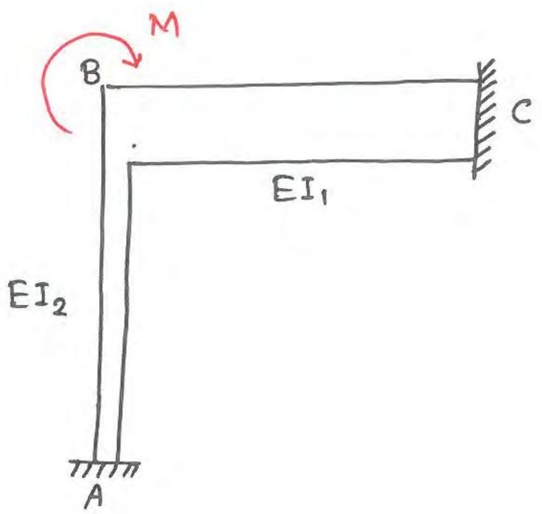
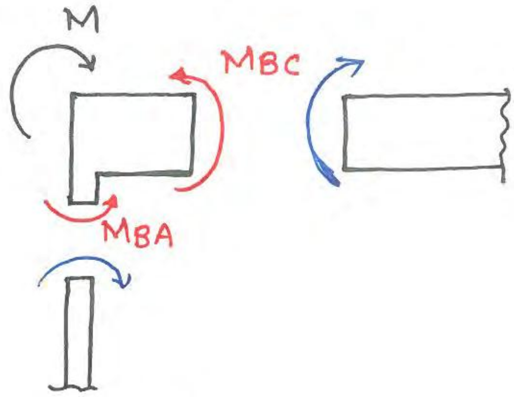
7 Moment Distribution Method

(Hardy-Cross Method)

7.1 Meaning of Moment Distribution:-





$EI_1 > EI_2$

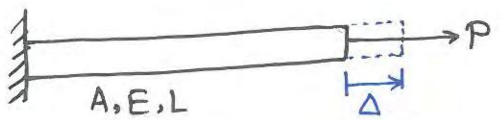
$\Rightarrow M_{BC} > M_{BA}$

It means, stiffer member attracts more moment.

7.2 Stiffness:

Force / moment required to produce unit displacement (deflection / rotation) is called stiffness.

7.2.1 Axial Stiffness:



$$\Delta = \frac{PL}{AE}$$

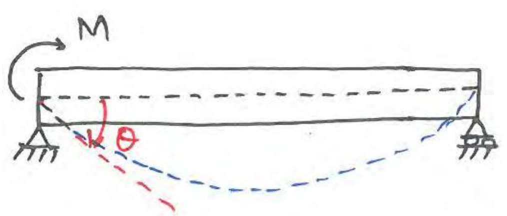
$$\Rightarrow P = \frac{AE}{L}\Delta$$

For $\Delta = 1$

$$\Rightarrow P = \frac{AE}{L} = \text{Stiffness } (k).$$

7.2.2 Bending Stiffness:

Case I: Simply supported Beam:



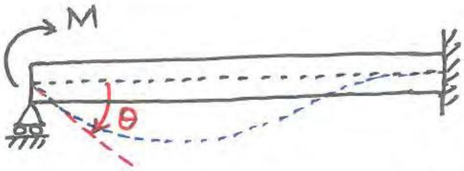
$$\theta = \frac{ML}{3EI}$$

$$\Rightarrow M = \frac{3EI}{L}\theta$$

For $\theta = 1$

$$\Rightarrow M = \frac{3EI}{L} = \text{Stiffness } (K).$$

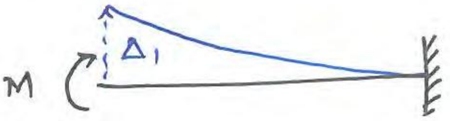
Case II: Propped Cantilever:-



Strain energy method is being used to calculate θ.

To apply strain energy method, method at all sections of beam need to be calculated. It means analysis need to be done first.

Analysis is being done by method of consistent deformation



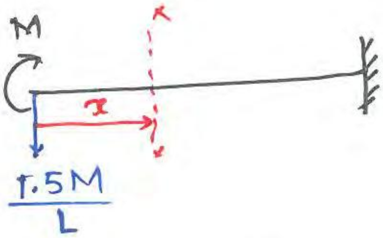
Compatibility condition:-

$$\Delta_1 + \Delta_2 = 0$$

$$\frac{ML^2}{2EI} + \frac{RL^3}{3EI} = 0$$

** $\boxed{R = -\frac{1.5M}{L}}$

Using strain energy method to calculate rotation.



$$M_x = M - \frac{1.5M}{L} \cdot x$$

$$\frac{\partial M_x}{\partial M} = 1 - \frac{1.5x}{L}$$

Now, $\frac{\partial U}{\partial M} = \theta \Rightarrow \int_0^L \frac{M_x \frac{\partial M_x}{\partial M} \, dx}{EI} = \theta$

$$\Rightarrow \int_0^L \frac{\left(M - \frac{1.5M}{L}x\right)\left(1 - \frac{1.5}{L}x\right) dx}{EI} = \theta$$

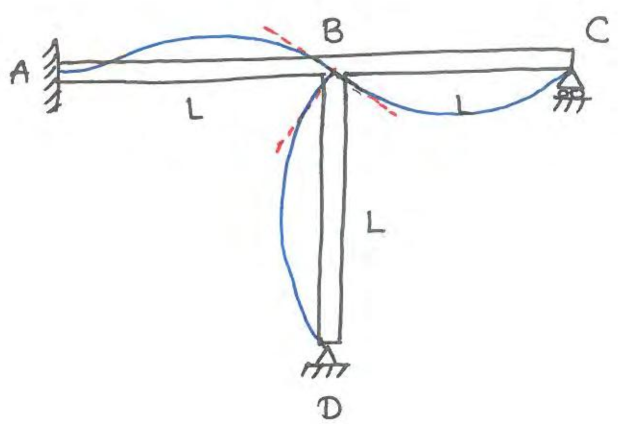
$$\Rightarrow \frac{ML}{4EI} = \theta$$

$$\Rightarrow M = \frac{4EI}{L}\theta$$

For $\theta = 1$

$$M = \frac{4EI}{L} = \text{Stiffness } (k).$$

Ex. Calculate rotational stiffness of joint B.

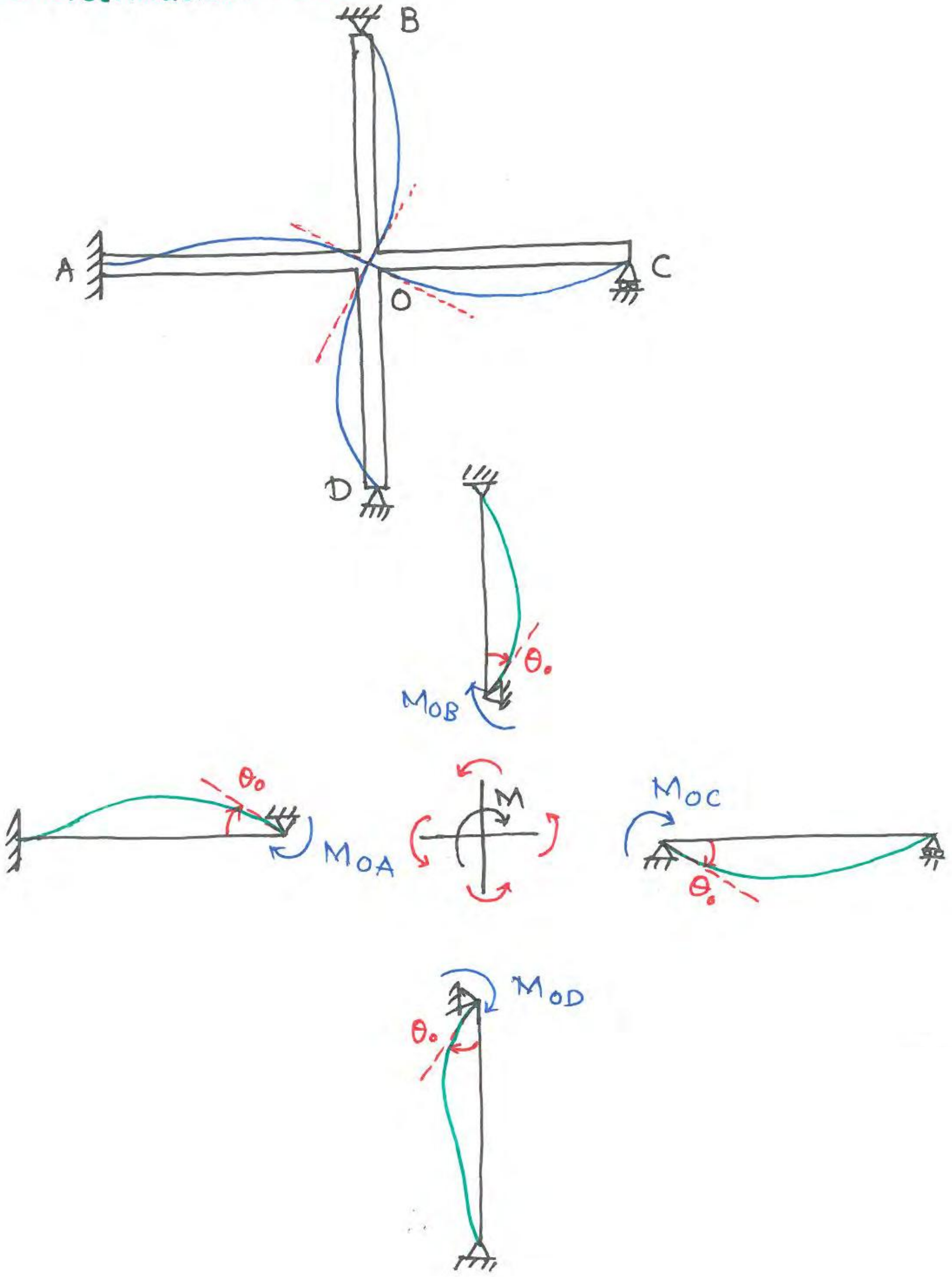


$$\Rightarrow K_B = K_{BA} + K_{BC} + K_{BD}$$

$$= \frac{4EI}{L} + \frac{3EI}{L} + \frac{3EI}{L}$$

$$= \frac{10EI}{L}$$

7.3 Distribution Factor:



$$M = M_{OA} + M_{OB} + M_{OC} + M_{OD}$$

$$M_{OA} = K_{OA} \cdot \theta_0$$

$$M_{OB} = K_{OB} \cdot \theta_0$$

$$M_{OC} = K_{OC} \cdot \theta_0$$

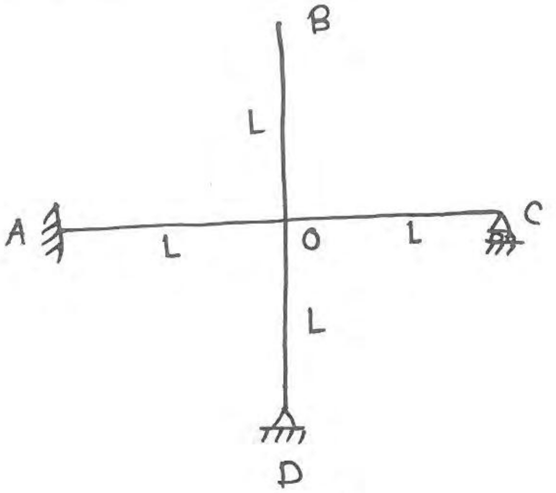
$$M_{OD} = K_{OD} \cdot \theta_0$$

$$DF_{OA} = \frac{M_{OA}}{M}$$

$$\Rightarrow DF_{OA} = \frac{K_{OA} \cancel{\theta_0}}{(K_{OA} + K_{OB} + K_{OC} + K_{OD}) \cancel{\theta_0}}$$

$$\Rightarrow \boxed{DF = \frac{\text{Stiffness of member}}{\text{Stiffness of Joint}}}$$

