10 Heron's Formula

Fastrack Revision

- **Area of Triangle:** The total space occupied inside the boundary of the triangle is said to be an area of triangle.

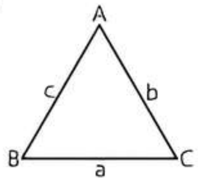
  Area of triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$

- **Perimeter of Triangle:** Sum of lengths of all three sides of a triangle.

  Perimeter, $2s = a + b + c$

  $\Rightarrow \quad s = \frac{a+b+c}{2}$

  where 's' is semi-perimeter; a, b and c are the three sides of a triangle.

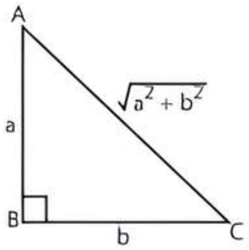


- **Right-angled Triangle:**

  It is a triangle with one right angle.

  1. Area $= \frac{1}{2} \times a \times b$
  2. Altitude $= a$
  3. Perimeter $= a + b + \sqrt{a^2 + b^2}$

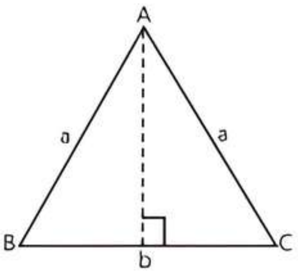
  where 'a' and 'b' are the sides that includes to the right angle.



- **Isosceles Triangle:** Triangle that has two equal sides and corresponding two equal angles.

  1. Area $= \frac{b}{4}\sqrt{4a^2 - b^2}$
  2. Perimeter $= 2a + b$
  3. Altitude $= \frac{1}{2}\sqrt{4a^2 - b^2}$

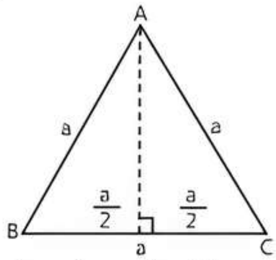
  where 'a' is length of two equal sides and 'b' is base.



- **Equilateral Triangle:** Triangle with all sides and all angles equal (each being 60°).

  1. Area $= \frac{\sqrt{3}}{4}a^2$
  2. Perimeter $= 3a$
  3. Altitude $= \frac{\sqrt{3}}{2}a$

  where 'a' is side.



- **Heron's Formula:** The formula given by Heron about the area of a triangle.

  Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

  where a, b and c are the sides of triangle and s is its semi-perimeter.

Knowledge BOOSTER

1. *The length of longest altitude is the perpendicular distance from the opposite vertex to the smallest side of a triangle.*
2. *The length of smallest altitude is the perpendicular distance from the opposite vertex to the largest side of a triangle.*
3. *Heron's formula is helpful when it is not possible to find the height of the triangle easily.*
4. *Heron's formula is applicable to all types of triangles whether it is a right triangle or an isosceles or an equilateral triangle.*

Practice Exercise

Multiple Choice Questions

Q 1. What is the area of an equilateral triangle with side 2 cm?

a. $\sqrt{6}\text{ cm}^2$ b. $\sqrt{3}\text{ cm}^2$ c. $\sqrt{8}\text{ cm}^2$ d. 4 cm^2

Q 2. If area of an equilateral triangle is $25\sqrt{3}\text{ cm}^2$, then semi-perimeter of a triangle is:

a. 20 cm b. 15 cm c. 25 cm d. 30 cm

Q 3. The area of a triangle is 150 cm^2 and its sides are in the ratio 3 : 4 : 5. What is its perimeter?

a. 10 cm b. 30 cm c. 45 cm d. 60 cm

Q 4. The base of a right triangle is 8 cm and hypotenuse is 10 cm. Its area will be:

a. 24 cm^2 b. 36 cm^2 c. 60 cm^2 d. 30 cm^2

Q 5. The sides of a triangle are 3 cm, 5 cm and 6 cm. What is its area?

a. $2\sqrt{3}\text{ cm}^2$ b. $2\sqrt{14}\text{ cm}^2$

c. $5\sqrt{12}\text{ cm}^2$ d. $2\sqrt{5}\text{ cm}^2$

Q 6. The area of an equilateral triangle with side $4\sqrt{3}$ cm is:

a. 4 cm^2
b. 5 cm^2
c. $3\sqrt{3}$ cm^2
d. $12\sqrt{3}$ cm^2

Q 7. Length of one of the equal sides of an isosceles triangle is 4 cm. If its base is 2 cm then what is its area?

a. $\sqrt{15}$ cm^2
b. $\sqrt{13}$ cm^2
c. $\sqrt{12}$ cm^2
d. $\sqrt{14}$ cm^2

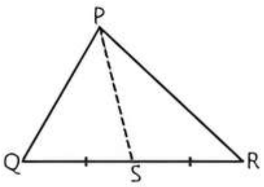
Q 8. If the perimeter of an equilateral triangle is 60 cm, then what is its area?

a. $200\sqrt{2}$ cm^2
b. $100\sqrt{2}$ cm^2
c. $100\sqrt{3}$ cm^2
d. $200\sqrt{3}$ cm^2

Q 9. If the sides of a triangle are 3 cm, 4 cm and 5 cm, then the length of the altitude to the largest side is:

a. 3 cm
b. 2.4 cm
c. 2.5 cm
d. 4 cm

Q 10. If PS is a median of ΔPQR and ar(ΔPSR) = 25 cm^2, then area of ΔPQR is:



a. 40 cm^2
b. 50 cm^2
c. 55 cm^2
d. 66 cm^2

Q 11. The sides of a triangle are 8 cm, 11 cm and 13 cm. What is its area?

a. $8\sqrt{30}$ cm^2
b. $4\sqrt{10}$ cm^2
c. $3\sqrt{100}$ cm^2
d. $6\sqrt{200}$ cm^2

Q 12. The sides of a triangle are 15 cm, 17 cm and 8 cm. What is its area?

a. 20 cm^2
b. 40 cm^2
c. 60 cm^2
d. 80 cm^2

Q 13. An isosceles right triangle has area 8 cm^2. The length of its hypotenuse is:

a. $\sqrt{32}$ cm
b. $\sqrt{16}$ cm
c. $\sqrt{48}$ cm
d. $\sqrt{24}$ cm

Q 14. The length of each side of an equilateral triangle having an area of $9\sqrt{3}$ cm^2 is:

a. 8 cm
b. 36 cm
c. 4 cm
d. 6 cm

Q 15. If the area of an equilateral triangle is $16\sqrt{3}$ cm^2 then the perimeter of triangle is:

a. 48 cm
b. 24 cm
c. 12 cm
d. 36 cm

Q 16. The sides of a triangle are 11 m, 60 m and 61 m. The altitude to the smallest side is:

a. 70 m
b. 40 m
c. 60 m
d. 80 m

Q 17. A ground is in the form of a triangle having sides 51m, 37 m and 20m. The cost of levelling the ground at the rate of ₹5 per m^2 is:

a. ₹ 1550
b. ₹ 1530
c. ₹ 1560
d. ₹ 1565

Assertion & Reason Type Questions

Directions (Q. Nos. 18-21): *In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct choice as:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
c. Assertion (A) is true but Reason (R) is false.
d. Assertion (A) is false but Reason (R) is true.

Q 18. Assertion (A): If the sides of a ΔABC are $a = 5$ cm, $b = 6$ cm and $c = 7$ cm, then area of ΔABC is $6\sqrt{6}$ cm^2.

Reason (R): The area of triangle having sides a, b and c with semi-perimeter s is given by

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

Q 19. Assertion (A): If the height of triangle is 9 cm and area is 144 cm^2, then its base is 30 cm.

Reason (R): Area of triangle can be determined by $\Delta = \frac{1}{2} \times \text{base} \times \text{height}$.

Q 20. Assertion (A): If the area of an equilateral triangle is $49\sqrt{3}$ cm^2, then the semi-perimeter of triangle is 42 cm.

Reason (R): If a, b and c are the sides of a triangle, then semi-perimeter of a ΔABC is:

$$s = \frac{a+b+c}{2}$$

Q 21. Assertion (A): The area of triangle PQR in which PQ = 5 cm, QR = 4 cm and PR = 7 cm, is $12\sqrt{2}$ cm^2.

Reason (R): The area of triangle having sides a, b and c with semi-perimeter 's' is given by $\sqrt{s(s+a)(s+b)(s+c)}$.

Fill in the Blanks Type Questions

Q 22. If the perimeter of an equilateral triangle is 30 m, the area is ....

Q 23. The side and altitude of an equilateral triangle are in the ratio ....

Q 24. The area of an isosceles triangle having base 3 cm and the length of one of the equal side as 6 cm is ....

Q 25. If base of a triangle is doubled then its area will be .... times of original area.

Q 26. The edges of a triangular board are 6 cm, 8 cm and 10 cm. The cost of painting it at the rate of 9 paise per cm^2 is ....

True/False Type Questions

Q 27. Heron's formula is not valid for finding area of all triangles.

