac{d^2y}{dx^2}=-\frac{1}{x^2}$ is **[MP PET 2003]**

(a) $y=\log x+c_1x+c_2$ (b) $y=-\log x+c_1x+c_2$ (c) $y=\frac{-1}{x}+c_1x+c_2$ (d) None of these

267. The solution of the differential equation $\cos^2 x\frac{d^2y}{dx^2}=1$ is

(a) $y=\log\cos x+cx$ (b) $y=\log\sec x+c_1x+c_2$ (c) $y=\log\sec x-c_1x+c_2$ (d) None of these

268. The solution of $y'-y=1$, $y(0)=-1$ is given by $y(x)=$ **[MP PET 2000]**

(a) $-\exp(x)$ (b) $-\exp(-x)$ (c) -1 (d) $\exp(x)-2$

269. The number of solutions of $y'=\frac{y+1}{x-1}, y(1)=2$ is **[MP PET 2000]**

(a) None (b) One (c) Two (d) Infinite

270. The solution of $y'=1+x+y^2+xy^2, y(0)=0$ is **[MP PET 2000]**

(a) $y^2=\exp\left(x+\frac{x^2}{2}\right)-1$ (b) $y^2=1+C\exp\left(x+\frac{x^2}{2}\right)$ (c) $y=\tan(C+x+x^2)$ (d) $y=\tan\left(x+\frac{x^2}{2}\right)$

271. $\frac{d^2y}{dx^2}=0$, then **[UPSEAT 1999]**

(a) $y=ax+b$ (b) $y^2=ax+b$ (c) $y=\log x$ (d) $y=e^x+C$

Advance Level

272. The solution of the equation $\frac{d^2y}{dx^2}=e^{-2x}$ **[AIEEE 2002]**

(a) $\frac{1}{4}e^{-2x}=y$ (b) $\frac{1}{4}e^{-2x}+cx+d=y$ (c) $\frac{1}{4}e^{-2x}+cx^2+d=y$ (d) $\frac{1}{4}e^{-2x}+c+d=y$

273. If $x^2+y^2=1$ then $\left(y'=\frac{dy}{dx}, y''=\frac{d^2y}{dx^2}\right)$ **[IIT Screening 2000]**

(a) $yy''-2(y')^2+1=0$ (b) $yy''+(y')^2+1=0$ (c) $yy''-(y')^2-1=0$ (d) $yy''+2(y')^2+1=0$

274. If $\frac{d^2y}{dx^2}+\sin x=0$, then the solution of the differential equation is **[Karnataka CET 2000]**

(a) $\sin x$ (b) $\cos x$ (c) $\tan x$ (d) $\log\sin x$

275. If $y^2=ax^2+bx+c$, then $y^3\frac{d^2y}{dx^2}$ is **[DCE 1999]**

(a) A constant (b) A function of x only (c) A function of y only (d) A function of x and y

276. If $\frac{dy}{dx}=e^{-2y}$ and $y=0$ when $x=5$, then value of x for $y=3$ is **[MP PET 2001]**

(a) e^5 (b) e^6+1 (c) $\frac{e^6+9}{2}$ (d) $\log_e 6$

277. The solution of the differential equation $y_1y_3=3y_2^2$ is

(a) $x=A_1y^2+A_2y+A_3$ (b) $x=A_1y+A_2$ (c) $x=A_1y^2+A_2y$ (d) None of these

278. Solution of the differential equation $\sin\frac{dy}{dx} = a$ with $y(0) = 1$ is **[Kurukshetra CEE 1998]**

(a) $\sin^{-1}\frac{(y-1)}{x} = a$ (b) $\sin\frac{(y-1)}{x} = a$ (c) $\sin\frac{(1-y)}{(1+x)} = a$ (d) $\sin\frac{y}{(x+1)} = a$