Ex. Calculate DF_{OD}

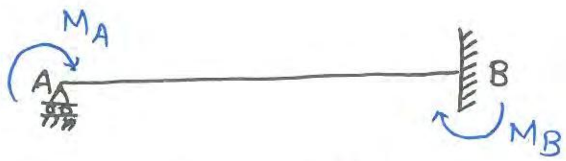


$$DF_{OD} = \frac{K_{OD}}{(K_{OA} + K_{OB} + K_{OC} + K_{OD})} = \frac{3EI/L}{\frac{4EI}{L} + 0 + \frac{3EI}{L} + \frac{3EI}{L}}$$

$$DF_{OD} = 0.3$$

7.4 Carry over factor:

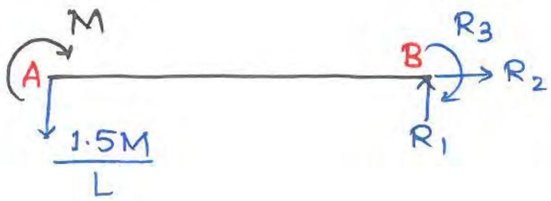
It is the ratio of moment transferred at far end to the moment applied at near end.



$$CO_{B/A} = \frac{M_B}{M_A}$$

Ex1. Calculate carry over factor for propped cantilever beam.

$\Rightarrow$ FBD



$$\Sigma M_B = 0$$

$$\Rightarrow M - \frac{1.5M}{L} \times L + R_3 = 0$$

$$\Rightarrow R_3 = M/2$$

$$CO_{B/A} = \frac{M_B}{M_A} = \frac{R_3}{M_A} = \frac{M/2}{M} = 1/2$$

$$CO_{B/A} = 1/2$$

Ex. 2 Calculate carry over factor for the beam given below.

⇒ FBD :-

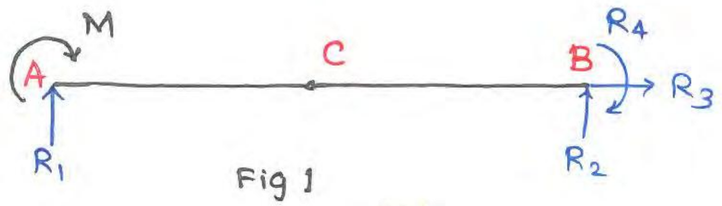


Fig 1

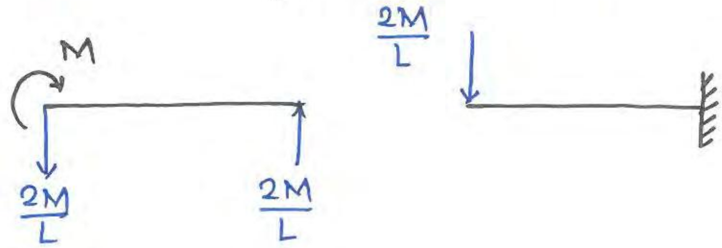


Fig. 2

from Fig. 1

$$M_C = 0 \quad (LHS)$$

$$\Rightarrow M + R_1 \times \frac{L}{2} = 0$$

$$\Rightarrow R_1 = -\frac{2M}{L}$$

$$\Sigma M_B = 0$$

$$\Rightarrow R_4 + M + R_1 \times L = 0$$

$$\Rightarrow R_4 = -M - R_1 \times L = -M + \frac{2M}{L} \times L$$

$$\Rightarrow R_4 = M$$

$$CO_{B/A} = \frac{M_B}{M_A}$$

$$= \frac{R_4}{M_A} = \frac{M}{M}$$

$$CO_{B/A} = 1$$

* Note:

Carry over factor may be positive, negative and greater than 1 also.

$$CO = \frac{b}{a} = \frac{b}{a}$$

For $b > a$

$CO > 1$

$$CO = \frac{M_B}{M_A} = \frac{-M}{M} = -1$$

7.5 Moment-Distribution Without Sway:

Step I: Calculate KI and identify them.

Step II Lock all KI and calculate FEM for each member

Step III: Calculate DF corrosponding to each KI

Step IV: Release 1 KI at a time and allow dispacement for unbalanced moment at that joint. Distribute unbalanced moment at that joint as per distribution factor.

Step V: Transfer carryover moment.

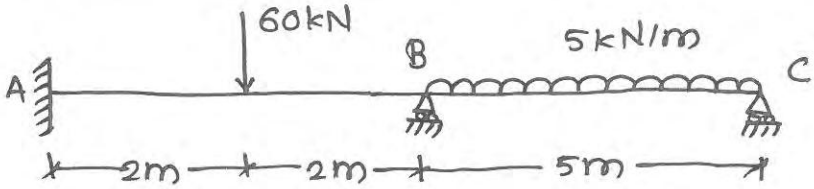
Step VI: Again check unbalanced moment at each KI. If any joint is unbalanced then repeat step IV to step VI still we find unbalanced moment at all joints within tolerence limit

Step VII: Add up all moments (FEM, distribution moment, carryover moment) at each end of member.

Step VIII: Draw BMD using end moments of step VII by method of superposition

Step IX: Calculate other reactions and draw SFD.

Ex. Analyze the ~~Frame~~ Beam given below:



Step I: KI:

$KI = 2 \quad (\theta_B, \theta_C)$

Step II: Locking KI and calculate FEM:

$$M_{FAB} = -\frac{PL}{8} = \frac{-60 \times 4}{8} = -30$$

$$M_{FBA} = \frac{PL}{8} = 30$$

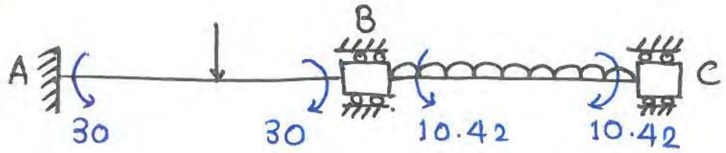
$$M_{FBC} = -\frac{wL^2}{12} = -\frac{5 \times 5^2}{12} = -10.42$$

$$M_{FCB} = \frac{wL^2}{12} = 10.42$$

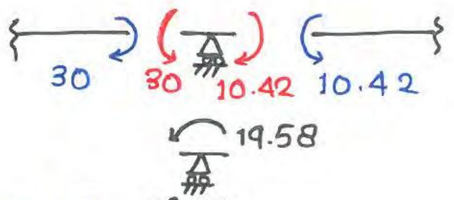
Step III: DF

Joint	Member	Stiffness	T.S.	D.F
B	BA	4EI/4	1.8EI	0.55
	BC	4EI/5		0.45
C	CB	4EI/5	0.8EI	1.

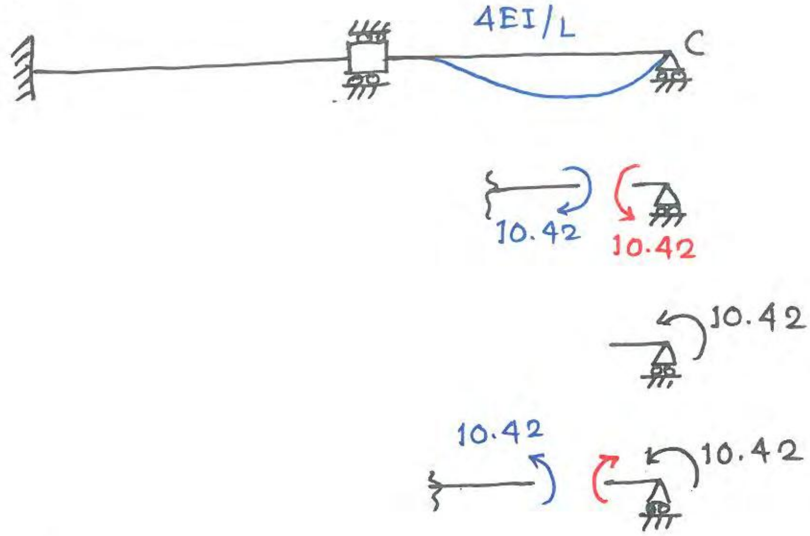
Step IV:



Release B.

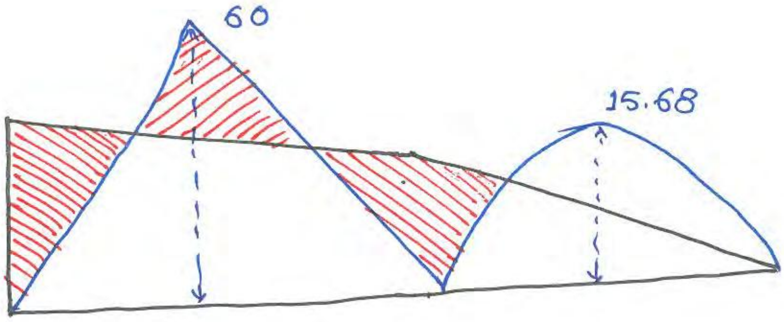


Release C



Joint	A	B		C
Member	AB	BA	BC	CB
DF	–	0.55	0.45	1
FEM	-30	30	-10.42	10.42
Bal		-10.77	-8.81	-10.42
CO	-5.38		-5.21	-4.40
Bal		2.87	2.34	4.40
CO	1.43		2.20	1.17
Bal		-1.21	-0.99	-1.17
CO	-0.60		-0.58	-0.49
Bal		0.32	0.26	0.49
CO	0.16		0.24	0.13
Bal		-0.13	-0.11	-0.13
CO	-0.06		-0.06	-0.05
Bal		0.03	0.03	0.05
Final	-34.45	21.1	-21.1	0

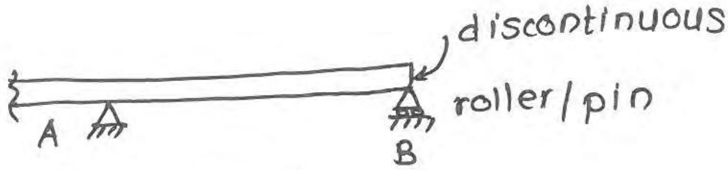
Step VIII: BMD



7.6 Modified MDM:

It is used if far end of member is discontinuous and pin/roller supported.