Q 28. The length of smallest altitude is the perpendicular distance from the opposite vertex to the largest side of a triangle.

Q 29. If each side of triangle is doubled, the perimeter will become 4 times.

Q 30. If p is the perimeter of the triangle having sides a, b and c, the area of triangle is

$$A=\frac{1}{2}\sqrt{p(p-2a)(p-2b)(p-2c)}.$$

Q 31. Area of triangle whose two sides are 8 m and 11 m and perimeter is 32 m is $8\sqrt{30}\ m^2$.

Solutions

1. (b) Given, side of an equilateral triangle $a = 2$ cm

$\therefore$ Area of an equilateral triangle $=\frac{\sqrt{3}}{4}(a)^2$

$$=\frac{\sqrt{3}}{4}\times(2)^2$$

$$=\sqrt{3}\ cm^2$$

2. (b) Given, area of an equilateral triangle $= 25\sqrt{3}$

$$\therefore \quad \frac{\sqrt{3}}{4}a^2 = 25\sqrt{3}$$

TR!CK

Area of an equilateral triangle $=\frac{\sqrt{3}}{4}(\text{side})^2$

$\Rightarrow \quad a^2 = 100 \quad \Rightarrow \quad a = 10$ cm

(Taking positive square root as side is always positive)

Now, perimeter of a triangle, $P = 3a$

$$= 3\times 10 = 30 \text{ cm}$$

$\therefore$ Semi-perimeter of a triangle, $s=\frac{P}{2}$

$$=\frac{30}{2}=15 \text{ cm}$$

3. (d) Let sides of a triangle be $3x$, $4x$ and $5x$. Then semi-perimeter of a triangle is

$$s=\frac{3x+4x+5x}{2}=\frac{12x}{2}$$

$$s = 6x$$

By using Heron's formula,

$$\Delta=\sqrt{s(s-a)(s-b)(s-c)}$$

$$\Rightarrow \quad 150=\sqrt{6x(6x-3x)(6x-4x)(6x-5x)}$$

$$=\sqrt{6x\times 3x\times 2x\times x} = x^2\sqrt{36} = 6x^2$$

$$\Rightarrow \quad 150 = 6x^2$$

$$\Rightarrow \quad x^2 = 25 \text{ cm}$$

$$\Rightarrow \quad x = 5 \text{ cm}$$

$\therefore$ The perimeter of a triangle $= a + b + c$

$$= 3x + 4x + 5x$$

$$= 12x$$

$$= 12\times 5 = 60 \text{ cm}$$

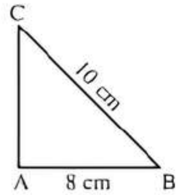
4. (a) By using Pythagoras theorem in ΔABC,

$$AC=\sqrt{(BC)^2-(AB)^2}$$

$$=\sqrt{(10)^2-(8)^2}$$

$$=\sqrt{100-64}$$

$$=\sqrt{36}=6 \text{ cm}$$



Area of $\Delta ABC=\frac{1}{2}\times AB\times AC$

$$=\frac{1}{2}\times 8\times 6=24 \text{ cm}^2$$

5. (b) Let sides of a triangle be

$a = 3$ cm, $b = 5$ cm and $c = 6$ cm

Then, semi-perimeter of a triangle is, $s=\frac{a+b+c}{2}$

$$=\frac{3+5+6}{2}$$

$$=\frac{14}{2}=7 \text{ cm}$$

By using Heron's formula,

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{7(7-3)(7-5)(7-6)}$$
$$= \sqrt{7 \times 4 \times 2 \times 1}$$
$$= 2\sqrt{14} \text{ cm}^2$$

6. (d)

TR!CK

Area of an equilateral triangle $= \frac{\sqrt{3}}{4}(\text{side})^2$

$$\therefore \text{Area of an equilateral triangle} = \frac{\sqrt{3}}{4}(4\sqrt{3})^2$$
$$= \frac{\sqrt{3}}{4} \times 16 \times 3$$
$$= 12\sqrt{3} \text{ cm}^2$$

7. (a) Given, equal sides of an isosceles triangle be $a = 4$ cm and base $b = 2$ cm

$$\therefore \text{Area of an isosceles triangle} = \frac{b}{4}\sqrt{4a^2 - b^2}$$
$$= \frac{2}{4}\sqrt{4(4)^2 - (2)^2}$$
$$= \frac{1}{2} \times 2\sqrt{16 - 1}$$
$$= \sqrt{15} \text{ cm}^2$$

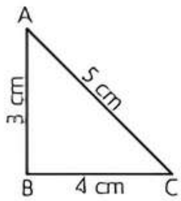
8. (c) Given perimeter of an equilateral is 60 cm.

$\therefore \quad 3a = 60 \Rightarrow a = 20$ cm.

$\therefore$ Area of an equilateral triangle

$$= \frac{\sqrt{3}}{4}(a)^2$$
$$= \frac{\sqrt{3}}{4} \times (20)^2$$
$$= \sqrt{3} \times 100$$
$$= 100\sqrt{3} \text{ cm}^2$$

9. (b) Let sides of a triangle be $a = 3$ cm, $b = 4$ cm and $c = 5$ cm.



Here $a^2 + b^2 = (3)^2 + (4)^2$
$= 9 + 16 = 25$
$= (5)^2 = (c)^2$

Thus, given sides of a triangle is right angled triangle.

Now, $\text{area of } \Delta ABC = \frac{1}{2} \times AB \times BC$

$$= \frac{1}{2} \times 3 \times 4$$
$$= 6 \text{ cm}^2$$

Here, the largest sides of a ΔABC is AC.

$$\text{Again area of } \Delta ABC = \frac{1}{2} \times AC \times \text{altitude}$$
$$\Rightarrow \quad 6 = \frac{1}{2} \times 5 \times \text{altitude}$$
$$\Rightarrow \quad \text{Altitude} = \frac{12}{5} = 2.4 \text{ cm}$$

10. (b) Given, ar $(\Delta PSR) = 25 \text{ cm}^2$

TiP

Median divides the triangle into two equal areas.

$$\therefore \text{Area of } \Delta PQR = 2 \times \text{ar}(\Delta PSR)$$
$$= 2 \times 25$$
$$= 50 \text{ cm}^2$$

11. (a) Let sides of a triangle are

$a = 8$ cm, $b = 11$ cm and $c = 13$ cm

Then, semi-perimeter of a triangle is

$$s = \frac{a+b+c}{2} = \frac{8+11+13}{2}$$
$$= \frac{32}{2} = 16 \text{ cm}$$

By using Heron's formula,

$$\text{Area of triangle, } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{16(16-8)(16-11)(16-13)}$$
$$= \sqrt{16 \times 8 \times 5 \times 3}$$
$$= 4 \times 2 \times \sqrt{30}$$
$$= 8\sqrt{30} \text{ cm}^2$$

12. (c) Let sides of a triangle are

$a = 15$ cm, $b = 17$ cm and $c = 8$ cm

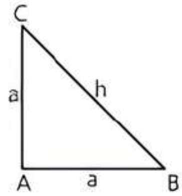
Then, semi-perimeter of a triangle,

$$s = \frac{a+b+c}{2}$$
$$= \frac{15+17+8}{2} = \frac{40}{2} = 20 \text{ cm}$$

$$\therefore \text{Area of triangle, } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{20(20-15)(20-17)(20-8)}$$
$$= \sqrt{20 \times 5 \times 3 \times 12} = 4 \times 5 \times 3$$
$$= 60 \text{ cm}^2$$

13. (a) Let equal sides of an isosceles triangle be a cm.

$\therefore$ Area of triangle ΔABC



$$= \frac{1}{2} \times AB \times AC$$

$$8 = \frac{1}{2} \times a \times a = \frac{a^2}{2}$$

$\Rightarrow \quad a^2 = 16$

$\Rightarrow \quad a = 4$ cm

In right triangle ΔABC, use Pythagoras theorem

$$BC = \sqrt{(AB)^2 + (BC)^2}$$

$$= \sqrt{(a)^2 + (a)^2} = \sqrt{a^2 + a^2}$$

$$= a\sqrt{2}$$

$$= 4\sqrt{2} \text{ cm} = \sqrt{32} \text{ cm}$$

14. (d) Given, area of an equilateral triangle is

$$\Delta = 9\sqrt{3} \text{ cm}^2$$

$$\therefore \quad \frac{\sqrt{3}}{4}(\text{side})^2 = 9\sqrt{3} \text{ cm}^2$$

$\Rightarrow \quad (\text{side})^2 = 36 \text{ cm}^2$

$\Rightarrow \quad \text{side} = 6$ cm

Hence, length of each side of an equilateral triangle is 6 cm.

15. (b) Given, area of an equilateral triangle is

$$A = 16\sqrt{3} \text{ cm}^2$$

$$\therefore \quad \frac{\sqrt{3}}{4}(\text{side})^2 = 16\sqrt{3} \text{ cm}^2$$

$\Rightarrow \quad (\text{side})^2 = 64 \text{ cm}^2$

$\Rightarrow \quad \text{side} = 8$ cm

TiP

The perimeter of an equilateral triangle is 3(side).

