Lecture - 15

Transverse Electric (TE), TM and TEM Modes:-

$$E(x,z,t)\ (x,y,z)$$
$$H(x,z,t)\ (x,y,z)$$

Using Maxwells equation

$$\nabla \times E = -j\omega\mu H \qquad \nabla \times H = j\omega\epsilon E$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

$$\begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \nabla \times E$$

→ All y derivatives are zero

→ All z derivatives are back the same function with $\bar{\gamma}$ scaling

$$\nabla \times E = -j\omega\mu H \qquad \nabla \times H = j\omega\epsilon E$$

$$\bar{\gamma} E_y = -j\omega\mu H_x \qquad \bar{\gamma} H_y = j\omega\epsilon E_x$$

$$-\bar{\gamma} E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \qquad -\bar{\gamma} H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y$$

$$\frac{\partial E_y}{\partial x} = -j\omega\mu H_z \qquad \frac{\partial H_y}{\partial x} = j\omega\epsilon E_z$$

$$H_x = \frac{-\bar{\gamma} E_y}{j\omega\mu}$$

$$-\bar{\gamma}\left(\frac{-\bar{\gamma} E_y}{j\omega\mu}\right) - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y$$

$$\bar{\gamma}^2 E_y - j\omega\mu \frac{\partial H_z}{\partial x} = -\omega^2 \mu\epsilon E_y$$

$$(\bar{\gamma}^2 + \omega^2\mu\epsilon) E_y = j\omega\mu \frac{\partial H_z}{\partial x}$$

(I) $$\frac{\partial H_z}{\partial x} = \frac{\gamma_x^2}{j\omega\mu} E_y$$

(II) $$\frac{\partial H_z}{\partial x} = \frac{-\gamma_x^2}{\bar{\gamma}} H_x$$

(III) $$\frac{\partial E_z}{\partial x} = \frac{-\gamma_x^2}{\bar{\gamma}} H_y$$

(IV) $$\frac{\partial E_z}{\partial x} = \frac{-\gamma_x^2}{\bar{\gamma}} E_x$$

<u>Note</u> :-

The axially directed field components determine the complete standing of the wave.

If $E_z = 0$ → physical connection

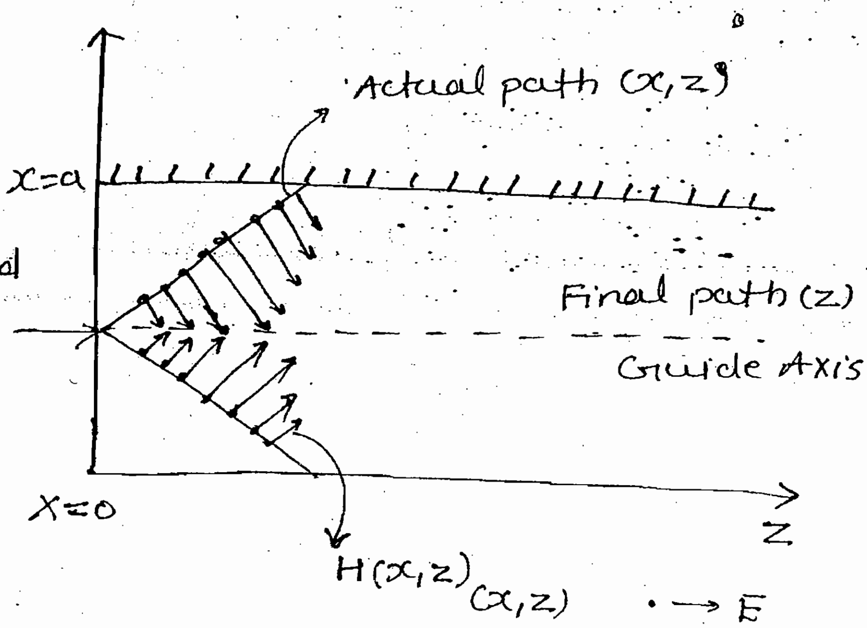
then $E_x = H_y = 0$ The waves becomes

$E(x, z, t)_y$

$H(\cancel{x}, \cancel{z}, t)_{(x,z)}$

E ⊥ Guide Axis

The wave is called as TE wave



If $H_z = 0$. then $H_x = E_y = 0$

the wave becomes $H(x,z,t)_y$

$E(x,z,t)_{x,z}$

~~E ⊥~~ $\boxed{H \perp \text{Guide Axis}}$

The wave is called as TM Waves

$\boxed{E \perp H \perp \text{Propogation}}$

TE Wave solutions in Parallel - Plane Waveguides :-

The waves has $(E_x = E_z = H_y = 0)$

$$E(x,z,t)_y = E_{y0} \cdot \sin\left(\frac{m\pi}{a}x\right) e^{-\overline{\gamma} z} e^{j\omega t} a_y$$

$$E(x,z,t)_x = H_{x0} \cdot \sin\left(\frac{m\pi}{a}x\right) e^{-\overline{\gamma} z} e^{j\omega t} a_x$$

$$H(x,z,t)_z = H_{z_0} \cos\left(\frac{m\pi}{a}x\right) e^{-\overline{\gamma} z} e^{j\omega t} a_z$$

It is a product (AND) solution of time, x & z

Harmonics

$E(t)/H(t) \longrightarrow e^{j\omega t}$ Source harmonic

$E(z)/H(z) \longrightarrow e^{-\overline{\gamma} z}$ Natural Harmonic

$E(x)/H(x) \longrightarrow$ Trignometric Harmonic

$E(x)_{tang} \longrightarrow$ 'Sin' Harmonic $\qquad \frac{\partial H_z}{\partial x} = (\ \ ) H_x$

$E(x)_y$ or $E(x)_z$ $\qquad = (\ \ ) E_y$

If $m = 0$ the TE wave does not exist and hence $m = 1$ is the least value required for propogation

TM Wave solutions in Parallel Plane Waveguides :-

($H_z = H_x = E_y = 0$)

$$H(x,z,t)_y = H_{y_0} \cos\left(\frac{m\pi}{a} x\right) e^{-\bar{\gamma} z} e^{j\omega t} a_y$$

$$E(x,z,t)_x = E_{x0} \cos\left(\frac{m\pi}{a} x\right) e^{-\bar{\gamma} z} e^{j\omega t} a_x$$

$$E(x,z,t)_z = E_{z0} \sin\left(\frac{m\pi}{a} x\right) e^{-\bar{\gamma} z} e^{j\omega t} a_z$$

If $m = 0$, $E_z = 0$

The wave becomes $E(z,t)_x$
$H(z,t)_y$

This wave is called as ~~TEM waves~~ TEM Waves with $E_z = H_z = 0$

Properties of TEM Waves :-

I) It has $m=0$ $\bar{\gamma} = \gamma = j\omega\sqrt{\mu\epsilon}$
- i.e. Propogates only along guide axis i.e. $\theta = 0$ → Always

II) It has $f_c = 0$ i.e. No cut off frequency

III) It has $m=0$ i.e. No multi-mode connection mechanism

Summary :-

$E(x,t)$, $H(x,t)$ → Wave at f_c, $\theta = 90°$, $\bar{\gamma} = 0$
(f_{c_1}, f_{c_2} ----- f_{c_m} exists)

$E(z,t)_x$, $H(z,t)_y$ → TEM wave, $\theta = 0°$ $\bar{\gamma} = \gamma = j\omega\sqrt{\mu\epsilon}$
$m = 0$, $f_c = 0$

3. $E(x,z,t)$, $H(x,z,t)$
→ TE → E_y, H_x, H_z → θ, f_c, mode, $\bar{V_p}$, $\bar{V_g}$
→ TM → H_y, E_x, E_z → θ, f_c, mode, $\bar{V_p}$, $\bar{V_g}$

Waveguides (Workbook) :-

I) $f = 22\ GHz$ $\sin\theta = \frac{f_c}{f}$ $f_c = \frac{mc}{2a}$

$f_{c_1} = \frac{1.38 \times 10^8}{2.25 \times 10^{-2}} = 6\ GHz$

1st Mode - $6\ GHz \longrightarrow \infty$

1st Mode $\sin\theta = \frac{6}{22}$

IInd Mode $12\ GHz \longrightarrow \infty$

III Mode $\sin\theta = \frac{18}{22}$

IIIrd Mode $18\ GHz \longrightarrow \infty$

II) $f = 40\ GHz$

$f_c = 36 \times 10^9 = \frac{3.3 \times 10^8}{2 \times a} \Rightarrow a = 1.25\ cm$

III) $\sin\theta = \frac{36}{40} = \frac{f_{c_3}}{f_3} = \frac{f_{c_7}}{f_7} = \frac{84}{f_7}$

3rd Mode $\longrightarrow 36$

7th Mode $\longrightarrow 84$

$f_7 = 93.3\ GHz$

IV) $f = 2\ GHz$

$f_{c_1} = \frac{1.3 \times 10^8}{\sqrt{9} \cdot 2 \times 3 \times 10^{-2}}$

$= \frac{5}{3} = 1.67\ GHz$

1st $\rightarrow 1.67 \rightarrow \infty$
2nd $\rightarrow 3.34 \rightarrow \infty$

$\sin\theta = \frac{5/3}{2} = \frac{5}{6}$

Note:-

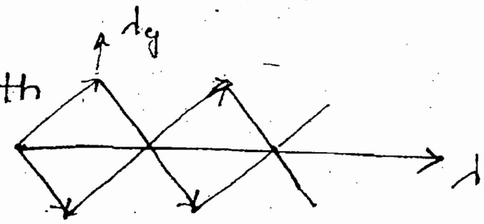
1. $\overline{\gamma}, \overline{\beta}, \overline{\lambda}, \overline{V_p}, \overline{V_g}$, TRANSVERSE $\longrightarrow$ w.r.t guide axis

$$\eta_{TE} = \frac{E_{trans}}{H_{trans}} = \frac{E_y}{H_x} = \frac{E_{total}}{H_{total}\cos\theta} = \frac{120\pi}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

9n:

$$\eta_{TM} = \frac{E_x}{H_y} = \frac{E_{total}\cos\theta}{H_{total}} = 120\pi\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

3. $$\bar{\lambda} = \frac{\lambda}{\cos\theta} = \lambda_g = \text{Guide Wavelength}$$



$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}}$$

$$1-\left(\frac{\lambda}{\lambda_c}\right)^2 = \left(\frac{\lambda}{\lambda_g}\right)^2$$

$$\boxed{\frac{1}{\lambda^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}}$$

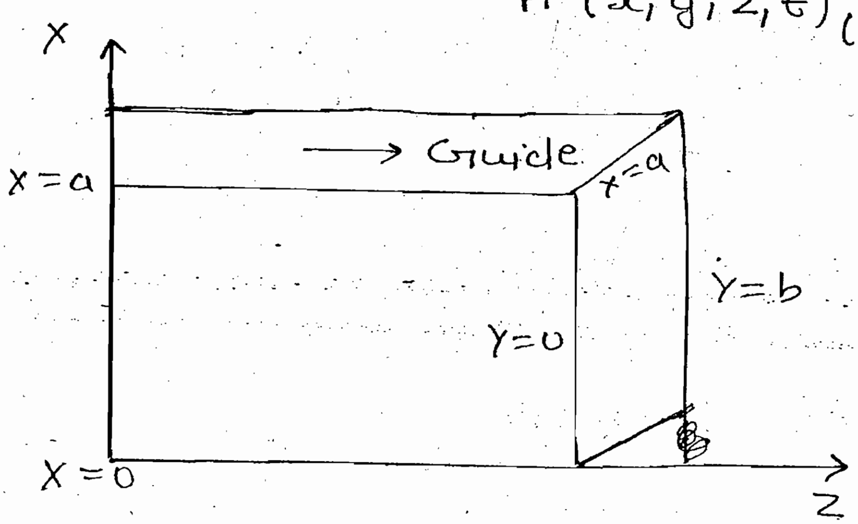
$\lambda \rightarrow \frac{c}{f}$ free space wavelength