By using modified MDM, size of distribution table is reduced



$$K_{BA} = \frac{3EI}{L}$$

$$K_{AB} = \frac{4EI}{L}$$

Ex. Analyze previous example using modified MDM.

Step I: KI

KI = same

since joint C is roller supported and discontinuous so modified MDM can be used.

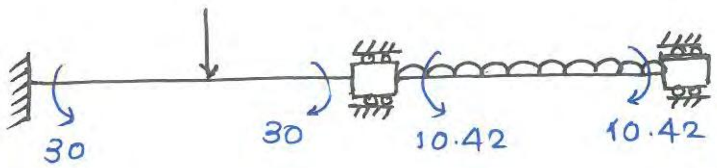
Step II: FEM

⇒ same

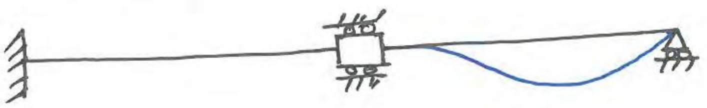
Step III: DF

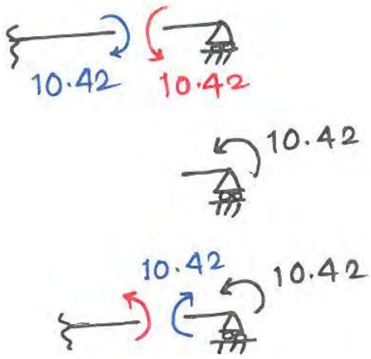
Joint	Member	Stiffness	T.S.	D.F.
B	BA	4EI/4	1.6 EI	0.63
	BC	3EI/5 (Mod.)		0.37
C	CB	4EI/5		1

FEM

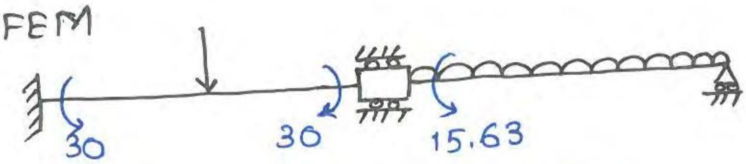


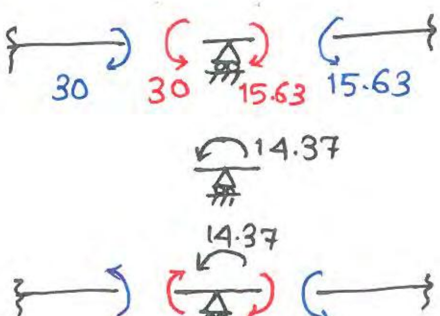
Release C permanently





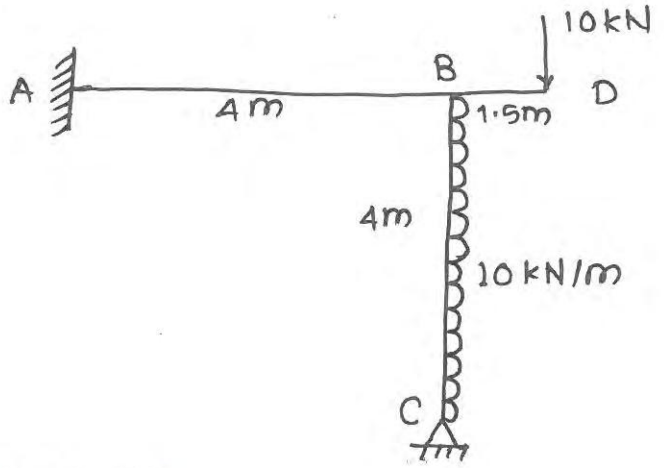
Mod. FEM





Joint	A	B		C
Member	AB	BA	BC	CB
DF	–	0.63	0.37	1
FEM	-30	30	-10.42	10.42
Release C				-10.42
CO			-5.21	
Mod. FEM	-30	30	-15.63	0
Bal		-9.05	-5.32	
CO	-4.52			
Bal	–	–	–	–
Final	-34.52	20.95	-20.95	0

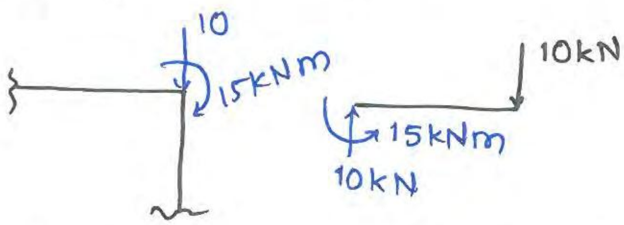
Ex.



Step I: KI

$KI = 4 \quad (\theta_B, \theta_C, \theta_D, \Delta_{YD})$

By removing overhang portion, KI can be reduced to 2.



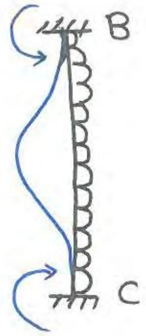
Since joint C is pin supported and discontinuous so modified MDM can be used.

Step II: FEM:

$$M_{FAB} = M_{FBA} = 0$$

$$M_{FBC} = -\frac{wL^2}{12} = \frac{-10 \times 4^2}{12} = -13.33$$

$$M_{FCB} = \frac{wL^2}{12} = 13.33$$

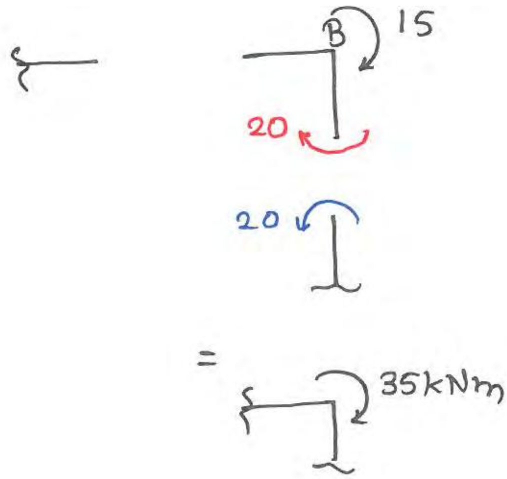


Step III: DF:

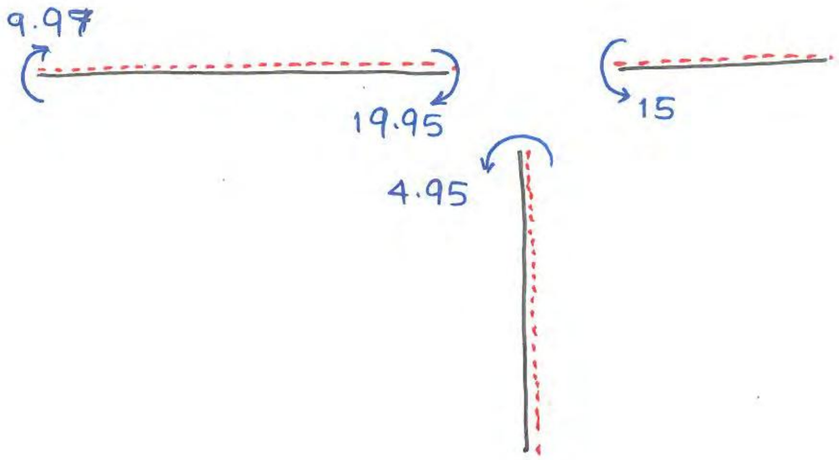
Joint	Member	Stiffness	T.S.	D.F.
B	BA	$4EI/4$	1.75EI	0.57
	BC	$3EI/4$		0.43
C	CB	$4EI/4$		1

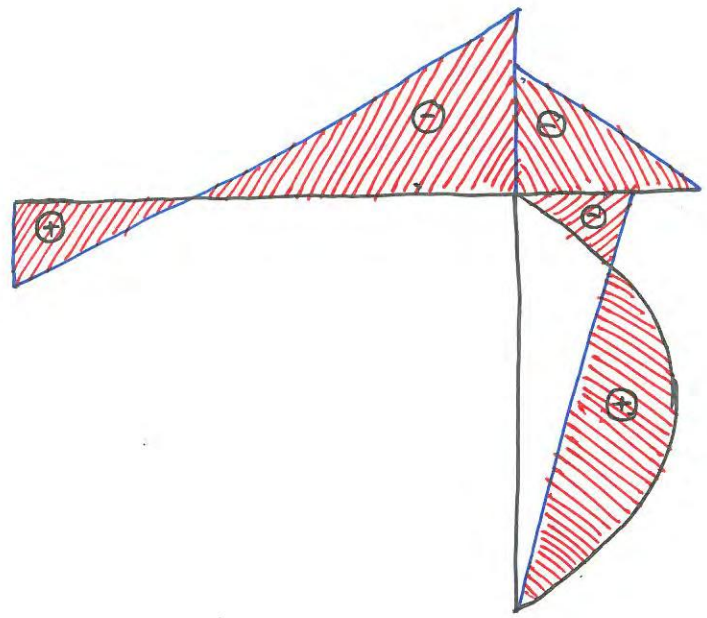
Step . :

Joint	A	B		C
Member	AB	BA	BC	CB
DF	–	0.57	0.43	1
FEM	0	0	−13.33	13.33
Release C				−13.33
CO			−6.67	
Mod.FEM	0	0	−20	0
Bal		19.95	15.05	
CO	9.97			
Bal	–	–	–	–
Final	9.97	19.95	−4.95	0

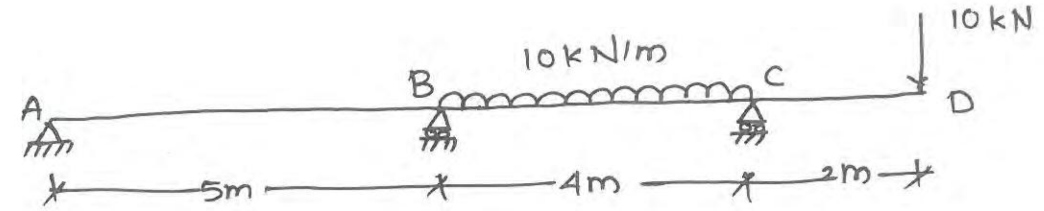


Step BMD:





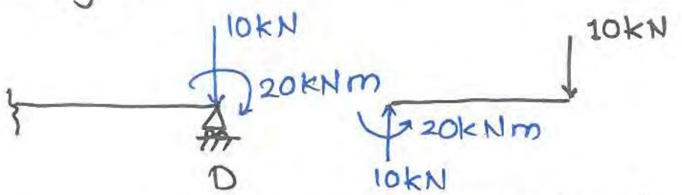
Ex



Step I: KI:-

KI = 5 $(\theta_A, \theta_B, \theta_C, \theta_D, \Delta_{YD})$

By removing overhang portion KI can be reduced to 3.



