Introduction to Physics

Exercise Solutions

Question 1: Find the dimensions of
(i) Linear Momentum
(ii) Frequency and
(iii) Pressure

Solution:
(i)
Linear momentum can be written as "mv"

Dimensions of Linear momentum: mv = $[MLT^{-1}]$

(ii) Frequency can be written as "1/T", where T is time.

Dimensions of Frequency: 1/T = $[M^0L^0T^{-1}]$

(iii) All units of pressure represent some ratio of force to area

So, dimensions of pressure: Force/Area = $\frac{[MLT^{-2}]}{[L^2]}$ = $[ML^{-1}T^{-2}]$

Question 2: Find the dimensions of
(a) Angular speed, ω
(b) Angular acceleration, α
(c) Torque, τ and
(d) Moment of Inertia, I

$$\omega = \frac{\theta_2 - \theta_1}{t_2 - t_1}, \quad \alpha = \frac{\omega_2 - \omega_1}{t_2 - t_1}, \quad \Gamma = F.r \text{ and } I = mr^2.$$

Some of the equations involving these quantities are.

Solution:

(a) Dimensions of Angular speed, ω

We know, $\omega = \theta/t$
Dimensions are $[M^0L^0T^{-1}]$

(b) Dimensions of Angular acceleration, α
We know, $\alpha = \omega/t$
So required dimensions are = $\frac{[M^0L^0T^{-1}]}{[T]}$ = $[M^0L^0T^{-2}]$
[using (a) result)]

(c) Dimensions of Torque, τ and

We know, $\tau = Fr$

So, $\tau = [MLT^{-2}][L] = [ML^2T^{-2}]$

(d) Dimensions of Moment of Inertia, I

Here $I = Mr^2 = [M][L^2] = [ML^2T^0]$

Question 3: Find the dimensions of
(a) Electric Field E
(b) Magnetic field B and
(c) Magnetic permeability μ_0
The relevant equations are

$$F = qE,\ F = qvB,\ \text{and}\ B = \frac{\mu_0 I}{2\pi a};$$

where F is force, q is charge, v is speed, I is current, and a is distance.

Solution:
(a) Dimensions of Electric Field E = F/q = $\frac{[MLT^{-2}]}{[TI]}$ = $[MLT^{-3}I^{-1}]$

(b) Dimensions of Magnetic field B = F/qv = $\frac{[MLT^{-2}]}{[TI][LT^{-1}]}$ = $[MT^{-2}I^{-1}]$

(c) Dimensions of Magnetic permeability μ_0 = (Bx2 πa)/I = $\frac{[MT^{-2}I^{-1}][L]}{[I]}$ = $[MLT^{-2} I^{-2}]$

Question 4: Find the dimensions of
(a) Electric dipole moment p and
(b) Magnetic dipole moment M.
The defining equations are p = qd and M = IA
Where d is distance, A is area, q is charge and I is current.

Solution:

a) Dimensions of Electric dipole moment p = ql = [IT] [L] = [LTI]
(b) Dimensions of Magnetic dipole moment M = IA = $[I][L^2]$ $[L^2I]$

Question 5: Find the dimensions of Planck's constant h from the equation E = hv where E is the energy and v is the frequency.

Solution:
Planck's constant can be written as, h = E/v
Where E = energy and v = frequency

=> h = E/v = $\frac{[ML^2T^{-2}]}{[T^{-1}]}$ = $[ML^2 T^{-1}]$

Question 6: Find the dimensions of
(a) the specific heat capacity c,
(b) the coefficient of linear expansion α and
(c) the gas constant R.
Some of the equations involving these quantities are

$$Q = mc(T_2 - T_1),\ l_t = l_0[1 + \alpha(T_2 - T_1)] \text{ and } PV = nRT.$$

Solution:
(a) Dimensions of specific heat capacity,

c = Q/mΔT = $\frac{[ML^2T^{-2}]}{[M][K]}$ = $[L^2 T^{-2}K^{-1}]$

(b) Dimensions of coefficient of linear expansion,

α = $\frac{L_1 - L_2}{L_0\ \Delta T} = \frac{[L]}{[L][R]} = K^{-1}$

(c) Dimensions of gas constant,

R = PV/nT = $\frac{[\mathrm{ML}^{-1}T^{-2}][L^3]}{[mol][\mathrm{K}]}$ = $[ML^2\ T^{-2}K^{-1}\ (mol)^{-1}]$

Question 7: Taking force, length and time to be the fundamental quantities find the dimensions of
(a) Density
(b) Pressure
(c) Momentum and
(d) Energy

Solution:
As per given instruction, considering force, length and time to be the fundamental quantities
(a) Density = mass/volume ...(1)

and, mass = Force/acceleration = (Force × time^2)/displacement.

