Comparing Quantities

Percentage Increase and Decrease

In many cases, it is better if we represent increase or decrease in quantities in percentage terms rather than describing them numerically.

Let us take the case where Manoj got a raise of Rs 5000 in his monthly salary. Now, we also have to consider his original salary in order to ascertain whether the raise was high or low. For example, if his original salary was Rs 10000, then this means that he got a raise of 50% in his monthly salary, which is quite high. However, let us suppose that his original monthly salary was Rs 50000. Now this would mean that Manoj got a raise of only 10%, which is quite low.

Thus, if we talk only about the increase or decrease in a quantity without referring to its original value, then it will cause confusion in some cases. However, this problem does not arise if we use percentages.

In the same way, we can find the percentage decrease, using the following formula.

$$\text{Percentage decrease} = \frac{\text{Decrease in quantity}}{\text{Original quantity}} \times 100$$

Let us consider an example to understand percentage decrease.

In an athletic event, 60 students of a school participated last year. This year, the number of students of that school taking part is decreased by 5%. Find the number of students taking part in the athletic event this year.

Here, we have percentage decrease = 5%

Original number of students = 60

$$\therefore \text{Percentage decrease} = \frac{\text{Decrease in number of students}}{\text{Original number of students}} \times 100$$

$$\Rightarrow 5 = \frac{\text{Decrease in number of students}}{60} \times 100$$

$$\Rightarrow \text{Decrease in number of students} = \frac{5 \times 60}{100}$$

$\Rightarrow$ Decrease in number of students = 3

Thus, number of students taking part this year = 60 – 3 = 57

Thus, 57 students participated this year.

Let us solve some examples to understand the concept better.

Example 1:

The price of a toy was decreased by 20%. If this meant a decrease of Rs 125 in its price, then what were the original and the reduced prices of that toy?

Solution:

Let the original price of the toy to be *x*.

Percentage decrease in the price of the toy = 20

Decrease in the price of the toy = Rs 125

In this case, $$\text{percentage decrease} = \frac{\text{Decrease in price of the toy}}{\text{Original price of the toy}} \times 100$$

$$\therefore 20 = \frac{\text{Rs } 125}{x} \times 100$$

$$x = \text{Rs}\left(\frac{125}{20} \times 100\right) = \text{Rs } 625$$

Thus, original price of the toy = Rs 625

And, reduced price of the toy = Rs 625 − Rs 125 = Rs 500

Example 2:

The population of a city in 2002 was 2.5 crores. If it increased by 15% in a year, then what was the population in 2003?

Solution:

Original population of the city = 2.5 crores

Percentage increase in population = 15 %

Let *x be* the increase in population of the city.

$$\text{percentage increase in population} = \frac{\text{Increase in population}}{\text{Original population}} \times 100$$

In this case,

$$\therefore 15 = \frac{x}{2.5 \text{ crores}} \times 100$$

$$\Rightarrow x = \frac{15 \times 2.5}{100} \text{ crores} = 0.375 \text{ crores}$$

Thus, the population increased by 0.375 crores over the year.

Thus, population of the town in 2003 = (2.5 + 0.375) crores = 2.875 crores

Example 3:

The enrolment at a school increased from 1400 to 1500 in one year. What is the percentage increase in the enrolment?

Solution:

Increase in the enrolment = 1500 - 1400 = 100

Percentage increase in the enrolment $= \frac{100}{1400} \times 100$

$$= \frac{100}{14}$$
$$= 7.14\%$$

Thus, the enrolment is increased by 7.14%.

Example 4:

The selling price of a DVD player was dropped by 20% in one year. If the selling price of the DVD player is Rs 8000 now, then find the selling price of the DVD player one year before.