Since joints A and C are pin/ roller supported and discontinuous so modified MDM can be used.

Step II: FEM:

$M_{FAB} = M_{FBA} = 0$

$M_{FBC} = -\frac{WL^2}{12} = -\frac{10\times 4^2}{12} = -13.33$

$M_{FCB} = \frac{WL^2}{12} = 13.33$

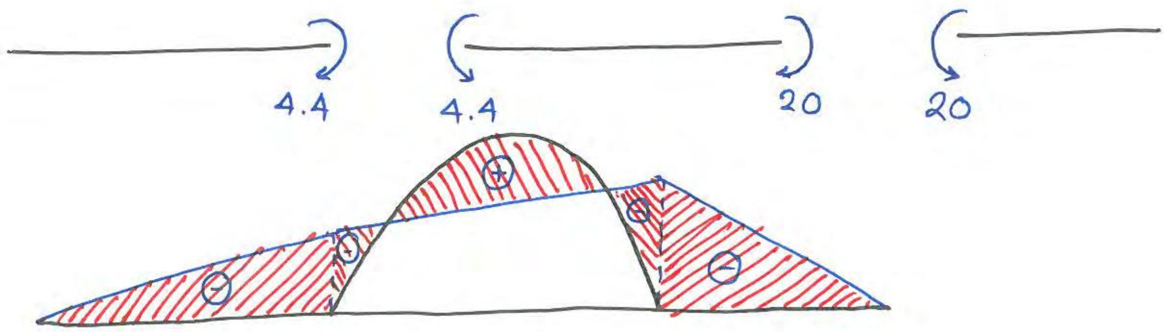
Step III: DF

Joint	Member	Stiffness	T.S.	D.F
A	AB	4EI/5	0.8EI	1
B	BA	3EI/5	1.35EI	0.44
	BC	3EI/4		0.56
C	CB	4EI/4	EI	1

Step :

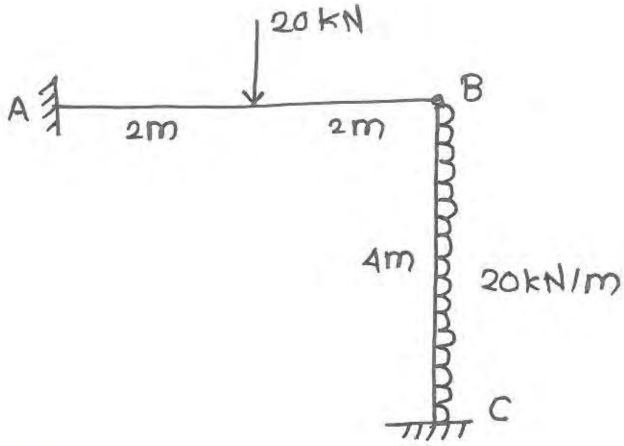
Joint	A	B		C
Member	AB	BA	BC	CB
DF	1	0.44	0.56	1
FEM	0	0	-13.33	13.33
Release A & C	0			(-13.33 + 20) = 6.67
CO		0	+3.33	
Mod. FEM	0	0	-10	20
Bal		4.4	5.6	
CO	–	–	–	–
Bal	–	–	–	–
Final	0	4.4	-4.4	20

Step BMD:



BMD.

Ex.



Step I: KI:

$KI = 2 \quad (\theta_{BA}, \theta_{BC})$

Step II: FEM:

$$M_{FAB} = -\frac{PL}{8} = -\frac{20 \times 4}{8} = -10$$

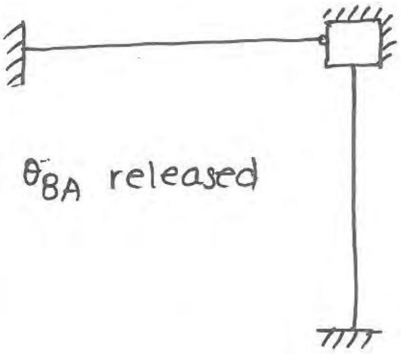
$$M_{FBA} = \frac{PL}{8} = 10$$

$$M_{FBC} = -\frac{wL^2}{12} = -\frac{20 \times 4^2}{12} = -26.67$$

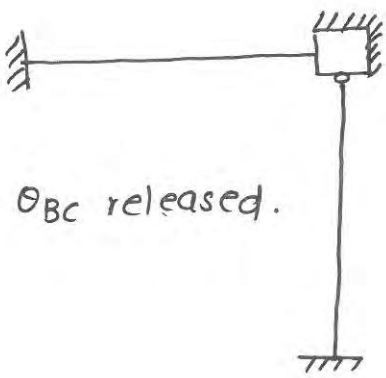
$$M_{FCB} = \frac{wL^2}{12} = 26.67$$

Step III: DF

Joint	Member	Stiffness	T.S.	D.F
B	BA	$4EI/4$	EI	1
	BC	$4EI/4$	EI	1



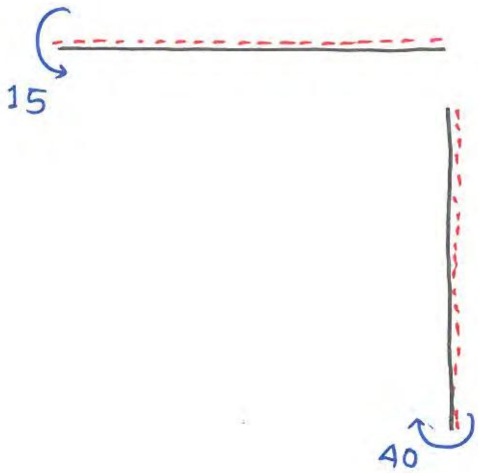
θ_{BA} released

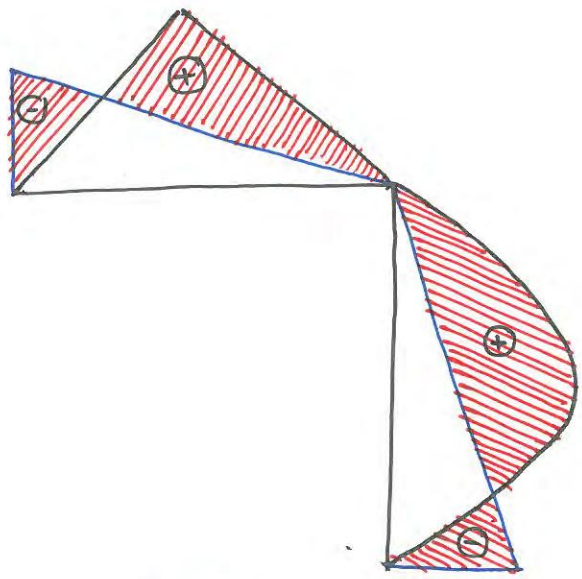


θ_{BC} released.

Joint	A	B		C
Member	AB	BA	BC	CB
DF	-	1	1	-
FEM	-10	10	-26.67	26.67
Bal		-10	26.67	
CO	-5			13.33
Bal	-	-	-	-
Final	-15	0	0	40

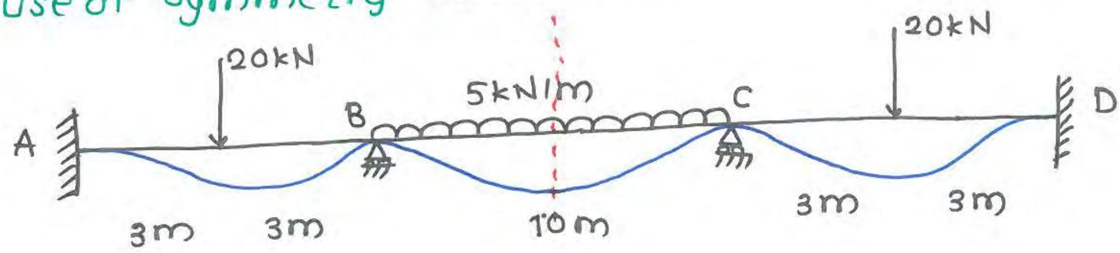
Step BMD.

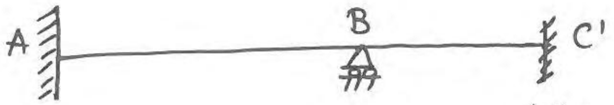




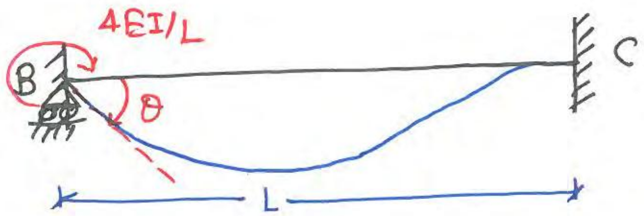
BMD.

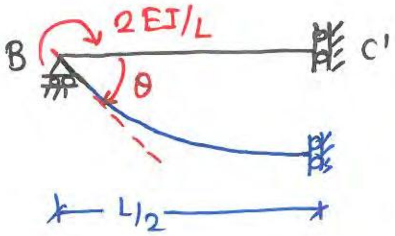
7.7 Use of Symmetry condition:





$$K_{BC'} = \frac{K_{BC}}{2}$$





Step I: KI:

$KI = 1 \quad (\theta_B)$

Step II: FEM.

$$M_{FAB} = -\frac{PL}{8} = -\frac{20 \times 6}{8} = -15$$

$$M_{FBA} = \frac{PL}{8} = \frac{20 \times 6}{8} = 15$$

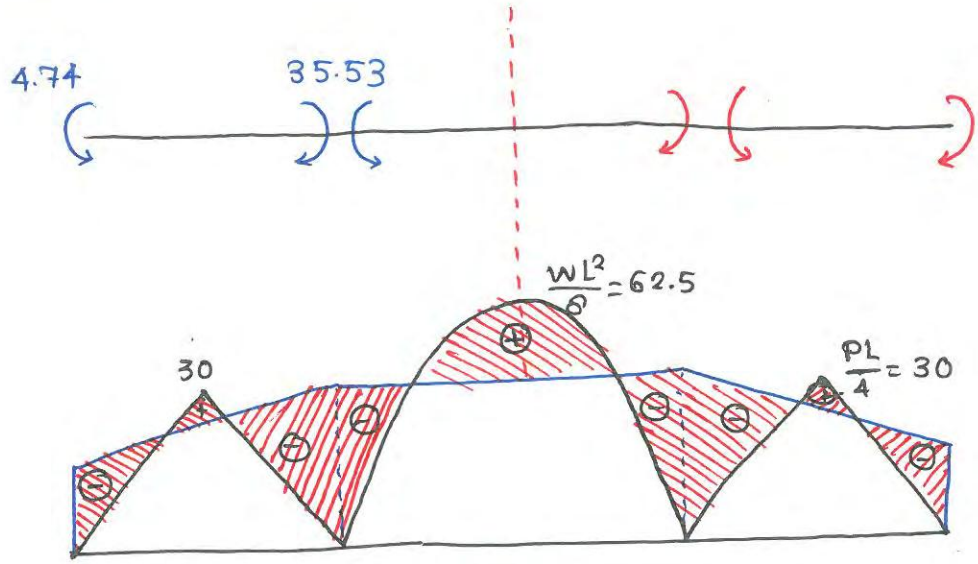
$$M_{FBC} = -\frac{wL^2}{12} = -\frac{5 \times 10^2}{12} = -41.67 \text{ kNm}$$