$\therefore$ Area of perimeter of an equilateral triangle

$$= 3(\text{side})$$

$$= 3 \times 8 = 24 \text{ cm}$$

16. (c) Let sides of a triangle be $a = 11$ m, $b = 60$ m and $c = 61$ m. Then, semi-perimeter of a triangle is

$$s = \frac{a+b+c}{2} = \frac{11+60+61}{2}$$

$$= \frac{132}{2} = 66 \text{ m}$$

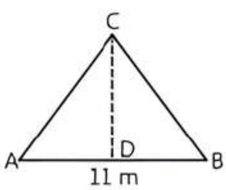
$\therefore$ Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{66(66-11)(66-60)(66-61)}$$

$$= \sqrt{66 \times 55 \times 6 \times 5}$$

$$= \sqrt{11 \times 6 \times 11 \times 5 \times 6 \times 5}$$

$$= 11 \times 6 \times 5 = 330 \text{ m}^2$$



In given triangle, the smallest side is 11 m.

$\therefore$ Area of triangle, ΔABC

$$= \frac{1}{2} \times AB \times \text{altitude}$$

$$\Rightarrow \quad 330 = \frac{1}{2} \times 11 \times \text{altitude}$$

$\Rightarrow$ Altitude = 60 m

17. (b) Let sides of a triangle be $a = 51$ m, $b = 37$ m and $c = 20$ m. Then, semi-perimeter of a triangle

$$s = \frac{a+b+c}{2} = \frac{51+37+20}{2}$$

$$= \frac{108}{2} = 54 \text{ m}$$

$\therefore$ Area of triangle, $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{54(54-51)(54-37)(54-20)}$$

$$= \sqrt{54 \times 3 \times 17 \times 34}$$

$$= \sqrt{3 \times 3 \times 3 \times 2 \times 3 \times 17 \times 17 \times 2}$$

$$= 3 \times 3 \times 2 \times 17 = 306 \text{ m}^2$$

Since, the cost of levelling the ground at the rate of ₹ 5 per m^2.

$\therefore$ The cost of levelling the ground having area 306 m^2

$$= 306 \times 5$$

$$= ₹\ 1530$$

18. (a) **Assertion (A):** Given sides are $a = 5$ cm, $b = 6$ cm and $c = 7$ cm.

Now, semi-perimeter of a ΔABC is $s = \dfrac{a+b+c}{2}$

$$= \frac{5+6+7}{2} = \frac{18}{2} = 9 \text{ cm}$$

$\therefore$ Area of triangle, $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{9(9-5)(9-6)(9-7)}$$

$$= \sqrt{9 \times 4 \times 3 \times 2}$$

$$= 3 \times 2\sqrt{3 \times 2}$$

$$= 6\sqrt{6} \text{ cm}^2$$

So, Assertion (A) is true.

Reason (R): It is true to say that the area of triangle is given by $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

19. (d) **Assertion (A):** Given, $h = 9$ cm

and $\quad$ Area $= 144$ cm^2

$\therefore$ Area of triangle $= \dfrac{1}{2} \times \text{base} \times \text{height}$

$\Rightarrow \quad 144 = \frac{1}{2} \times \text{base} \times 9$

$\Rightarrow \quad \text{base} = \frac{144 \times 2}{9} = 32 \text{ cm}$

So, Assertion (A) is false.

Reason (R): It is true to say that area of triangle can be determined by

$$\Delta = \frac{1}{2} \times \text{base} \times \text{height}$$

Hence, Assertion (A) is false but Reason (R) is true.

20. (d) **Assertion (A):** Given area of an equilateral triangle is $\Delta = 49\sqrt{3}$ cm^2

$\therefore \quad \frac{\sqrt{3}}{4}(\text{side})^2 = 49\sqrt{3}$

$\Rightarrow \quad (\text{Side})^2 = (7 \times 2)^2$

$\Rightarrow \quad \text{Side} = 14 \text{ cm}$

$\therefore$ The perimeter of an equilateral triangle is

$$3a = 3 \times 14 = 42 \text{ cm}$$

Thus, the semi-perimeter of an equilateral triangle is

$$\frac{3a}{2} = \frac{42}{2} = 21 \text{ cm}$$

So, Assertion (A) is false.

Reason (R): It is true to say that the semi-perimeter of a triangle is given by $s = \frac{a+b+c}{2}$

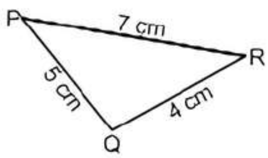
Hence, Assertion (A) is false but Reason (R) is true.

21. (c) **Assertion (A):** In ΔPQR, sides

are $a = PQ = 5$ cm, $b = QR = 4$ cm

and $c = PR = 7$ cm

Now, semi-perimeter of



$\Delta PQR, s = \frac{a+b+c}{2}$

$= \frac{5+4+7}{2} = \frac{16}{2}$

$= 8$ cm.

$\therefore$ Area of $\Delta PQR = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{8(8-5)(8-4)(8-7)}$

$= \sqrt{8 \times 3 \times 4 \times 3} = 3 \times 4\sqrt{2}$

$= 12\sqrt{2}$ cm^2

So, Assertion (A) is true.

Reason (R): It is false

Hence, Assertion (A) is true but Reason (R) is false.

22. The perimeter of an equilateral triangle is 30 m.

$\Rightarrow 3(\text{side}) = 30 \quad \Rightarrow \quad \text{side} = 10$ cm

$\therefore$ The area of an equilateral triangle is $\frac{\sqrt{3}}{4}(\text{side})^2$

$= \frac{\sqrt{3}}{4}(10)^2 = \frac{\sqrt{3}}{4} \times 100$

$= 25\sqrt{3}$ m^2

23. Let side of an equilateral triangle be a cm.

Then, altitude of an equilateral triangle is $\frac{\sqrt{3}}{2}a$.

$\therefore$ Required ratio $= \frac{a}{(\sqrt{3}/2)\,a} = \frac{2}{\sqrt{3}}$

24. Given equal sides of an isosceles triangle is $a = 6$ cm and base side is $b = 3$ cm.

$\therefore$ Area of an isosceles triangle

$= \frac{b}{4}\sqrt{4a^2 - b^2}$

$= \frac{3}{4}\sqrt{4(6)^2 - (3)^2}$

$= \frac{3}{4}\sqrt{4 \times 36 - 9} = \frac{3 \times 3}{4}\sqrt{16 - 1}$

$= \frac{9}{4}\sqrt{15}$ cm^2

25. Let base and altitude of a triangle be b and h respectively. Then,

area of $\Delta = \frac{1}{2} \times b \times h$

If base of a triangle is doubled *i.e.*, $b_1 = 2b$ then,

area of $\Delta_1 = \frac{1}{2} \times b_1 \times h = \frac{1}{2} \times 2b \times h = bh$

$\Rightarrow \quad \Delta_1 = 2\Delta$

Hence, if base of a triangle is doubled then its area will be double of original area.

26. Let edges of a triangular board be $a = 6$ cm, $b = 8$ cm and $c = 10$ cm

Then semi-perimeter of a triangle,

$$s = \frac{a+b+c}{2}$$

$$= \frac{6+8+10}{2} = \frac{24}{2} = 12 \text{ cm}$$

$\therefore$ Area of triangle, $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{12(12-6)(12-8)(12-10)}$

$= \sqrt{12 \times 6 \times 4 \times 2} = 12 \times 2 = 24$ cm^2

$\because$ The cost of painting 1 cm^2 = ₹ 0.09

$\therefore$ The cost of painting 24 cm^2 = 24 × 0.09

= ₹ 2.16

27. False

28. True

29. False,

Let a, b and c are the sides of a ΔABC, then perimeter of a triangle, $p = a + b + c$

Suppose each sides a triangle is double, then perimeter is $p_1 = 2a + 2b + 2c$

$= 2(a + b + c) = 2p$

30. True,

The semi-perimeter of given triangle is

$$s = \frac{a+b+c}{2}$$

$\Rightarrow \quad 2s = a + b + c$

$\Rightarrow \quad p = a + b + c$

$\therefore$ Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{\frac{p}{2}\left(\frac{p}{2}-a\right)\left(\frac{p}{2}-b\right)\left(\frac{p}{2}-c\right)}$$

$$= \frac{1}{2}\sqrt{p(p-2a)(p-2b)(p-2c)}$$

31. True,

Let $a = 8$ m, $b = 11$ m and $2s = 32$ m

$\Rightarrow \quad a + b + c = 32$

$\Rightarrow \quad 8 + 11 + c = 32 \quad \Rightarrow \quad c = 13$ m

$\therefore$ Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{16(16-8)(16-11)(16-13)}$$

$$\left[\because s = \frac{32}{2} = 16 \text{ m}\right]$$

$$= \sqrt{16 \times 8 \times 5 \times 3}$$

$$= 4 \times 2\sqrt{30} \text{ m}^2 = 8\sqrt{30} \text{ m}^2$$

Case Study Based Questions

Case Study 1

World sandwich day is celebrated every year on 3 November. The sandwich got it's name from John Montagu in the 18th century. On the occasion of sandwich day, a food manufacturing company decided to make a record by making the biggest triangular sandwich.

