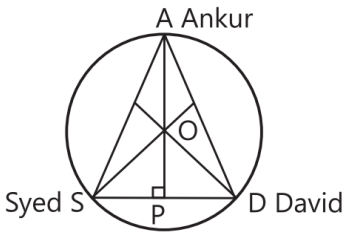
Circles

Case Study Based Questions

Case Study 1

A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are standing at equal distance on its boundary each having a toy telephone in his hands to talk each other.

In mathematically; A, S and D are the position of Ankur, Syed and David are sitting at equal distance and O be the position of the centroid of a triangle.



On the basis of the above information, solve the following questions:

Q 1. The length of AP is:

a. 20 m b. 30 m

c. 15 m d. 25 m

Q 2. The distance between any two boys is:

a. $20\sqrt{3}$ m b. $10\sqrt{3}$ m

c. $15\sqrt{3}$ m d. 20 m

Q 3. The angle between AS and SD is:

a. 65° b. 60° c. 70° d. 75°

Q 4. $\angle$SOD is equal to:

a. 70° b. 60° c. 120° d. 80°

Q 5. $\angle$OSP is equal to:

a. 50° b. 60° c. 30° d. 70°

Solutions

1. (b) Let Ankur, Syed and David be represented by A, S and D respectively.

Let $AS = SD = AD = 2a$ m

Since, O is the centroid, so it divides a median in the ratio 2 : 1.

So, $\frac{AO}{OP} = \frac{2}{1}$

$\Rightarrow \frac{20}{OP} = \frac{2}{1}$ [$\because$ AO = Radius = 20 m]

$\Rightarrow OP = 10$ m

$\therefore AP = AO + OP = 20 + 10 = 30$ m

So, option (b) is correct.

2. (a) In right angled ΔAPS, by Pythagoras theorem,

$AS^2 = AP^2 + PS^2$

$\Rightarrow (2a)^2 = (30)^2 + a^2 \Rightarrow 4a^2 - a^2 = 900$

$\Rightarrow 3a^2 = 900 \Rightarrow a^2 = 300$

$\Rightarrow a = 10\sqrt{3}$ m

So, $2a = 2 \times 10\sqrt{3} = 20\sqrt{3}$ m

Hence, the distance between any two boys is $20\sqrt{3}$ m.

So, option (a) is correct.

3. (b) Since, all three boys are standing at equal distance. Therefore, Δ ASD is an equilateral triangle.

So, angle between AS and SD is 60°.

So, option (b) is correct.

4. (c) As we know that, the intersection point of altitudes is at the centre of circumcentre of circle. The angle subtended by an arc at the centre is double the angle subtended by it any point on the remaining part of the circle.

$\therefore \angle SOD = 2 \angle SAD$

$= 2 \times 60°$

[$\because \angle SAD = \angle ASD = \angle ADS = 60°$]

$= 120°$

So, option (c) is correct.

5. (c) Since, $\angle SOD = 120°$

$\therefore \quad \angle SOP = \frac{1}{2} \times 120° = 60°$ [$\because$ AP is a median]

Now, In ΔOSP,

$\angle SOP + \angle OSP + \angle OPS = 180°$

$\Rightarrow \quad 60° + \angle OSP + 90° = 180°$ [$\because$ AP $\perp$ SD]

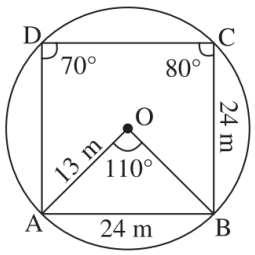
$\Rightarrow \quad \angle OSP = 180° - 150° = 30°$

So, option (c) is correct.

Case Study 2

I have a circular garden in the outside of the city. Municipality decided to put some benches, so that the people can sit there and can have some fresh air.

In the given figure, there are two benches of same colour A and B, which are placed at a distance AB = 24 m. Similarly, two other benches of same colour C and D are also placed at a same distance of 24 m from each other.



On the basis of the above information, solve the following questions:

Q 1. The measure of $\angle BOC$ is:

a. 120° b. 55° c. 110° d. 60°

Q 2. The measure of $\angle ABC$ is:

a. 130° b. 120° c. 105° d. 110°

Q 3. The perpendicular distance from centre O to the chord AB is:

a. 6 m b. 5 m c. 10 m d. 12 m

Q 4. The value of angle ∠BAO is:

a. 60° b. 35° c. 70° d. 80°

Q 5. The measure of ∠DAO is:

a. 70° b. 65° c. 80° d. 85°

Solutions

1. (c) Given, ∠AOB = 110°

In the given figure, AB and BC are of equal chords. Therefore, they subtend equal angles at the centre.

