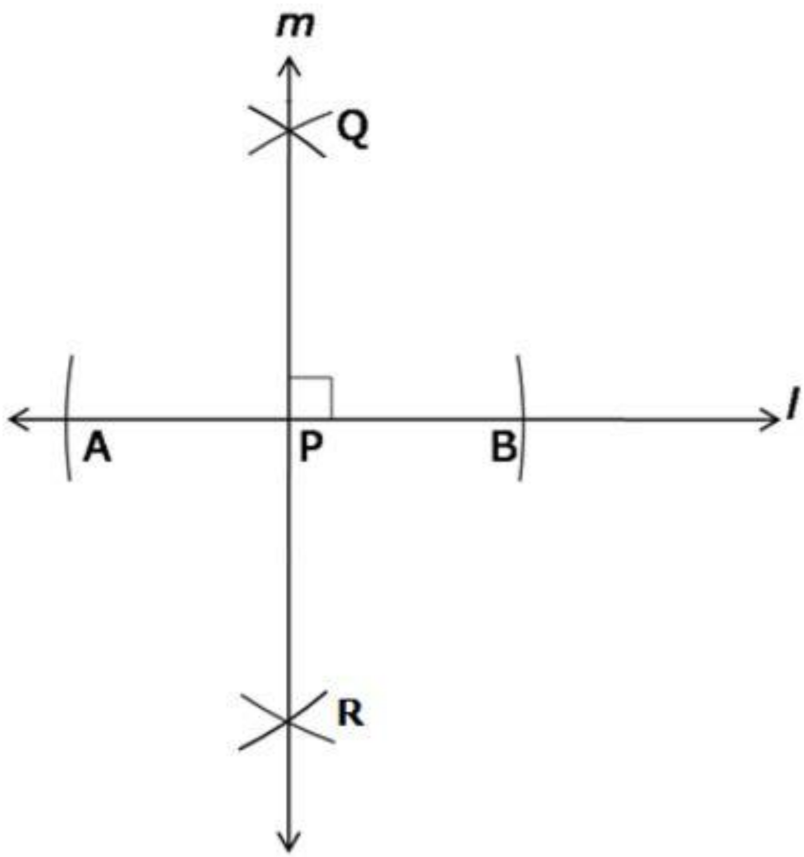
Geometrical Constructions

Exercise – 7.1

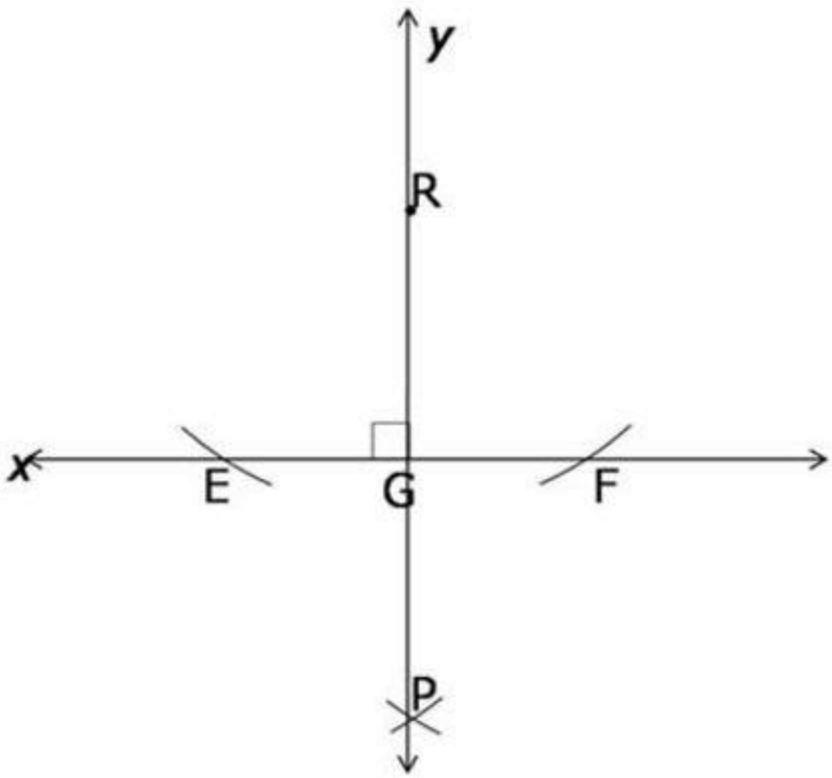
Solution 1:



Steps of construction:

1. Draw line l, take any point P on it.
2. Taking P as the centre and any radius, draw two arcs of circle on line l.
3. Name the points of intersection of line l and arcs as A and B.
4. Taking A and B as the centres and radius more than half of AB, draw arcs of circle above and below the line l.
5. Mark the points of intersection as Q and R.
6. Draw line $m \perp$ line l by joining the points Q and R
7. Hence, the required line $m \perp$ line l has been constructed.

Solution 2:

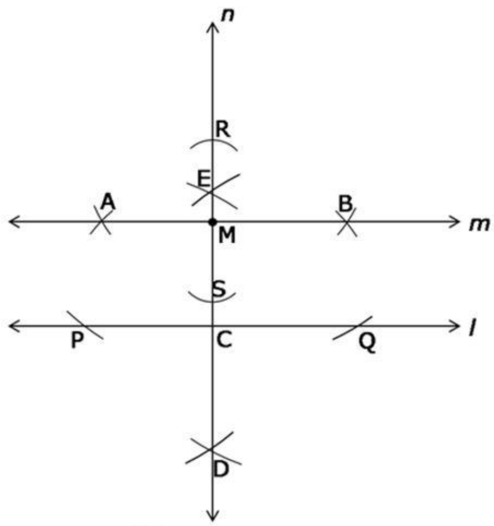


Steps of construction:

1. Draw line *x*. Take a point R outside the line.
2. Taking R as the centre, draw two arcs of circle cutting line x at point E and F.
3. Taking E and F as centres and the radius more than half of EF, draw two arcs of circle intersecting each other below the line *x* and opposite to point R.
4. Name the point of intersection as P.
5. Draw the line *y* passing through points P and R.

Hence, line $y \perp$ line x passing through point R, has been constructed.

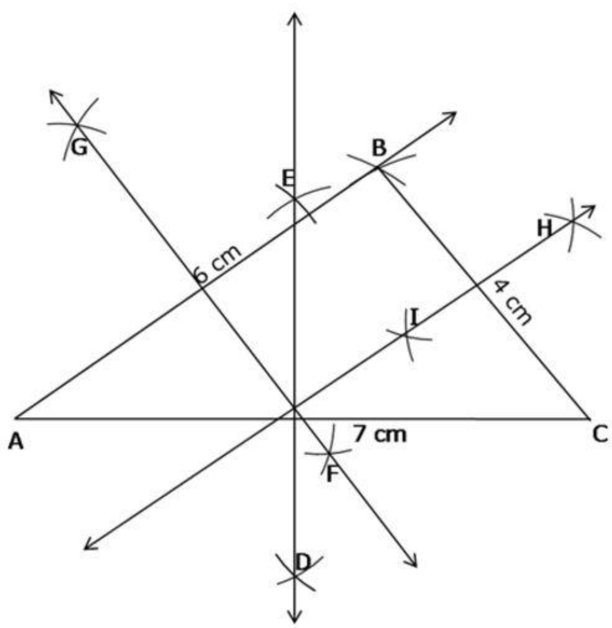
Solution 3:



Steps of construction:

1. Draw line l, take any point M outside it.
2. Taking M as the centre and any radius, draw two arcs of circle on line l.
3. Name the points of intersection of the arc and line l as P and Q.
4. Taking P and Q as the centres, and radius more than half of PQ, draw arcs of circle above and below the line l.
5. Name the points of intersection as E and D.
6. Draw line $n \perp$ line l by joining the points E and D.
7. Taking M as the centre and any radius, draw arcs of circle above and below point M on line n. Name the points of intersection as R and S.
8. Taking R and S as centres and radius more than half of RS, draw arcs of circle to intersect each other on both the sides of the line n at points A and B.
9. Draw line $m \perp$ line n by joining A and B.
   Line $l \perp$ line n, line $m \perp$ line n, Hence line m and line l are parallel to each other
10. Hence, the required line $m \parallel$ line l has been constructed.