Suppose sides of sandwich are 7 cm, 8 cm and 9 cm.

On the basis of the above information, solve the following questions:

Q 1. Heron's formula is used for finding the:

a. area of circle
b. area of triangle
c. area of cuboid
d. area of cone

Q 2. The perimeter of a triangle is:

a. 28 cm
b. 24 cm
c. 26 cm
d. 30 cm

Q 3. The area of sand wich is:

a. 13 cm^2
b. $12\sqrt{5} \text{ cm}^2$
c. $3\sqrt{5} \text{ cm}^2$
d. $7\sqrt{9} \text{ cm}^2$

Q 4. The length of altitude to the smallest side of a triangle is:

a. $\frac{24\sqrt{5}}{7}$ cm
b. $\frac{25\sqrt{5}}{7}$ cm
c. $\frac{23}{7}$ cm
d. $\frac{13}{7}$ cm

Q 5. The length of altitude to the largest side of a triangle is:

a. $4\sqrt{5}$ cm
b. $\frac{8\sqrt{5}}{3}$ cm
c. $\frac{4}{3}\sqrt{5}$ cm
d. $4\sqrt{7}$ cm

Solutions

1. (b) Heron's formula is used for finding the area of triangle.

So, option (b) is correct.

2. (b) The perimeter of a triangle is 7 + 8 + 9 *i.e.*, 24 cm.

So, option (b) is correct.

3. (b) Since, sandwich is in the shape of triangle.

Let $a = 7$ cm, $b = 8$ cm and $c = 9$ cm.

Then, semi-perimeter of a triangle is:

$$s = \frac{a+b+c}{2} = \frac{7+8+9}{2}$$

$$= \frac{24}{2} = 12 \text{ cm}$$

By using Heron's formula,

Area of triangle, $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{12(12-7)(12-8)(12-9)}$$

$$= \sqrt{12 \times 5 \times 4 \times 3}$$

$$= 12\sqrt{5} \text{ cm}^2$$

So, option (b) is correct.

4. (a) In given sides of a triangle, the smallest side is 7 cm, which we consider the base of the triangle.

Let h be the length of altitude of a triangle.

TR!CK

Area of triangle $= \frac{1}{2} \times$ *base* $\times$ *altitude*

$\therefore$ Area of triangle $= \frac{1}{2} \times 7 \times h$

$$\Rightarrow \quad 12\sqrt{5} = \frac{7}{2} \times h$$

$$\Rightarrow \quad h = \frac{24\sqrt{5}}{7} \text{cm}$$

So, option (a) is correct.

5. (b) In given sides of triangle, the largest side is 9 cm, which we consider the base of the triangle. Let h_1 be the length of altitude of a triangle. Then area of triangle,

$$A = \frac{1}{2} \times \text{base} \times \text{altitude}$$

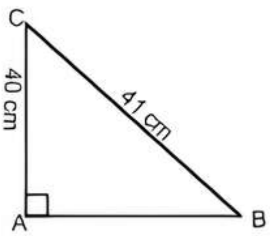
$$\Rightarrow \quad 12\sqrt{5} = \frac{1}{2} \times 9 \times h_1$$

$$\Rightarrow \quad h_1 = \frac{12\sqrt{5} \times 2}{9} = \frac{8\sqrt{5}}{3} \text{ cm}$$

So, option (b) is correct.

Case Study 2

Mr. Silvestar is a property dealer. He bought a triangle shaped field such that the biggest measures 41 cm. The measurement of sides of the field is as shown in the figure.



On the basis of the above information, solve the following questions:

Q 1. The length of AB in the triangle ABC is:

a. 10 cm b. 9 cm
c. 12 cm d. 15 cm

Q 2. Area of triangle ABC is:

a. 190 cm^2 b. 185 cm^2
c. 180 cm^2 d. 200 cm^2

Q 3. The perimeter of a triangle is:

a. 85 cm b. 93 cm c. 90 cm d. 95 cm

Q 4. If the cost of painting a field is ₹1.20 per cm^2, then the total cost of painting the field is:

a. ₹ 220 b. ₹ 216 c. ₹ 230 d. ₹ 250

Q 5. Identify the correct statement:

a. The length of longest altitude is the perpendicular distance from the opposite vertex to the largest side of a triangle.

b. Heron's formula is helpful when it is not possible to find the height of the triangle easily.

c. The length of smallest altitude is the perpendicular distance from the opposite vertex to the smallest side of a triangle.

d. Area of an equilateral triangle is $\frac{\sqrt{3}}{2}(\text{side})^2$.

Solutions

1. (b) In right angled ΔABC, use Pythagoras theorem,

$$AB = \sqrt{(BC)^2 - (AC)^2}$$
$$= \sqrt{(41)^2 - (40)^2} = \sqrt{1681 - 1600}$$
$$= \sqrt{81} = 9 \text{ cm}$$

So, option (b) is correct.

2. (c) Area of $\Delta ABC = \frac{1}{2} \times AB \times AC$

$$= \frac{1}{2} \times 9 \times 40$$
$$= 180 \text{ cm}^2$$

So, option (c) is correct.

3. (c) The perimeter of a triangle is AB + BC + CA

$$= 9 + 41 + 40 = 90 \text{ cm}$$

So, option (c) is correct.

4. (b) Since, cost of 1 cm^2 field is ₹ 1.20

$\therefore$ Cost of 180 cm^2 field = ₹ 180 × 1.20

= ₹ 216

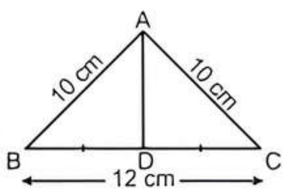
So, option (b) is correct.

5. (b) The correct statement is 'Heron's formula is helpful when it is not possible to find the height of triangle easily.

So, option (b) is correct.

Case Study 3

International kite festival in Gujarat also known as Uttarayan is one of the biggest festival celebrated in Gujarat. It is celebrated on the auspicious day of makar sankranti every year. It is a sign for the farmers about the beginning of summer season. On the day of Uttarayan, Suresh a 16 year old boy wants to fly kite. He ordered a triangle shape kite whose sides are 10 cm, 10cm and 12 cm.



On the basis of the above information, solve the following questions:

Q 1. The perimeter of a ΔABC is:

a. 22 cm b. 16 cm
c. 32 cm d. 24 cm

Q 2. Find the area of triangle ABC.

a. 40 cm^2 b. 48 cm^2

c. 49 cm^2 d. 50 cm^2

Q 3. The length of AD is:

a. 7 cm

b. 8 cm

c. 5 cm

d. Can not be determined

Q 4. The area of ΔABD is:

a. 18 cm^2 b. 20 cm^2

c. 24 cm^2 d. 23 cm^2

Q 5. If cost of paper is ₹ 1.50 per cm^2, then the cost of making a kite is:

a. ₹ 75 b. ₹ 72

c. ₹ 80 d. ₹ 90

Solutions

1. (c) The perimeter of a ΔABC is AB + BC + AC

$= 10 + 12 + 10$

$= 32$ cm

So, option (c) is correct.

2. (b) Here, $a = 10$ cm and $b = 12$ cm

∴ Area of an isosceles triangle

$$= \frac{b}{4}\sqrt{4a^2 - b^2}$$

$$= \frac{12}{4}\sqrt{4(10)^2 - (12)^2}$$

$$= 3\sqrt{400 - 144}$$

$$= 3\sqrt{256} = 3 \times 16$$

$$= 48 \text{ cm}^2$$

So, option (b) is correct.

3. (b) In an isosceles triangle, median AD is equal to the altitude of a triangle.

∴ Area of Δ ABC $= \frac{1}{2} \times BC \times AD$

$$48 = \frac{1}{2} \times 12 \times AD$$

⇒ AD = 8 cm

So, option (b) is correct.

4. (c)

TiP

Median divides the triangle into two equal areas.

∴ Area of $\Delta ABD = \frac{1}{2} \times$ area of ΔABC

$$= \frac{1}{2} \times 48 = 24 \text{cm}^2$$

So, option (c) is correct.

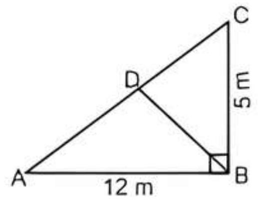
5. (b) ∴ The cost of making a kite = 1.50 × 48

= ₹ 72

So, option (b) is correct.

Case Study 4

Mayank bought a triangle shape field and wants to grow potato and wheat on his field. He divided his field by joining opposite sides. On the largest park he grew wheat and on the rest part he grew potato. The dimensions of a park are shown in the park.