$\lambda_c \rightarrow \frac{2a}{m}$ cut-off wavelength

Rectangular Waveguides:-

The waves are $E(x, y, z, t)\,(x, y, z)$

$H(x, y, z, t)\,(x, y, z)$



→ It is a single conductor hollow structure used to confine EM wave in 2 dimension using 4 walls

$x = 0 \quad x = a$

$y = 0 \quad y = b$

It has $\gamma_x = \frac{m\pi}{a}$

$\gamma_y = \frac{n\pi}{b}$

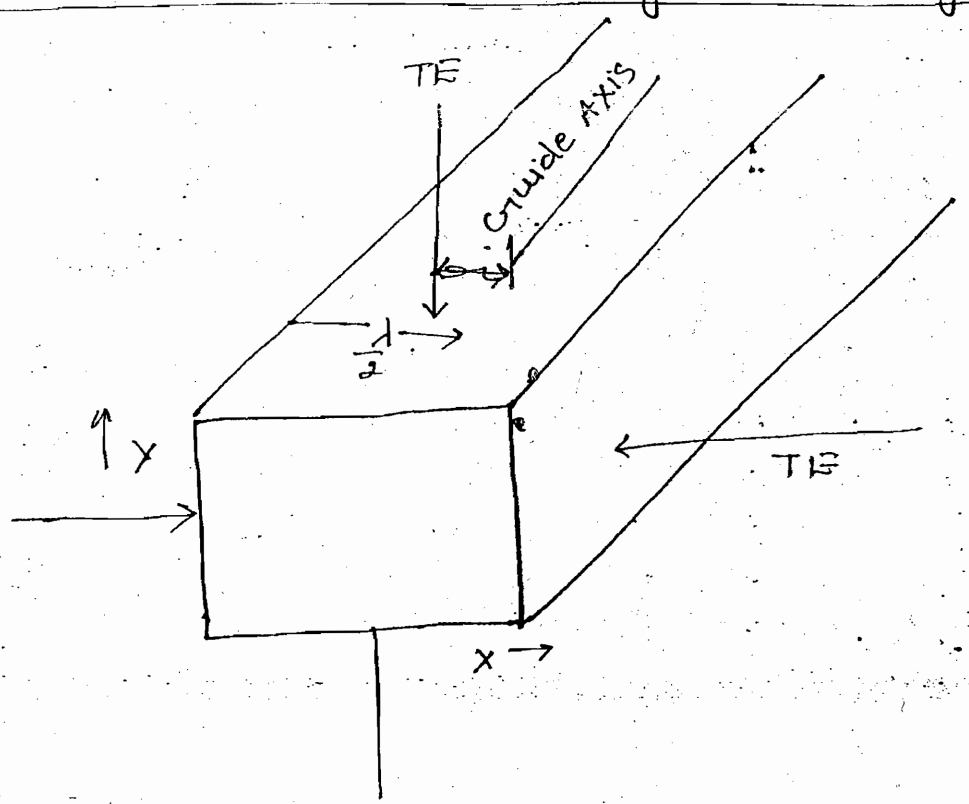
Applying $\nabla^2 E = -\omega^2 \mu \epsilon E$

$$\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \epsilon}$$

$$\omega_c = \text{cut-off frequency} = \left(\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}\right) c$$

$$f_c = \left(\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\right) \frac{c}{2}$$

Modes and feeds in Rectangular Waveguides:-



E_y or E_x but $E_z = 0$

A horizontal or vertical field always gives a suitable E field but not along the guide axis

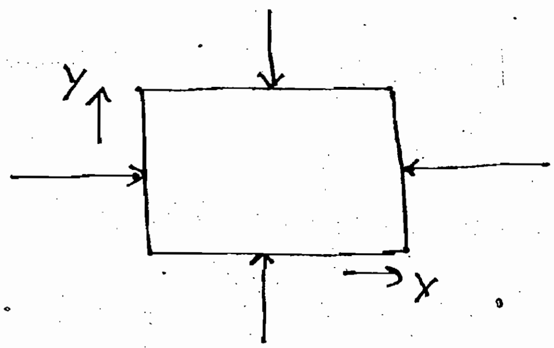
The no. of out of phase feed connections decides the integers m and n.

The modes are always designated as TE_{mn}

H_x \$ H_y but $H_z = 0$

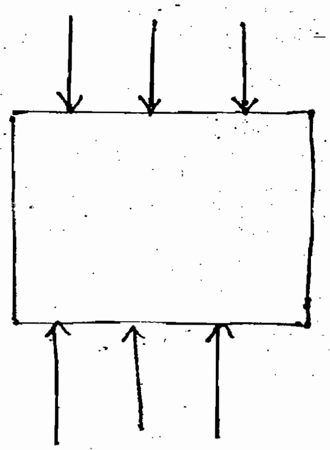
A lateral or axial feed always gives a suitable H field but not along the guide axis

(I)



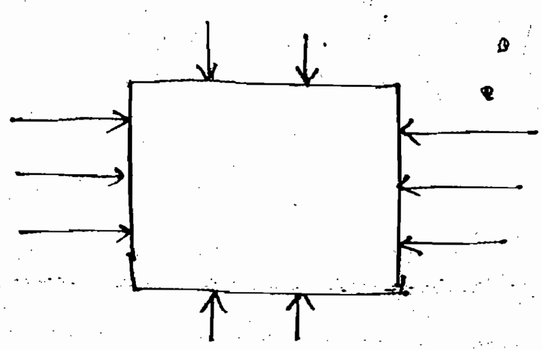
TE_{11}

(II)



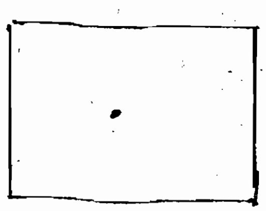
TE_{03}

(III)



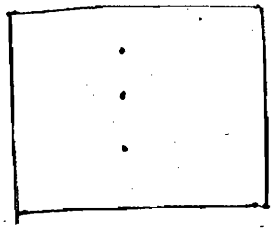
~~TE_{32}~~ TE_{23}

(IV)



TM_{11}

(v)



TM_{13}

(vi)

TM_{42}

Note:-

TM_{M0} or TM_{0N} modes are evenescent and donot exist in rectangular waveguides.

Such a connection mechanism doesn't exist.

(1) $f_c = \left(\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\right) \frac{c}{2}$

For both TE & TM are same so TE_{74}, TM_{74} are same cutoff called degenerate mode

Two different modes having the same f_c but different connection mechanism) are said to be degenerate modes.

(III) For $m=1$, $n=0$

or $m=0$, $n=1$

i.e. TE_{10} & TE_{01} mode have the least f_c

$f_c\,(TE_{10}) = \frac{c}{2a}$

$f_c\,(TE_{01}) = \frac{c}{2b}$

If $a>b$ TE_{10} is dominent mode

$a<b$ TE_{01} is dominent mode

→ Broadside dimensions decides the dominent mode

→ Narrowside dimensions decides the maximum operable voltage and power handling ability

TM Wave Solutions in Rectangular Waveguide ($H_z = 0$) :-

The wave is $E(x,y,z,t)_{(x,y,z)}$
$H(x,y,z,t)_{(x,y)}$

$$E(x,y,z,t)_z = E_{z0} \cdot \sin\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-\bar{\gamma} z} e^{j\omega t} a_z$$

$$E(x,y,z,t)_x = E_{x0}\left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-\bar{\gamma} z} e^{j\omega t} a_x$$

$$E(x,y,z,t)_y = E_{y0}\left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-\bar{\gamma} z} e^{j\omega t} a_y$$

$$H(x,y,z,t)_x = H_{x0}\left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-\bar{\gamma} z} e^{j\omega t} a_x$$

$$H(x,y,z,t)_y = H_{y0}\left(\frac{n\pi}{a}\right) \cos\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-\bar{\gamma} z} e^{j\omega t} a_y$$

It is a product solution of x, y, z, t harmonics.

$E(t)/H(t) \longrightarrow e^{j\omega t} \longrightarrow$ Source harmonic

$E(z)/H(z) \longrightarrow e^{-\gamma z} \longrightarrow$ Natural Harmonic

$H/E(x \text{ or } y) \longrightarrow$ Trignometric harmonic

$\left.\begin{array}{l} E(x)_y \text{ or } E(x)_z \\ E(y)_x \text{ or } E(y)_z \end{array}\right]$ Tangential harmonic $\Big\}$ 'sin' Harmonics

If $m = 0$ & $n \neq 0$ or $m \neq 0$ & $n = 0$

TM waves donot exist i.e. they are even sent modes

TE Wave Solutions in Rectangular Waveguide ($E_z = 0$):-

The wave is $E(x,y,z,t)_{(x,y)}$

$H(x,y,z,t)_{(x,y,z)}$

$$H(x,y,z,t)_z = H_{z0} \cdot \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\bar{\gamma}z} \cdot e^{j\omega t} a_z$$

$$E(x,y,z,t)_x = E_{x0} \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\bar{\gamma}z} e^{j\omega t} a_x$$

$$E(x,y,z,t)_y = E_{y0} \left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\bar{\gamma}z} e^{j\omega t} a_y$$

$$H(x,y,z,t)_x = H_{x0} \left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\bar{\gamma}z} e^{j\omega t} a_x$$

$$H(x,y,z,t)_y = H_{y0} \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\bar{\gamma}z} e^{j\omega t} a_y$$

If $m \neq 0$ & $n = 0$

The TE_{m0} wave exists $E(x,z,t)_y$

$H(x,z,t)_{(x,z)}$

If $m = 0$ & $n \neq 0$

The TE_{0n} wave exists $E(y,z,t)_x$

$H(y,z,t)_{(y,z)}$

If $E_z = H_z = 0$ the wave cannot exist

or $m = n = 0$

i.e. TEM waves donot exist in rectangular waveguide.