Solution:

Let *x* be the selling price of the DVD player before one year. The selling price of the DVD player was dropped by 20%. Therefore, now the selling price of the DVD is (100 - 20)% of *x* = 80% of *x*

$\therefore$ 80% of x = Rs 8000

$$\frac{80}{100} \times x = \text{Rs } 8000$$
$$x = \text{Rs } \frac{8000 \times 100}{80}$$
$$x = \text{Rs } 10000$$

Thus, the selling price of the DVD player before one year was Rs 10000.

Estimation In Percentages

Some times we have to calculate percentages very quickly. If we try to find the exact value of percentage, then it would take a lot of time. Therefore, in such cases, we estimate the percentage.

Let us consider an example. Suppose you want to purchase a house. A property dealer told you that the price of this house was Rs 500000 last year and its price was increased by 19.6% in one year.

Can you calculate the price of the house this year?

Yes, we can calculate the price of the house very quickly using estimation technique.

19.6% is very close to 20%.

Therefore, instead of calculating 19.6%, we calculate 20% of Rs 500000.

$$\begin{aligned} 20\% \text{ of Rs } 500000 &= \text{Rs} \frac{20}{100} \times 500000 \\ &= \text{Rs } 100000 \end{aligned}$$

Therefore, price of the house this year = Rs 500000 + Rs 100000

= Rs 600000

Thus, the price of the house now is approximately Rs 600000.

Let us solve one more example based on the estimation of percentages.

Example 1:

What is 15% of 3108?

Solution:

Here, we can round off the value 3108 to 3000.

∴ 15% of 3000

$$= \frac{15}{100} \times 3000$$
$$= 450$$

Thus, 15% of 3108 is approximately equal to 450.

Sales Tax (ST) And Value Added Tax (VAT)

Sales tax is the amount that is added to the price of an item. It is charged by the government from the customers.

The tax which customers pay these days is known as Value Added Tax (VAT).

The sales tax or value added tax is generally given as a percentage. Therefore, we can calculate our bill amount using the concept of percentages.

Example 1:

The cost of a shirt including 12% VAT is Rs 896. Find the cost of the shirt without including VAT.

Solution:

Let x be the cost of the shirt without including VAT.

We know that,

Cost of shirt including VAT = Selling price + VAT

Rs 896 = x + 12% of x

$$\text{Rs } 896 = x + \left(\frac{12}{100} \times x \right)$$

$$\text{Rs } 896 = \frac{112x}{100}$$

$$x = \text{Rs}\frac{896 \times 100}{112}$$

$$x = \text{Rs } 800$$

Thus, the selling price of the shirt without including VAT is Rs 800.

Example 2:

The original cost of a food item is Rs 175. What is the bill amount, if the sales tax is charged as 5%?

Solution:

Original cost of the food item = Rs 175

Sales tax = 5% of Rs 175 = $\frac{5}{100} \times \text{Rs } 175$ = Rs 8.75

Bill amount = Rs 175 + Rs 8.75 = Rs 183.75

Thus, the bill amount is Rs 183.75.

Concept of Profit Percent and Loss Percent

In buying and selling articles, sometimes there is loss and sometimes profit. We can also write profit and loss as a percentage. Profit percent or loss percent is always calculated on the cost price of the article.

In the same way, we can find loss percent.

Remember the following formulae.

$$\text{Profit \%} = \frac{\text{Profit}}{\text{C.P}} \times 100$$

$$\text{Loss \%} = \frac{\text{Loss}}{\text{C.P.}} \times 100$$

$$\text{S.P.} = \frac{(100 + \text{Profit}\,\%)}{100} \times \text{C.P.}$$

$$\text{S.P.} = \frac{(100 - \text{Loss}\,\%)}{100} \times \text{C.P.}$$

$$\text{C.P.} = \frac{100}{(100 + \text{Profit}\,\%)} \times \text{S.P.}$$

$$\text{C.P.} = \frac{100}{(100 - \text{Loss}\,\%)} \times \text{S.P.}$$

Now, let us solve some examples.

Example 1:

Apurva sold his bike for Rs 30000 at a loss of 40%. At what price did Apurva buy the bike?