On the basis of the above information, solve the following questions:

Q 1. Find the length of AC in a ΔABC.

Q 2. Find the area of ΔABC.

Q 3. If the cost of ploughing park is ₹ 5 per cm^2, then find the total cost of ploughing the park.

Solutions

1. In right angled ΔABC, use Pythagoras theorem,

$$AC = \sqrt{(AB)^2 + (BC)^2} = \sqrt{(12)^2 + (5)^2}$$

$$= \sqrt{144 + 25} = \sqrt{169} = 13 \text{ m}$$

Hence, length of AC is 13 m.

2. Area of ΔABC

$$= \frac{1}{2} \times AB \times BC$$

$$= \frac{1}{2} \times 12 \times 5 = 30 \text{ m}^2$$

3. Since, the total area of the park = 30 m^2

∵ The cost of ploughing the park in 1 m^2 = 5

∴ The cost of ploughing the park in 30 m^2

= 5 × 30

= ₹ 150

Very Short Answer Type Questions

Q 1. The perimeter of a triangle is 36 cm and its sides are in the ratio of $a : b : c = 3 : 4 : 5$, then find the sides a, b and c.

Q 2. Calculate the side of an isosceles right triangle whose hypotenuse is $5\sqrt{2}$ cm.

Q 3. Find the area of an isosceles triangle having base 3 cm and length of one of equal sides 4 cm.

Q 4. Find the area of a ΔABC, such that $AB = BC = 4$ cm and $\angle B = 90°$.

Q 5. An isosceles right triangle has area 200 cm^2. Find the length of its hypotenuse.

Q 6. The area of an equilateral triangle is $2\sqrt{3}$ cm^2. Find its side.

Q 7. The sides of a triangle are 7 cm, 24 cm and 25 cm. What will be its area?

Q 8. The base and corresponding altitude of triangle are 10 cm and 7 cm, respectively. Find its area.

Short Answer Type-I Questions

Q 1. If the length of median of an equilateral triangle is x cm, then find its area.

Q 2. Find the area of triangle whose two sides are 195 m and 180 m and the perimeter is 450 m.

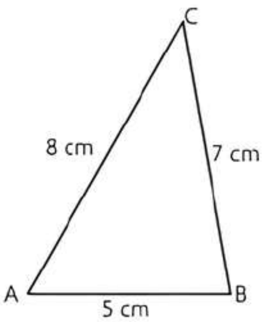
Q 3. The longest side of a right angled triangle is 125 m and one of the remaining two sides is 100 m. Find its area using Heron's formula.

Q 4. The perimeter of an isosceles triangle is 32 cm. The ratio of the equal sides to its base is 3 : 2. Find the area of the triangle.

Q 5. The sides of a triangle are x, $x+1$, $2x-1$ and its area is $x\sqrt{10}$. What is the value of x?

Q 6. The cost of levelling a ground in the form of a triangle having the sides 51 m, 37 m and 20 m at rate of ₹ 3 per m^2 is ₹ 918. State whether the statement is true or false and justify your answer.

Q 7. In figure, find the perimeter and area of ΔABC.



Q 8. Suman has a piece of land, which is in the shape of isosceles triangle ABC. She wants her two sons to work on the land and produce different crops. She divides land in two equal parts by drawing a vertex A to the mid-point of BC. If its perimeter is 320 m and one of longest side is of length 120 m. How much area each of them will get for his crops?

Short Answer Type-II Questions

Q 1. The perimeter of a triangular field is 420 m and its sides are in the ratio of 6 : 7 : 8. Find the area of triangular field.

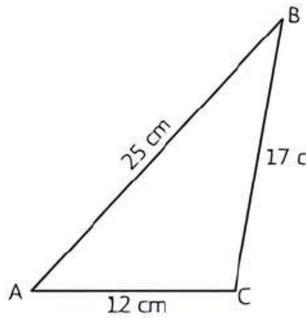
Q 2. The sides of a ΔABC are 6 cm, 8 cm and the angle between the first two sides is a right angle. Find length AC and area of triangle.
Also, determine the area of an equilateral triangle, which is drawn from the side AB.

Q 3. If each side of a triangle is doubled, then find the ratio of area of new triangle thus formed and the given triangle.

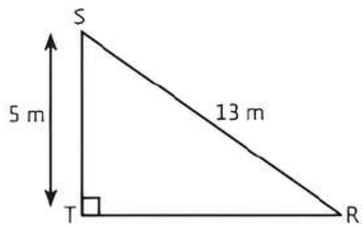
Q 4. The perimeter of a triangular field is 144 m and its sides are in the ratio 3 : 4 : 5. Find the length of perpendicular from the opposite vertex of side whose length is 60 m.

Q 5. A triangular park has sides 120 m, 80 m and 50 m. A gardener has to put a fence all around it and also plant grass inside. How much area does he need to plant? Find the cost of fencing it with barbed wire at the rate of ₹ 20 per metre having space 3 m wide for a gate on one side.

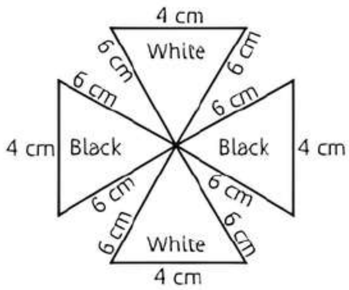
Q 6. Find the area of triangle given in the figure. Also, find the maximum length of altitude.



Q 7. Find the area of ΔRST in figure below. Also, determine the area of another triangle, which is drawn from the side SR and its remaining sides are 7 m and 8 m.



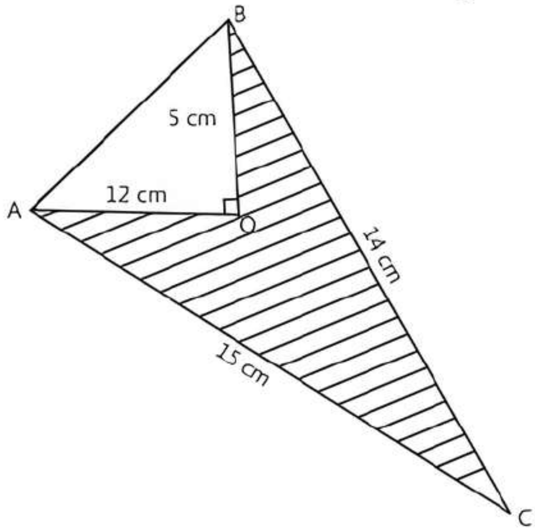
Q 8. Black and white coloured triangular sheets are used to make a toy as shown in the figure. Find the total area of each black and white colour sheets used for making the toy.



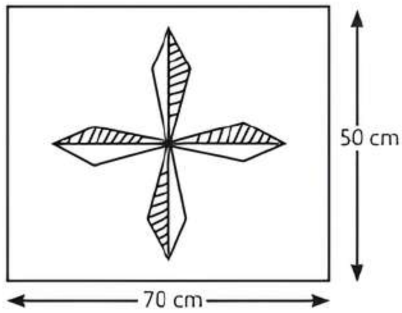
Long Answer Type Questions

Q 1. From a point in the interior of an equilateral triangle, perpendiculars are drawn on the three sides. The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm. Find the area of an equilateral triangle.

Q 2. Calculate the area of the shaded region.

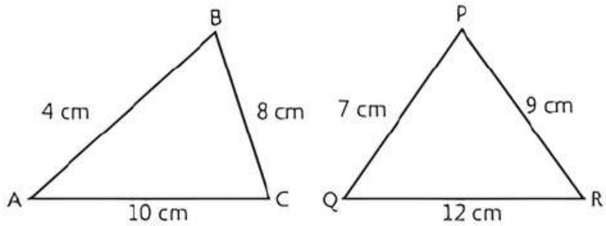


Q 3. A design is made on a rectangular tile of dimensions 50 cm × 70 cm. The design shows 8 triangles, each of sides 26 cm, 17 cm and 25 cm. Find the total area of the design.



Q 4. Find the area of an isosceles triangle, whose base is 1 cm and each equal side is 5 cm. Also determine the altitude and area of an equilateral triangle, which is drawn from one of the equal side of a given triangle.

Q 5. In the following figure, which figure has maximum area of triangle.



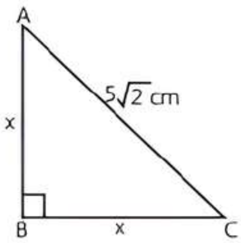
(i) (ii)

Solutions

Very Short Answer Type Questions

1. Let $a = 3x$, $b = 4x$ and $c = 5x$ be the sides of triangles.
Given, perimeter of triangle = 36 cm
$\Rightarrow \quad a + b + c = 36$
$\therefore \quad 3x + 4x + 5x = 36$
$\Rightarrow \quad 12x = 36$
$\Rightarrow \quad x = 3$
Thus, $a = 3 \times 3 = 9$,
$b = 4 \times 3 = 12$
and $c = 5 \times 3 = 15$
Hence, sides of the triangle are 9 cm, 12 cm and 15 cm.