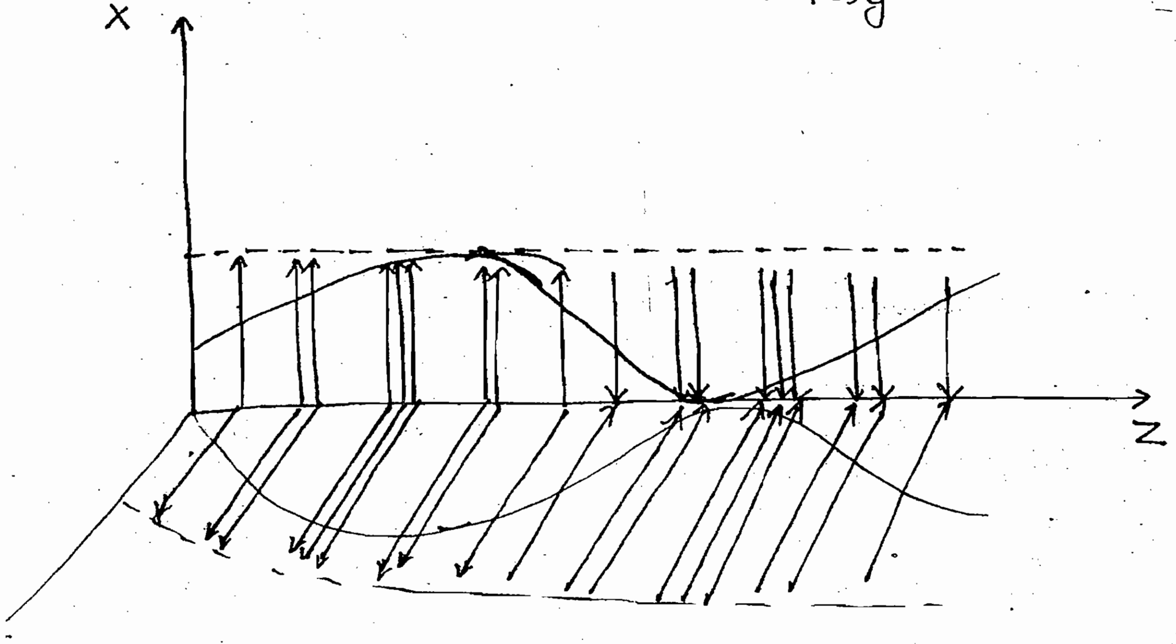
In general TEM waves donot exist in all single conductor guides

TEM needs two distinct conductor for its existence.

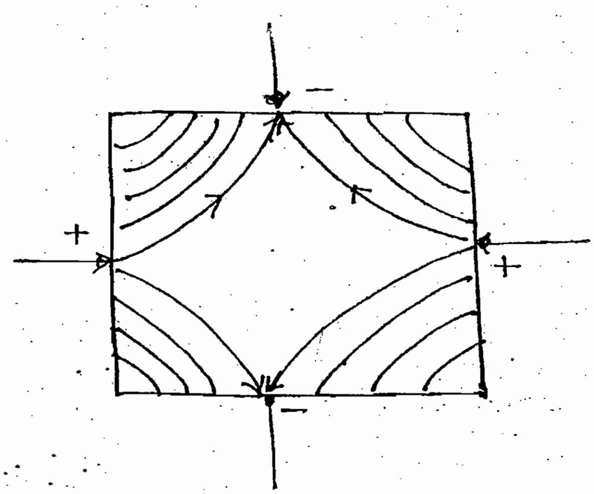
Field line Representations of guided waves:-

(1) Uniform plane Wave - $E(z,t)_x$

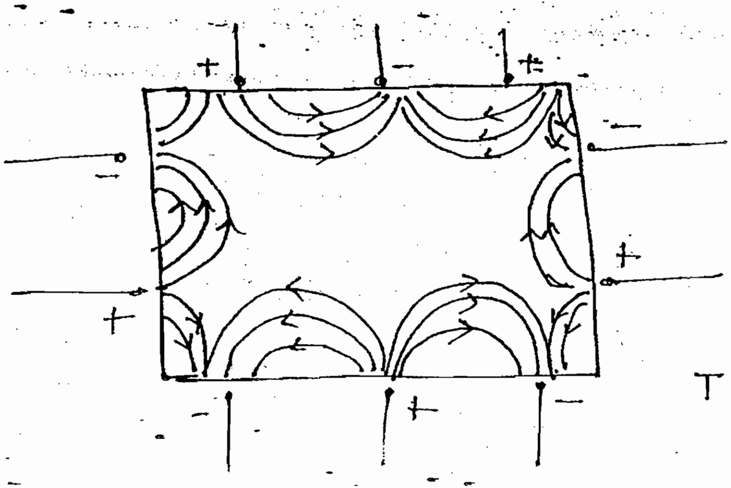
$H(z,t)_y$



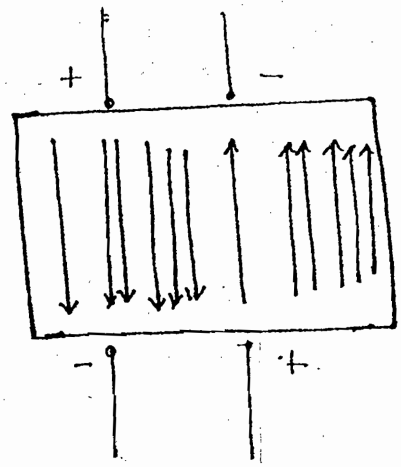
TE Waves:- (E_x, E_y)



TE_{11}

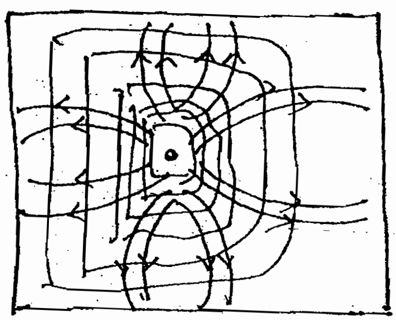


TE_{32}



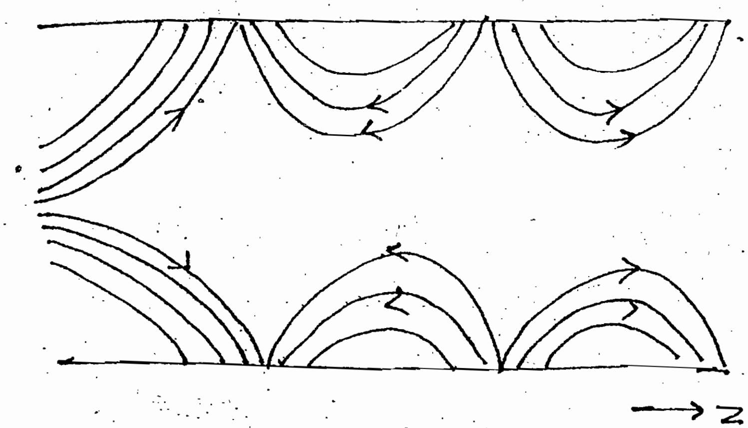
TE_{20}

TM Waves (E_x, E_y, E_z):-

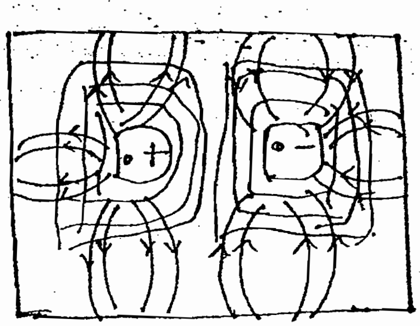


~~TE~~

TM_{11}



TM_{21}:-



TM_{21}

10K

With a guide axis out of board and both E & H fields traced on the board the wave is called as TEM waves else TE or TM

eg:- Co-axial cable and all other low frequency Transmission line. So TEM waves is low frequency energy transfer format

TM/TE $\rightarrow$ High frequency transmission waveguide

Workbook!-

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}$$

$$\frac{3\times10^8}{2.5\times10^9} = 12cm$$

TE_{10} $\rightarrow$ $2a = 20cm$

$$\lambda_g =$$

6. $$f_c = \frac{1\times3\times10^8}{\sqrt{4}\times2\times3\times10^{-2}}$$

$$= 2.5 GH$$

7. $v_p > c$

3. $$f_c = 10\times10^9 = \frac{1\times3\times10^8}{2\times a}$$

$a = 1.5cm$

$a \cdot b > 2$

$a > b$

9. $$\eta_{TE} = \frac{120\pi}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$= \frac{377}{\sqrt{1-\left(\frac{90}{30}\right)^2}}$$

$$= 400\Omega$$

$$f_c = \frac{c}{2a} = 10GHz$$

0. $E(x,z,t)_y$

$\rightarrow$ Not depends on y

$\Rightarrow n=0$

$\sin\left(\frac{2\pi}{a}x\right)$

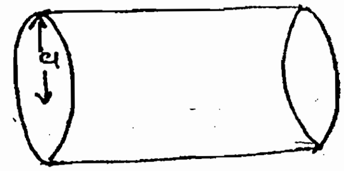
$\sin(\omega t - \beta z)$

TE_{m0}

TE_{20}

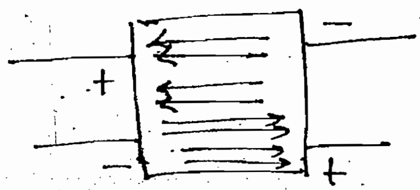
11. TEM exists → single conductor

(B)



12. No cut off freq. → TEM exists   Ans - A

13. TE_{02}



14. $f_c = \frac{3\times10^8}{2\times2\times10^{-2}} = \frac{3\times10^8}{2\times\sqrt{\epsilon_R}\times1\times10^{-2}} \Rightarrow \epsilon_R = 4$

15 **Note:-** Parallel wave guide

TEM — $(0-\infty)$

TE_1/TM_1 — $(f_{c_1}-\infty)$

TE_2/TM_2 — $(f_{c_2}-\infty)$

15. ① $m=1$ $n=0$
② $m=0$ $n=1$
③ $m=1$ $n=1$

$$f_c = \left(\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}\right)\frac{c}{2}$$

4×3 cm

① → TE_{10} $f_{c_1} = \frac{c}{2a} = 3.75$ GHz $(3.75\to\infty)$ GHz

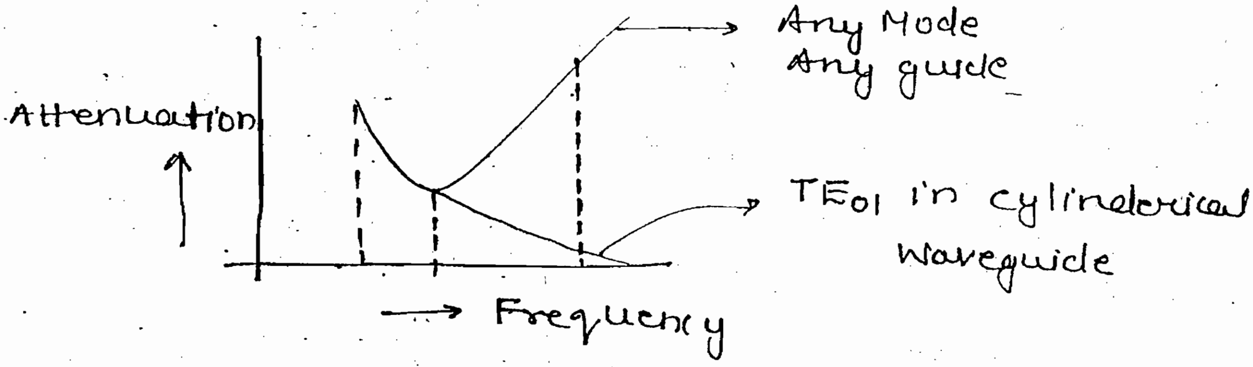
TE_{01} $f_{c_2} = \frac{c}{2b} = 5$ GHz $(5\text{ GHz}\to\infty)$

$\frac{TE_{11}}{TM_{11}}$ $f_{c_3} = 6.25$ GHz $(6.25\text{ GHz}\to\infty)$

From (3.75-5) GHz there is strictly single mode operation.

