

CHAPTER

7

Term-II

INTEGRALS

Syllabus

- **Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts. Evaluation of simple integrals of the following types and problems based on them.**

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}}, \int \frac{px + q}{ax^2 + bx + c} dx,$$

$$\int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx, \int \sqrt{a^2 \pm x^2} dx, \int \sqrt{x^2 - a^2} dx$$

Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.



STAND ALONE MCQs

(1 Mark each)

Q. 1. $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta} dx$ is equal to

- (A) $2(\sin x + x \cos \theta) + C$
 (B) $2(\sin x - x \cos \theta) + C$
 (C) $2(\sin x + 2x \cos \theta) + C$
 (D) $2(\sin x - 2x \cos \theta) + C$

Ans. Option (A) is correct.

Explanation : Let,

$$\begin{aligned} I &= \int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta} dx \\ &= \int \frac{(2\cos^2 x - 1 - 2\cos^2 \theta + 1)}{\cos x - \cos \theta} dx \\ &= 2 \int \frac{(\cos x + \cos \theta)(\cos x - \cos \theta)}{(\cos x - \cos \theta)} dx \\ &= 2 \int (\cos x + \cos \theta) dx \\ &= 2 \sin x + 2x \cos \theta + C \end{aligned}$$

Q. 2. The value of $\int_{-\pi/2}^{\pi/2} (x^3 + x \cos x + \tan^5 x + 1) dx$ is

- (A) 0
 (B) 2
 (C) π
 (D) 1

Ans. Option (C) is correct.

Explanation : Let,

$$\begin{aligned} I &= \int_{-\pi/2}^{\pi/2} (x^3 + x \cos x + \tan^5 x + 1) dx \\ &= \int_{-\pi/2}^{\pi/2} x^3 dx + \int_{-\pi/2}^{\pi/2} x \cos x + \int_{-\pi/2}^{\pi/2} \tan^5 x dx + \int_{-\pi/2}^{\pi/2} 1 \cdot dx \end{aligned}$$

It is known that if $f(x)$ is an even function, then

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$$

and if $f(x)$ is an odd function, then

$$\int_{-a}^a f(x) dx = 0$$

$$\therefore I = 0 + 0 + 0 + 2 \int_0^{\pi/2} 1 \cdot dx$$

$$= 2 \left[x \right]_0^{\pi/2} = \frac{2\pi}{2} = \pi$$

Q. 3. $\int \frac{dx}{e^x + e^{-x}}$ is equal to

- (A) $\tan^{-1}(e^x) + C$
 (B) $\tan^{-1}(e^{-x}) + C$
 (C) $\log(e^x - e^{-x}) + C$
 (D) $\log(e^x + e^{-x}) + C$

Ans. Option (A) is correct.

Explanation:

$$\begin{aligned} \text{Let } I &= \int \frac{dx}{e^x + e^{-x}} dx \\ &= \int \frac{e^x}{e^{2x} + 1} dx \end{aligned}$$

$$\begin{aligned} \text{Also, let } e^x &= t \\ \Rightarrow e^x dx &= dt \\ \Rightarrow I &= \int \frac{dt}{1+t^2} \\ &= \tan^{-1} t + C \\ &= \tan^{-1}(e^x) + C \end{aligned}$$

Q. 4. $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$ is equal to

- (A) $\frac{-1}{\sin x + \cos x} + C$
 (B) $\log |\sin x + \cos x| + C$
 (C) $\log |\sin x - \cos x| + C$
 (D) $\frac{1}{(\sin x + \cos x)^2}$

Ans. Option (B) is correct.

Explanation:

$$\begin{aligned} \text{Let } I &= \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx \\ I &= \int \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)^2} dx \\ &= \int \frac{(\cos x + \sin x)(\cos x - \sin x)}{(\cos x + \sin x)^2} dx \\ &= \int \frac{\cos x - \sin x}{\cos x + \sin x} dx \\ \text{Let } \cos x + \sin x &= t \\ \Rightarrow (\cos x - \sin x) dx &= dt \\ \Rightarrow I &= \int \frac{dt}{t} \\ &= \log|t| + C \\ &= \log|\cos x + \sin x| + C \end{aligned}$$

Q. 5. If $f(a + b - x) = f(x)$, then $\int_a^b xf(x)dx$ is equal to

- (A) $\frac{a+b}{2} \int_a^b f(b-x)dx$ (B) $\frac{a+b}{2} \int_a^b f(b+x)dx$
 (C) $\frac{b-a}{2} \int_a^b f(x)dx$ (D) $\frac{a+b}{2} \int_a^b f(x)dx$

Ans. Option (D) is correct.

Explanation:

$$\begin{aligned} \text{Let } I &= \int_a^b xf(x)dx \quad \dots(i) \\ I &= \int_a^b (a+b-x)f(a+b-x)dx \\ &\quad \left[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx \right] \\ \Rightarrow I &= \int_a^b (a+b-x)f(x)dx \\ \Rightarrow I &= (a+b) \int_a^b f(x)dx - I \quad [\text{Using Eq. (i)}] \\ \Rightarrow I + I &= (a+b) \int_a^b f(x)dx \\ \Rightarrow 2I &= (a+b) \int_a^b f(x)dx \\ \Rightarrow I &= \left(\frac{a+b}{2} \right) \int_a^b f(x)dx \end{aligned}$$

Q. 6. The value of $\int_0^1 \tan^{-1} \left(\frac{2x-1}{1+x-x^2} \right) dx$ is

- (A) 1 (B) 0
 (C) -1 (D) $\frac{\pi}{4}$

Ans. Option (B) is correct.