Step III: DF:

Joint	Member	Stiffness	T.S.	D.F.
B	BA	$4EI/6$	$0.87EI$	0.77
	BC'	$\frac{K_{BC}}{2} = \frac{4EI/10}{2}$		0.23

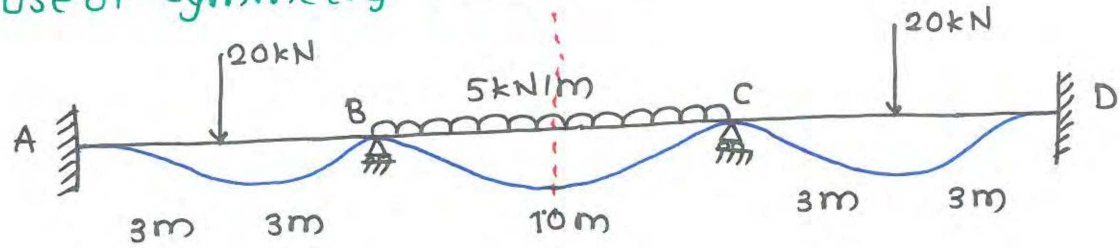
Joint	A	B	
Member	AB	BA	BC'
DF	–	0.77	0.23
FEM	−15	15	−41.67
Bal		20.53	6.13
CO	10.26		
Bal	–	–	–
Final	−4.74	35.53	−35.53

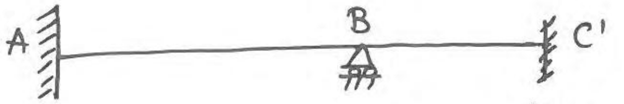
Step BMD.



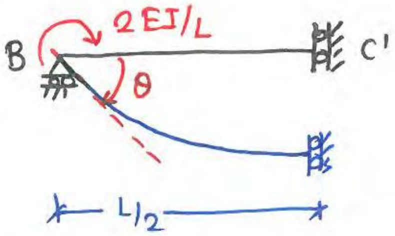
BMD.

7.7 Use of Symmetry condition:





$$K_{BC'} = \frac{K_{BC}}{2}$$



Step I: KI:

KI = 1 (θ_B)

Step II: FEM.

$$M_{FAB} = -\frac{PL}{8} = -\frac{20 \times 6}{8} = -15$$

$$M_{FBA} = \frac{PL}{8} = \frac{20 \times 6}{8} = 15$$

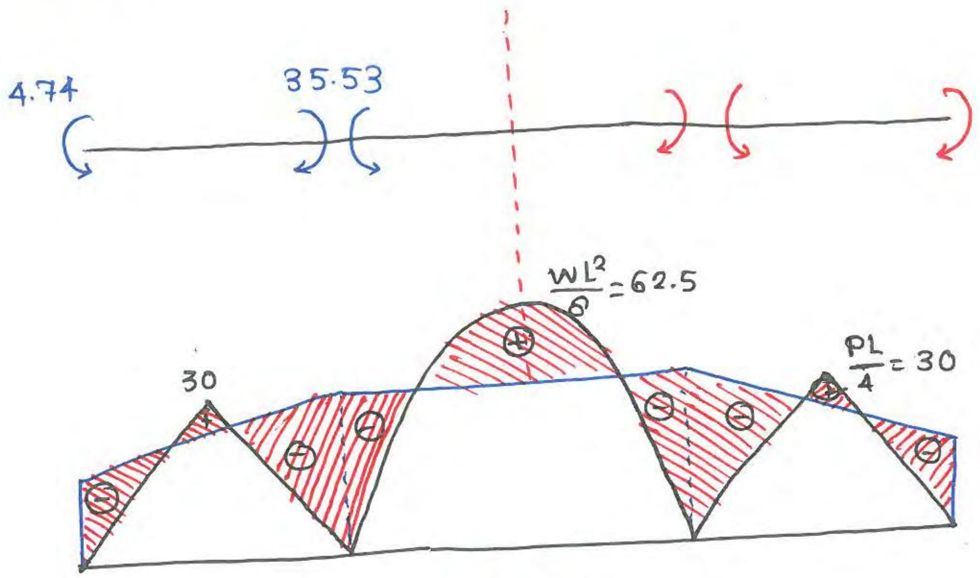
$$M_{FBC} = -\frac{wL^2}{12} = -\frac{5 \times 10^2}{12} = -41.67 \text{ kNm}$$

Step III: DF:

Joint	Member	Stiffness	T.S.	D.F.
B	BA	$4EI/6$	0.87EI	0.77
	BC'	$\frac{K_{BC}}{2} = \frac{4EI/10}{2}$		0.23

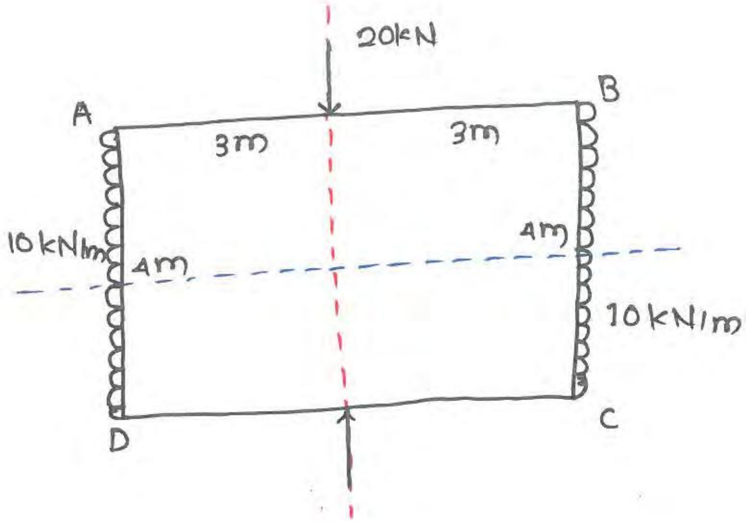
Joint	A	B	
Member	AB	BA	BC'
DF	–	0.77	0.23
FEM	−15	15	−41.67
Bal		20.53	6.13
CO	10.26		
Bal	–	–	–
Final	−4.74	35.53	−35.53

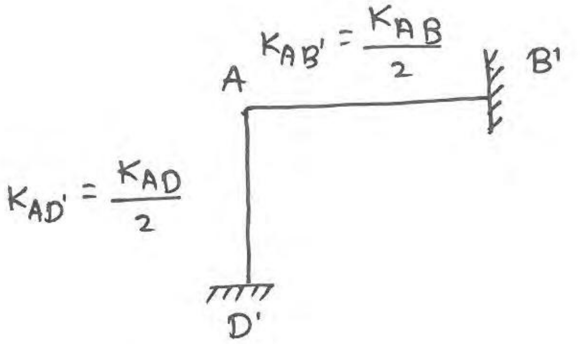
Step BMD.



BMD.

Ex





Step I: KI :-

$KI = 1 \quad (\theta_A)$

Step II: FEM:

$$M_{FAB} = -\frac{PL}{8} = -\frac{20 \times 6}{8} = -15$$

$$M_{FAD} = \frac{wL^2}{12} = \frac{10 \times 4^2}{12} = 13.33$$

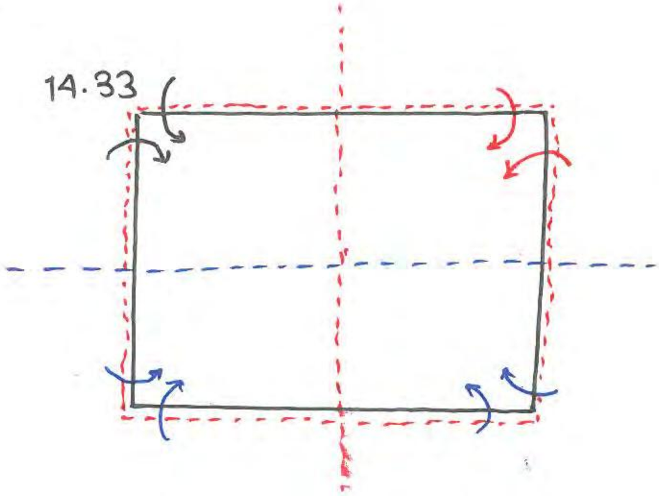
Step III: D.F.

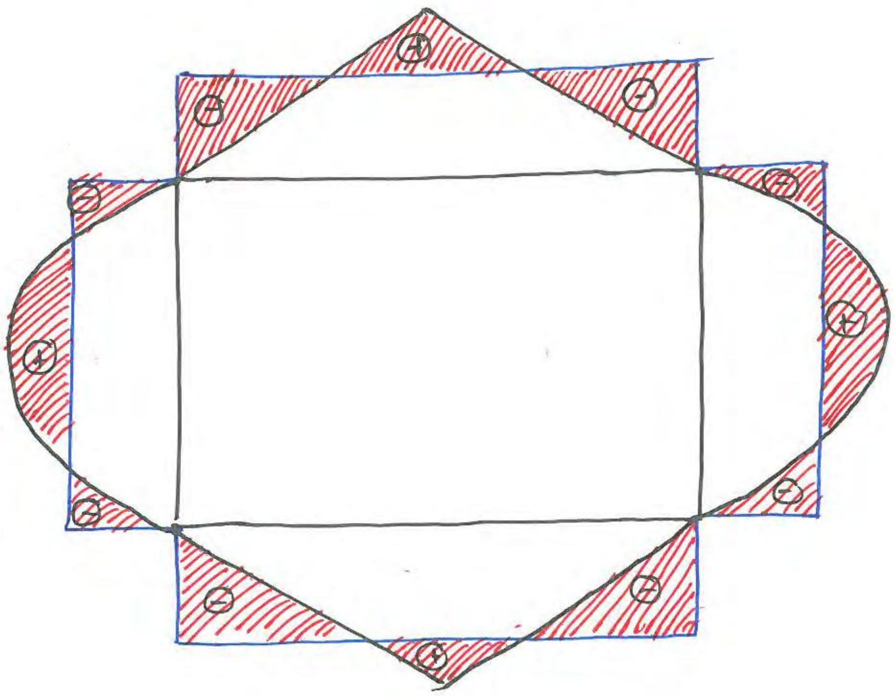
Joint	Member	Stiffness	T.S.	D.F
A	AB'	$\frac{K_{AB}}{2} = \frac{(4EI/6)}{2}$	0.83EI	0.40
	AD'	$\frac{K_{AD}}{2} = \frac{(4EI/4)}{2}$		0.60