Solution:

Let the price at which Apurva bought the bike be x.

C.P. = x

S.P. = Rs 30000

Loss = C.P. − S.P. = Rs (x − 30000)

We know that, Loss % $= \frac{\text{Loss}}{\text{C.P.}} \times 100$

$$40 = \frac{(x - 30000)}{x} \times 100$$

$40x = 100x - 3000000$

$100x - 40x = 3000000$

$60x = 3000000$

$$x = \frac{3000000}{60}$$

x = Rs 50000

Thus, Apurva bought the bike for Rs 50000.

Example 2:

Javed sold a refrigerator and a washing machine for Rs 15000 and Rs 10000 respectively. He made a gain of 25% on the refrigerator and a loss of 20% on the washing machine. Find his overall gain or loss.

Solution:

Let the cost price of refrigerator be x and that of washing machine be y.

S.P. of refrigerator = Rs 15000

Gain on refrigerator = S.P. − C.P. = Rs (15000 − x)

We know that, gain % $= \frac{\text{Gain}}{\text{C.P.}} \times 100$

$$25 = \frac{(15000 - x)}{x} \times 100$$

$25x = (1500000 - 100x)$

$125x = 1500000$

$$x = \frac{1500000}{125}$$

x = Rs 12000

C.P. of refrigerator = Rs 12000

Now, selling price of washing machine = Rs 10000

Loss on washing machine = Rs (y − 10000)

We know that, Loss% $= \frac{\text{Loss}}{\text{C.P.}} \times 100$

$$20 = \frac{(y - 10000)}{y} \times 100$$

$20y = 100y - 1000000$

$100y - 20y = 1000000$

$80y = 1000000$

$$y = \frac{1000000}{80}$$

y = Rs 12500

C.P. of washing machine = Rs 12500

Now, total cost price of refrigerator and washing machine is, (Rs 12000 + Rs 12500)

= Rs 24500

Total S.P. = Rs 15000 + Rs 10000 = Rs 25000

Overall gain = Rs 25000 − Rs 24500 = Rs 500

Thus, there is an overall gain of Rs 500 on the selling of the refrigerator and the washing machine.

Example 3:

Rahul purchased a television for Rs 20000 and its transportation cost was Rs 100. For how much should the television be sold so that he makes a profit of 7%?

Solution:

C.P. of the television = Rs 20000

Overhead charges = Rs 100

∴ Total cost of TV = Rs 20000 + Rs 100 = Rs 20100

Profit % = 7%

We know that, Profit % $= \frac{\text{Profit}}{\text{C.P.}} \times 100$

$$7 = \frac{\text{Profit}}{20100} \times 100$$

7 × 201 = Profit

∴ Profit = Rs 1407

Now,

S.P. = C.P. + Profit = Rs 20100 + Rs 1407 = Rs 21507

Thus, the television should be sold for Rs 21507.

Example 4:

Arun bought an umbrella for Rs 125 and sold it for a profit of Rs 20. What was the selling price of the umbrella and the profit percent of the transaction?

Solution:

Cost price (C.P.) of the umbrella = Rs 125

Profit made in the transaction = Rs 20

∴ Selling price (S.P.) of the umbrella = C.P. + Profit = Rs 125 + Rs 20 = Rs 145

Hence, profit percent of the transaction $= \frac{\text{Profit}}{\text{C.P.}} \times 100 = \frac{20}{125} \times 100 = 16$ %

Example 5:

Julie bought a washing machine for Rs 20000. She then sold it at 10% profit. At what price did she sell the washing machine?

Solution:

Profit per cent = 10%

Cost price (C.P.) of the washing machine = Rs 20000

Profit per cent of the transaction $= \frac{\text{Profit}}{\text{C.P.}} \times 100 = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$

$$\therefore 10 = \frac{\text{S.P.} - 20000}{20000} \times 100$$

$$\text{S.P.} - 20000 = \frac{10 \times 20000}{100}$$

$$\text{S.P.} - 20000 = 2000$$

$$\text{S.P.} = 20000 + 2000 = 22000$$

Hence, Julie sold the washing machine for Rs 22000.