Explanation :

$$\begin{aligned} \text{Let } I &= \int_0^1 \tan^{-1} \left(\frac{2x-1}{1+x-x^2} \right) dx \\ \Rightarrow I &= \int_0^1 \tan^{-1} \left(\frac{x-(1-x)}{1+x(1-x)} \right) dx \\ \Rightarrow I &= \int_0^1 [\tan^{-1} x - \tan^{-1}(1-x)] dx \quad \dots(i) \\ \Rightarrow I &= \int_0^1 [\tan^{-1}(1-x) - \tan^{-1}(1-1+x)] dx \\ \Rightarrow I &= \int_0^1 [\tan^{-1}(1-x) - \tan^{-1}(x)] dx \\ \Rightarrow I &= \int_0^1 [\tan^{-1}(1-x) - \tan^{-1}(x)] dx \quad \dots(ii) \end{aligned}$$

Adding equations (i) and (ii), we obtain

$$\begin{aligned} 2I &= \int_0^1 (\tan^{-1} x + \tan^{-1}(1-x) \\ &\quad - \tan^{-1}(1-x) - \tan^{-1} x) dx \\ \Rightarrow 2I &= 0 \\ \Rightarrow I &= 0 \end{aligned}$$

Q. 7. $\frac{dx}{\sin(x-a) \sin(x-b)}$ is equal to

- (A) $\sin(b-a) \log \left| \frac{\sin(x-b)}{\sin(x-a)} \right| + C$
 (B) $\operatorname{cosec}(b-a) \log \left| \frac{\sin(x-a)}{\sin(x-b)} \right| + C$
 (C) $\operatorname{cosec}(b-a) \log \left| \frac{\sin(x-b)}{\sin(x-a)} \right| + C$
 (D) $\sin(b-a) \log \left| \frac{\sin(x-a)}{\sin(x-b)} \right| + C$

Ans. Option (C) is correct.

Explanation : Let,

$$\begin{aligned}
 I &= \int \frac{dx}{\sin(x-a)\sin(x-b)} \\
 &= \frac{1}{\sin(b-a)} \int \frac{\sin(b-a)}{\sin(x-a)\sin(x-b)} dx \\
 &= \frac{1}{\sin(b-a)} \int \frac{\sin(x-a-x+b)}{\sin(x-a)\sin(x-b)} dx \\
 &= \frac{1}{\sin(b-a)} \int \frac{\sin\{(x-a)-(x-b)\}}{\sin(x-a)\sin(x-b)} dx \\
 &= \frac{1}{\sin(b-a)} \int \frac{\sin(x-a)\cos(x-b) - \cos(x-a)\sin(x-b)}{\sin(x-a)\sin(x-b)} dx \\
 &= \frac{1}{\sin(b-a)} \int [\cot(x-b) - \cot(x-a)] dx \\
 &= \frac{1}{\sin(b-a)} [\log|\sin(x-b)| - \log|\sin(x-a)|] + C \\
 &= \operatorname{cosec}(b-a) \log \left| \frac{\sin(x-b)}{\sin(x-a)} \right| + C
 \end{aligned}$$

Q. 8. $\int \sqrt{1+x^2} dx$ is equal to

- (A) $\frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\log\left|x + \sqrt{1+x^2}\right| + C$
 (B) $\frac{2}{3}(1+x^2)^{3/2} + C$
 (C) $\frac{2}{3}x(1+x^2)^{3/2} + C$
 (D) $\frac{x^2}{2}\sqrt{1+x^2} + \frac{1}{2}x^2 \log\left|x + \sqrt{1+x^2}\right| + C$

Ans. Option (A) is correct.

Explanation : It is known that,

$$\begin{aligned}
 \int \sqrt{a^2+x^2} dx &= \frac{x}{2}\sqrt{a^2+x^2} \\
 &\quad + \frac{a^2}{2} \log\left|x + \sqrt{x^2+a^2}\right| + C \\
 \therefore \int \sqrt{1+x^2} dx &= \frac{x}{2}\sqrt{1+x^2} \\
 &\quad + \frac{1}{2} \log\left|x + \sqrt{1+x^2}\right| + C
 \end{aligned}$$

Q. 9. $\int \frac{xdx}{(x-1)(x-2)}$ equals

- (A) $\log\left|\frac{(x-1)^2}{x-2}\right| + C$
 (B) $\log\left|\frac{(x-2)^2}{x-1}\right| + C$
 (C) $\log\left|\left(\frac{x-1}{x-2}\right)^2\right| + C$
 (D) $\log|(x-1)(x-2)| + C$

Ans. Option (B) is correct.

Explanation :

$$\begin{aligned}
 \text{Let } \frac{x}{(x-1)(x-2)} &= \frac{A}{(x-1)} + \frac{B}{(x-2)} \\
 x &= A(x-2) + B(x-1) \quad \dots(i)
 \end{aligned}$$

Substituting $x = 1$ and 2 in Eq. (i), we obtain

$$A = -1 \text{ and } B = 2$$

$$\begin{aligned}
 \therefore \frac{x}{(x-1)(x-2)} &= -\frac{1}{(x-1)} + \frac{2}{(x-2)} \\
 \Rightarrow \int \frac{x}{(x-1)(x-2)} dx &= \int \left\{ \frac{-1}{(x-1)} + \frac{2}{(x-2)} \right\} dx \\
 &= -\log|x-1| + 2\log|x-2| + C \\
 &= \log\left|\frac{(x-2)^2}{x-1}\right| + C
 \end{aligned}$$

Q. 10. If $f(x) = \int_0^x t \sin t dt$, then $f'(x)$ is

- (A) $\cos x + x \sin x$
 (B) $x \sin x$
 (C) $x \cos x$
 (D) $\sin x + x \cos x$

Ans. Option (B) is correct.

Explanation :

$$f(x) = \int_0^x t \sin t dt$$

Integrating by parts, we obtain

$$\begin{aligned}
 f(x) &= t \int_0^x \sin t dt - \int_0^x \left\{ \left(\frac{d}{dt} t \right) \int \sin t dt \right\} dt \\
 &= [t(-\cos t)]_0^x - \int_0^x (-\cos t) dt \\
 &= [-t \cos t + \sin t]_0^x \\
 &= -x \cos x + \sin x \\
 \Rightarrow f'(x) &= -[x(-\sin x)] + \cos x + \cos x \\
 &= x \sin x - \cos x + \cos x \\
 &= x \sin x
 \end{aligned}$$

Q. 11. $\int \tan^{-1} \sqrt{x} dx$ is equal to

- (A) $(x+1)\tan^{-1} \sqrt{x} - \sqrt{x} + C$
 (B) $x \tan^{-1} \sqrt{x} - \sqrt{x} + C$
 (C) $\sqrt{x} - x \tan^{-1} \sqrt{x} + C$
 (D) $\sqrt{x} - (x+1) \tan^{-1} \sqrt{x} + C$

Ans. Option (A) is correct.