2. Let the side of an isosceles right triangle be x cm.
Then, by Pythagoras theorem,
$$x^2 + x^2 = (5\sqrt{2})^2$$
$\Rightarrow \quad 2x^2 = 50$
$\Rightarrow \quad x^2 = 25$
$\Rightarrow \quad x = 5$ cm
Hence, the side is 5 cm.



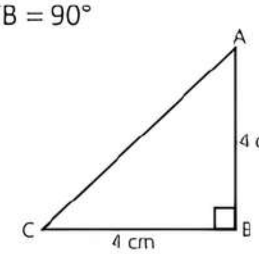
3.

TR!CK

Area of an isosceles triangle $= \frac{b}{4}\sqrt{4a^2 - b^2}$ *, where b is base and a is one of the equal sides.*

$$\text{Area of isosceles triangle} = \frac{b}{4}\sqrt{4a^2 - b^2}$$
$$= \frac{3}{4}\sqrt{4 \times 4^2 - 3^2} = \frac{3}{4}\sqrt{4 \times 16 - 9}$$
$$= \frac{3}{4}\sqrt{64 - 9} = \frac{3}{4}\sqrt{55} \text{ cm}$$
($\because$ $b = 3$ cm and $a = 4$ cm)

4. Given, AB = BC = 4 cm
and $\angle B = 90°$



Area of $\Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times BC \times AB$

$= \frac{1}{2} \times 4 \times 4 = 8 \text{ cm}^2$

5. Given, area of an isosceles right triangle

$= 200 \text{ cm}^2$

Let each of equal sides be a cm.

$\Rightarrow \quad \frac{1}{2} \times (a)^2 = 200$

($\because$ In an isosceles right triangle, base = height)

$\Rightarrow \quad a^2 = 400 \quad \Rightarrow \quad a = \sqrt{400} = 20 \text{ cm}$

$\therefore$ Hypotenuse $= \sqrt{a^2 + a^2}$

[By Pythagoras theorem]

$= \sqrt{(20)^2 + (20)^2}$

$= \sqrt{400 + 400}$

$= \sqrt{800} = 20\sqrt{2} \text{ cm}$

6. We know that,

$\frac{\sqrt{3}}{4} \times (\text{Side})^2 = \text{Area of an equilateral triangle}$

$\Rightarrow \quad \frac{\sqrt{3}}{4} \times (\text{Side})^2 = 2\sqrt{3} \quad [\because \text{Area} = 2\sqrt{3} \text{ cm}^2]$

$\Rightarrow \quad (\text{Side})^2 = 2\sqrt{3} \times \frac{4}{\sqrt{3}} = 8$

$\Rightarrow \quad \text{Side} = \sqrt{8} = 2\sqrt{2} \text{ cm}$

7. We know that, $s = \frac{a + b + c}{2}$

$\Rightarrow \quad s = \frac{7 + 24 + 25}{2} = \frac{56}{2} = 28 \text{ cm}$

($\because a = 7$ cm, $b = 24$ cm and $c = 25$ cm)

$\therefore$ Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{28(28-7)(28-24)(28-25)}$

$= \sqrt{28 \times 21 \times 4 \times 3}$

$= \sqrt{7 \times 4 \times 7 \times 3 \times 4 \times 3}$

$= 84 \text{ cm}^2$

8. Given, base of triangle = 10 cm

and corresponding altitude = 7 cm

Area of triangle

$= \frac{1}{2} \times \text{Base} \times \text{Corresponding altitude}$

$= \frac{1}{2} \times 10 \times 7 = 35 \text{ cm}^2$

Short Answer Type-I Questions

1. Let each equal sides of equilateral triangle be 2y. Median is also a perpendicular bisector in equilateral triangle.

In right ΔADB, by Pythagoras theorem,

$\Rightarrow \quad AB^2 = AD^2 + BD^2$

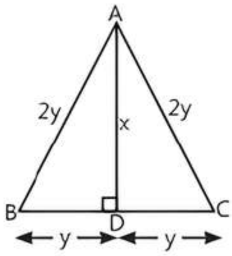
$\Rightarrow \quad (2y)^2 = x^2 + y^2$

$\Rightarrow \quad 4y^2 = x^2 + y^2$

$\Rightarrow \quad x^2 = 3y^2$

$\Rightarrow \quad x = \sqrt{3}\, y$

or $\quad y = \frac{x}{\sqrt{3}}$



Area of given triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times 2y \times x = y \times x$

$= \frac{x}{\sqrt{3}} \times x = \frac{x^2}{\sqrt{3}} \text{cm}^2$

2. Let a, b, c be the sides of given triangle.

Given, $a = 195$ m, $b = 180$ m and perimeter of triangle = 450 m

We know that, $2s = a + b + c$

$\Rightarrow \quad a + b + c = 450$

$\Rightarrow \quad 195 + 180 + c = 450$

$\Rightarrow \quad 375 + c = 450$

$\Rightarrow \quad c = 450 - 375 = 75 \text{ m}$

Semi-perimeter $(s) = \frac{a + b + c}{2} = \frac{450}{2} = 225$

$\therefore$ Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{225(225-195)(225-180)(225-75)}$

$= \sqrt{225 \times 30 \times 45 \times 150}$

$= \sqrt{2^2 \times 3^6 \times 5^6} = 2 \times 3^3 \times 5^3$

$= 6750 \text{ m}^2$

3. Given, the two sides of a right-angled triangle are 125 m and 100 m.

$\therefore$ Third side $= \sqrt{(125)^2 - (100)^2}$

$= \sqrt{(125 + 100)(125 - 100)}$

$= \sqrt{225 \times 25} = 75 \text{ m}$

Now, semi-perimeter,

$s = \frac{125 + 100 + 75}{2} = \frac{300}{2} = 150 \text{ m}$

$\therefore$ Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{150 \times (150-125)(150-100)(150-75)}$

$= \sqrt{150 \times 25 \times 50 \times 75}$

$= \sqrt{50 \times 3 \times 25 \times 50 \times 25 \times 3}$

$= 50 \times 25 \times 3 = 3750 \text{ m}^2$

4. Let each of the equal sides of isosceles triangle $= 3x$ cm and base of isosceles triangle $= 2x$ cm

$\therefore$ Perimeter of an isosceles triangle

$= 3x + 3x + 2x$ [Given]

$\Rightarrow \quad 32 = 8x \quad \Rightarrow \quad x = 4$

$\therefore$ Sides are 3×4, 3×4 and 2×4,

i.e., 12 cm, 12 cm and 8 cm.

Let $a = 12$ cm, $b = 12$ cm and $c = 8$ cm

Then, $s = \dfrac{a+b+c}{2} = \dfrac{12+12+8}{2} = \dfrac{32}{2} = 16$ cm

$\therefore$ Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16(16-12)(16-12)(16-8)}$

$= \sqrt{16 \times 4 \times 4 \times 8} = 32\sqrt{2}\ \text{cm}^2$

5. Given, sides of a triangle are x, $x+1$ and $2x-1$.

Now, semi-perimeter

$s = \dfrac{x+(x+1)+(2x-1)}{2} = \dfrac{4x}{2} = 2x$ units

$\therefore$ Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{2x(2x-x)(2x-x-1)(2x-2x+1)}$

$= \sqrt{2x \cdot x \cdot (x-1) \cdot 1}$

$= x\sqrt{2(x-1)}$ sq. units

But area of triangle $= x\sqrt{10}$ sq. units [Given]

$\Rightarrow \quad x\sqrt{2(x-1)} = x\sqrt{10} \quad \Rightarrow \quad 2(x-1) = 10$

$\Rightarrow \quad x - 1 = 5$

$\Rightarrow \quad x = 6$ units

6. Let the sides of a triangular ground are

$a = 51$ m, $b = 37$ m and $c = 20$ m

Now, $s = \dfrac{a+b+c}{2} = \dfrac{51+37+20}{2}$

$= \dfrac{108}{2} = 54$ cm

$\therefore$ Area of triangular ground

$= \sqrt{s(s-a)(s-b)(s-c)}$ [By Heron's formula]

$= \sqrt{54(54-51)(54-37)(54-20)}$

$= \sqrt{54 \times 3 \times 17 \times 34} = \sqrt{93636} = 306\ \text{m}^2$

So, cost of levelling the ground

$= ₹\ 3 \times 306 = ₹\ 918$

Hence, the given statement is true.

7. Let a, b and c be the sides of the Δ ABC,

where, $a = 5$ cm, $b = 7$cm and $c = 8$ cm

Now, perimeter of a triangle

$2s = a + b + c = 5 + 7 + 8$

$= 20$ cm

Here, semi-perimeter of a triangle,

$s = \dfrac{2s}{2} = \dfrac{20}{2} = 10$ cm

$\therefore$ Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{10(10-5)(10-7)(10-8)}$

$= \sqrt{10 \times 5 \times 3 \times 2} = 10\sqrt{3}\ \text{cm}^2$

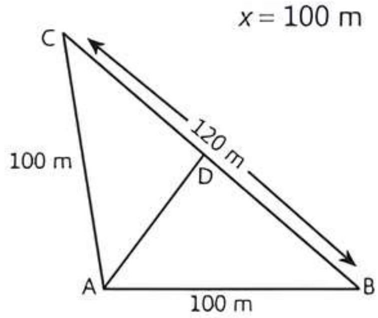
8. Let each equal side of an isosceles triangle be x m.