Example 6:

If the cost price of 16 chocolates is equal to selling price of 12 chocolates, then find the profit percent.

Solution:

Let the C.P. of each chocolate be Re 1. Then the C.P. of 16 chocolates will be Rs 16.

By the given data, S.P. of 12 chocolates = C.P. of 16 chocolates = Rs 16

$\therefore$ S.P. of 1 chocolate = Rs $\frac{16}{12}$

Profit = S.P. – C.P. = $\text{Rs}\frac{16}{12} - \text{Re}1 = \text{Re}\frac{4}{12} = \text{Re}\frac{1}{3}$

Thus, there is a profit of Rs $\frac{1}{3}$ on each chocolate.

Profit % = $\frac{\text{Profit}}{\text{C.P.}} \times 100\% = \frac{\text{Re}\frac{1}{3}}{\text{Re}1} \times 100\% = \frac{100}{3}\% = 33\frac{1}{3}\%$

Thus, the profit percent is $33\frac{1}{3}\%$.

Example 7:

On selling a bicycle for Rs 5,600, a dealer loses 20%. For how much should he sell it to gain 15%?

Solution:

Selling price of the bicycle is Rs 5,600 and the loss is 20%. Therefore,

$$\text{C.P.} = \frac{100}{(100 - \text{loss}\%)} \times \text{S.P.}$$

$$= \frac{100}{(100-20)} \times \text{Rs } 5{,}600$$

$$= \frac{100}{80} \times \text{Rs } 5{,}600$$

$$= \text{Rs } 7{,}000$$

Expected profit = 15%

$$\text{S.P.} = \frac{(100+\text{profit}\%)}{100} \times \text{C.P.}$$
$$= \frac{(100+15)}{100} \times \text{Rs } 7{,}000$$
$$= \frac{115}{100} \times \text{Rs } 7{,}000$$
$$= \text{Rs } 8{,}050$$

Thus, the selling price of the bicycle to gain 15% is Rs 8,050.

Example 8:

The cost price of a laptop is Rs 24,000. An additional Rs 1000 was spent on installing a software. If it is sold at 15% profit, then find the selling price of the laptop.
Solution:
Cost price of the laptop = Rs 24,000 + Rs 1,000 (overhead charges) = Rs 25,000
The laptop is sold at a profit of 15%. Therefore,

$$\text{S.P.} = \frac{(100+\text{Profit}\%)}{100} \times \text{C.P.}$$
$$= \frac{(100+15)}{100} \times \text{Rs } 25{,}000$$
$$= \text{Rs } \frac{115}{100} \times 25{,}000$$
$$= \text{Rs } 28{,}750$$

Thus, the selling price of the laptop is Rs 28, 750.

Concept of Discount

We have come across situations where some shops give **discount (rebate)** and we can obtain the article for less than the **marked price** or **catalogue price** of the article. Let us consider such a situation.

A book shop offers a discount of 20% on the purchase of any book. If the marked price of a book is Rs 350, then what will be the cash discount and the sale price of the book?

In the above example, we calculated the sale price of the book when its marked price was Rs 350 and the discount was 20%. What will be the sale price of the book, if the above bookshop offers yet another discount of 25% after giving the discount of 20%?