(1)=> Density = {(Force × time^2)/displacement}/Volume

Dimensions of Density = $[FL^{-4}T^2]$

(b)
Pressure = F/A
Dimensions of A = $[L^2]$
Dimensions of Pressure = $[FL^{-2}]$

(c)
Momentum = mv = (force/acceleration) x velocity
= $[F/LT^{-2}]$ x $[LT^{-1}]$ = [FT]

Dimensions of Momentum [FT]

(d)
Energy = ½ mv^2 = Force/acceleration x $(\text{velocity})^2$

= $\frac{[F]}{[LT^{-2}]}\ [LT^{-1}]^2 = [FL]$

Dimensions of Energy = [FL]

Question 8: Suppose the acceleration due to gravity at a place is 10 m/s^2. Find its value in $cm/(minute)^2$.

Solution:
Given: acceleration due to gravity at a place is 10 m/s^2
Convert units into cm/min^2

Here, g = 10 m/sec^2 = 36 x 10^5 cm/min^2

Question 9: The average speed of a snail is 0.020 miles/hour and that of a leopard is 70 miles/hour. Convert these speeds in SI units.

Solution:
Average speed of a snail = 0.020 miles/hour (Given)
Average speed of a leopard = 70 miles/hour (Given)

In SI Units:
0.020 miles/hour = (0.02x1.6x1000)/3600 = 0.0089 m/s
[Using 1 mile = 1.6 km = 1600m]
And,
70 miles/hr = (70x1.6x1000)/3600 = 31 m/s

Question 10: The height of mercury column in a barometer in a Calcutta laboratory was recorded to be 75 cm. Calculate this pressure in SI and CGS units using the following data.
Specific gravity of mercury = 13.6
Density of water = 10^3 kg/m^3,

gravity, g = 9.8 m/s^2 at Calcutta.
Pressure = hρg in usual symbols.

Solution:
The height of mercury column in a barometer in a Calcutta laboratory was recorded to be 75 cm. (Given)
Say, h = 75 cm
Calculate pressure in SI and CGS units.
Pressure = hρg = 10×10^4 N/m^2 approx

In C.G.S. units, Pressure = 10×10^5 $dyne/cm^2$

Question 11: Express the power of a 100 watt bulb in CGS unit.

Solution:
Write power 100 watt in CGS units.

In S.I. units: 100 watt = 100 Joule/sec
In C.G.S. unit = 10^9 erg/sec

Question 12: The normal duration of I.Sc. Physics practical period in Indian colleges is 100 minutes. Express this period in microcenturies. 1 microcentury = $10^{-6} \times 100$ years. How many microcenturies did you sleep yesterday?

Solution:
Given: 1 microcentury = $10^{-6} \times 100$ years.
1 year = 365 x 24 x 60 min
Or 1 microcentury = $10^{-4} \times 365 \times 24 \times 60$ min
So, 100 min = $10^5/52560$ = 1.9 microcentury

Question 13: The surface tension of water is 72 dyne/cm. Convert it in SI units.

Solution:
Given: surface tension of water is 72 dyne/cm

In S.I units: 72 dyne/cm = 0.072 N/m

Question 14: The kinetic energy K of a rotating body depends on its moment of inertia I and its angular speed ω. Assuming the relation to be $K = kI^a \omega^b$ where k is a dimensionless constant, find a and b. Moment of inertia of a sphere about its dimeter is $2/5\ Mr^2$.

Solution:
$K = kI^a \omega^b$; where k is a dimensionless constant, K = kinetic energy and ω = angular speed
To find: a and b
Now, $K = [ML^2T^{-2}]$

$I^a = [ML^2]^a$ and $\omega^b = [T^{-1}]^b$

$\Rightarrow [ML^2T^{-2}] = [ML^2]^a [T^{-1}]^b$
[using principle of homogeneity of dimension]

Equating the dimensions, we get
$2a = 2 \Rightarrow a = 1$
$-b = -2 \Rightarrow b = 2$

Question 15: Theory of relativity reveals that mass can be converted into energy. The energy E so obtained is proportional to certain powers of mass m and the speed c of light. Guess a relation among the quantities using the method of dimensions.

Solution:
The relationship between energy, mass and speed of light is,
$E \propto M^aC^b$
Where M = mass and C = speed of light
or $E = K\ M^aC^b$ ......(1)
where K = constant of proportionality

Find the dimensions of (1)

$[ML^2T^{-2}] = [M]^a [LT^{-1}]^b$

By comparing values, we have

a = 1 and b = 2

So, we have required relation is $E = KMC^2$

Question 16: Let I = current through a conductor, R = its resistance and V = potential difference across its ends. According to Ohm's law, product of two of these quantities equals the third. Obtain Ohm's law from dimensional analysis. Dimensional formulae for R and V are $[ML^2I^{-2}T^{-3}]$ and $[ML^2T^{-3}I^{-1}]$ respectively.