$\therefore \quad \angle BOC = \angle AOB = 110°$

So, option (c) is correct.

2. (d) Since, points A, B, C and D lie on the circle. Therefore, it forms a cyclic quadrilateral.

$\Rightarrow \quad \angle ABC + \angle ADC = 180°$

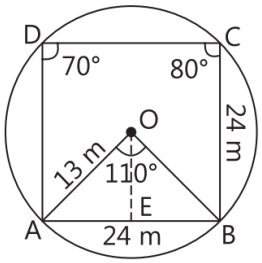
$\Rightarrow \quad \angle ABC + 70° = 180°$

$\therefore \quad \angle ABC = 110°$

So, option (d) is correct.

3. (b) Draw a perpendicular line from centre O to the chord AB, which bisects the chord.

$\therefore \quad AE = BE = 12 \text{ m}$



In right angle ΔOEA at E, use Pythagoras theorem

$$OE = \sqrt{(OA)^2 - (AE)^2}$$

$$= \sqrt{(13)^2 - (12)^2} = \sqrt{169 - 144}$$

$$= \sqrt{25} = 5 \text{ m}$$

So, option (b) is correct.

4. (b) In ΔAOB,

$OA = OB$ [Radii of a circle]

$\Rightarrow \quad \angle ABO = \angle BAO$

[Angles opposite to equal sides are equal]

Use angle, sum property of a ΔAOB,

$\angle ABO + \angle BAO + \angle AOB = 180°$

$\Rightarrow \quad \angle BAO + \angle BAO + 110° = 180°$

$\Rightarrow \quad 2\angle BAO = 70°$

$\therefore \quad \angle BAO = 35°$

So, option (b) is correct.

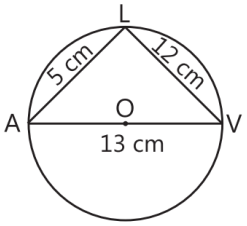
5. (b) Since, A and C are opposite points of a cyclic quadrilateral. Therefore

$$\angle BAD + \angle DCB = 180°$$
$$\Rightarrow \quad \angle BAD = 180° - 80° = 100°$$
$$\Rightarrow \quad \angle BAO + \angle DAO = 100°$$
$$\Rightarrow \quad 35° + \angle DAO = 100°$$
$$\Rightarrow \quad \angle DAO = 65°$$

So, option (b) is correct.

Case Study 3

Three boys Anshul, Vivek and Lalit are playing a game by standing on the boundary of a of a circle of diameter 13 cm with centre O. Anshul throws a ball to Vivek, Vivek to Lalit and Lalit to Anshul and so on. The distance between Anshul and Lalit is 5 cm and between Vivek and Lalit is 12 cm.



On the basis of the given information, solve the following questions:

Q 1. Measure $\angle$ ALV:

a. 60° b. 90° c. 50° d. 40°

Q 2. The length of the longest chord is:

a. 12 cm b. 5 cm c. 13 cm d. 16 cm

Q 3. If $\angle$ LAV = 50°, then $\angle$ AVL is:

a. 140° b. 50° c. 40° d. 100°

Q 4. The area covered by these three persons is:

a. 20 cm^2 b. 30 cm^2 c. 25 cm^2 d. 35 cm^2

Q 5. Equal chords of a circle subtends equal angle at:

a. centre
b. end point of diameter
c. other than centre
d. None of the above

Solutions

1. (b) Given, AV is a diameter of a circle. So, angle subtended by diameter to the circumference of the circle is 90°.

$$\therefore \quad \angle ALV = 90°$$

So, option (b) is correct.

2. (c) The length of the longest chord in a circle is equal to the length of diameter.

$\therefore$ Length of longest chord = length of diameter

= 13 cm

So, option (c) is correct.

3. (c) In Δ AVL,

$\angle LAV + \angle AVL + \angle ALV = 180°$

[by angle sum property of a triangle]

$\Rightarrow \quad 50° + \angle AVL + 90° = 180° \qquad [\because \angle ALV = 90°]$

$\therefore \qquad \angle AVL = 40°$

So, option (c) is correct.

4. (b) The area covered by these three persons is equal to the area of Δ AVL.