In this case, the bookshop offers two successive discounts (one is 20% and the other is 25% after giving the discount of 20%) on Rs 350. So, first of all, we calculate the sale price of the book when M.P. = Rs 350 and discount % = 20%. After that, we take this sale price as

the M.P., on which the discount of 25% is offered and then, we calculate the sale price as follows:

For the 1st part of the successive discounts:

M.P. (marked price) = Rs 350, discount (d)% = 20%

So, the selling price (*S.P.*) of the book after 1st discounts is:

$$\text{S.P.} = \frac{100-d}{100} \times \text{M.P.}$$

$$\Rightarrow \text{S.P.} = \frac{100-20}{100} \times \text{Rs } 350 = \text{Rs } 280$$

For the 2nd part of the successive discounts:

M.P. (marked price) = Rs 280, discount (d)% = 25%

So, the selling price (S.P.) of the book after 2nd successive discounts is:

$$\text{S.P.} = \frac{100-d}{100} \times \text{M.P.}$$

$$\Rightarrow \text{S.P.} = \frac{100-25}{100} \times \text{Rs } 280 = \text{Rs } 210$$

So, the bookshop sells the given book for Rs 210.

For successive discounts on an item, we can generalise a formula as given below:

If the successive discount %, d_1%, d_2%, d_3% ... are given, then

S.P. = $$\text{M.P.} \times \left(\frac{100-d_1}{100}\right) \times \left(\frac{100-d_2}{100}\right) \times \left(\frac{100-d_3}{100}\right) \times \ldots.$$

We can also find the marked price, if the discount and the sale price are known.

Now, let us solve some more examples to understand the concept better.

Example 1:

The price of a movie ticket is Rs 200, but there is a discount of 15% on the tickets of the morning show. Calculate the discount and cost of tickets of the morning show.

Solution:

Actual cost of a movie ticket = Rs 200

Discount = 15% of Rs 200

$$= \text{Rs}\frac{15}{100} \times 200$$

= Rs 30

Cost of a ticket of the morning show

= Rs 200 − Rs 30

= Rs 170

Thus, the cost of a ticket of morning show is Rs 170 and cash discount is Rs 30.

Example 2:

A shop offers a discount of 25% on each purchase of item made of leather. A customer purchases a leather belt for Rs 375. What is the marked price of the belt?

Solution:

Let the marked price of the leather belt be x.

Discount = 25% of x

$$= \frac{25}{100} \times x$$

$$= \frac{x}{4}$$

We know that,

Sale price = M.P. − Discount

$$\text{Rs } 375 = x - \frac{x}{4}$$

$$\text{Rs } 375 = \frac{4x - x}{4}$$

$$\frac{3x}{4} = \text{Rs } 375$$

$$x = \text{Rs}\frac{375 \times 4}{3}$$

x = Rs 500

Thus, the marked price of the belt is Rs 500.

Example 3:

For the stock sale at the end of a season, a garment shop offers 50% and then 40% on the garments. What is the marked price of a shirt if the shop offers a total discount of Rs 840 after giving two successive discounts?

Solution:

Let the marked price of the shirt be Rs x.

In two successive discounts, d_1% = 50% and d_2% = 40%.

We know that

$$\begin{aligned}\text{S.P.} &= \left(\frac{100 - d_1}{100}\right) \times \left(\frac{100 - d_2}{100}\right) \times \text{M.P.} \\ &= \left(\frac{100 - 50}{100}\right) \times \left(\frac{100 - 40}{100}\right) \times x \\ &= \frac{50}{100} \times \frac{60}{100} \times x \\ &= \frac{3x}{10}\end{aligned}$$

We know that

Discount = M.P. − S.P.

$$\Rightarrow 840 = x - \frac{3x}{10}$$

$$\Rightarrow \frac{7x}{10} = 840$$

$$\Rightarrow x = \frac{840 \times 10}{7} = 1200$$

Hence, the marked price of the shirt is Rs 1200.

Example 4:

A shop keeper buys an article for Rs 750. He marks it at 10% above the cost price. If he sells it at 8% discount, then find the selling price.

Solution:
Cost price of the article = Rs 750
Profit = 10% of Rs 750 $= \frac{10}{100} \times \text{Rs}\,750 = \text{Rs}\,75$
Marked price = Cost price + Profit = Rs 750 + Rs 75 = Rs 825
Discount = 8%
For Rs 825, discount $= \frac{8}{100} \times \text{Rs}\ 825 = \text{Rs}\,66$
∴ Selling price = Marked price – Discount = Rs 825 – Rs 66 = Rs 759
Thus, the selling price of article is Rs 759.