Solution 4:

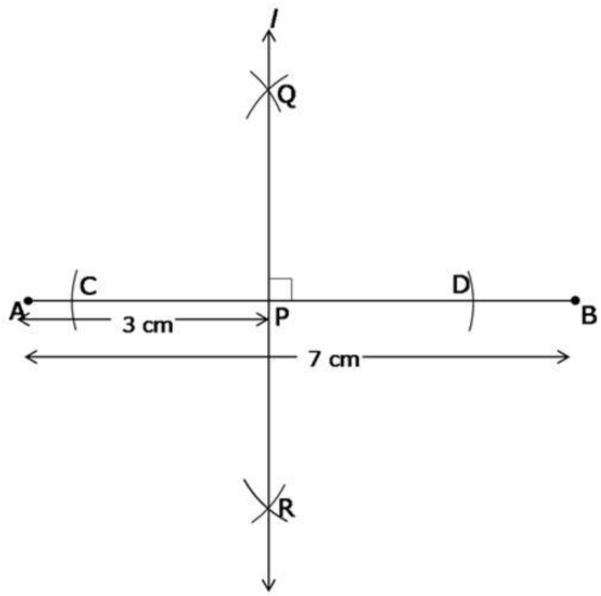


Steps of construction:

1. Draw seg AC = 7 cm.
2. Taking A as the centre, and radius = 6 cm, draw an arc of circle above the seg AC.
3. Taking C as the centre, and radius = 4 cm, draw an arc of circle above the seg AC intersecting the previous arc.
4. Mark the point of intersection as B.
5. Construct ΔABC by joining A and B, B and C.
6. Taking A and C as the centres, and radius greater than half of AC, draw arcs of circle above and below $\overline{AC}$ intersecting each other at E and D.
7. Draw perpendicular bisectors of $\overline{BC}$ and $\overline{AB}$ in the same manner.

The perpendicular bisectors of $\overline{AC}$, $\overline{BC}$ and $\overline{AB}$ intersect at one point.

Solution 5:



Steps of construction:

1.Draw $\overline{AB}$ of length 7 cm, take a point P, such that AP = 3 cm.

2.Taking P as the centre, draw two arcs of circle on $\overline{AB}$ to intersect $\overline{AB}$ at points C and D.

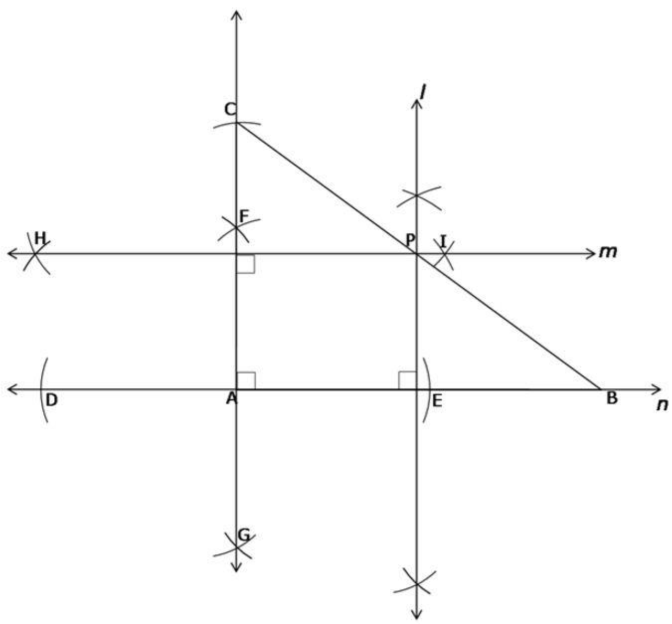
3.Taking C and D as the centres and radius greater than half of CD draw arcs of circle above and below $\overline{AB}$.

4. Mark the points of intersection as Q and R.

5. Draw line $l \perp \overline{AB}$ by joining the points Q and R.

Hence, l is the required line perpendicular to $\overline{AB}$ through point P.