Solution:

Given:

Dimensional formulae for R = $[ML^2I^{-2}T^{-3}]$ and

Dimensional formulae for V = $[ML^2T^{-3}I^{-1}]$

Therefore,

$[ML^2T^{-3}I^{-1}] = [ML^2I^{-2}T^{-3}]\ [I]$

=> V = IR

Question 17: The frequency of vibration of a string depends on the length L between the nodes, the tension F in the string and its mass per unit length m. Guess the expression for its frequency from dimensional analysis.

Solution:

L = length, M = mass and F = Force

Here, $f = KL^aF^bM^c$ …(1)

Dimension of frequency, f = $[T^{-1}]$ or $[M^0L^0T^{-1}]$

Dimension of length, L = [L]

Dimension of mass, M = $[ML^{-1}]$

Dimension of force, F = $[MLT^{-2}]$

(1)=> $[M^0L^0T^{-1}] = K\ [L]^a\ [MLT^{-2}]^b\ [ML^{-1}]^c$

Equating both sides, we get

b + c = 0

-c + a + b = 0

-2b = -1

Solving above three equations , we have

a = -1, b = ½ and c = -1/2

Therefore, frequency is

$f = KL^{-1}F^{1/2}M^{-1/2}$

$$f = KL^{-1}F^{1/2}M^{-1/2} = \frac{K}{L}\sqrt{\frac{F}{M}}$$

Question 18: Test if the following equations are dimensionally correct:

(a) $h = \frac{2S\cos\theta}{\rho r g}$, (b) $v = \sqrt{\frac{P}{\rho}}$,

(c) $V = \frac{\pi P r^4 t}{8\eta l}$, (d) $\nu = \frac{1}{2\pi}\sqrt{\frac{mgl}{I}}$;

where h = height, S = surface tension, ρ = density, P = pressure, V = volume, η = coefficient of viscosity, v = frequency and I = moment of inertia

Solution:

(a) Dimension of h = [L]

Dimension of S = F/l = $MLT^{-2}/L = [MT^{-2}]$
Dimension of density = M/V = $[ML^{-3}T^{0}]$
Dimension of radius = [L]
Dimension of gravity = $[LT^{-2}]$
Now,

$$\frac{2S\cos\theta}{\rho r g} = \frac{[MT^{-2}]}{[ML^{-3}T^{0}][L][LT^{-2}]} = [M^{0}L^{1}T^{0}] = [L]$$

Relation is correct.

(b) Let velocity = v = $\sqrt{(p/\rho)}$ ….(1)
Dimension of v = $[LT^{-1}]$
Dimension of p = F/A = $[ML^{-1}T^{-2}]$

Dimension of ρ = m/v = $[ML^{-3}]$

Substituting dimensions in (1), we have

$$\sqrt{\frac{p}{\rho}} = \sqrt{\frac{[ML^{-1}T^{-2}]}{[ML^{-3}]}} = [L^2T^{-2}]^{1/2} = [LT^{-1}]$$

Therefore, relation is correct.

(c)
Dimension of V = $[L^3]$
Dimension of p = $[ML^{-1}T^{-2}]$
Dimension of r^4 = $[L^4]$
Dimension of t = $[T]$
Dimension of η = $[ML^{-1}T^{-1}]$

$$\frac{\pi p r^4 t}{8\eta l} = \frac{[ML^{-1}T^{-2}][L^4][T]}{[ML^{-1}T^{-1}][L]}$$

Therefore, relation is correct.

(d)
Dimension of v = $[T^{-1}]$
Dimension of m = $[M]$
Dimension of g = $[LT^{-2}]$
Dimension of l = $[L]$
Dimension of inertia = $[ML^2]$

$$\sqrt{(mgl/I)} = \sqrt{\frac{[M][LT^{-2}][L]}{[ML^2]}} = [T^{-1}]$$

Therefore, relation is correct.

Question 19: Let x and a stand for distance. Is below equation dimensionally correct?

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \frac{1}{a}\sin^{-1}\frac{a}{x}$$

Solution:

Dimensions of a = [L]
Dimensions of x = [L]

LHS

$$\int \frac{dx}{\sqrt{(a^2 - x^2)}} = \int \frac{L}{\sqrt{(L^2 - L^2)}} = [L^0]$$

RHS

$$\frac{1}{a}\sin^{-1}\left(\frac{a}{x}\right) = [L^{-1}]$$

$$\int \frac{dx}{\sqrt{(a^2 - x^2)}} \neq \frac{1}{a}\sin^{-1}\left(\frac{a}{x}\right)$$