Example 5:
A cloth seller marks a dress at 35% above the cost price and allows a discount of 10%. What profit does he make in selling the dress?

Solution:
Suppose the cost price of the dress material is Rs 100. Since the seller marks it at 35% above the C.P., the marked price would be Rs 100 + Rs 35 = Rs 135.
Discount of 10% on this marked price = 10% of Rs 135 $= \frac{10}{100} \times \text{Rs}\,135 = \text{Rs}\,13.50$
∴ Selling price = Rs 135 – Rs 13.50 = Rs 121.50. So,
Profit = S.P. – C.P. = Rs 121.50 – Rs 100 = Rs 21.50
Hence, profit percent = $\frac{\text{Profit}}{\text{C.P.}} \times 100 = \frac{21.50}{100} \times 100 = \text{Rs}\ 21.50$
Thus, the cloth seller makes a profit of 21.50% on selling the dress.

Example 6:
The catalogue price of a book is Rs 450. If a shopkeeper offers some discount on the book and sells it at Rs 405. What is the rate of discount?

Solution:
Marked price of the book = Rs 450
Price after discount = Rs 405
Discount = Marked price – Price after discount
= Rs 450 – Rs 405
= Rs 45

Rate of discount = $\frac{\text{Discount} \times 100}{\text{Marked price}}$

$= \frac{45 \times 100}{450}$

= 10 %

Compound Interest Using the Concept of Simple Interest

When we deposit some money in a bank, the extra money paid by the bank is called interest.

We know how to calculate simple interest. However, the interest paid by the banks or post offices and charged by the money lenders is not of this kind. The interest paid or charged in such cases is compound interest.

Let us know the method of finding compound interest by taking an example.

Deepak borrowed Rs 16000 from Manish at a rate of 5% per annum for two years. Can you calculate the amount paid by Deepak as compound interest?

Now, let us solve one more example to understand the concept better.

Example:

Javed invested Rs 15000 for three years in a bank at the rate of 10% per annum compounded annually. On the other hand, Sujata invested the same sum in some other bank at the same rate for the same duration but on simple interest. Who will earn more interest?

Solution:

It is given that,

Principal = Rs 15000

Rate of interest = 10% per annum

Time = 3 years

Let us first calculate compound interest.

Interest for the 1st year $= \text{Rs } \frac{15000 \times 10}{100}$

= Rs 1500

Amount at the end of 1st year

= Rs 15000 + Rs 1500

= Rs 16500

Interest for 2nd year $= \text{Rs } \frac{16500 \times 10}{100}$

= Rs 1650

Amount at the end of 2nd year

= Rs 16500 + Rs 1650

= Rs 18150

Interest for 3rd year $= \text{Rs } \frac{18150 \times 10}{100}$

= Rs 1815

∴ Total interest received by Javed after 3 years

= Rs 1500 + Rs 1650 + Rs 1815

= Rs 4965

Simple interest received by Sujata after 3 years

$= \text{Rs } \frac{15000 \times 10 \times 3}{100}$

= Rs 4500

Thus, Javed received more interest than Sujata.

Formula for Calculating Compound Interest

Supriya invested Rs 75000 in a bank at the rate of 10% per annum compounded annually. What is the amount received by her after 2 years?

We know the method of calculating compound interest using the concept of simple interest. However, this method is very lengthy as we have to calculate the interest and then amount for each year one by one.

We can also find the amount directly using a formula. Before solving the given problem, let us know about the formula.

When the interest is compounded annually, the amount after *n* years is given by

$$A = P\left(1+\frac{R}{100}\right)^n$$

where P is the principal and R is the rate of interest per annum.

Now, using this formula, let us calculate the amount received by Supriya after 2 years.