Explanation : Let,

$$\begin{aligned}
 I &= \int 1 \cdot \tan^{-1} \sqrt{x} dx \\
 &= \tan^{-1} \sqrt{x} \cdot x - \frac{1}{2} \int \frac{1}{(1+x)} \cdot \frac{2}{\sqrt{x}} dx \\
 &= x \tan^{-1} \sqrt{x} - \frac{1}{2} \int \frac{2}{\sqrt{x}(1+x)} dx
 \end{aligned}$$

Put $x = t^2$

$$\Rightarrow dx = 2t dt$$

$$\begin{aligned}\therefore I &= x \tan^{-1} \sqrt{x} - \int \frac{t}{t(1+t^2)} dt \\ &= x \tan^{-1} \sqrt{x} - \int \frac{t^2}{1+t^2} dt \\ &= x \tan^{-1} \sqrt{x} - \int \left(1 - \frac{1}{1+t^2}\right) dt \\ &= x \tan^{-1} \sqrt{x} - \sqrt{x} + \tan^{-1} t + C \\ &= x \tan^{-1} \sqrt{x} - \sqrt{x} + \tan^{-1} \sqrt{x} + C \\ &\quad - (x+1) \tan^{-1} \sqrt{x} - \sqrt{x} + C\end{aligned}$$

Q. 12. $\int x^2 e^{x^3} dx$ is equal to

- (A) $\frac{1}{3} e^{x^3} + C$ (B) $\frac{1}{3} e^{x^2} + C$
(C) $\frac{1}{2} e^{x^3} + C$ (D) $\frac{1}{2} e^{x^2} + C$

Ans. Option (A) is correct.

Explanation:

$$\text{Let } I = \int x^2 e^{x^3} dx$$

$$\text{Also, let } x^3 = t$$

$$\Rightarrow 3x^2 dx = dt$$

$$\begin{aligned}\Rightarrow I &= \frac{1}{3} \int e^t dt \\ &= \frac{1}{3} (e^t) + C \\ &= \frac{1}{3} e^{x^3} + C\end{aligned}$$

Q. 13. $\int e^x \sec x (1 + \tan x) dx$ is equal to

- (A) $e^x \cos x + C$ (B) $e^x \sec x + C$
(C) $e^x \sin x + C$ (D) $e^x \tan x + C$

Ans. Option (B) is correct.

Explanation : $\int e^x \sec x (1 + \tan x) dx$

Let

$$I = \int e^x \sec x (1 + \tan x) dx = \int e^x (\sec x + \sec x \tan x) dx$$

$$\text{Also, let } \sec x = f(x) \Rightarrow \sec x \tan x = f'(x)$$

$$\text{It is known that, } \int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$$

$$\therefore I = e^x \sec x + C$$

Q. 14. $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$ is equal to

- (A) $\tan x + \cot x + C$
(B) $\tan x + \operatorname{cosec} x + C$
(C) $-\tan x + \cot x + C$
(D) $\tan x + \sec x + C$

Ans. Option (A) is correct.

Explanation:

$$\begin{aligned}\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx &= \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} - \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx \\ &= \int (\sec^2 x - \operatorname{cosec}^2 x) dx \\ &= \tan x + \cot x + C\end{aligned}$$

Q. 15. $\int \frac{dx}{\sin^2 x \cos^2 x}$ is equal to

- (A) $\tan x + \cot x + C$ (B) $\tan x - \cot x + C$
(C) $\tan x \cot x + C$ (D) $\tan x - \cot 2x + C$

Ans. Option (B) is correct.

Explanation:

$$\begin{aligned}\text{Let } I &= \int \frac{dx}{\sin^2 x \cos^2 x} \\ &= \int \frac{1}{\sin^2 x \cos^2 x} dx \\ &= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx \\ &= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} dx + \int \frac{\cos^2 x}{\sin^2 x \cos^2 x} dx \\ &= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx \\ &= \tan x - \cot x + C\end{aligned}$$

Q. 16. $\int \frac{x^9}{(4x^2 + 1)^6} dx$ is equal to

- (A) $\frac{1}{5x} \left(4 + \frac{1}{x^2}\right)^{-5} + C$ (B) $\frac{1}{5} \left(4 + \frac{1}{x^2}\right)^{-5} + C$
(C) $\frac{1}{10x} (1+4)^{-5} + C$ (D) $\frac{1}{10} \left(\frac{1}{x^2} + 4\right)^{-5} + C$

Ans. Option (D) is correct.

Explanation : Let

$$\begin{aligned}I &= \int \frac{x^9}{(4x^2 + 1)^6} dx \\ &= \int \frac{x^9}{x^{12} \left(4 + \frac{1}{x^2}\right)^6} dx \\ &= \int \frac{dx}{x^3 \left(4 + \frac{1}{x^2}\right)^6}\end{aligned}$$

$$\text{Put } 4 + \frac{1}{x^2} = t$$

$$\Rightarrow \frac{-2}{x^3} dx = dt$$

$$\Rightarrow \frac{1}{x^3} dx = -\frac{1}{2} dt$$

$$\begin{aligned}\therefore I &= -\frac{1}{2} \int \frac{dt}{t^6} \\ &= -\frac{1}{2} \left[\frac{t^{-6+1}}{-6+1} \right] + C\end{aligned}$$

$$= \frac{1}{10} \left[\frac{1}{t^5} \right] + C$$

$$= \frac{1}{10} \left(4 + \frac{1}{x^2} \right)^{-5} + C$$

Q. 17. $\int \frac{x^3}{x+1}$ is equal to

- (A) $x + \frac{x^2}{2} + \frac{x^3}{3} - \log|1-x| + C$
- (B) $x + \frac{x^2}{2} - \frac{x^3}{3} - \log|1-x| + C$
- (C) $x - \frac{x^2}{2} - \frac{x^3}{3} - \log|1+x| + C$
- (D) $x - \frac{x^2}{2} + \frac{x^3}{3} - \log|1+x| + C$

Ans. Option (D) is correct.