Perimeter of triangle field $= 320$

$\therefore \quad x + x + 120 = 320$

$\Rightarrow \quad 2x = 200$

$\Rightarrow \quad x = 100$ m



Let $a = b = 100$ m and $c = 120$ cm. Then, semi perimeter of triangle, $s = \dfrac{320}{2} = 160$ m

$\therefore$ Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{160(160-100)(160-100)(160-120)}$

$= \sqrt{160 \times 60 \times 60 \times 40} = 80 \times 60 = 4800\ \text{m}^2$

Since, AD is a median, so its divide the area into two parts.

$\therefore$ Area of each part $= \dfrac{4800}{2} = 2400\text{m}^2$

Hence, area of land allotted to each son for their crops is $2400\ \text{m}^2$ each.

Short Answer Type-II Questions

1. Given, perimeter of triangular field is 420 m and sides are in the ratio of 6 : 7 : 8.

Let the sides of triangular field be $a = 6x$, $b = 7x$ and $c = 8x$.

$\therefore$ Perimeter of triangular field $= a + b + c$

$\Rightarrow \quad 420 = 6x + 7x + 8x \quad \Rightarrow \quad 420 = 21x$

$\Rightarrow \quad x = \dfrac{420}{21} = 20$ m

$\therefore$ Sides of triangular field

$a = 6 \times 20 = 120$ m

$b = 7 \times 20 = 140$ m

$c = 8 \times 20 = 160$ m

Now, semi-perimeter,

$s = \dfrac{a+b+c}{2}$

$= \dfrac{120+140+160}{2} = \dfrac{420}{2} = 210$

$\therefore$ Area of triangular field

$= \sqrt{s(s-a)(s-b)(s-c)}$ [By Heron's formula]

$= \sqrt{210(210-120)(210-140)(210-160)}$

$= \sqrt{210 \times 90 \times 70 \times 50}$

$= 100\sqrt{7 \times 3 \times 3^2 \times 7 \times 5}$

$= 100 \times 7 \times 3 \times \sqrt{15} = 2100\sqrt{15}\ \text{m}^2$

Hence, area of triangular field is $2100\sqrt{15}\ \text{m}^2$.

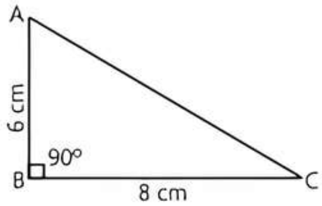
2. Given, ΔABC is a triangle right-angled at B.

So, in ΔABC,

$AC^2 = AB^2 + BC^2$ [By Pythagoras theorem]

$\Rightarrow \quad AC^2 = 6^2 + 8^2 = 36 + 64 = 100$

$\therefore \quad AC = 10\,cm$



Area of $\Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times BC \times AB$

$= \frac{1}{2} \times 8 \times 6 = 24\ cm^2$

Now, area of an equilateral triangle, which is drawn from the side AB, is $\frac{\sqrt{3}}{4}(\text{side AB})^2$

$$= \frac{\sqrt{3}}{4} \times (6)^2$$

$$= \frac{\sqrt{3}}{4} \times 36 = 9\sqrt{3}\,cm^2$$

3. Let a, b, c be the sides of the given triangle and s be its semi-perimeter.

Then $\quad s = \frac{a+b+c}{2}$ ...(1)

$\therefore$ Area of the given triangle

$$= \sqrt{s(s-a)(s-b)(s-c)} = \Delta\,(\text{say})$$

Since, each side of triangle is doubled, therefore sides of new triangle will be $2a$, $2b$ and $2c$.

Let s' be the semi-perimeter of new triangle.

$\therefore \quad s' = \frac{2a+2b+2c}{2} = a+b+c$ ...(2)

From eqs. (1) and (2), we get

$$s' = 2s$$

Now, area of new triangle

$$= \sqrt{s'(s'-2a)(s'-2b)(s'-2c)}$$

$$= \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$$

$$= \sqrt{16s(s-a)(s-b)(s-c)}$$

$$= 4\sqrt{s(s-a)(s-b)(s-c)} = 4\Delta$$

Hence, the required ratio is 4 : 1.

4. Let the sides of triangle be $3x$, $4x$ and $5x$.

$\because$ The perimeter of triangular field = 144 m [Given]

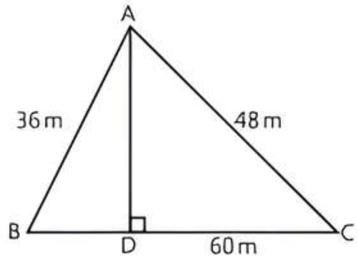
$\Rightarrow \quad$ Sum of all three sides = 144

$\therefore \quad 3x + 4x + 5x = 144$

$\Rightarrow \quad 12x = 144$

$\Rightarrow \quad x = \frac{144}{12} = 12\,m$

$\therefore$ Sides of triangle are $3 \times 12 = 36\,m$, $4 \times 12 = 48\,m$ and $5 \times 12 = 60\,m$



Let $\quad a = 36$ m, $b = 48$ m and $c = 60$ m

$\therefore \quad s = \frac{a+b+c}{2} = \frac{36+48+60}{2}$

$= \frac{144}{2} = 72\,m$

Now, area of ΔABC

$= \sqrt{s(s-a)(s-b)(s-c)}$ [By Heron's formula]

$= \sqrt{72(72-36)(72-48)(72-60)}$

$= \sqrt{72(36)(24)(12)}$

$= \sqrt{2^3 \times 3^2 \times 2^2 \times 3^2 \times 2^3 \times 3 \times 2^2 \times 3}$

$= \sqrt{2^{10} \times 3^6} = 2^5 \times 3^3 = 864\,m^2$ ...(1)

Also, area of $\Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Altitude}$

$= \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 60 \times AD = 30 \times AD$

$\therefore \quad 30 \times AD = 864$ [From eq. (1)]

$\Rightarrow \quad AD = \frac{864}{30} = 28.8\,m$

Hence, the length of perpendicular is 28.8 m.

5. Let the sides of triangular park are $a = 120$ m, $b = 80$ m and $c = 50$ m.

$\therefore s = \frac{a+b+c}{2} = \frac{120+80+50}{2} = \frac{250}{2} = 125\,m$

Area of triangular park $= \sqrt{s(s-a)(s-b)(s-c)}$

[By Heron's formula]

$= \sqrt{125(125-120)(125-80)(125-50)}$

$= \sqrt{125 \times 5 \times 45 \times 75}$

$= 375\sqrt{15} = 375 \times 3.87$

$= 1451.25\,m^2$

Perimeter of the park = Sum of its all sides

$= 120 + 80 + 50$

$= 250$ m

Space for a gate = 3 m [Given]

Length of the wire needed for fencing

= Perimeter of the park – Space for a gate

= 250 – 3 = 247 m

$\therefore$ Cost of fencing with barbed wire at the rate of ₹20 per metre = 247 × 20

= ₹4940

6. Given, sides of ΔABC are $AC = a = 12$ cm, $BC = b = 17$ cm and $AB = c = 25$ cm

$\therefore$ Semi-perimeter of ΔABC, $s = \dfrac{a+b+c}{2}$

$= \dfrac{12+17+25}{2} = \dfrac{54}{2} = 27$ cm

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$ [By Heron's formula]

$= \sqrt{27(27-12)(27-17)(27-25)}$

$= \sqrt{27 \times 15 \times 10 \times 2}$

$= 3^2 \times 5 \times 2 = 90 \text{ cm}^2$

TiP

The maximum length of the altitude is the perpendicular distance from vertex to the smallest side of a triangle.

In the given figure, the smallest side is 12 cm.

$\because$ Area of $\Delta ABC = \dfrac{1}{2} \times AC \times \text{Altitude}$

$90 = \dfrac{1}{2} \times 12 \times \text{Altitude}$

$\Rightarrow$ Altitude $= \dfrac{90 \times 2}{12} = 15$ cm

Hence, maximum length of altitude is 15 cm.

COMMON ERROR

Sometimes, students finding the maximum length of altitude by considering the perpendicular distance from vertex to the largest side of a triangle. So adequate practice is required.

7. In right-angled ΔSTR,

$(SR)^2 = (ST)^2 + (TR)^2$ [By Pythagoras theorem]

$\Rightarrow (13)^2 = (5)^2 + (TR)^2$

$\Rightarrow (TR)^2 = 169 - 25$

$\Rightarrow (TR)^2 = 144 \Rightarrow TR = 12$ m ...(1)

Now, area of right-angled ΔSTR

$= \dfrac{1}{2} \times \text{Base} \times \text{Height}$

$= \dfrac{1}{2} \times TR \times ST = \dfrac{1}{2} \times 12 \times 5 = 30 \text{ m}^2$

To determine the area of another triangle having sides 13m, 7m and 8m.