In the given problem,

Principal, P = Rs 75000

Rate of interest, R = 10% per annum

Number of years, *n* = 2 years

By the formula, we obtain

$$A = 75000\left(1+\frac{10}{100}\right)^2$$

$$A = 75000\left(\frac{11}{10}\right)^2$$

$$A = 75000\times\frac{11}{10}\times\frac{11}{10}$$

A = 750 × 121

A = Rs 90750

Thus, Supriya received Rs 90750 after 2 years.

Using this formula, we can find any one of the values - amount, time, rate of interest or principal, if the other three values are known to us.

For example, the compound interest on a certain sum invested for two years at the rate of 8% p.a. compounded annually is Rs 3328. The sum can be calculated as:

Let x be the sum.

Amount = Principal + Interest

Amount = Rs (x + 3328)

Now using the formula, we obtain

$$A = P\left(1+\frac{R}{100}\right)^{n}$$

$$\Rightarrow (x + 3328) = x\left(1+\frac{8}{100}\right)^{2}$$

$$\Rightarrow (x + 3328) = x\left(\frac{27}{25}\right)^{2}$$

$$\Rightarrow (x + 3328) = x\times\frac{27}{25}\times\frac{27}{25}$$

$$\Rightarrow x+3328=\frac{729x}{625}$$

$$\Rightarrow 625x + 2080000 = 729x$$

$$\Rightarrow 729x - 625x = 2080000$$

$$\Rightarrow 104\,x = 2080000$$

$$\Rightarrow x=\frac{2080000}{104}$$

$\Rightarrow x = 20000$

Thus, the sum is Rs 20000.

Now, let us solve some more examples.

Example 1:

Anurag borrowed Rs 50000 from his friend at the rate of 4% p.a. compounded annually. After some time, he repaid Rs 56243.20 to his friend. For how many years did Anurag borrow the money?

Solution:

Principal, P = Rs 50000

Rate, R = 4% p.a.

Amount, A = Rs 56243.2

Let n be the number of years for which Anurag borrowed the money.

Using the formula, $A = P\left(1+\frac{R}{100}\right)^n$,

$\Rightarrow$ Rs 56243.2 = Rs $50000\left(1+\frac{4}{100}\right)^n$

$$\Rightarrow \frac{56243.2}{50000} = \left(1+\frac{4}{100}\right)^n$$

$$\Rightarrow \frac{562432}{500000} = \left(1+\frac{1}{25}\right)^n$$

$$\Rightarrow \frac{17576}{15625} = \left(\frac{26}{25}\right)^n$$

$$\Rightarrow \left(\frac{26}{25}\right)^3 = \left(\frac{26}{25}\right)^n$$

$$\Rightarrow n = 3$$

Thus, Anurag borrowed the money for three years.

Example 2:

Find the rate at which Rs 700 becomes Rs 847 in two years, when the interest is compounded annually.

Solution:

Let R be the rate of interest.

Principal, P = Rs 700

Amount, A = Rs 847

Number of years, n = 2

Now, using the formula, $A = P\left(1+\frac{R}{100}\right)^n$, we obtain

$$\Rightarrow 847 = 700\left(1+\frac{R}{100}\right)^2$$

$$\Rightarrow \frac{847}{700} = \left(1+\frac{R}{100}\right)^2$$

$$\Rightarrow \frac{121}{100} = \left(1+\frac{R}{100}\right)^2$$

$$\Rightarrow \left(\frac{11}{10}\right)^2 = \left(1+\frac{R}{100}\right)^2$$

$$\Rightarrow \frac{11}{10} = 1+\frac{R}{100}$$

$$\Rightarrow \frac{11}{10} - 1 = \frac{R}{100}$$

$$\Rightarrow \frac{1}{10} = \frac{R}{100}$$

$$\Rightarrow R = 10\%$$

Thus, the rate of interest is 10%.