Explanation : Let,

$$I = \int \frac{x^3}{x+1} dx$$

$$= \int \left((x^2 - x + 1) - \frac{1}{(x+1)} \right) dx$$

$$= \frac{x^3}{3} - \frac{x^2}{2} + x - \log|x+1| + C$$

Q. 18. If $\int \frac{x^3 dx}{\sqrt{1+x^2}} = a(1+x^2)^{3/2} + b\sqrt{1+x^2} + C$, then

- (A) $a = \frac{1}{3}, b = 1$ (B) $a = \frac{-1}{3}, b = 1$
- (C) $a = \frac{-1}{3}, b = -1$ (D) $a = \frac{1}{3}, b = -1$

Ans. Option (D) is correct.

Explanation: Let,

$$I = \int \frac{x^3}{\sqrt{1+x^2}} dx$$

$$= a(1+x^2)^{3/2} + b\sqrt{1+x^2} + C$$

$$\therefore I = \int \frac{x^3}{\sqrt{1+x^2}} dx$$

$$= \int \frac{x^2 \cdot x}{\sqrt{1+x^2}} dx$$

$$\text{Put } 1+x^2 = t^2$$

$$\Rightarrow 2x dx = 2t dt$$

$$\therefore I = \int \frac{t(t^2-1)}{t} dt$$

$$= \frac{t^3}{3} - t + C$$

$$= \frac{1}{3}(1+x^2)^{3/2} - \sqrt{1+x^2} + C$$

$$\therefore a = \frac{1}{3} \text{ and } b = -1$$

Q. 19. $\int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos 2x}$ is equal to

- (A) 1 (B) 2
- (C) 3 (D) 4

Ans. Option (A) is correct.

Explanation : Let

$$I = \int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos 2x}$$

$$= \int_{-\pi/4}^{\pi/4} \frac{dx}{2\cos^2 x}$$

$$= \frac{1}{2} \int_{-\pi/4}^{\pi/4} \sec^2 x dx$$

$$= \int_0^{\pi/4} \sec^2 x dx$$

$$= [\tan x]_0^{\pi/4}$$

$$= 1$$

Q. 20. $\int \frac{dx}{x^2+2x+2}$ equals

- (A) $x \tan^{-1}(x+1) + C$ (B) $\tan^{-1}(x+1) + C$
- (C) $(x+1) x \tan^{-1} + C$ (D) $\tan^{-1} + C$

Ans. Option (B) is correct.

Explanation :

$$\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x^2+2x+1)+1}$$

$$= \int \frac{1}{(x+1)^2+(1)^2} dx$$

$$= [\tan^{-1}(x+1)] + C$$

Q. 21. $\int_0^{\pi/2} \sqrt{1-\sin 2x} dx$ is equal to

- (A) $2\sqrt{2}$ (B) $2(\sqrt{2}+1)$
- (C) 2 (D) $2(\sqrt{2}-1)$

Ans. Option (D) is correct.

Explanation: Let

$$I = \int_0^{\pi/2} \sqrt{1-\sin 2x} dx$$

$$= \int_0^{\pi/4} \sqrt{(\cos x - \sin x)^2} dx$$

$$+ \int_{\pi/4}^{\pi/2} \sqrt{(\sin x - \cos x)^2} dx$$

$$= [\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{\pi/2}$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 0 - 1 + \left(-0 - 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right)$$

$$= 2\sqrt{2} - 2$$

$$= 2(\sqrt{2}-1)$$

Q. 22. The anti-derivative of $\left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)$ equals

- (A) $\frac{1}{3}x^{1/3} + 2x^{1/2} + C$ (B) $\frac{2}{3}x^{2/3} + \frac{1}{2}x^2 + C$
- (C) $\frac{2}{3}x^{2/3} + 2x^{1/2} + C$ (D) $\frac{3}{2}x^{3/2} + \frac{1}{2}x^{1/2} + C$

Ans. Option (C) is correct.

Explanation:

$$\begin{aligned}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx &= \int x^{1/2}dx + \int x^{-1/2}dx \\ &= \frac{x^{3/2}}{\frac{3}{2}} + \frac{x^{1/2}}{\frac{1}{2}} + C \\ &= \frac{2}{3}x^{3/2} + 2x^{1/2} + C\end{aligned}$$

Q. 23. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ equals

- (A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$
(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{12}$

Ans. Option (D) is correct.

Explanation:

$$\int \frac{dx}{1+x^2} = \tan^{-1} x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}\int_1^{\sqrt{3}} \frac{dx}{1+x^2} &= F(\sqrt{3}) - F(1) \\ &= \tan^{-1} \sqrt{3} - \tan^{-1} 1 \\ &= \frac{\pi}{3} - \frac{\pi}{4} \\ &= \frac{\pi}{12}\end{aligned}$$

Q. 24. $\int_0^{2/3} \frac{dx}{4+9x^2}$ equals

- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{12}$
(C) $\frac{\pi}{24}$ (D) $\frac{\pi}{4}$

Ans. Option (C) is correct.

Explanation :

$$\int \frac{dx}{4+9x^2} = \int \frac{dx}{(2)^2 + (3x)^2}$$

Put $3x = t$

$\Rightarrow 3dx = dt$

$$\begin{aligned}\therefore \int \frac{dx}{(2)^2 + (3x)^2} &= \frac{1}{3} \int \frac{dt}{(2)^2 + (t)^2} \\ &= \frac{1}{3} \left[\frac{1}{2} \tan^{-1} \frac{t}{2} \right] \\ &= \frac{1}{6} \tan^{-1} \left(\frac{3x}{2} \right) \\ &= F(x)\end{aligned}$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}\int_0^{2/3} \frac{dx}{4+9x^2} &= F\left(\frac{2}{3}\right) - F(0) \\ &= \frac{1}{6} \tan^{-1} \left[\frac{3}{2} \times \frac{2}{3} \right] - \frac{1}{6} \tan^{-1}(0) \\ &= \frac{1}{6} \tan^{-1}(1) \\ &= \frac{1}{6} \tan^{-1} \left[\tan \frac{\pi}{4} \right] \\ &= \frac{\pi}{24}\end{aligned}$$

Q. 25. If $\frac{d}{dx} f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$. Then $f(x)$ is

- (A) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ (B) $x^3 + \frac{1}{x^4} + \frac{129}{8}$
(C) $x^4 + \frac{1}{x^3} + \frac{129}{8}$ (D) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

Ans. Option (A) is correct.