Note:-

Practically Graph :-



With practical non-ideal conducting walls, higher freq. are not preferred in lower modes

16. → D

17. → TE_{10} only

$$\frac{c}{2a} < \frac{c}{\lambda} < \frac{c}{2b}$$

$$\underset{TE_{10}}{f_c} < f < \underset{TE_{01}}{f_c}$$

$$2a > \lambda > 2b$$

ANTENNAS :-

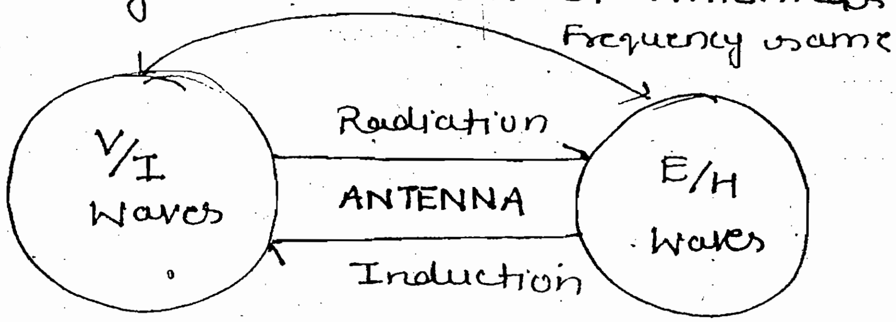
→ Hertzian Dipole / Halfwave Dipole

→ Basic Terms and Definitions → 50%

→ Antenna Arrays

→ FRIIS – Free Space Propogation

→ Study / classification of Antennas



Hertzian Dipole as a Basic Radiating Element :-

It is a dl length $I(t) = I_m \sin\omega t$ (oscillatory or Harmonic) current element with $dl << \lambda$

(I) $$A(t) = \frac{\mu I(t) dl}{4\pi r}$$

(II) $$B(t) = \mu H(t) = \nabla \times \bar{A}$$

(III) $$\nabla \times H = \epsilon \frac{\partial E}{\partial t}$$ (IV) $$E(t) = \frac{1}{\epsilon}\int (\nabla \times H)\, dt$$

Every harmonic current produces a time / space harmonic E & H around it. This EM wave generation is called as radiation

Expression for Radiated fields of Hertzian Dipole :-

Radi-ated wave

$$E(r,\theta,\phi,t)_\theta = \left(\frac{I_m dl \cdot \sin\theta \cdot \omega}{4\pi\epsilon c^2 r}\right) \sin\omega t \cdot e^{-j\beta r} a_\theta$$

$$H(r,\theta,\phi,t)_\phi = \left(\frac{I_m dl \sin\theta \, \omega}{4\pi c r}\right) \sin\omega t \cdot e^{-j\beta r} a_\phi$$

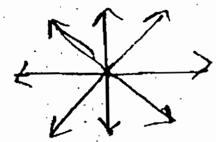
10 p

W → $E(z,t)_x = E_0 \, e^{j\omega t} e^{-\gamma z} \, a_x$

Properties of Radiated Waves:-

i) They travel radially outward and hence their amplitudes decrease as $\frac{1}{r}$ due to their power density decreases as $\frac{1}{r^2}$

→ Amplitude dec. but does not attenuate.



$$E_\theta \times H_\phi = P_r$$

$$\frac{E_\theta}{H_\phi} = \frac{1}{\epsilon c} = \frac{1}{\epsilon \sqrt{\frac{1}{\mu \epsilon}}} = \sqrt{\frac{\mu}{\epsilon}} = 120\pi \ \Omega$$

(ii) The amplitude of the radiated wave is not omni-directional and depends on θ and ϕ i.e. Radiation is directive in nature

(iii) The amplitude of the radiated wave always depends on $\left(\frac{dl}{\lambda}\right)$ ratio

i.e. frequency decides radiated power

Total Power Radiated from a Hertzian Dipole :-

$$W_r = \oint P_{avg} \cdot ds$$

$$= \oint_{\text{Sphere}} \frac{1}{2} \frac{E_0^2}{\eta} a_r \cdot ds \cdot a_r$$

$$W_r = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{1}{2} \left(\frac{I_m \, dl \sin\theta \, \omega}{4\pi \epsilon c^2 r} \right)^2 \frac{1}{120\pi} r^2 \sin\theta \, d\theta \, d\phi$$

$$\left(\omega = \frac{2\pi c}{\lambda} \right)$$

$$W_r = I^2_{rms} \cdot 80\pi^2 \left(\frac{dl}{\lambda}\right)^2$$

The expression is similar to a resistor dissipating power as heat i.e. every antenna radiates power in the form of EM wave

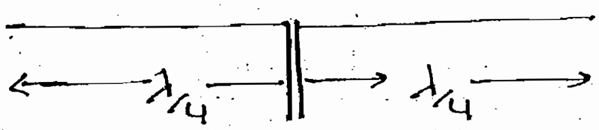
R_r = Radiation resistance of Hertzian Dipole

$$= 80\pi^2 \left(\frac{dl}{\lambda}\right)^2$$

→ R_r is a measure of radiated power for a given input current i.e. it should be as possible for a practical antenna

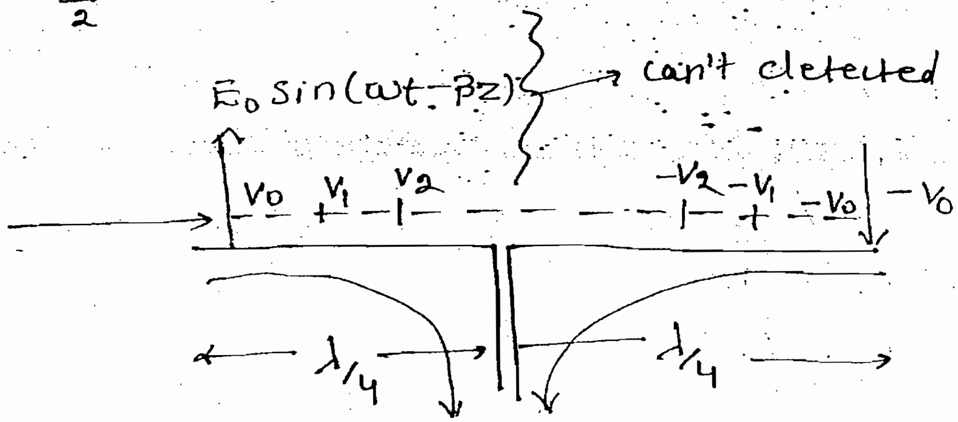
Halfwave dipole as a fundamental Antenna :-

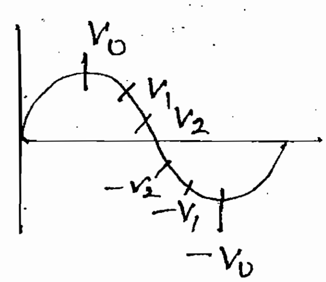
→ A centre feed $\lambda/2$ - dipole is a transmission opened out by $\lambda/4$ on either sides



→ The length of antenna is strictly depends on the frequency of operation

FM	GSM
$f = 100\,MHz$	$f = 1800\,MHz$
$\lambda = 3m$	$\lambda =$ few cm
$\frac{\lambda}{2} = 1.5m$	





Note :-

When an EM waves having right frequency and right polarization travels along the axis of the antenna, it induced V_0 voltage at one end and progressively decrease voltage such that it is $-V_0$ at the other end. Hence the word half wave dipole

> These voltages drive a current towards centre which is maximum in the line such that

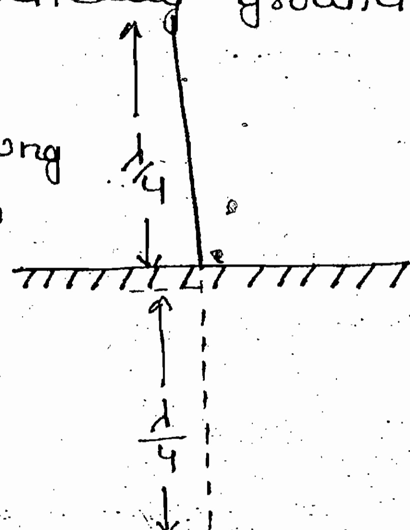
$$\text{Assymptotic } V/I \text{ distribution} \left\{ \begin{array}{l} |V(z)| = V_0 \sin \beta z \\ |I(z)| = I_0 \cos \beta z \end{array} \right] \longrightarrow z = -\frac{\lambda}{4} \text{ to } \frac{\lambda}{4}$$

* V_0 = -ve at other end bec. of phase diff = 180°

Quaterwave Monopole as a Practical Antenna :-

→ It is a low frequency vertically grounded single wire of $\lambda/4$ length

→ It is a base feed and along with its image works like a harmonic dipole.



Summary :-

A halfway dipole is an array of Hertzian dipoles with dl from $-\lambda/4$ to $\lambda/4$ and

$$\underbrace{I_m = |I(z)| = I_0 \cos \beta z}_{\text{Assymptotic}}$$

Radiation Expressions for Halfwave Dipole :-

$$E(r,\theta,\phi,t)_\theta = \left(\frac{60 I_0}{r} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right) \cdot \sin\omega t \cdot e^{-j\beta r} a_\theta$$

$$H(r,\theta,\phi,t)_\phi = \left(\frac{I_0}{2\pi r} \frac{\cos\left(\pi/_2 \cos\theta\right)}{\sin\theta} \right) \sin\omega t \cdot e^{-j\beta r} a_\phi$$

Total power radiated from Halfwave Dipole

$$W_r = \int \frac{1}{2} \frac{E_0^2}{\eta} \cdot d_r$$

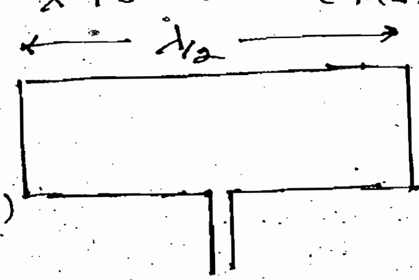
$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{1}{2} \left(\frac{60 I_0}{r} \cdot \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right)^2 \cdot \frac{1}{120\pi} r^2 \sin\theta d\theta d\phi$$

$$= I_{rms}^2 (73)$$

R_r for a Halfwave Dipole $= 73\Omega$

R_r for quarterwave monopole $= 36.5\Omega$

R_r for a folded dipole $= 2^2 \times 73\Omega = 292\Omega$



R_r for n-Dipoles $= n^2 \times 73$

$(n^2 \times 273)$

Basic Terms and Definitions :-

1) Isotropic Antenna :-

It radiates power in all directions uniformly. Its E field pattern is independent of θ and ϕ

eg:- Broadcast antenna

(ii) Radiation Power Density :-

It is the strength of the radiated EM wave in any direction at any distance from the antenna

$$\frac{\text{Power}}{\text{Area}} \text{ or } \text{Watts}/m^2 = \frac{dW_r}{ds}$$

= Poynting vector of EM wave

$$= \frac{1}{2}\frac{E_0^2}{\eta}(r, \theta, \phi)$$

$$= V(r, \theta, \phi)$$

Imp

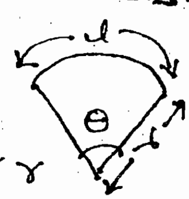
iii) Radiation Power Intensity :-

It is the strength of the radiated EM wave in any direction from the antenna

$$\frac{\text{Power}}{\text{direction}} = \frac{\text{Power/Solid angle}}{}$$

= with steradian $= \dfrac{dW_r}{d\Omega}$

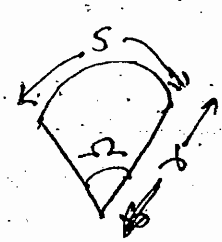
$l = \theta r$



If $\theta = 1$ then $l = r$

If $\theta = 6.28$ radian then $C = 6.28\,r$

$S = \Omega r^2$



$\Omega = 1$ Steradian

$S = r^2$

$\Omega = 12.56$ steradian

Total surface area $= 12.56\,r^2$

Any small incremental area, $ds = r^2 d\Omega = r^2 \sin\theta \, d\theta \, d\phi$

$d\Omega = \sin\theta \, d\theta \, d\phi$

$$\frac{dW_r}{d\Omega} = \frac{dW_r}{ds} \cdot \frac{ds}{d\Omega}$$

$$= V(r, \theta, \phi) \, r^2 = \Psi(\theta, \phi)$$

eg:- Isotropic Antenna

$$\frac{dW_r}{ds} = \frac{W_r}{4\pi r^2} = V_{avg}.$$

$$\frac{dW_r}{d\Omega} = \frac{W_r}{4\pi} = \Psi_{avg}.$$

IV) <u>Gain of an Antenna</u>:-

$G_D \longrightarrow$ Directive gain.