Problems Where the Interest is not Compounded Annually

We know how to find the compound interest using the following formula:

Amount, $A = P\left(1+\frac{R}{100}\right)^n$

Here, P is the principal, R is the rate of interest per annum, and n is the number of years.

Let us now try to solve an example.

What is the amount received after $1\frac{1}{2}$ years when Rs 20000 are invested at the rate of 6% p.a. compounded half yearly?

Do you find any difference between this question and the questions you have solved before?

Yes, in this example, the interest is compounded half yearly. In the questions we have solved before, the interest was compounded annually.

The time interval after which the interest is compounded is known as the conversion period.

In the given example, the conversion period is six months as the interest is compounded half yearly.

To solve such questions where the interest is not compounded annually, we have to change the rate and time accordingly.

In the given example, the time period is given as $1\frac{1}{2}$ years.

Here, the conversion period is six months.

$\therefore$ Number of conversion periods = 3

Rate of interest = 6% per annum

= 3% per half yearly

Now, we can solve this question using the same formula.

$$A = P\left(1+\frac{R}{100}\right)^n$$

Where,

n is the number of conversion periods and *R* is the rate per conversion period.

$$A = 20000\left(1+\frac{3}{100}\right)^3$$

$$A = 20000\left(\frac{103}{100}\right)^3$$

$$A = 20000\times\frac{103}{100}\times\frac{103}{100}\times\frac{103}{100}$$

$$A = \text{Rs } 21854.54$$

Hence, the amount received after $1\frac{1}{2}$ years is Rs 21854.54.

Before solving more examples, let us see the following table that will help us find the number of conversion periods and the rate of interest per conversion period.

Let *R* be the rate of interest per annum and *t* be the number of years.

Then,

When the interest is compounded	**Number of conversion periods (n)**	**Rate of interest per conversion period (r)**
Half yearly	t × 2	$\frac{R}{2}$
Quarterly	t × 4	$\frac{R}{4}$
Monthly	t × 12	$\frac{R}{12}$

Now, let us solve some more examples.

Example 1:

The compound interest on a certain sum invested for one year at the rate of 10% per annum compounded half yearly is Rs 5125. Find the sum.

Solution:

Let P be the sum.

Compound interest = Rs 5125

Amount, A = P + 5125

Number of conversion periods, n = 1 × 2 = 2

Rate of interest per conversion period $= \frac{10}{2}\% = 5\%$

We know that,

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$\Rightarrow (P+5125) = P\left(1+\frac{5}{100}\right)^2$$

$$\Rightarrow (P+5125) = P\left(\frac{21}{20}\right)^2$$

$$\Rightarrow (P+5125) = \frac{441}{400}P$$

$$\Rightarrow 400P + 2050000 = 441P$$

$$\Rightarrow 441P - 400P = 2050000$$

$$\Rightarrow 41P = 2050000$$

$$\Rightarrow P = \frac{2050000}{41}$$

$$\Rightarrow P = 50000$$

Hence, the sum is Rs 50000.

Example 2:

Lalit borrowed Rs 125000 from his friend at the rate of 8% per annum compounded quarterly. After some time, he re-paid Rs 132651 to his friend. For how long did Lalit borrow the money?

Solution:

Principal, P = Rs 125000

Amount, A = Rs 132651

Let n be the number of conversion periods.

Rate of interest per conversion period, $r = \frac{8}{4}\% = 2\%$

We know that,

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$132651 = 125000\left(1 + \frac{2}{100}\right)^n$$

$$\frac{132651}{125000} = \left(1 + \frac{1}{50}\right)^n$$

$$\left(\frac{51}{50}\right)^3 = \left(\frac{51}{50}\right)^n$$

$$\Rightarrow n = 3$$

∴ Number of conversion periods = 3

Here, the interest is compounded quarterly and we know that, one quarter = 3 months

∴ 1 conversion period = one quarter = 3 months

∴3 conversion periods = 3 × 3 months = 9 months

Hence, Lalit borrowed the money for a period of 9 months.