Solution 6:



Steps of construction:

1. Draw line n and mark points A and B on it.
2. From point A taking any radius draw arcs of circle intersecting line n at points D and E.
3. Taking D and E as centres with radius greater than half of DE draw arcs of circle above and below the line n
4. Mark the points of intersection as F and G.
5. Join points F and G and extend the line $\overleftrightarrow{FG}$ on both the sides. $\overleftrightarrow{FG} \perp$ line n, hence $m\angle FAB = 90^\circ$.
6. Taking A as the centre with a required radius, Mark an arc on $\overleftrightarrow{FG}$ with a compass to intersect at C.
7. Join C and B to draw a right angled triangle ABC.
8. Taking A and C as the centres and radius more than half of AC, draw two arcs of circle on both the sides of $\overline{AC}$
9. Mark the points of intersection of the arcs as H and I.
10. Draw the perpendicular bisector line m by joining the points H and I.
11. Construct the perpendicular bisector line l of $\overline{AB}$ in the similar manner.
12. Mark the point of intersection of the bisectors as P.

Exercise – 7.2

Solution 1:

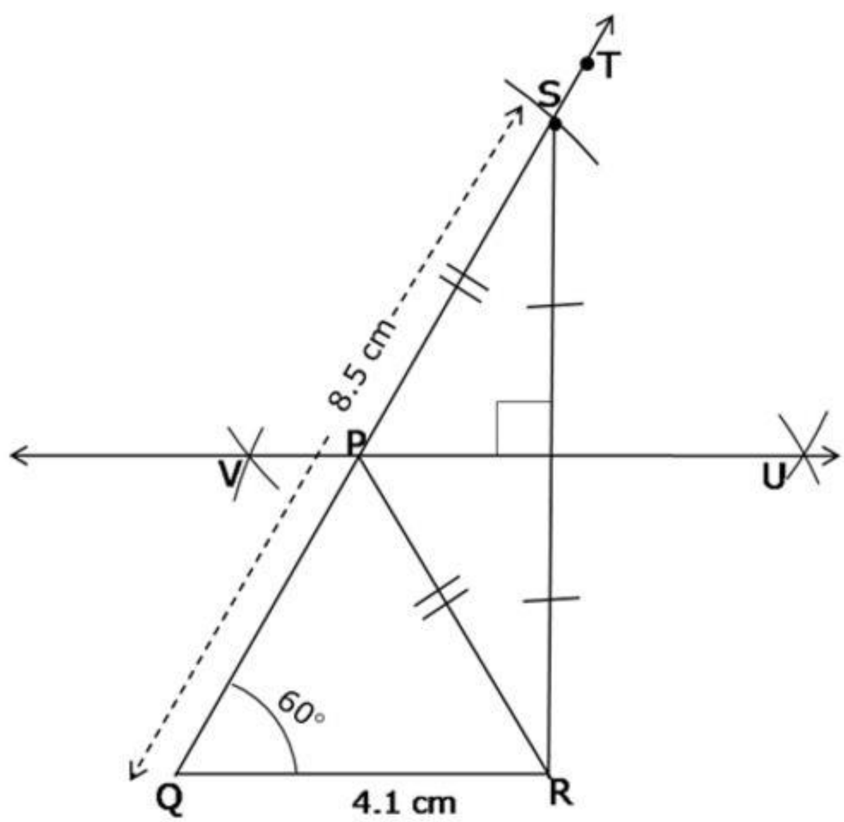
Steps of construction:

1. Draw $\overline{QR}$ of length 4.1 cm.
2. Draw $\overrightarrow{QT}$ such that $m\angle RQT = 60^\circ$
3. With centre Q and radius 8.5 cm draw an arc which intersects $\overrightarrow{QT}$ at point S.
4. Join R and S.
5. Keeping R and S as centres and radius greater than half of $\overline{RS}$, draw arcs of circle on both the sides of $\overline{RS}$.
6. Mark the points of intersection of arcs as V and U.
7. Join $\overline{UV}$ which is the perpendicular bisector of $\overline{RS}$.
8. The perpendicular bisector of $\overline{RS}$ intersects QS at point P.
9. Now by Perpendicular Bisector Theorem, PS = PR

$QS = PQ + PS = 8.5$ cm

$\therefore PQ + PR = 8.5$ cm $(\because PS = PR)$

Hence, ΔPQR is the required triangle.