$G_P \longrightarrow$ Power gain

$D \longrightarrow$ Directivity

(a) <u>Directive Gain (G_D)</u>:-

The radiation intensity of the antenna in a given direction to the radiation intensity of isotropic ~~ao~~ antenna

$$G_D = \frac{\Psi(\theta, \phi)}{\Psi_{avg}} = \frac{4\pi \Psi(\theta, \phi)}{W_r} = \frac{4\pi \Psi(\theta, \phi)}{\int \Psi(\theta, \phi) \, d\Omega}$$

$G_D > 1$ or $G_D < 1 \longrightarrow$ depends on direction

(b) <u>Power Gain</u>:-

$$G_P = \frac{4\pi \cdot \Psi(\theta, \phi)}{W_N}$$

W_r = Total o/p power

W_N = Total i/p power

$$G_p = \frac{4\pi \psi(\theta, \phi)}{W_r} \cdot \frac{W_r}{W_N}$$

$$= G_D \times \text{Efficiency of Radiation}$$

$$\text{Efficiency} = \frac{W_r}{W_r + W_l} = \frac{R_r}{R_r + R_l}$$

$$\text{Directivity} = G_D\Big|_{max}$$

$\boxed{D \geq 1}$ → Always

$D = 1$ for isotropic antenna

Radiation Pattern of an Antenna:-

It is a polar plot of radiation intensity indicating various directions around the antenna where radiation is finitely strong

HPBW — Half Power Beam width

BWFN — Beam Width Full Nulls

eg:- $|E| = \frac{k\sin\theta}{r}$

$\psi = \psi_0 \sin^2\theta$

$\theta = 0°/180°$

$|E| = 0$ → Null points

→ θ_{NP}

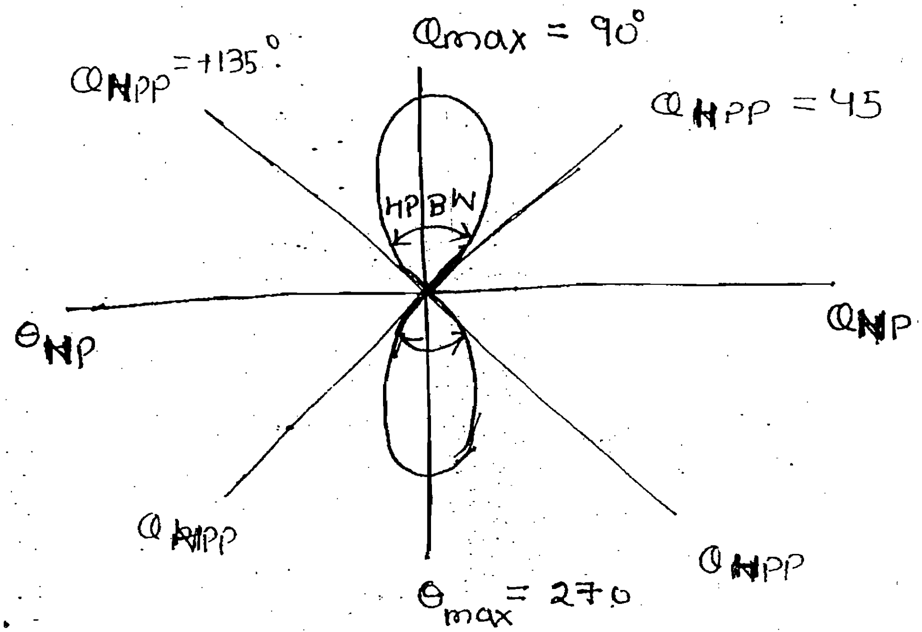
$\theta = 90^\circ/270^\circ$

$|E| = E_{max}$

$\rightarrow \theta_{max}$

$\theta = 45^\circ/135^\circ/225^\circ/315^\circ$

$|E| = \frac{E_{max}}{\sqrt{2}}$



$$\theta_{HPBW} = \text{HPP is next HPP in the maxima} \quad \text{(Half power pt.)}$$

$$= 135^\circ - 45^\circ = 90^\circ$$

The antenna considered has a ϕ independent pattern and hence a circular top view of the beam (for all ϕ)

In general

$$\theta_{HPBW} \times \phi_{HPBW} = \Omega_A \quad \text{Beam solid angle}$$

$$= \text{steradian} = \text{radian}^2$$

In circular

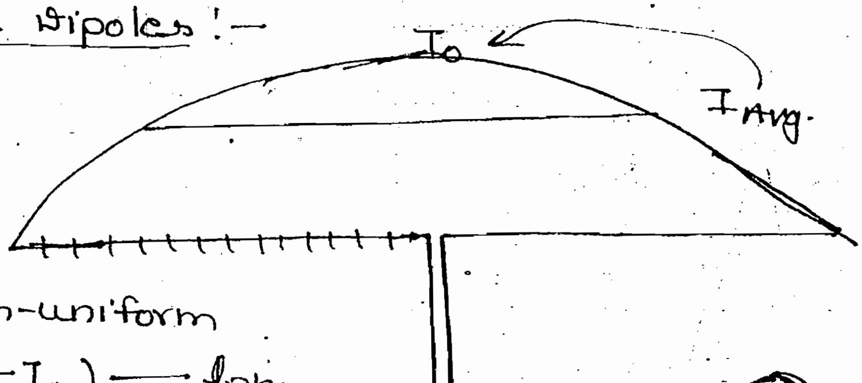
$$\left(\theta_{HPBW}\right)^2 = \Omega_A$$

$$D \propto \frac{1}{\Omega_A}$$

$$\Rightarrow \boxed{D = \frac{4\pi}{\Omega_A}}$$

(VI) Effective length of an antenna :-

(a) For Halfwave Dipoles :-



Non-uniform

W_r —— $(0 - I_0)$ —— l_{phy}

W_r —— I_{avg} —— l_{eff}

Note :-

→ An antenna radiates W_r power over its physical length and non-uniform currents everywhere. Effective length is a length required to radiate same power assuming uniform currents everywhere

$$I_{avg} = \frac{1}{\pi}\int_0^{\pi} I_0 \sin t \, dt = \frac{2I_0}{\pi}$$

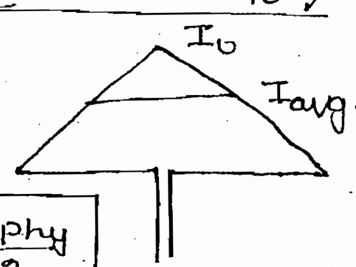
$$\boxed{l_{eff} = \frac{2 l_{phy}}{\pi}}$$

(b) For electrically short Dipoles $\left(l_{phy} < \frac{\lambda}{10} \right)$:-

$|I(z)| \propto z$

Linear current

$$I_{avg} = \frac{I_0}{2}$$



$$\boxed{l_{eff} = \frac{l_{phy}}{2}}$$

c) <u>For Hertzian Dipole</u> $\left(l_{phy} < \frac{\lambda}{25} \right)$:—

$$l_{phy} = l_{eff.} = dl$$

$$I_{avg} = I_m \longrightarrow \text{Uniform currents}$$

For Hertzian Dipole $R_r = 80\pi^2 \left(\frac{dl}{\lambda} \right)^2$

For any Dipole, $R_r = 80\pi^2 \left(\frac{l_{eff}}{\lambda} \right)^2$

For Electrically short Dipoles $R_r = 80\pi^2 \left(\frac{l}{2\lambda} \right)^2$

$$= 20\pi^2 \left(\frac{l}{\lambda} \right)^2$$

" " Monopoles $R_r = 10\pi^2 \left(\frac{l}{\lambda} \right)^2$

Monopoles over conducting earth $R_r = 10\pi^2 \left(\frac{2h}{\lambda} \right)^2$

$$= 40\pi^2 \left(\frac{h}{\lambda} \right)^2$$

For halfwave dipole $= 80\pi^2 \left(\frac{2l}{\pi\lambda} \right)^2$

$$= 320\pi^2 \left(\frac{l}{\lambda} \right)^2 = 80\,\Omega$$

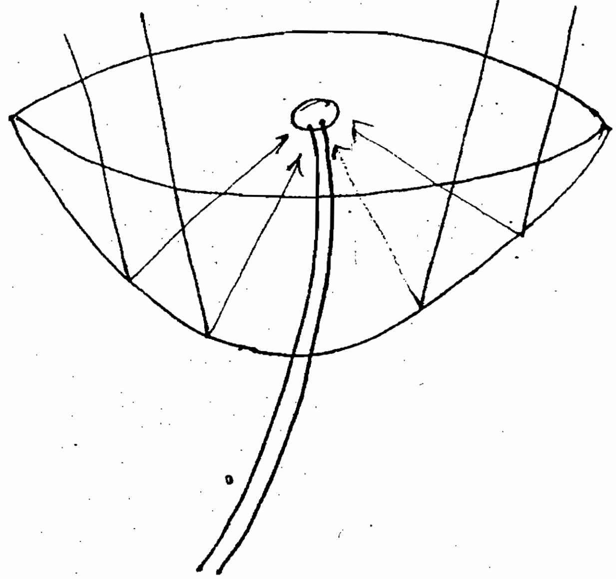
VII) <u>Capture Area or Effecting Area (A_c)</u> :—

In microwave antenna l = few cm

r = few 1000 km

A_C = capture Area = $\dfrac{\text{Power induced}}{\text{Poynting Vector}}$

$$\boxed{A_C = \frac{\lambda^2}{4\pi} D}$$



Workbook :-

$$D \rightarrow G_D\big|_{max} \longrightarrow \Psi(\theta,\phi) \longrightarrow U(r,\theta,\phi) \longrightarrow E(r,\theta,\phi)$$