Explanation : It is given that,

$$\frac{d}{dx} f(x) = 4x^3 - \frac{3}{x^4}$$

$\therefore$ Anti-derivative of $4x^3 - \frac{3}{x^4} = f(x)$

$$\therefore f(x) = \int 4x^3 - \frac{3}{x^4} dx$$

$$f(x) = 4 \int x^3 dx - 3 \int (x^{-4}) dx$$

$$\therefore f(x) = 4 \left(\frac{x^4}{4} \right) - 3 \left(\frac{x^{-3}}{-3} \right) + C$$

$$f(x) = x^4 + \frac{1}{x^3} + C$$

Also,

$$f(2) = 0$$

$$\begin{aligned}\therefore f(2) &= (2)^4 + \frac{1}{(2)^3} + C \\ &= 0\end{aligned}$$

$$\Rightarrow 16 + \frac{1}{8} + C = 0$$

$$\Rightarrow C = -\left(16 + \frac{1}{8}\right)$$

$$\Rightarrow C = \frac{-129}{8}$$

$$\therefore f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$$



ASSERTION AND REASON BASED MCQs

(1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:

- (A) Both A and R are true and R is the correct explanation of A
- (B) Both A and R are true but R is NOT the correct explanation of A
- (C) A is true but R is false
- (D) A is false and R is True

Q. 1. Assertion (A): $\int \frac{dx}{x^2 + 2x + 3} = \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x+1}{\sqrt{2}} \right) + c$

Reason (R): $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c$

Ans. Option (A) is correct.

Explanation:

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c.$$

This is a standard integral and hence R is true.

$$\begin{aligned} \int \frac{dx}{x^2 + 2x + 3} &= \int \frac{dx}{(x+1)^2 + (\sqrt{2})^2} \\ &= \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x+1}{\sqrt{2}} \right) + c \end{aligned}$$

Hence A is true and R is the correct explanation for A.

Q. 2. Assertion(A): $\int e^x [\sin x - \cos x] dx = e^x \sin x + C$

Reason (R): $\int e^x [f(x) + f'(x)] dx = e^x f(x) + c$

Ans. Option (D) is correct.

Explanation:

$$\begin{aligned} \int e^x [f(x) + f'(x)] dx &= \int e^x f(x) dx + \int e^x f'(x) dx \\ &= f(x)e^x - \int f'(x)e^x dx + \int f'(x)e^x dx \\ &= e^x f(x) + c \end{aligned}$$

Hence R is true.

$$\begin{aligned} \int e^x (\sin x - \cos x) dx &= e^x (-\cos x) + c \\ &= -e^x \cos x + c \end{aligned}$$

$$\left[\because \frac{d}{dx} (-\cos x) = \sin x \right]$$

Hence A is false.

Q. 3. Assertion (A): $\int x^x (1 + \log x) dx = x^x + c$

Reason (R): $\frac{d}{dx} (x^x) = x^x (1 + \log x)$

Ans. Option (A) is correct.

Explanation : Let $y = x^x$

$$\Rightarrow \log y = x \log x$$

Differentiating w.r.t. x

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = x \left(\frac{1}{x} \right) + \log x (1)$$

$$\frac{dy}{dx} = y(1 + \log x)$$

$$= x^x (1 + \log x)$$

Hence R is true.

Since $\frac{d}{dx} (x^x) = x^x (1 + \log x)$

$$\int x^x (1 + \log x) dx = x^x + c$$

Using the concept of anti-derivative, A is true.
R is the correct explanation for A.

Q. 4. Assertion (A): $\int x^2 dx = \frac{x^3}{3} + c$

Reason (R): $\int e^{x^2} dx = e^{x^{3/3}} + c$

Ans. Option (C) is correct.

Explanation:

Since $\int x^n dx = \frac{x^{n+1}}{n+1} + c,$

$$\begin{aligned} \int x^2 dx &= \frac{x^{2+1}}{2+1} + c \\ &= \frac{x^3}{3} + c \end{aligned}$$

$\therefore$ A is true.

$\int e^{x^2} dx$ is a function

which can not be integrated.

$\therefore$ R is false.

Q. 5. Assertion (A): $\int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x} dx = \frac{\pi}{4}$

Reason (R): $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx = \frac{\pi}{4}$

Ans. Option (A) is correct.

Explanation:

$$\text{Let } I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx \quad \dots(i)$$

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$\therefore I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2}-x\right) dx}{\sin\left(\frac{\pi}{2}-x\right) + \cos\left(\frac{\pi}{2}-x\right)}$$

$$I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} dx \quad \dots(ii)$$

Adding equations (i) + (ii),

$$\begin{aligned} \Rightarrow 2I &= \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx \\ &= \int_0^{\pi/2} 1 dx \\ &= [x]_0^{\pi/2} \\ &= \frac{\pi}{2} \\ \therefore I &= \frac{\pi}{4} \end{aligned}$$

Hence R is true.

From (ii), A is also true.

R is the correct explanation for A.

Q. 6. Assertion (A): $\int_{-3}^3 (x^3 + 5) dx = 30$

Reason (R): $f(x) = x^3 + 5$ is an odd function.

Ans. Option (C) is correct.

Explanation:

$$\begin{aligned} \text{Let } f(x) &= x^3 + 5 \\ f(-x) &= (-x)^3 + 5 \\ &= -x^3 + 5 \end{aligned}$$

$f(x)$ is neither even nor odd. Hence R is false.

$$\begin{aligned} \int_{-3}^3 x^3 dx &= 0 \quad [\because x^3 \text{ is odd}] \\ \int_{-3}^3 5 dx &= 5[x]_{-3}^3 = 30 \end{aligned}$$

$$\therefore \int_{-3}^3 (x^3 + 5) dx = 0 + 30 = 30$$

Hence A is true.

Q. 7. Assertion (A): $\frac{d}{dx} \left[\int_0^{x^2} \frac{dt}{t^2 + 4} \right] = \frac{2x}{x^4 + 4}$

Reason (R): $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c$

Ans. Option (A) is correct.

Explanation:

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c.$$

This is a standard integral and hence true.