Step :-

Joint	A	
Member	AB'	AD'
DF	0.4	0.6
FEM	-15	13.33
Bal	0.67	1
CO	–	–
Bal	–	–
Final	-14.33	14.33

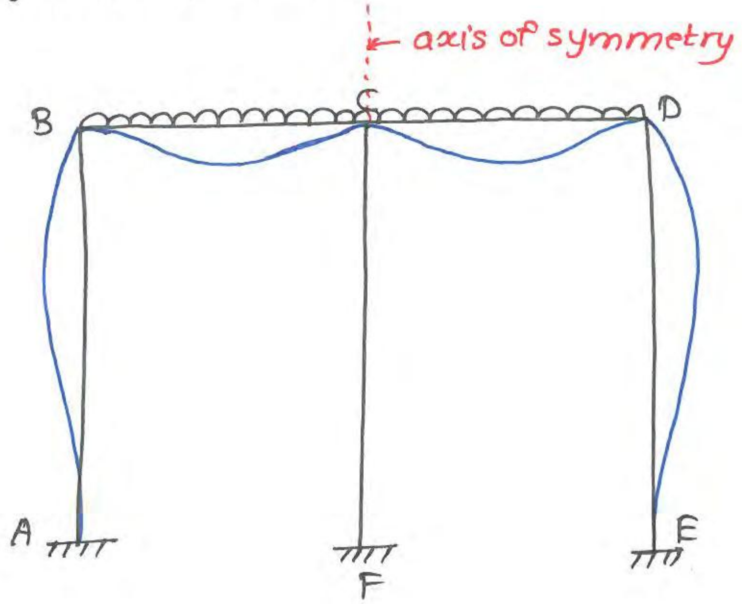
Step :- BMD





* Note:

Above concept of use of symmetry condition is not applicable if there is no sway in member at axis of symmetry. For e.g., structure given below can not be analyzed by above procedure.



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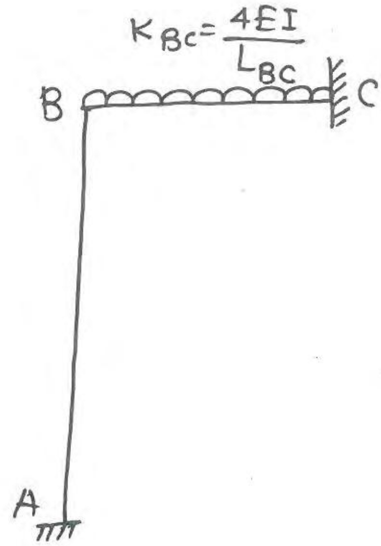
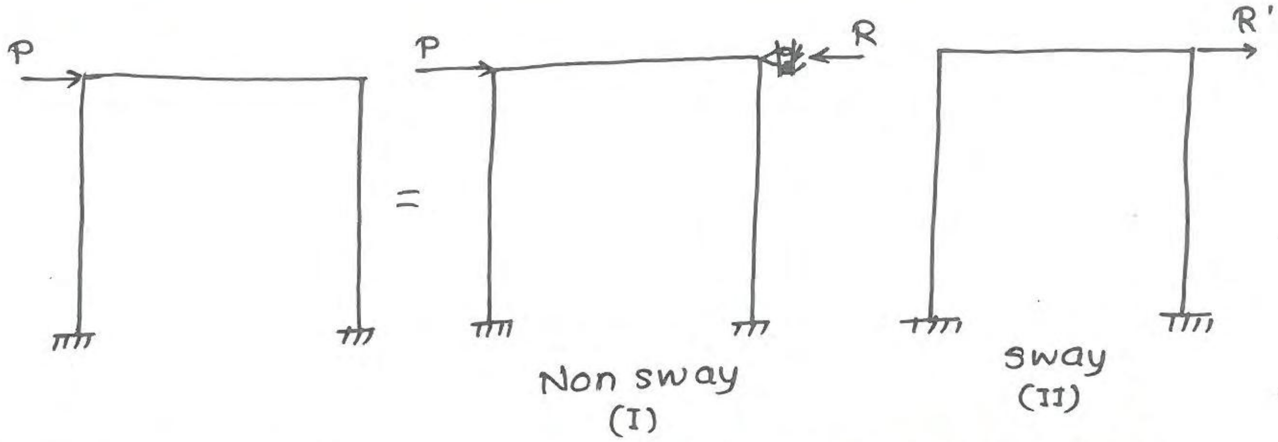


7.8 Moment Distribution with sway:

#

StepI: Calculate KI and identify them. If any KI is translational then this is the case of sway analysis.

StepII: Divide the structure in two parts as given below.



StepIII: Perform Non-sway analysis for structure I and calculate all end moments.

StepIV:- Write equilibrium equation corresponding to translational KI and calculate R.

StepV: Calculate FEM for sway Δ of a structure ~~to~~ II and get ratio of all FEM.

StepVI: Select any arbitary value of FEM as per ratio of stepV and perform moment distribution for sway case.

StepVII: Again write equilibrium equation corresponding to sway and calculate R' which is responsible to produce end moments calculated in stepVI

StepVIII: Calculate correction factor $\left(\frac{R}{R'}\right)$

StepIX: Calculate final end moment.

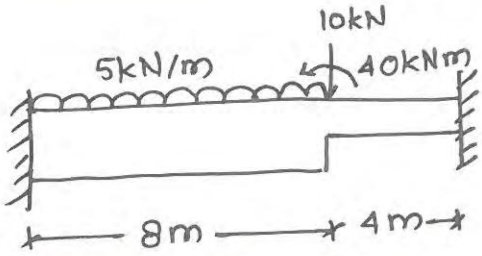
$$M_{Final} = M_{Non\ sway} + \frac{R}{R'} M_{sway}.$$

$$R' \longrightarrow M_{sway}$$
$$1 \longrightarrow \frac{M_{sway}}{R'}$$
$$R \longrightarrow \frac{R}{R'} M_{sway}$$

StepX: Draw BMD using moments of stepIX by method of superposition.

StepXI: Calculate other reactions and draw SFD.

Ex.

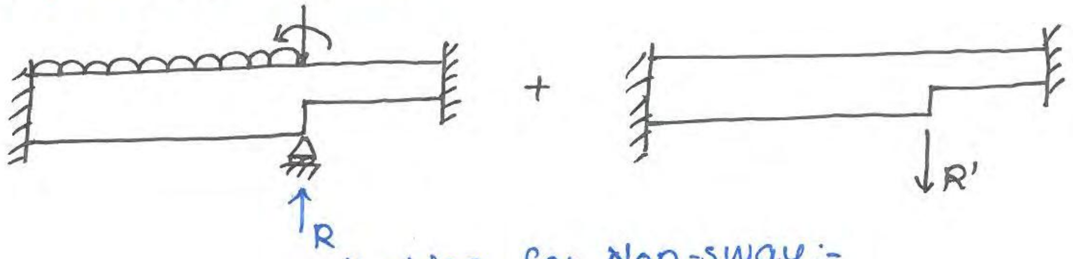


Since cross-section of beam is changing so a joint is being considered at junction.

Step I: KI:-

$KI = 2 \quad (\theta_B, \Delta_{YB} = \Delta (\downarrow))$

Step II: Divide the structure:



Step III: Moment distribution for Non-sway:-

$KI = 1 \quad (\theta_B)$

$$M_{FAB} = -\frac{wL^2}{12} = -\frac{5 \times 8^2}{12} = -26.67$$

$$M_{FBA} = \frac{wL^2}{12} = 26.67$$

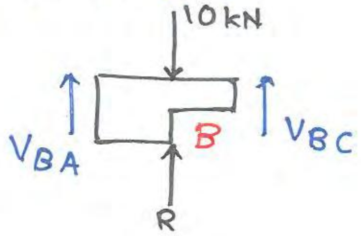
$$M_{FBC} = M_{FCB} = 0$$

DF:-

Joint	Member	Stiffness	T.S.	D.F
B	BA	$4E(2I)/8$	2EI	0.5
	BC	$4EI/4$		0.5

Joint	A	B		C
Member	AB	BA	BC	CB
DF	–	0.5	0.5	–
FEM	−26.67	26.67	0	0
Bal		−33.33	−33.33	
CO	−16.67			−16.67
Bal	–	–	–	–
Final	−43.34	−6.67	−33.33	−16.67

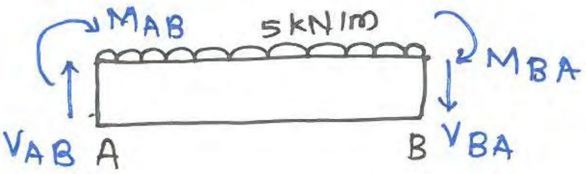
Step IV:- Calculation of R:



$$\Sigma F_y = 0$$

$$\Rightarrow V_{BA} + V_{BC} - 10 = 0 \quad \cdots\cdots (i)$$

For V_{BA} :-



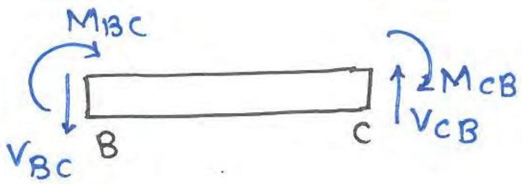
$$\Sigma M_A = 0$$

$$\Rightarrow V_{BA} = -\left(\frac{M_{AB} + M_{BA} + 5\times8\times4}{8}\right)$$

$$= -\left(\frac{-43.34 - 6.67 + 160}{8}\right)$$

$$= -13.75 \text{ kN}$$

For V_{BC} :-



$$\Sigma M_C = 0$$

$$\Rightarrow V_{BC} = \frac{M_{BC} + M_{CB}}{4}$$

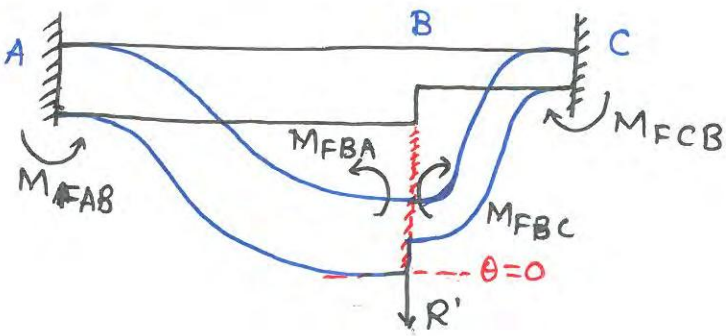
$$= \frac{-33.33 - 16.67}{\cancel{12.5}\ 4}$$

$$= -12.5 \text{ kN}$$

from eq^n (i)

$$R = 36.25 \text{ kN.}$$

Step V:



$M_{FAB} : M_{FBA} : M_{FBC} : M_{FCB}$

$$\Rightarrow \frac{-6E(2I)\Delta}{8^2} : \frac{-6E(2I)\Delta}{8^2} : \frac{6EI\Delta}{4^2} : \frac{6EI\Delta}{4^2}$$