Hertzian Dipole, $|E| = \dfrac{k\sin\theta}{r}$

$$U(r,\theta,\phi) = \frac{1}{2}\frac{k^2\sin^2\theta}{r^2\cdot\eta}$$

$$\Psi(\theta,\phi) = \frac{1}{2}\frac{k^2\sin^2\theta}{\eta}$$

$$G_D = 4\pi^2 \frac{1}{2}\frac{k^2\sin^2\theta}{\eta}$$

$$\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{1}{2} k^2 \frac{\sin^2\theta}{\eta} \sin\theta \, d\theta \cdot d\phi \quad = \underline{2\sin^2\theta}$$

$$\int_{\theta=0}^{\pi} \sin^3\theta \, d\theta = \frac{4}{3}$$

$$\Leftarrow \qquad = \frac{3}{2}\sin^2\theta$$

$D = GD|_{max} = 1.5$

For Halfwave Dipole, $D = 1.63$

$4 - \frac{\lambda}{2}$ Dipoles

$W_r = 4 I_{rms}^2 \cdot 73 = 4 \left(\frac{0.5}{\sqrt{2}}\right)^2 73 = 36.5$ Watts

3. $D = \frac{4\pi}{\Omega_A}$

$10 \log D = 44$ dB

$\Rightarrow D = \frac{4\pi}{(\theta_{HPBW})^2}$

$\Rightarrow D = 10^{4.4}$

$10^{4.4} = \frac{4 \times 3.14 \times (57)^2}{(\theta_{HPBW})^2}$

4. Efficiency $= \frac{R_r}{R_r + R_d}$

R_r for $\lambda/8$ Dipole

$R_r \; \lambda/2$ — 73Ω

$R_r \; \lambda/4$ — 36.5Ω

$R_r \; \lambda/8$ — $18.25\,\Omega$

Efficiency $= \frac{18.25}{19.75} = 89\%$

5. $\lambda \rightarrow 492$ m

$d \rightarrow 124$ m

$\frac{\lambda}{4} = 123$ m

$R_r = 36.5\Omega$

6. $10 \log 4 = 6$ dB

$G = 4$

$W_{IN} = 1$ mW

lossless

$W_r = W_{IN}$

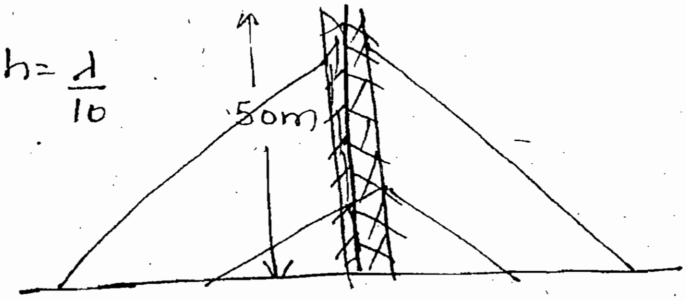
$\rightarrow$ Ans- (B)

7. (B)

$h = 50m$

$f = 60 kHz$ $\quad h = \frac{\lambda}{10}$

$\lambda = \frac{3 \times 10^8}{6 \times 10^5} = 500m$



$R_r = 40\pi^2 \left(\frac{50}{500}\right)^2$

$\cong 4\Omega = \frac{2\pi^2}{5}$

125 — $\lambda/4$ — 36.5Ω

50m — $\frac{\lambda}{10}$ — 4Ω

$W_r = I_{rms}^2 R_r$

9. $\psi_{avg} \doteq \frac{100}{4\pi} = \frac{25}{\pi} = 7.96$

$V_{avg} = \frac{100}{4\pi \times (10 \times 10^3)^2} = 7.96 \times 10^{-8}$

$= 0.08 \mu W$

10. $\psi \propto W_r \propto R_r \propto \frac{1}{\lambda^2} \times f^2$

strongly

$f' = f/2$

$\psi' = \psi/4$

11. $D = ?$ $\quad D = ?\ dB$

$$D = \frac{4\pi\ \psi(\theta, \phi)_{max}}{W_r}$$

$$= \frac{4\pi \cdot 150}{0.9\ (W_{IN})} = \frac{4\pi \cdot 150}{36\pi} = 16.67 = D$$

$10 \log 16.67 = 11.76\ dB$

$D = 11.76\ dB$

2\. 1 A — 1m — using

$\rightarrow I_{rms}$

$f = 10 MHz$

$\lambda = \frac{3 \times 10^8}{10 \times 10^6} \times 30m$

$l = \frac{\lambda}{30}$

$W_r = I_{rms}^2 R_r$

$R_r = 80\pi^2 \left(\frac{1}{30}\right)^2 = 0.88\Omega$

$W_r = 0.88$

$\left(\frac{dl}{\lambda}\right) \begin{cases} \lambda/2 \rightarrow 73\Omega \\ \frac{\lambda}{4} \rightarrow 36.5\Omega \\ \frac{\lambda}{10} \rightarrow 4\Omega \\ \frac{\lambda}{30} \rightarrow 0.88\Omega \end{cases}$

13\. $\theta_{HPBW} = 90^0$

14\. A

15\. Uniform currents

$l = 5m$

$\lambda = 100m$

$R_r = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2$

$= 80\pi^2 \left(\frac{\lambda}{20\lambda}\right)^2 \cong 2\Omega$

16\. A

18\. $l = 1.5m$

$f = 100 MHz$

$\lambda = 3m$

$l = \frac{\lambda}{2}$

$l = 15m$

$f = 10 MHz$

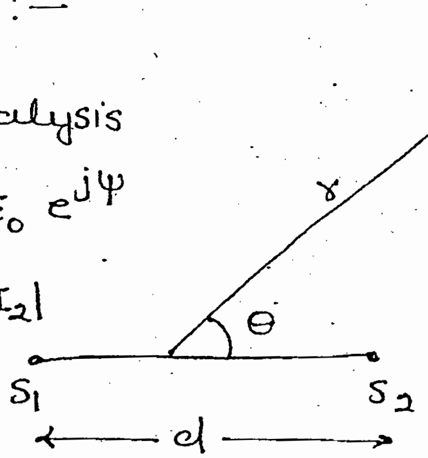
$\lambda = 30m$

$l = \frac{\lambda}{2}$

Halfwave Dipole

ANTENNA ARRAYS :-

2-Element Isotropic Array :-



$r >> d$ Far-zone analysis

$$E_r = E_1 + E_2 = E_0 + kE_0 e^{j\psi}$$

where $k = 1$ if $|I_1| = |I_2|$

ψ = Phase diff. b/w the two fields

= current having phase difference (α) + path difference giving phase

Path difference $= d\cos\theta$

phase diff. $= \frac{2\pi}{\lambda} d\cos\theta = \beta d\cos\theta$

$$\boxed{\psi = \alpha + \beta d\cos\theta}$$

$$E_T = E_0(1 + \cos\psi + j\sin\psi)$$

$$|E_r| = |E_0|\sqrt{(1+\cos\psi)^2 + \sin^2\psi}$$

$$= |E_0|\sqrt{2 + 2\cos\psi}$$

$$|E_T| = 2E_0 \cos(\psi/2)$$

Case-(1) :-

$d = \frac{\lambda}{2}$, $\alpha = 0$

$$\psi = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2}\cos\theta$$

$$|E_T| = 2E_0 \cos\left(\frac{\pi}{2}\cos\theta\right)$$

If $\theta = 90°/270° \longrightarrow \theta_{max}$

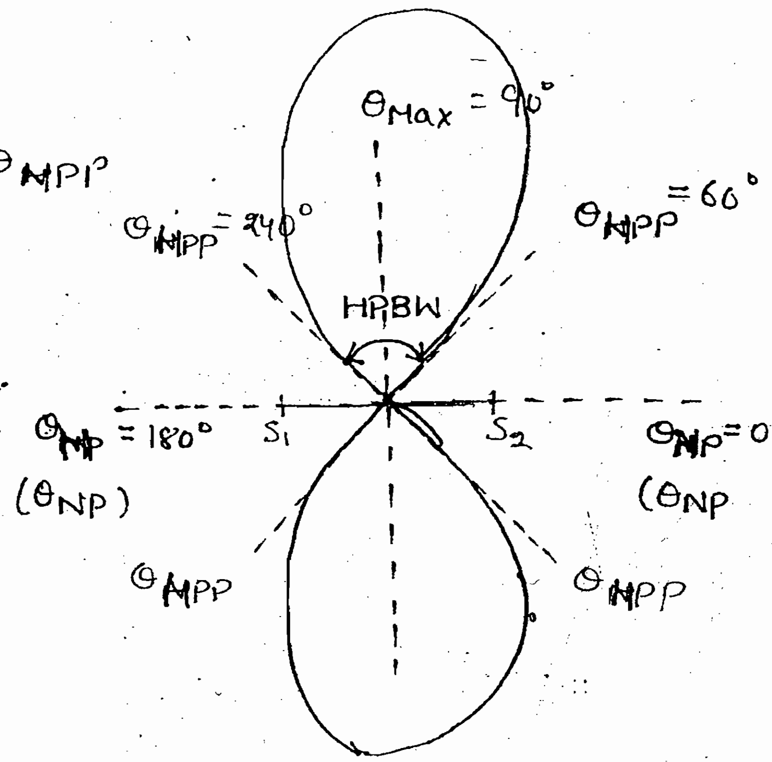
$$|E_T| = 2E_0 = E_{max}$$

If $\theta = 0^\circ / 180^\circ \longrightarrow \theta_{NP}$

$|E_T| = 0$

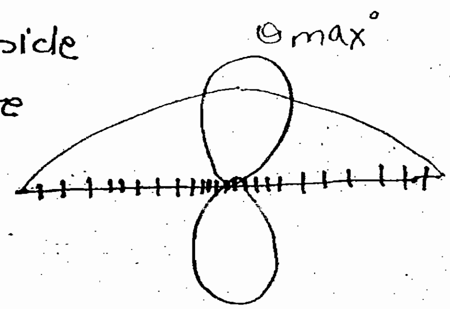
If $\theta = 60^\circ / 120^\circ / 240^\circ / 300^\circ \longrightarrow \theta_{HPP}$

$|E_T| = \frac{E_{max}}{\sqrt{2}}$



Note :—

The currents on either side inphase and max. at the centre and halfwave dipole is also broad side pattern