Solution 2:

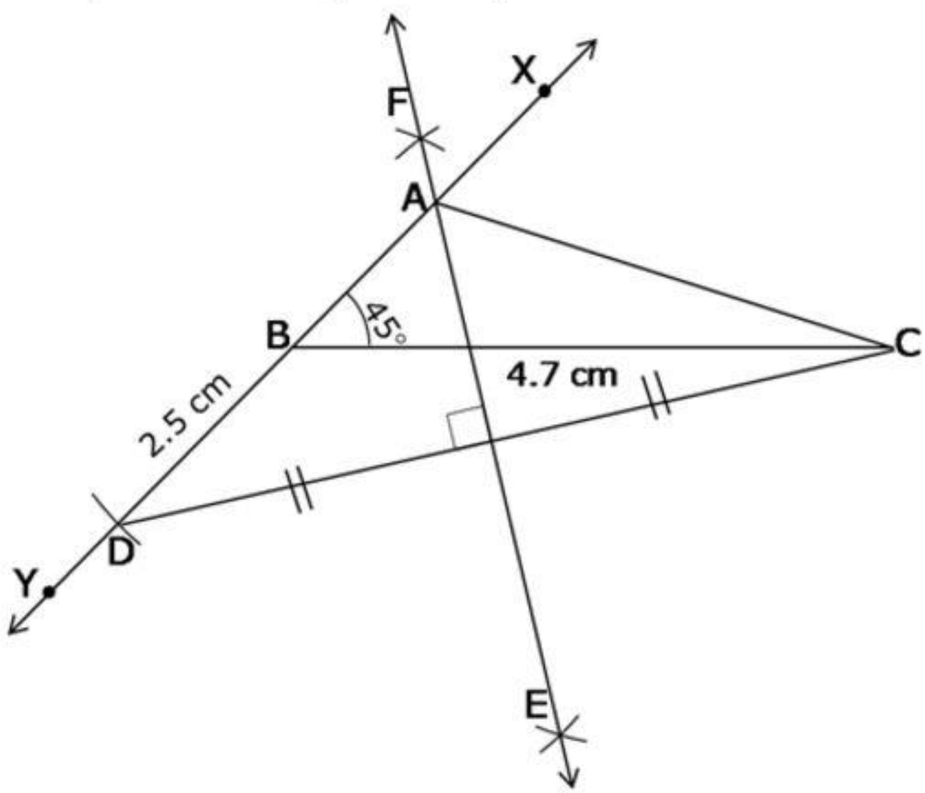
Steps of construction:

1. Draw $\overline{BC}$ of length 4.7 cm.
2. Draw $\overrightarrow{BX}$, passing through point B, such that m∠CBX = 45°
3. Draw a ray $\overrightarrow{BY}$ opposite to $\overrightarrow{BX}$
4. With centre B and radius 2.5 cm, draw an arc which intersects $\overleftrightarrow{XY}$ at point D.
5. Join D and C.
6. Keeping C and D as centres and radius greater than half of CD, draw arcs of circle above and below the line segment CD.
7. Mark the points of intersection of arcs as F and E.
8. Join $\overline{EF}$ which is the perpendicular bisector of $\overline{CD}$.
9. Mark the point of intersection of $\overline{EF}$ and $\overrightarrow{BX}$ as A
10. Join A and C.
11. Now by the Perpendicular Bisector Theorem, AD = AC

    BD = 2.5 cm

    $\therefore$ AC - AB = AD - AB = 2.5 cm ($\because$ AC = AD)

Hence, ΔABC is the required triangle.



Solution 3:

Steps of construction:

1.Draw $\overline{YZ}$ of length 7.4 cm.

2.From point Y draw $\overrightarrow{YR}$, such that m∠ZYR = 45°

3. ∵XY – XZ = 2.9 cm, XY > XZ

∴ With centre Y and radius 2.9 cm draw an arc which intersects $\overrightarrow{YR}$ at point W.

4. Join W and Z.

5. Draw the perpendicular bisector ST of $\overline{WZ}$.

6.Mark the point of intersection of $\overline{WZ}$ and $\overrightarrow{YR}$ as X.