$\therefore$ Area of Δ AVL $= \frac{1}{2} \times AL \times LV$

$= \frac{1}{2} \times 5 \times 12 = 30 \text{ cm}^2$

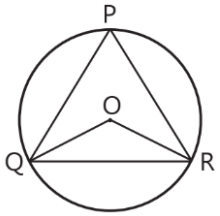
So, option (b) is correct.

5. (a) Equal chords of a circle subtends equal angle at the centre.

So, option (a) is correct.

Case Study 4

Government of India is working regularly for the growth of handicapped persons. For these three STD booths situated at point P, Q and R are as shown in the figure, which are operated by handicapped persons. These three booths are equidistant from each other as shown in the figure.



On the basis of the above information, solve the following questions:

Q1. Which type of ΔPQR is in the given figure?

Q2. Measure $\angle QOR$.

Q3. Find the value of ∠OQR.

Q4. Is it true that points P, Q, and R lie on the circle?

Solutions

1. Given P, Q and R are equidistant. It means their distances are equal.

So, ΔPQR is an equilateral triangle.

2. Since, Δ PQR is an equilateral triangle.

$\therefore \quad \angle PQR = \angle PRQ = \angle QPR = 60°$

The angle subtended by an arc at the centre is double the angle subtended by it any point on the remaining part of the circle.

$\therefore \angle QOR = 2\angle QPR = 2 \times 60° = 120°$

3. In Δ OQR,

$OQ = OR$ [Radii of a circle)

$\Rightarrow \quad \angle ORQ = \angle OQR$ [Angles opposite to equal sides of a triangle are equal]

Using angle sum property of a triangle,

$\angle OQR + \angle ORQ + \angle QOR = 180°$

$\Rightarrow \quad \angle OQR + \angle OQR + 120° = 180°$

$\Rightarrow \quad 2\angle OQR = 60°$

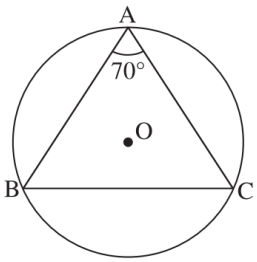
$\therefore \quad \angle OQR = 30°$

4. Yes, it is true that points P, Q and R lie on the circle.

Case Study 5

Narendra Modi Stadium also known as Motera Stadium is situated in Ahmedabad, Gujarat. Currently it is the largest stadium in the world with a capacity of 1,32,000 viewers. In the field there are 2 ends known as bowler end and striker end.

Geometrically, O is the centre of a circle, ∠ BAC = 70°, which is shown in figure.



On the basis of the above information, solve the following questions:

Q1. Find the angle subtended by the chord BC at the centre.

Q2. Find ∠OBC + ∠OCB.

Q3. If OB = 5 cm and BC = 6 cm, then find the distance from centre O to the chord BC.

Q4. If point D lies on the circle between B and C, then find ∠BDC.

Solutions

1. Given, $\angle BAC = 70°$

The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$\therefore \quad \angle BOC = 2\angle BAC$

$= 2 \times 70° = 140°$

2. In Δ OBC, use angle sum property of a triangle,

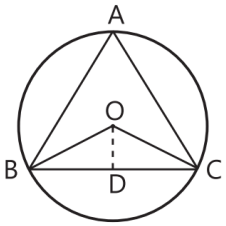
$\angle OBC + \angle OCB + \angle BOC = 180°$

$\Rightarrow \quad \angle OBC + \angle OCB + 140° = 180°$

$\Rightarrow \quad \angle OBC + \angle OCB = 40°$

3. As we know that, perpendicular drawn from centre O of circle to the chord BC bisects the chord.

$\therefore$ BD $= \frac{1}{2}$ BC $= \frac{1}{2} \times 6 = 3$ cm

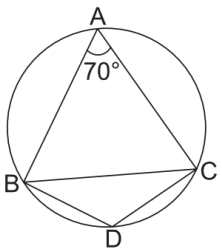


In right angled Δ ODB, use Pythagoras theorem

$OD = \sqrt{(OB)^2 - (BD)^2}$

$= \sqrt{(5)^2 - (3)^2} = \sqrt{25 - 9} = \sqrt{16} = 4$ cm

4. Here, we see that four points A, B, C and D lie on a circle, so it forms a cyclic quadrilateral.



$\therefore \quad \angle BAC + \angle BDC = 180°$

$\Rightarrow \quad 70° + \angle BDC = 180°$

$\therefore \quad \angle BDC = 110°$