Case — (ii) :—

$d = \frac{\lambda}{2}$, $\alpha = \pi$

$\Psi = \pi + \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cos\theta$

$|E_T| = 2E_0 \sin\left(\frac{\pi}{2}\cos\theta\right)$

If $\theta = 90^\circ / 270^\circ \longrightarrow \theta_{NP}$

$|E_T| = 0$

If $\theta = 0^\circ / 180^\circ \longrightarrow \theta_{max}$

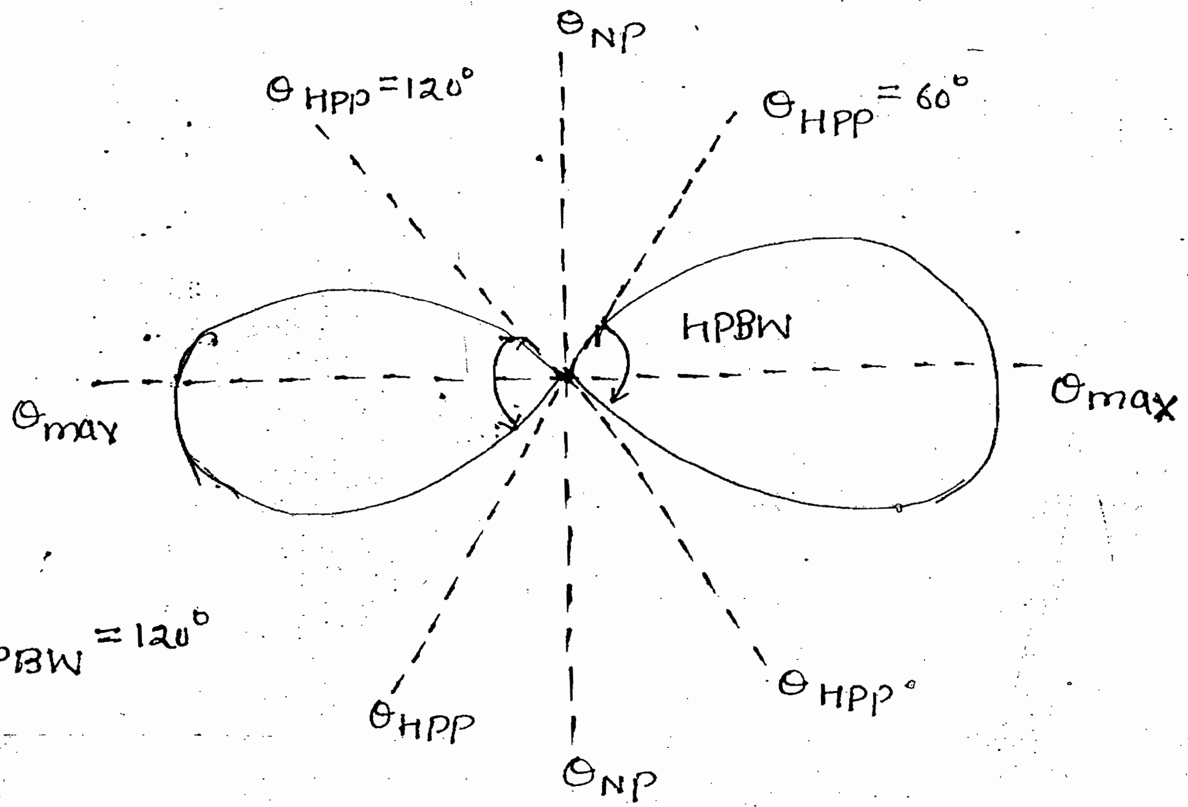
$|E_T| = E_{max} = 2E_0$

If $\theta = 60^\circ / 120^\circ / 240^\circ / 300^\circ \longrightarrow \theta_{HPP}$

$|E_T| = \frac{E_{max}}{\sqrt{2}}$

This is called as End-fire Array whose

$\theta_{max} = 0°/180°$



$\theta_{HPBW} = 120°$

<u>Case-(III):-</u>

$d = \lambda, \alpha = 0°$

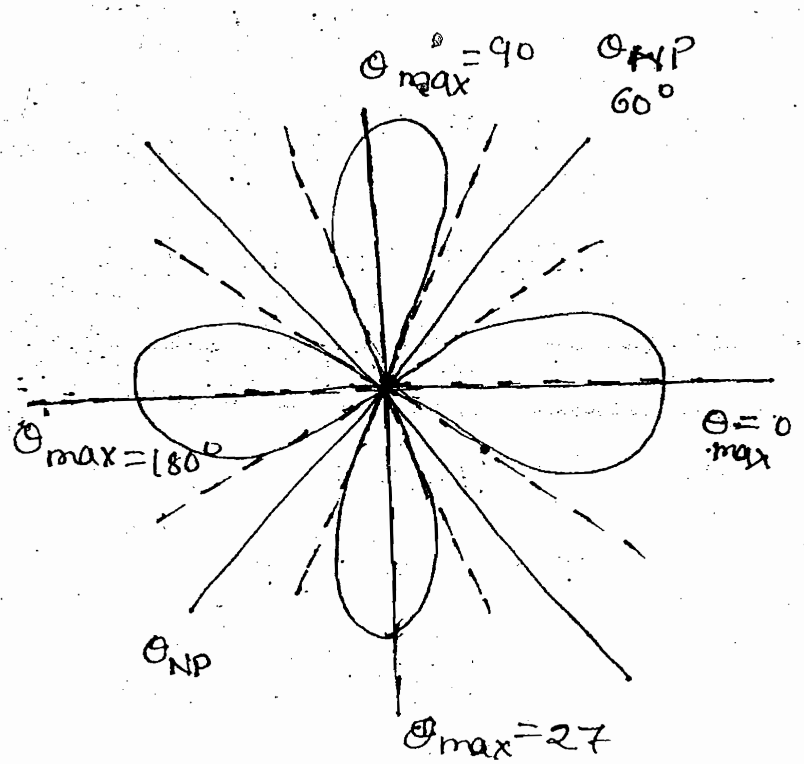
$E_T = 2E_0 \cos(\pi \cos\theta)$

If $\theta = 0°/90°/180°/270°$
$\longrightarrow \theta_{max}$

If $\theta = 0°/90°/180°/270°$
$\longrightarrow \theta_{max}$

If $\theta = 60°/120°/240°/300°$
$\longrightarrow \theta_{NP}$

$\cos\theta = \pm\frac{1}{4}$ or $\pm\frac{3}{4}$



In general θ_{max} can be designed towards a specific direction

With $\psi \to 0$

$$\alpha + \beta d \cos\theta_{max} = 0$$

$$\boxed{\cos\theta_{max} = \frac{-\alpha}{\beta d}}$$

For broadside array,

$$\theta_{max} = 90^\circ / 270^\circ$$

$$\Rightarrow \boxed{\alpha = 0}$$

Inphase currents

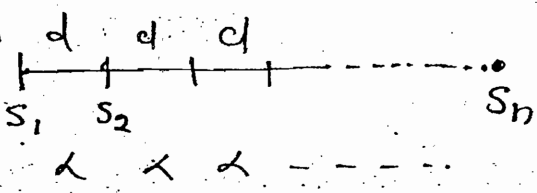
For end-fire array

$$\theta_{max} = 0^\circ / 180^\circ \quad \Rightarrow \boxed{\alpha = \pm \beta d}$$

$\cancel{\theta_{max}}$ $\cancel{\alpha = \pm}$

Extension:-

For n elements, uniform, linear isotropic sources



$$E_T = E_0 (1 + e^{j\psi} + e^{j2\psi} + \dots e^{j(n-1)\psi}$$

G.P with $a = 1$ $r = e^{j\psi}$

$$E_T = \frac{E_0 (1 - e^{jn\phi})}{(1 - e^{j\psi})}$$

$$\boxed{|E_T| = \frac{E_0 \sin(n \psi/2)}{\sin(\psi/2)}}$$

If $\psi \to 0$

$$|E_T| = E_0 \frac{n(\psi/2)}{\psi/2} = nE_0$$

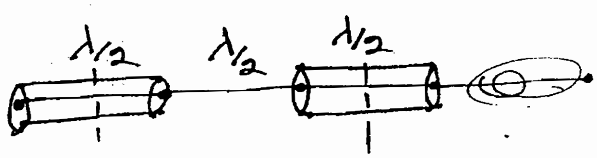
$$= E_{max}$$

If $n = 2$

$$|E_T| = 2E_0 \cos(\psi/2)$$

~~Workbook~~

9. Theorem of Multiplication of Patterns (Symm. Arrays) 2



Step-1 :-

Identify the two points that symm. makes the group and Identify Eu unit pattern.

UNIT :-

$d = \frac{\lambda}{2}$, $\alpha = 0$

2- Element Array

$$E_u = \cos\left(\frac{\pi}{2}\cos\theta\right)$$

Step 2 :-

Identify the group formed from the two points and its pattern

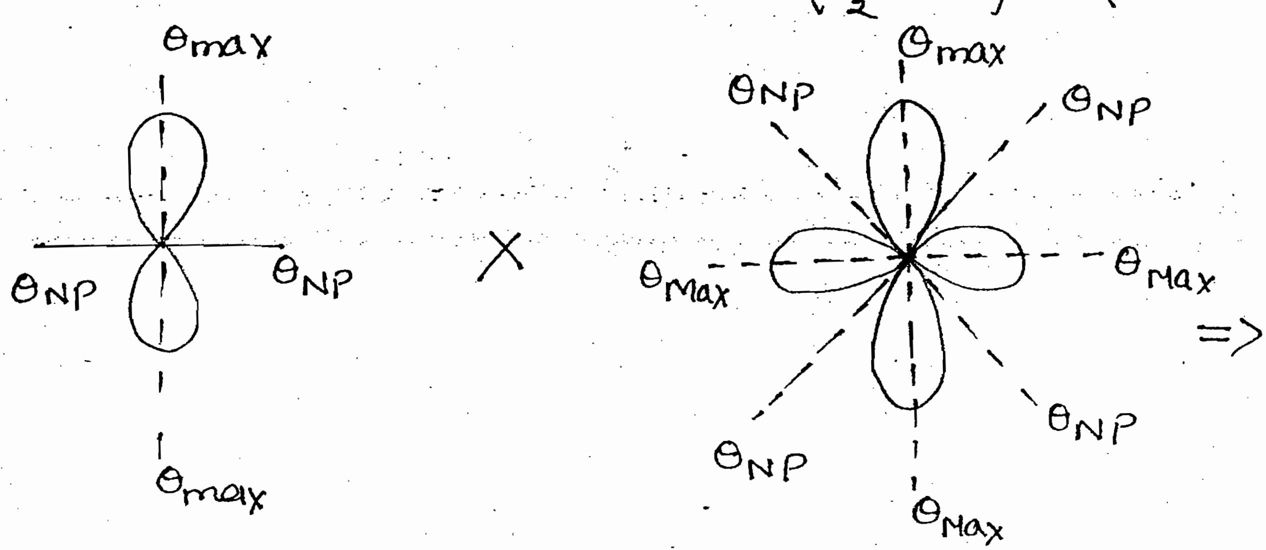
GROUP :- $d = \lambda$ $\alpha = 0$

$$E_g = \cos(\pi \cos\theta)$$

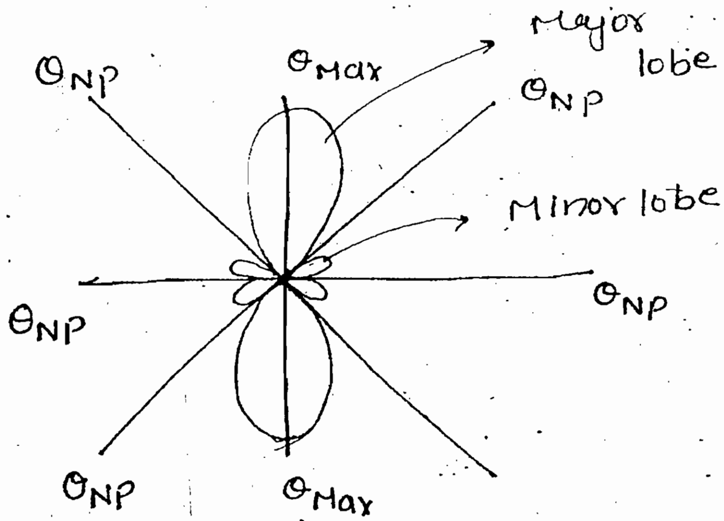
Step 3 :-

Resultant Pattern = Unit Pattern x Group Pattern

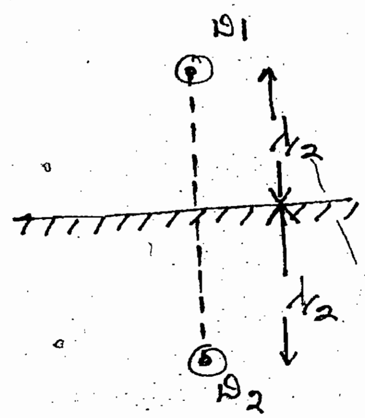
$$= \cos\left(\frac{\pi}{2}\cos\theta\right)\cos(\pi\cos\theta)$$