So R is true.

$$\begin{aligned} \int_0^{x^2} \frac{dt}{t^2 + 4} &= \left[\frac{1}{2} \tan^{-1} \left(\frac{t}{2} \right) \right]_0^{x^2} \\ &= \frac{1}{2} \tan^{-1} \left(\frac{x^2}{2} \right) \end{aligned}$$

$$\begin{aligned} \frac{d}{dx} \left[\int_0^{x^2} \frac{dt}{t^2 + 4} \right] &= \frac{d}{dx} \left[\frac{1}{2} \tan^{-1} \left(\frac{x^2}{2} \right) \right] \\ &= \frac{1}{2} \times \frac{1}{1 + \frac{x^4}{4}} \times \frac{2x}{2} \\ &= \frac{x}{2} \times \frac{4}{4 + x^4} \\ &= \frac{2x}{4 + x^4} \end{aligned}$$

Hence A is true and R is the correct explanation for A.

Q. 8. Assertion (A): $\int_{-1}^1 (x^3 + \sin x + 2) dx = 0$

Reason (R):

$$\int_{-a}^a f(x) dx = \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f(x) \text{ is an even function} \\ & \text{i.e., } f(-x) = f(x) \\ 0, & \text{if } f(x) \text{ is an odd function} \\ & \text{i.e., } f(-x) = -f(x) \end{cases}$$

Ans. Option (D) is correct.

Explanation:

$$\int_{-a}^a f(x) dx = \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f(x) \text{ is an even function} \\ & \text{i.e., } f(-x) = f(x) \\ 0, & \text{if } f(x) \text{ is an odd function} \\ & \text{i.e., } f(-x) = -f(x) \end{cases}$$

This is a property of the definite integrals and hence R is true.

$$\begin{aligned} &\int_{-1}^1 (x^3 + \sin x + 2) dx \\ &= \int_{-1}^1 x^3 dx + \int_{-1}^1 \sin x dx + \int_{-1}^1 2 dx \\ &\quad \downarrow \quad \quad \quad \downarrow \\ &\quad \text{Odd function} \quad \quad \text{Even function} \\ &= 0 + 2[x]_{-1}^1 \\ &= 2 \times 2 \\ &= 4 \end{aligned}$$

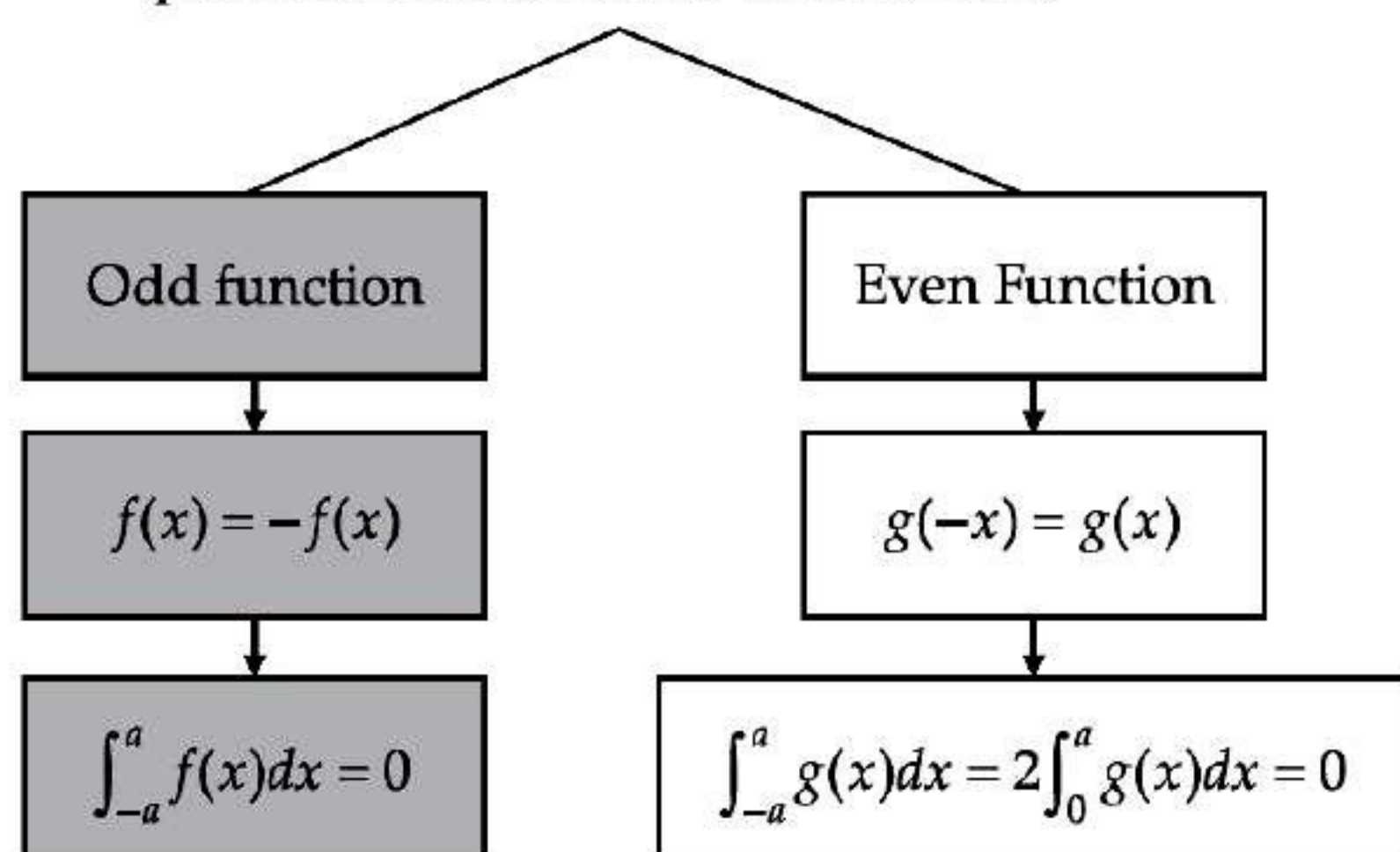
Hence A is false.



CASE-BASED MCQs

Attempt any four sub-parts from each question.
Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:



Q. 1. $\int_{-1}^1 x^{99} dx = \underline{\hspace{2cm}}$.

- (A) 0 (B) 1
(C) -1 (D) 2

Ans. Option (A) is correct.