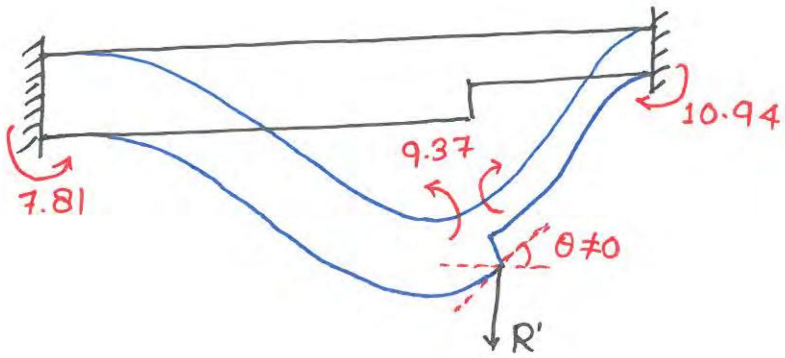
$$\Rightarrow \frac{-2}{8^2} : \frac{-2}{8^2} : \frac{1}{4^2} : \frac{1}{4^2}$$

$$\Rightarrow -0.03125 : -0.03125 : 0.0625 : 0.0625$$

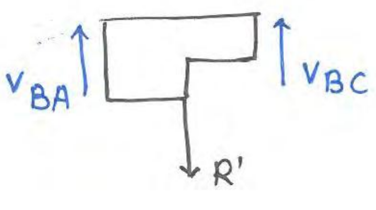
Step VI: Multiplying by any arbitary value (assuming 200)

$$\Rightarrow -6.25 : -6.25 : 12.5 : 12.5$$

Joint	A	B		C
Member	AB	BA	BC	CB
DF	–	0.5	0.5	–
FEM	-6.25	-6.25	12.5	12.5
Bal		-3.12	-3.12	
CO	-1.56			-1.56
Bal	–	–	–	–
Final	-7.81	-9.37	9.37	10.94



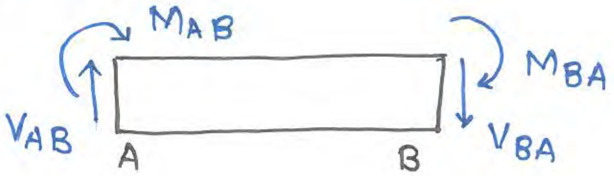
Step VII: Calculation of R'



$\Sigma F_y = 0$

$$\Rightarrow V_{BA} + V_{BC} - R' = 0 \quad \cdots (ii)$$

For V_{BA} :-



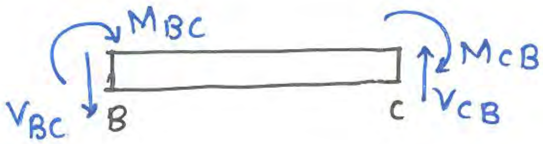
$$\Sigma M_A = 0$$

$$\Rightarrow V_{BA} = -\frac{(M_{AB} + M_{BA})}{8}$$

$$= -\frac{(-7.81 - 9.37)}{8}$$

$$V_{BA} = 2.15 \text{ kN}$$

For V_{BC} :-



$$\Sigma M_C = 0$$

$$\Rightarrow V_{BC} = \frac{M_{BC} + M_{CB}}{4}$$

$$= \frac{9.37 + 10.94}{4}$$

$$= 5.08 \text{ kN}$$

from eq^n (ii)

$$R' = 7.23 \text{ kN.}$$

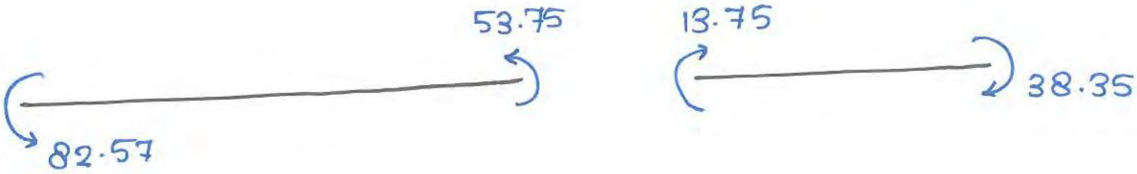
Step VIII: Calculate correction Factor:

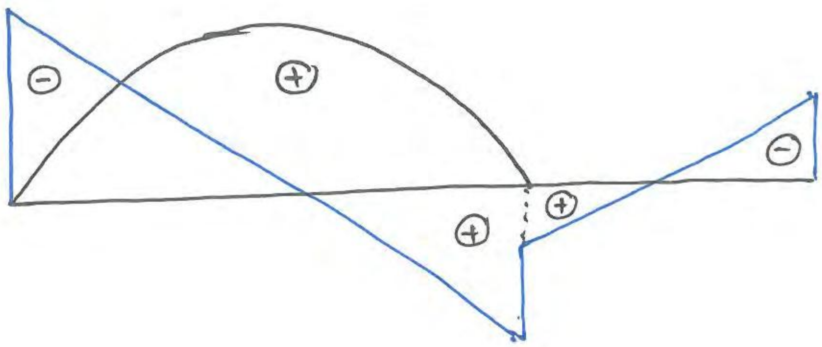
$$\frac{R}{R'} = \frac{36.25}{7.23}$$

Step IX: Final Moments:

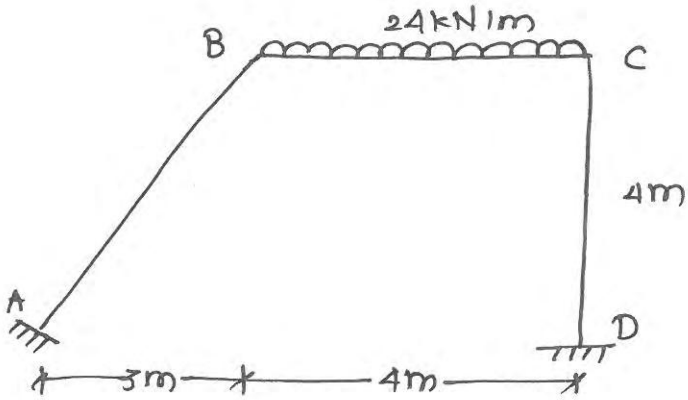
Members	AB	BA	BC	CB
$M_{Nonsway}$	-43.34	-6.67	-33.33	-16.67
M_{sway}	-7.81	-9.37	9.37	10.94
$M_{Final} = M_{N.S.} + \frac{R}{R'} M_S$	-82.57	-53.75	13.75	38.35

Step X: BMD:





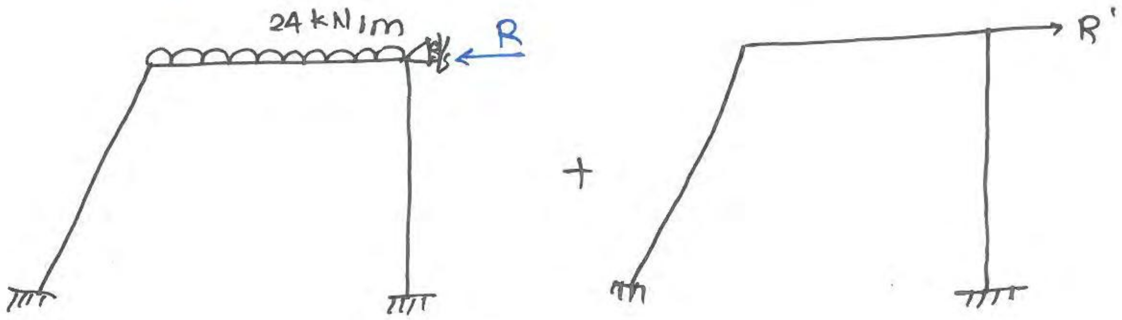
Ex



Step I:- KI:

$$KI = 3 \quad (\theta_B, \theta_C, \Delta_{xB} = \Delta\ (\rightarrow))$$

Step II: Dividing the structure



Step III: Non sway analysis :-

$KI = 2 \ (\theta_B, \theta_C)$

$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$

$M_{FBC} = -\frac{WL^2}{12} = -\frac{24 \times 4^2}{12} = -32$

$M_{FCB} = \frac{WL^2}{12} = 32$

DF :-

Joint	Member	Stiffness	T.S.	D.F.
B	BA	4EI/5	1.8EI	0.44
	BC	4EI/4		0.56
C	CB	4EI/4	2EI	0.5
	CD	4EI/4		0.5

Moment Distribution:-

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	–	0.44	0.56	0.5	0.5	–
FEM	0	0	-32	32	0	0
Bal						
Final	9.41	18.9	-18.9	22.06	-22.06	-10.96

Step III: Non sway analysis :-

$KI = 2\ (\theta_B, \theta_C)$

$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$

$M_{FBC} = -\frac{WL^2}{12} = -\frac{24 \times 4^2}{12} = -32$

$M_{FCB} = \frac{WL^2}{12} = 32$

DF :-

Joint	Member	Stiffness	T.S.	D.F.
B	BA	4EI/5	1.8EI	0.44
	BC	4EI/4		0.56
C	CB	4EI/4	2EI	0.5
	CD	4EI/4		0.5

Moment Distribution:-

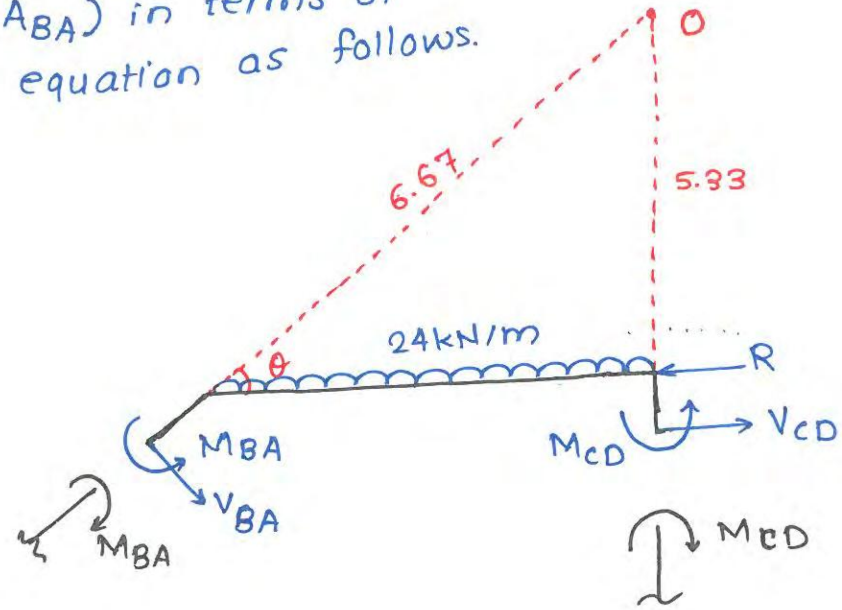
Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	–	0.44	0.56	0.5	0.5	–
FEM	0	0	-32	32	0	0
Bal						
Final	9.41	18.9	-18.9	22.06	-22.06	-10.96