$\Rightarrow$



<u>Workbook :-</u>



19. c

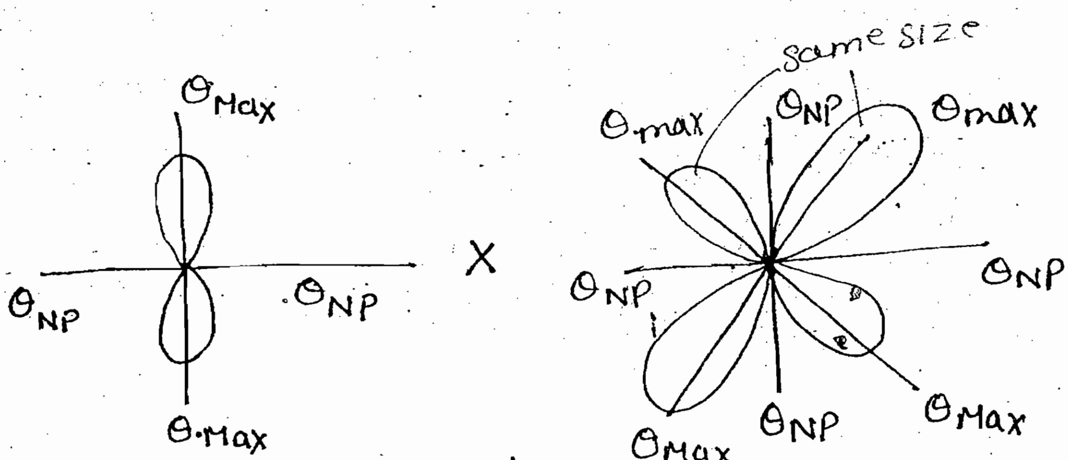
20. Unit - Dipole Antenna

$$E_u = \cos\left(\frac{\pi}{2}\cos\theta\right)$$

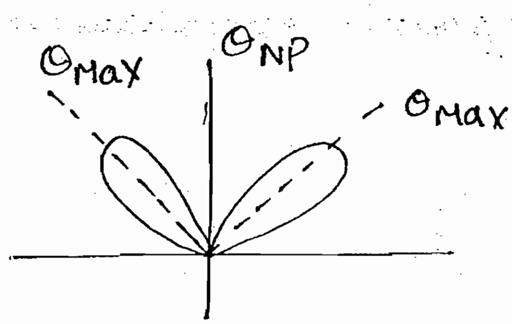
GROUP → $d = \lambda$

$\alpha = \pi$ - 2 Dipoles

$$E_g = \sin(\pi\cos\theta)$$

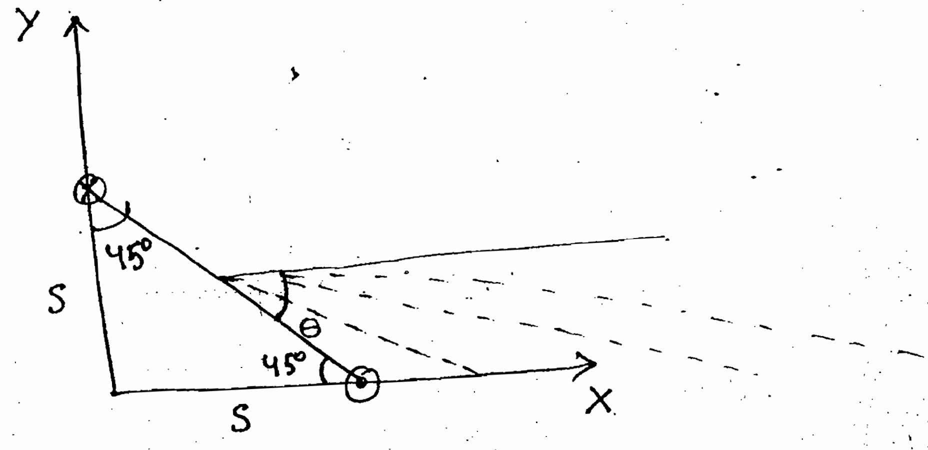


$\Downarrow$



<u>Ans- B</u>

$\theta = \frac{\pi}{2}$ Plane $\longrightarrow$ $z = 0$ $r\cos\theta$ $\longrightarrow$ XY Plane



$d = \sqrt{2}s$, $\alpha = \pi$

$\theta = 45°$

$$\frac{E_T}{E_0} = 2\cos\left(\frac{\alpha + \beta d\cos\theta}{2}\right)$$

$$= 2\cos\left(\pi + \frac{2\pi}{\lambda}\cdot\frac{d}{2}\cdot\frac{1}{\sqrt{2}}\right)$$

$$= 2\sin\left(\frac{\pi s}{\lambda}\right)$$

2\. $\theta_{max} = 60°$ off end-fire

$\alpha + \beta d\cos 60° = 0$

$\alpha = -\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}\cdot\frac{1}{2} = -\frac{\pi}{4}$

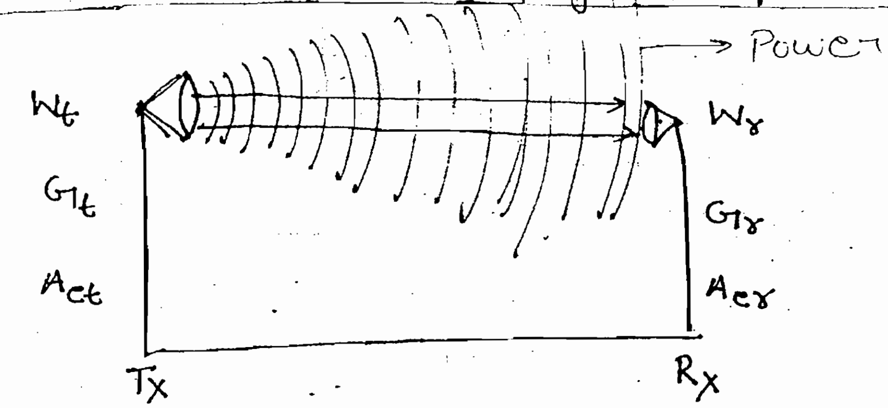
23\. Unambigous

Direction finding

— 4 lobe pattern

$d = \lambda$ $\alpha = 0°$

FRIIS - free Space Propogation Equation:-



$$\text{Power density at Rx} = \frac{W_t}{4\pi d^2} \cdot G_t$$

$$W_r \text{ at Rx} = \frac{W_t \cdot G_t \cdot A_{er}}{4\pi d^2}$$

$$A_{er} = \frac{\lambda^2 G_r}{4\pi}$$

$$W_r = \frac{W_t \cdot G_t \cdot G_r}{\left(\frac{4\pi d}{\lambda}\right)^2}$$

$$\left(\frac{4\pi d}{\lambda}\right)^2 = L_s = \text{Loss due to spatial dispersion.}$$

$$W_r\,(dBW) = W_t\,(dBW) + G_t\,(dB) + G_r\,(dB) - L_s\,(dB)$$

$$\frac{W_t G_t}{4\pi d^2} = \frac{1}{2}\frac{E_0^2}{\eta} = \frac{E_{rms}^2}{120\pi}$$

$$\boxed{E_{rms} = \frac{\sqrt{30\, W_t \cdot G_t}}{d}}$$

Workbook :-

17. $W_r\,(dBW) = 10\,dBW + 10\,dB + G_r(dB) - 100\,dB$

Tx - circularly polarized

Rx - Linearly polarised.

$\frac{1}{2}$ - Power received

due to polarization mismatch

$G_r = \frac{1}{2}$ $\qquad 10\log\frac{1}{2} = -3\,dB$

$G = 2 = 3\,dB$

$\longrightarrow$ doubled

24. $W_r = \frac{W_t \cdot G_t \cdot A_{er}}{4\pi d^2} = 0.8\,W$

25. $W_r = \frac{W_t \cdot (G)^2}{\left(\frac{4\pi d}{\lambda}\right)^2}$ $\qquad d = 30\,km$

$\lambda = \frac{3\times10^8}{1\times10^9} = 0.3\,m$

$W_r = -30\,dBm$ for $W_t = 1W$

$\left(\frac{W_r}{W_t} = -30\,dBm\right)$

$\left(\frac{W_r}{W_t} = -30\,dB\right)\times10^{-3}$

$\left(10\log\frac{W_r}{W_t} = -30\right)\times10^{-3}$

$\left(\frac{W_r}{W_t} = 10^{-3}\right)\cdot10^{-3}$ $\qquad$ Ans - D

$\frac{W_r}{W_t} = 10^{-6}$

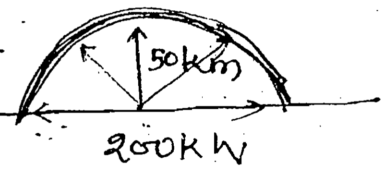
26.

$$E_{rms} = \frac{\sqrt{30 \cdot 1000 \cdot 1.63}}{10 \times 10^3}$$

27.

$$P_{avg} = \frac{W_t}{2\pi d^2} a_r$$

$$= \frac{200 \times 10^3}{2\pi \times (50 \times 10^3)^2} \cdot a_r$$



$\longrightarrow \vartheta$

28.

E_{rms} — 5kms —— d

E'_{rms} — $\frac{E_{rms}}{\sqrt{2}}$ —— $d' = ?$

Further more ?

to detect 3dB decrease

3dB in power — $\frac{1}{2}$ — power

— $\frac{1}{\sqrt{2}}$ — field

$$E \propto \frac{1}{d}$$

$$E' = \frac{E}{\sqrt{2}}$$

$$d' = \sqrt{2}d = \sqrt{2} \times 5 \text{ kms} = 7 \text{ km}$$

Further more = 2 km

29.

$E \propto \frac{1}{d}$

Conventional :-

$l = 10\,cm$ $\quad f = 60\,MHz$

$\lambda = \frac{3\times10^8}{60\times10^6} \cong 5m$ $\quad l = \frac{\lambda}{50} \rightarrow$ Hertzian dipole

$R_r = 80\pi^2\left(\frac{dl}{\lambda}\right)^2$

$W_r = I_{rms}^2 R_r \quad \text{--- (1)}$

$I_{rms} = \frac{10mA}{\sqrt{2}}$

$\text{Efficiency} = \frac{R_r}{R_r + 0.1}$

$E_{rms} = \frac{1\overset{\text{milli}}{m}V}{\underset{\text{meter}}{m}} = \frac{\sqrt{30 \cdot W_t \cdot G_t}}{d}$

$\Rightarrow 10^{-3} = \frac{\sqrt{30 \cdot W_t \cdot 1.5}}{d}$

At $\frac{\lambda}{50}$ lengths both monopole and dipole have same effect

→ $f = 1500\,MHz$ $\quad l = 10\,cm$

$\lambda = 20\,cm$ $\quad l = \frac{\lambda}{2} \rightarrow$ halfwave dipole

$R_r = 73$

$E_{rms} = \frac{\sqrt{30 \cdot W_t \cdot 1.63}}{d}$

$\downarrow$

$= ?$