Explanation:

$$\int_{-1}^1 x^{99} dx = 0, \text{ since } x^{99} \text{ is an odd function.}$$

Q. 2. $\int_{-\pi}^{\pi} x \cos x dx = \underline{\hspace{2cm}}$.

- (A) 1 (B) 0
(C) -1 (D) $\frac{\pi}{2}$

Ans. Option (B) is correct.

Explanation:

$$\int_{-\pi}^{\pi} x \cos x dx = 0, \text{ since } x \cos x \text{ is an odd function.}$$

Q. 3. $\int_{-\pi/2}^{\pi/2} \sin^3 x dx = \underline{\hspace{2cm}}$.

- (A) 1 (B) 0
(C) -1 (D) π

Ans. Option (B) is correct.

Explanation:

$$\int_{-\pi/2}^{\pi/2} \sin^3 x dx = 0, \text{ since } \sin^3 x \text{ is an odd function.}$$

Q. 4. $\int_{-\pi}^{\pi} x \sin x dx = \underline{\hspace{2cm}}$.

- (A) π (B) 0
(C) 2π (D) $\frac{\pi}{2}$

Ans. Option (C) is correct.

Explanation: Since, $x \sin x$ is an even function

$$\begin{aligned} \int_{-\pi}^{\pi} x \sin x dx &= 2 \int_0^{\pi} x \sin x dx \\ &= 2 \left[-x \cos x + \int (1 \times \cos x dx) \right]_0^{\pi} \\ &= 2 \left[-x \cos x + \sin x \right]_0^{\pi} \\ &= 2 \left[(\pi + 0) - (0 + 0) \right] \\ &= 2\pi \end{aligned}$$

Q. 5. $\int_{-\pi}^{\pi} \tan x \sec^2 x dx = \underline{\hspace{2cm}}$.

- (A) 1 (B) -1
(C) 0 (D) 2

Ans. Option (C) is correct.

Explanation:

$$\int_{-\pi}^{\pi} \tan x \sec^2 x dx = 0, \text{ Since it is an odd function}$$

II. Read the following text and answer the following questions on the basis of the same:

$$\begin{aligned} \int e^x [f(x) + f'(x)] dx &= \int e^x f(x) dx + \int e^x f'(x) dx \\ &= f(x)e^x - \int f'(x)e^x dx \\ &\quad + \int f'(x)e^x dx \\ &= e^x f(x) + c \end{aligned}$$

Q. 1. $\int e^x (\sin x + \cos x) dx = \underline{\hspace{2cm}}$.

- (A) $e^x \cos x + c$ (B) $e^x \sin x + c$
(C) $e^x + c$ (D) $e^x (-\cos x + \sin x) + c$

Ans. Option (B) is correct.

Explanation:

$$\int e^x \left(\frac{\sin x}{f(x)} + \frac{\cos x}{f'(x)} \right) dx = e^x \sin x + c$$

Q. 2. $\int e^x \left(\frac{x-1}{x^2} \right) dx = \underline{\hspace{2cm}}$.

- (A) $e^x + c$ (B) $\frac{e^x}{x} + c$
(C) $\frac{e^x}{x^2} + c$ (D) $\frac{-e^x}{x^2} + c$

Ans. Option (B) is correct.

Explanation:

$$\begin{aligned} \int e^x \left(\frac{x-1}{x^2} \right) dx &= \int e^x \left(\frac{1}{\frac{x}{f(x)}} - \frac{1}{\frac{x^2}{f'(x)}} \right) dx \\ &= \frac{e^x}{x} + c \end{aligned}$$

- Q. 3. $\int e^x(x+1)dx = \underline{\hspace{2cm}}$.
 (A) $xe^x + c$ (B) $e^x + c$
 (C) $e^{-x} + c$ (D) None of these

Ans. Option (A) is correct.

Explanation:

$$\int e^x \left(\frac{x}{f(x)} + \frac{1}{f'(x)} \right) dx = xe^x + c$$

- Q. 4. $\int_0^\pi e^x(\tan x + \sec^2 x)dx = \underline{\hspace{2cm}}$.
 (A) 0 (B) 1
 (C) -1 (D) $-e^\pi$

Ans. Option (A) is correct.

Explanation:

$$\int_0^\pi e^x(\tan x + \sec^2 x)dx = \left[e^x \tan x \right]_0^\pi = 0$$

- Q. 5. $\int \frac{xe^x}{(1+x)^2} dx = \underline{\hspace{2cm}}$.
 (A) $xe^x + c$ (B) $\frac{e^x}{(x+1)^2} + c$
 (C) $\frac{xe^x}{x+1} + c$ (D) $\frac{e^x}{x+1} + c$

Ans. Option (D) is correct.

Explanation:

$$\begin{aligned} \int e^x \left[\frac{(x+1)-1}{(x+1)^2} \right] dx &= \int e^x \left[\frac{1}{x+1} - \frac{1}{(x+1)^2} \right] dx \\ &= \frac{e^x}{x+1} + c \end{aligned}$$

III. Read the following text and answer the following questions on the basis of the same:

Let's say that we want to evaluate $\int [P(x)/Q(x)] dx$, where $P(x)/Q(x)$ is a proper rational fraction. In such cases, it is possible to write the integrand as a sum of simpler rational functions by using partial fraction decomposition. Post this, integration can be carried out easily. The following image indicates some simple partial fractions which can be associated with various rational functions:

S. No.	Form of the rational function	Form of the partial fraction
1.	$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a} + \frac{B}{x-b}$
2.	$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
3.	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$

4.	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
5.	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$
where, x^2+bx+c cannot be factorised further		

In the above table, A, B and C are real numbers to be determined suitably.

- Q. 1. $\int \frac{dx}{(x+1)(x+2)}$
 (A) $\log \left| \frac{x+1}{x+2} \right| + C$ (B) $\log \left| \frac{x-1}{x+2} \right| + C$
 (C) $\log \left| \frac{x+1}{x-2} \right| + C$ (D) $\log \left| \frac{x+2}{x+1} \right| + C$

Ans. Option (A) is correct.