7. Draw $\overline{XZ}$, by joining X and Z.

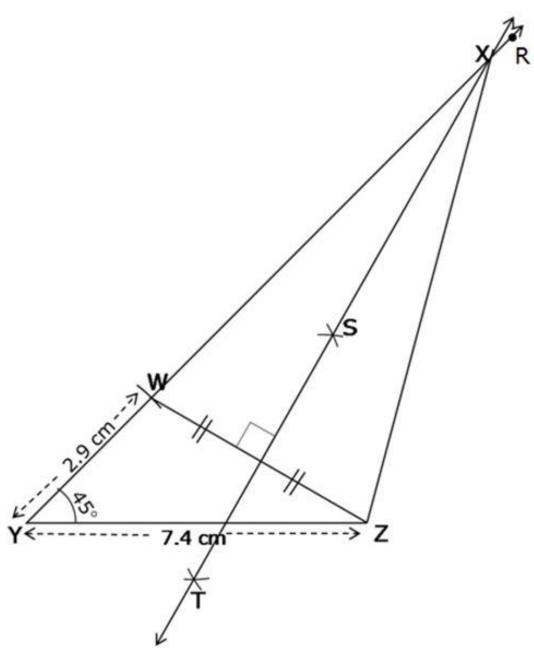
9. Now by the Perpendicular Bisector Theorem, XW = XZ

YW = 2.9 cm

∴ XY - XZ = XY - XW = 2.9 cm (∵ XW = XZ)

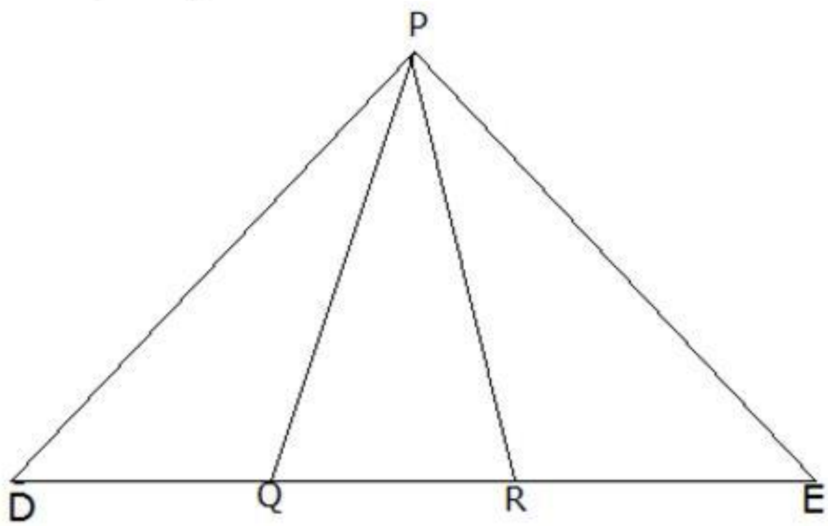
Hence, ΔXYZ is the required triangle.

* - Rectified as XY - XZ = 2.9 cm



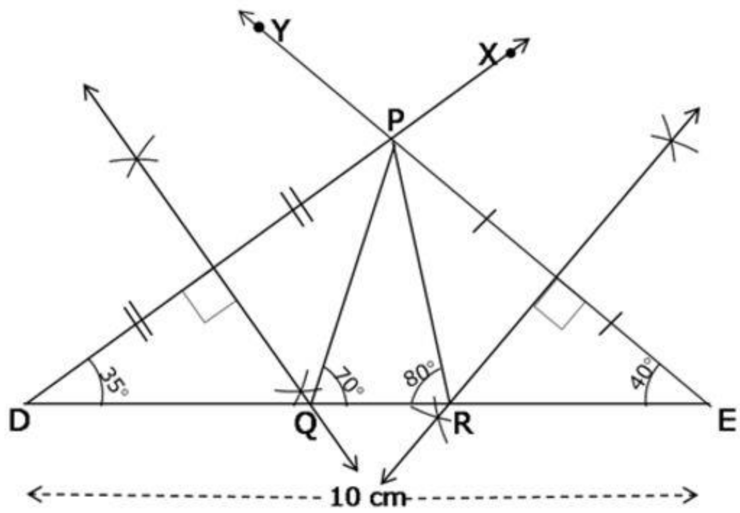
Solution 4:

Here instead of lengths of each side of the triangle, the perimeter PQ + QR + PR = 10 cm is given.
In order to construct the triangle with the given measurements, we need to draw a rough diagram.
Let us draw DE = PQ + QR + PR.
Let P be a point above DE and Q and R are points on DE.
Relation between $\angle PQR$ and $\angle PRQ$ with ΔPQD and ΔPRE:
$\angle PQR$ and $\angle PRQ$ are the exterior angles of ΔPQD and ΔPRE respectively.
Hence, by Remote Interior Angles Theorem
$2\angle PDE = \angle PQR$ and $2\angle PED = \angle PRQ$
$\therefore m\angle PQR = 70°, \therefore m\angle PDE = 70° \div 2 = 35°$
And
$\therefore m\angle PRQ = 80°, \therefore m\angle PED = 80° \div 2 = 40°$



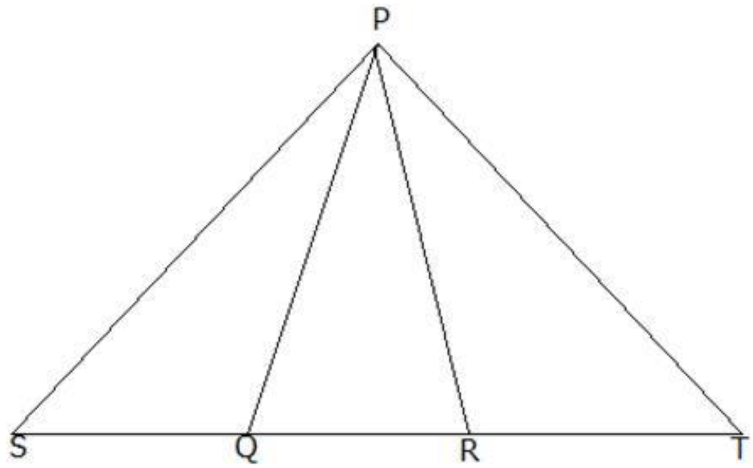
Steps of construction:

1. Draw $\overline{DE}$ of length 10 cm.
2. From point D draw $\overrightarrow{DX}$, such that $m\angle XDE = 35°$
3. From point E draw $\overrightarrow{EY}$, such that $m\angle DEY = 40°$.
4. Let P be the point of intersection of $\overrightarrow{DX}$ and $\overrightarrow{EY}$.
5. Draw the perpendicular bisectors of $\overline{PD}$ and $\overline{PE}$, which intersect $\overline{DE}$ in points Q and R respectively.
6. Join PQ and PR.
7. Hence, ΔPQR is the required triangle.



Solution 5:

Here instead of the lengths of each side of the triangle, the perimeter PQ + QR + PR = 9.5 cm is given.
In order to construct the triangle with the given measurements, we need to draw a rough diagram.
Let us draw ST = PQ + QR + PR.
Let P be a point above ST and Q and R be points on ST.
Relation between $\angle PQR$ and $\angle PRQ$ with ΔPQS and ΔPRT:
$\angle PQR$ and $\angle PRQ$ are the exterior angles of ΔPQS and ΔPRT respectively.
Hence, by Remote Interior Angles Theorem
$2\angle PST = \angle PQR$ and $2\angle PTS = \angle PRQ$
$\therefore$ $m\angle PQR = 80^\circ$, $\therefore$ $m\angle PST = 80^\circ \div 2 = 40^\circ$
And
$\therefore$ $m\angle PRQ = 80^\circ$, $\therefore$ $m\angle PTS = 80^\circ \div 2 = 40^\circ$



Steps of construction:

1.Draw $\overline{ST}$ of length 9.5 cm.

2.From point S draw $\overrightarrow{SX}$, such that m∠XST = 40°

3.From point T draw $\overrightarrow{TY}$, such that m∠YTS = 40°

4. Let P be the point of intersection of $\overrightarrow{SX}$ and $\overrightarrow{TY}$

5.Draw the perpendicular bisectors of $\overline{PS}$ and $\overline{PT}$.

6. The perpendicular bisectors intersect $\overline{ST}$ in points Q and R respectively.

7. Join PQ and PR.

8. Hence, ΔPQR is the required triangle.

It can be observed that ΔPQR is an isosceles triangle.