Step IV: Calculation of R :-

$$\Sigma F_x = 0$$

$$\Rightarrow V_{BA}\sin\theta + V_{CD} - R - A_{BA}\cos\theta = 0.$$

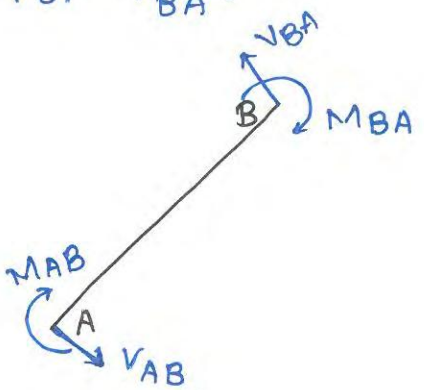
Since it is difficult to convert axial force of member (A_{BA}) in terms of end moments so writing shear equation as follows.



$$\Sigma M_O = 0$$

$$\Rightarrow -M_{BA} - V_{BA}\times 6.67 - M_{CD} - V_{CD}\times 5.33 + R\times 5.33 - 24\times 4\times 2 = 0 \quad \text{------(i)}$$

For V_{BA} :-



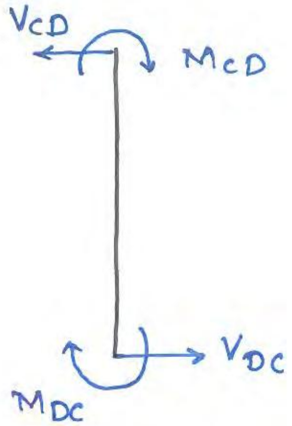
$$\Sigma M_A = 0$$

$$\Rightarrow V_{BA} = \frac{M_{AB} + M_{BA}}{5}$$

$$= \frac{9.41 + 18.9}{5}$$

$$V_{BA} = 5.66 \text{ kN}.$$

For V_{CD} :-



$$\Sigma M_D = 0$$

$$\Rightarrow V_{CD} = \frac{M_{CD} + M_{DC}}{4}$$

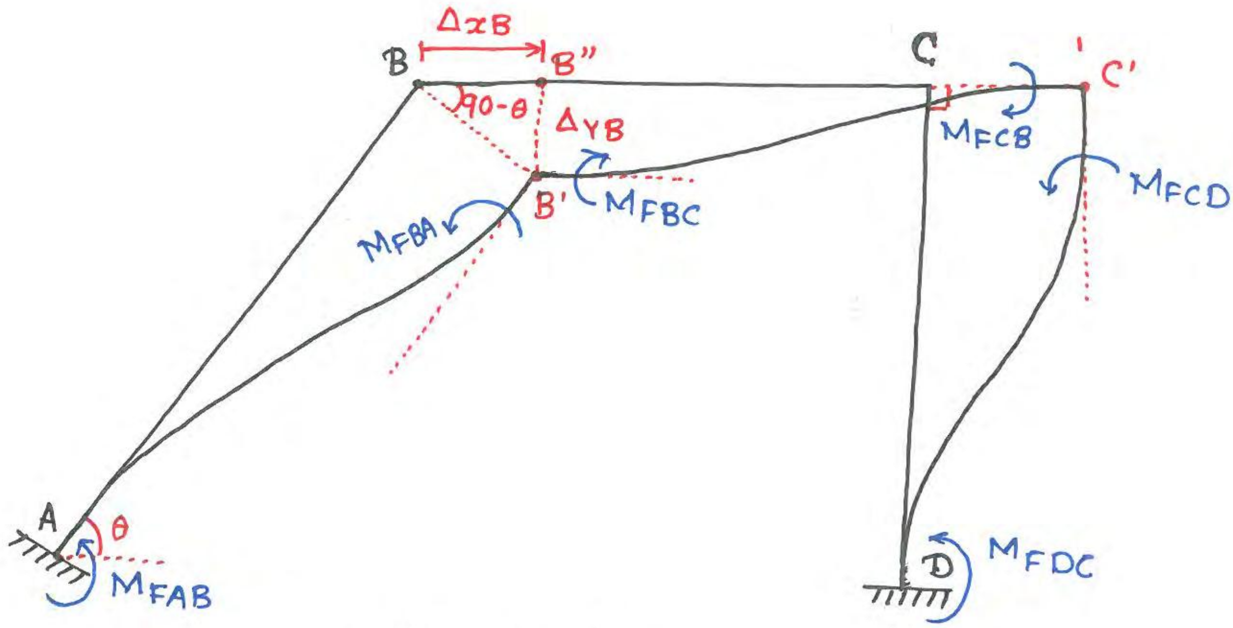
$$= \frac{-22.06 - 10.96}{4}$$

$$= -8.25 \text{ kN}$$

from equation (i)

$$R = 34.27 \text{kN}$$

Step V :-



$$BB'' = \Delta x_B = \Delta$$

$$BB' = \frac{\Delta x_B}{\sin\theta}$$

$$= \frac{5\Delta}{4}$$

$$B'B'' = \Delta x_B \cot\theta$$

$$= \frac{3\Delta}{4}$$

$$CC' = \Delta x_C = \Delta x_B = \Delta.$$

$M_{FAB} : M_{FBA} : M_{FBC} : M_{FCB} : M_{FCD} : M_{FDC}$

$$\Rightarrow \frac{-6EI(BB')}{5^2} : \frac{-6EI(BB')}{5^2} : \frac{6EI(B'B'')}{4^2} : \frac{6EI(B'B'')}{4^2} : \frac{-6EI(CC'')}{4^2} : \frac{-6EI(CC'')}{4^2}$$

$$\Rightarrow \frac{-5\Delta/4}{5^2} : \frac{-5\Delta/4}{5^2} : \frac{3\Delta/4}{4^2} : \frac{3\Delta/4}{4^2} : \frac{-\Delta}{4^2} : \frac{-\Delta}{4^2}$$

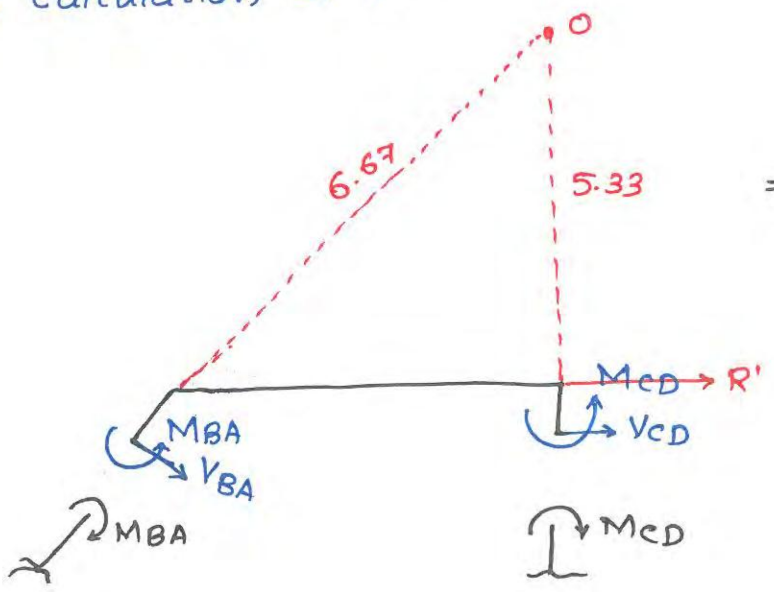
$$\Rightarrow -0.05 : -0.05 : 0.047 : 0.047 : -0.0625 : -0.0625$$

Step VI: Multiplying by any arbitary value (assuming 600)

$$\Rightarrow -30 : -30 : 28.2 : 28.2 : -37.5 : -37.5$$

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	–	0.44	0.56	0.5	0.5	–
FEM	-30	-30	28.2	28.2	-37.5	-37.5
Final	-30.3	-30.17	30.17	32.72	-32.72	-35.10.

Step VII: Calculation of R':-



$\Sigma M_O = 0$

$\Rightarrow -V_{BA} \times 6.67 - M_{BA}$

$- V_{CD} \times 5.33 - M_{CD}$

$- R' \times 5.33 = 0 \cdots (i)$

Since there is no loading on member BA and CD so expression derived for V_{BA} and V_{CD} earlier will remain same.

$$V_{BA} = \frac{M_{AB} + M_{BA}}{5} = \frac{-30.3 - 30.17}{5} = -12.04\,kN$$

$$V_{CD} = \frac{M_{CD} + M_{DC}}{4} = \frac{-32.72 - 35.10}{4} = -16.95\,kN.$$

from equation (ii) $\Rightarrow$

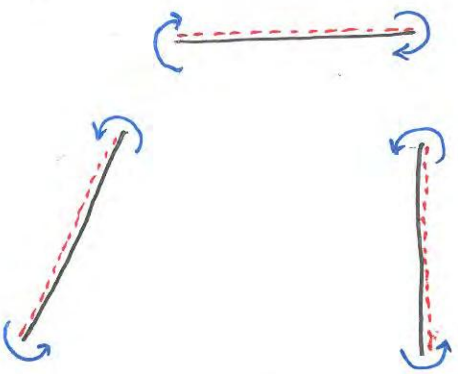
$$R' = 43.81\,kN.$$

Step VIII: Correction factor:-

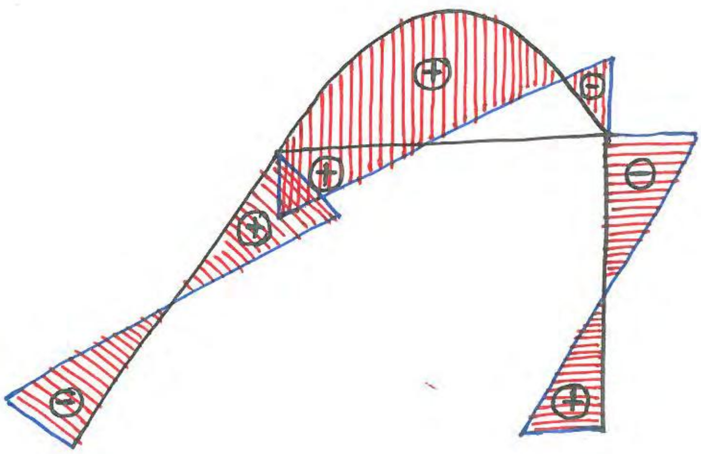
$$\text{Correction factor} = \frac{R}{R'} = \frac{34.27}{43.81} = 0.78$$

Member	AB	BA	BC	CB	CD	DC
$M_{Non\ sway}$						
M_{sway}						
$M_{final} = M_{N.S} + \frac{R}{R'} M_S$	-14.21	-4.83	4.83	47.81	-47.81	-38.53.

Step IX: BMD:



Ref. face → Outside.



BMD