279. If $y = ax^{n+1} + bx^{-n}$, then $x^2\frac{d^2y}{dx^2}$ equals to **[Rajasthan PET 2001]**

(a) $n(n-1)y$ (b) $n(n+1)y$ (c) ny (d) n^2y

280. The solution of $\frac{d^2y}{dx^2} = \cos x - \sin x$ is

(a) $y = -\cos x + \sin x + c_1x + c_2$ (b) $y = -\cos x - \sin x + c_1x + c_2$

(c) $y = \cos x - \sin x + c_1x^2 + c_2x$ (d) $y = \cos x + \sin x + c_1x^2 + c_2x$

281. The solution of $\frac{d^2y}{dx^2} = \sec^2 x + xe^x$ is **[DSSE 1985]**

(a) $y = \log(\sec x) + (x-2)e^x + c_1x + c_2$ (b) $y = \log(\sec x) + (x+2)e^x + c_1x + c_2$

(c) $y = \log(\sec x) - (x+2)e^x + c_1x + c_2$ (d) None of these

282. The general solution of the differential equation $\frac{dy}{dx} + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right)$ is **[MP PET 2001]**

(a) $\log\tan\left(\frac{y}{2}\right) = c - 2\sin x$ (b) $\log\tan\left(\frac{y}{4}\right) = c - 2\sin\left(\frac{x}{2}\right)$

(c) $\log\tan\left(\frac{y}{2} + \frac{\pi}{4}\right) = c - 2\sin x$ (d) $\log\tan\left(\frac{y}{4} + \frac{\pi}{4}\right) = c - 2\sin\left(\frac{x}{2}\right)$

283. A solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$ is **[IIT 1999; Karnataka CET 2002]**

(a) $y = 2$ (b) $y = 2x$ (c) $y = 2x - 4$ (d) $y = 2x^2 - 4$

284. If $\phi(x) = \int \{\phi(x)\}^{-2}dx$ and $\phi(1) = 0$ then $\phi(x) =$

(a) $\{2(x-1)\}^{1/4}$ (b) $\{5(x-2)\}^{1/5}$ (c) $\{3(x-1)\}^{1/3}$ (d) None of these

285. Solution of the differential equation $\sin y\frac{dy}{dx} = \cos y(1 - x\cos y)$ is

(a) $\sec y = x - 1 - ce^x$ (b) $\sec y = x + 1 + ce^x$ (c) $y = x + e^x + c$ (d) None of these

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Answer Sheet

Assignment (Basic and Advance Level)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b	c	a	a	d	d	a	c	b	d	a	c	d	d	d	b	d	a	b	c
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
a	a	a	d	a	a	c	a	a	c,d	c	a	c	c	b	a	c	b	c	c
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
a	c	c	a	b	a	a	a	a	a	c	d	d	b	c	c	b	c	d	b
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
a	c	a	a	b	b	b	a	c	b	c	a	b	a	c	d	c	b	a	a
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
a	c	a	c	a,c	a	a	a	c	a	a	b	a	b	a	b	b	a	b	a
101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
c	a	c	d	c	c	b	a	a	a	c	d	b	b	d	d	b	c	a	a
121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
a	c	d	b	a	a	b	d	c	b	a,b,c	b	a	d	b	c	a	a	c	a
141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160
a	a	a	d	d	c	a	b	b	c	c	a	b	b	c	d	b	c	b	d
161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
d	b	a	a	b	c	a	d	a	a	b	c	b	a	a	d	a	a	a	a
181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200
c	a	a	b	b	b	a	b	a	b	a	b	a	c	c	c	b	d	b	b
201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220
a	d	b	a	b	c	c	a	b	a	c	c	a	b	a	b	a	a	d	b
221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240
d	a	c	d	c	d	a	b	a	c	a	a	b	d	a	a,b,c	a,b	a,b,c,d	a,b	b,c
241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260
a	b	c	a	b	a	c	b	c	c	b	c	d	b	b	b	c	c	b	b
261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280
c	c	a	c	b	a	b,c	c	a	d	a	b	b	a	a	c	a	b	b	a
281	282	283	284	285															
a	c	c	c	b															