Explanation: We write,

$$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} \quad \dots(i)$$

where, real number A and B are to be determined suitably. This gives

$$1 = A(x+2) + B(x+1)$$

Equating the coefficients of x and the constant term, we get

$$A+B=0$$

$$\text{and } 2A+B=1$$

Solving these equations, we get $A=1$ and $B=-1$.

Thus, the integrand is given by

$$\frac{1}{(x+1)(x+2)} = \frac{1}{x+1} - \frac{1}{x+2}$$

Therefore,

$$\begin{aligned} \int \frac{dx}{(x+1)(x+2)} &= \int \frac{dx}{x+1} - \int \frac{dx}{x+2} \\ &= \log|x+1| - \log|x+2| + C \\ &= \log \left| \frac{x+1}{x+2} \right| + C \end{aligned}$$

- Q. 2. Integration of $\frac{x}{(x+1)(x+2)}$

- (A) $\log \frac{(x+1)^2}{(x+2)} + C$ (B) $\log \frac{(x+2)^2}{(x+1)} + C$
 (C) $\log \frac{(x)^2}{(x+1)} + C$ (D) $\log \frac{(x-2)^2}{(x+1)} + C$

Ans. Option (B) is correct.

Explanation:

$$\begin{aligned} \text{Let } \frac{x}{(x+1)(x+2)} &= \frac{A}{x+1} + \frac{B}{x+2} \\ \Rightarrow x &= A(x+2) + B(x+1) \end{aligned}$$

Equating the coefficients of x and constant term, we obtain

$$A + B = 1$$

$$2A + B = 0$$

On solving, we obtain

$$A = -1 \text{ and } B = 2$$

$$\therefore \frac{x}{(x+1)(x+2)} = \frac{-1}{(x+1)} + \frac{2}{(x+2)}$$

$$\begin{aligned} \Rightarrow \int \frac{x}{(x+1)(x+2)} dx &= \int \frac{-1}{(x+1)} + \frac{2}{(x+2)} dx \\ &= -\log|x+1| + 2\log|x+2| + C \\ &= \log(x+2)^2 - \log|x+1| + C \\ &= \log \frac{(x+2)^2}{(x+1)} + C \end{aligned}$$

Q. 3. $\int \frac{1}{x^2-9} dx$

- (A) $\frac{1}{6} \log \left| \frac{x-3}{x+3} \right| + C$ (B) $\frac{1}{6} \log \left| \frac{x-2}{x+3} \right| + C$
 (C) $\frac{1}{6} \log \left| \frac{x+3}{x-3} \right| + C$ (D) $\frac{1}{3} \log \left| \frac{x-3}{x+3} \right| + C$

Ans. Option (A) is correct.

Explanation:

$$\begin{aligned} \text{Let } \frac{1}{(x+3)(x-3)} &= \frac{A}{(x+3)} + \frac{B}{(x-3)} \\ 1 &= A(x-3) + B(x+3) \end{aligned}$$

Equating the coefficients of x and constant term, we obtain

$$A + B = 0$$

$$-3A + 3B = 1$$

On solving, we obtain

$$A = -\frac{1}{6} \text{ and } B = \frac{1}{6}$$

$$\therefore \frac{1}{(x+3)(x-3)} = \frac{-1}{6(x+3)} + \frac{1}{6(x-3)}$$

$$\begin{aligned} \Rightarrow \int \frac{1}{(x^2-9)} dx &= \int \left(\frac{-1}{6(x+3)} + \frac{1}{6(x-3)} \right) dx \\ &= -\frac{1}{6} \log|x+3| + \frac{1}{6} \log|x-3| + C \\ &= \frac{1}{6} \log \left| \frac{(x-3)}{(x+3)} \right| + C \end{aligned}$$

Q. 4. $\int \frac{1}{e^x-1} dx =$

- (A) $\log \left| \frac{e^x-1}{2} \right| + C$ (B) $\log \left| \frac{e^x-1}{2e^x} \right| + C$
 (C) $\log \left| \frac{e^x-1}{2x} \right| + C$ (D) $\log \left| \frac{e^x-1}{e^x} \right| + C$

Ans. Option (D) is correct.

Explanation:

$$\text{Let } e^x = t$$

$$\Rightarrow e^x dx = dt$$

$$\begin{aligned} \Rightarrow \int \frac{1}{e^x-1} dx &= \int \frac{1}{t-1} \times \frac{dt}{t} \\ &= \int \frac{1}{t(t-1)} dt \end{aligned}$$

$$\begin{aligned} \text{Let } \frac{1}{t(t-1)} &= \frac{A}{t} + \frac{B}{t-1} \\ 1 &= A(t-1) + Bt \quad \dots(i) \end{aligned}$$

Substituting $t = 1$ and $t = 0$ in equation (i), we obtain

$$A = -1 \text{ and } B = 1$$

$$\therefore \frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\begin{aligned} \Rightarrow \int \frac{1}{t(t-1)} dt &= \log \left| \frac{t-1}{t} \right| + C \\ &= \log \left| \frac{e^x-1}{e^x} \right| + C \end{aligned}$$

Q. 5. $\int \frac{dx}{x(x^2+1)} =$

- (A) $\log|x| + \frac{1}{2} \log|x^2+1| + C$
 (B) $\log|x| - \frac{1}{4} \log|x^2+1| + C$
 (C) $\log|x| - \frac{1}{2} \log|x^2+1| + C$
 (D) $\log|x| - \frac{1}{3} \log|x^2-1| + C$

Ans. Option (C) is correct.

Explanation:

$$\begin{aligned} \text{Let } \frac{1}{x(x^2+1)} &= \frac{A}{x} + \frac{Bx+C}{x^2+1} \\ 1 &= A(x^2+1) + (Bx+C)x \end{aligned}$$

Equating the coefficients of x^2 , x and constant term, we obtain

$$A + B = 0$$

$$C = 0$$

$$A = 1$$

On solving these equations, we obtain

$$A = 1, B = -1, \text{ and } C = 0$$

$$\therefore \frac{1}{x(x^2+1)} = \frac{1}{x} - \frac{x}{x^2+1}$$

$$\begin{aligned} \Rightarrow \int \frac{1}{x(x^2+1)} dx &= \int \left\{ \frac{1}{x} - \frac{x}{x^2+1} \right\} dx \\ &= \log|x| - \frac{1}{2} \log|x^2+1| + C \end{aligned}$$