Let $a = 13$m, $b = 7$m and $c = 8$m

Then semi-perimeter of triangle,

$s = \dfrac{a+b+c}{2} = \dfrac{13+7+8}{2} = \dfrac{28}{2} = 14$m

$\therefore$ Area of triangle $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{14(14-13)(14-7)(14-8)}$

$= \sqrt{14 \times 1 \times 7 \times 6}$

$= 14\sqrt{3} \text{ m}^2$

8. Given, sides of a triangular sheet are $a = 6$ cm, $b = 6$ cm and $c = 4$ cm

$\therefore s = \dfrac{a+b+c}{2}$

$= \dfrac{6+6+4}{2} = \dfrac{16}{2} = 8$ cm

Area of a triangular sheet $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{8(8-6)(8-6)(8-4)}$

$= \sqrt{8 \times 2 \times 2 \times 4} = 8\sqrt{2} \text{ cm}^2$

Now, area of two black triangles $= 8\sqrt{2} \times 2$

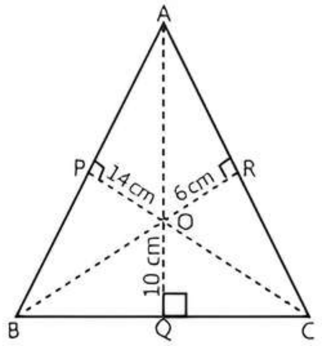
$= 16\sqrt{2} \text{ cm}^2$

And area of two white triangles $= 16\sqrt{2} \text{ cm}^2$

Hence, the areas of black and white colour sheets used for making the toy are $16\sqrt{2}$ cm^2 each.

Long Answer Type Questions

1. Let ABC be an equilateral triangle, O be its interior point and AQ, BR and CP are perpendiculars drawn through point O on the three sides.



Given, OP = 14 cm, OQ = 10 cm and OR = 6 cm

Let the sides of an equilateral triangle be a cm each.

$\therefore AB = BC = CA = a$ cm

TR!CK

Area of triangle $= \dfrac{1}{2} \times$ *Base* $\times$ *Height*

Area of $\Delta OAB = \dfrac{1}{2} \times AB \times OP$

$= \dfrac{1}{2} \times a \times 14 = 7a \text{ cm}^2$ ...(1)

Area of $\Delta OBC = \dfrac{1}{2} \times BC \times OQ$

$= \dfrac{1}{2} \times a \times 10 = 5a \text{ cm}^2$ ...(2)

Area of $\Delta OAC = \dfrac{1}{2} \times AC \times OR$

$= \dfrac{1}{2} \times a \times 6 = 3a \text{ cm}^2$ ...(3)

$\therefore$ Area of ΔABC = Area of $(\Delta OAB + \Delta OBC + \Delta OAC)$

$= 7a + 5a + 3a = 15a \text{ cm}^2$ ...(4)

Semi-perimeter, $s = \frac{a+a+a}{2} = \frac{3a}{2}$ cm

$\therefore$ Area of an equilateral ΔABC

$= \sqrt{s(s-a)(s-b)(s-c)}$ [By Heron's formula]

$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$

$= \sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}} = \frac{\sqrt{3}}{4}a^2$ ...(5)

From eqs. (4) and (5),

$\frac{\sqrt{3}}{4}a^2 = 15a$

$\Rightarrow \quad a = \frac{15 \times 4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{60\sqrt{3}}{3} = 20\sqrt{3}$ cm

Putting $a = 20\sqrt{3}$ in eq. (5), we get

Area of equilateral ΔABC

$= \frac{\sqrt{3}}{4}(20\sqrt{3})^2 = \frac{\sqrt{3}}{4} \times 400 \times 3$

$= 300\sqrt{3}$ cm^2

Hence, the area of an equilateral triangle is

$= 300\sqrt{3}$ cm^2

2. Area of right-angled $\Delta AOB = \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times OA \times OB$

$= \frac{1}{2} \times 12 \times 5 = 30$ cm^2

Again, in right-angled ΔAOB,

$AB^2 = OA^2 + OB^2$ [By Pythagoras theorem]

$\Rightarrow AB^2 = 12^2 + 5^2 = 144 + 25 = 169$

$\Rightarrow AB = \sqrt{169} = 13$ cm

Now in ΔABC,

$BC = a = 14$ cm, $CA = b = 15$ cm

and $AB = c = 13$ cm

$\therefore \quad s = \frac{a+b+c}{2} = \frac{14+15+13}{2}$

$= \frac{42}{2} = 21$ cm

Now, area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

[By Heron's formula]

$= \sqrt{21(21-14)(21-15)(21-13)}$

$= \sqrt{21 \times 7 \times 6 \times 8}$

$= 2 \times 2 \times 3 \times 7 = 84$ cm^2

$\therefore$ Area of shaded region

= Area of ΔABC – Area of ΔAOB

$= 84$ cm$^2 - 30$ cm$^2 = 54$ cm^2

3. Let the sides of each equal triangles of design are $a = 25$ cm, $b = 17$ cm and $c = 26$ cm.

Semi-perimeter,

$s = \frac{a+b+c}{2} = \frac{25+17+26}{2} = \frac{68}{2} = 34$ cm

Area of one triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

[By Heron's formula]

$= \sqrt{34(34-25)(34-17)(34-26)}$

$= \sqrt{34 \times 9 \times 17 \times 8}$

$= 17 \times 3 \times 2 \times 2 = 204$ cm^2

$\therefore$ Total area of eight triangles

$= 204 \times 8 = 1632$ cm^2

Area of the design = Total area of eight triangles

$= 1632$ cm^2

4. Let $a = 5$ cm, $b = 5$ cm, $c = 1$ cm

$\therefore s = \frac{a+b+c}{2} = \frac{5+5+1}{2}$ cm $= \frac{11}{2}$ cm $= 5.5$ cm

Area of the triangle ABC

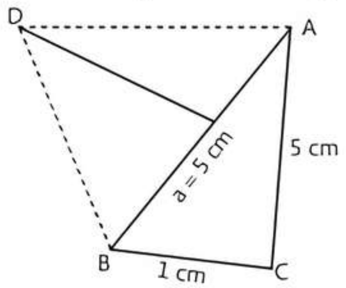
$= \sqrt{s(s-a)(s-b)(s-c)}$ [By Heron's formula]

$= \sqrt{5.5(5.5-5)(5.5-5)(5.5-1)}$ cm^2

$= \sqrt{5.5 \times 0.5 \times 0.5 \times 4.5}$ cm^2

$= \sqrt{6.1875}$ cm$^2 = 2.49$ cm^2

Here, side of an equilateral triangle ABD is 5 cm



$\therefore$ The altitude of an equilateral triangle ABD is

$\frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{2} \times 5$

$= \frac{5\sqrt{3}}{2}$ cm

And the area of an equilateral triangle,

$A = \frac{\sqrt{3}}{4}(a)^2$

$= \frac{\sqrt{3}}{4}(5)^2$

$= \frac{25}{4}\sqrt{3}$ cm^2

5. (i) Let $a = 4$ cm, $b = 8$ cm and $c = 10$ cm

Then, semi perimeter of a triangle ABC is

$s = \frac{a+b+c}{2}$

$= \frac{4+8+10}{2}$

$= \frac{22}{2} = 11$ cm

$\therefore$ Area of triangle ABC is

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{11(11-4)(11-8)(11-10)}$

$= \sqrt{11 \times 7 \times 3 \times 1}$

$= \sqrt{231} = 15.2 \text{ cm}^2$

(ii) Let A = 7 cm, B = 9 cm and C = 12 cm

Then, semi perimeter of a triangle PQR is

$$S = \frac{A+B+C}{2}$$

$$= \frac{7+9+12}{2} = \frac{28}{2}$$

$= 14$cm

∴ Area of ΔPQR is

$$\Delta' = \sqrt{S(S-A)(S-B)(S-C)}$$

$= \sqrt{14(14-7)(14-9)(14-12)}$

$= \sqrt{14 \times 7 \times 5 \times 2}$

$= 14\sqrt{5}$

$= 14 \times 2.24$

$= 31.36 \text{ cm}^2$

It is clear that ar (ΔPQR) > ar (ΔABC).

Hence, figure (ii) has maximum area.

Chapter Test

Multiple Choice Questions

Q 1. The lengths of the three sides of a triangular field are 40 m, 24 m and 32m respectively. The area of the triangle is:

a. 384 m^2 b. 350 m^2
c. 370 m^2 d. 440 m^2

Q 2. An isosceles right triangle has area 32cm^2. The length of its hypotenuse is:

a. $8\sqrt{3}$ cm b. 8 cm c. $8\sqrt{2}$ cm d. 16 cm

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct choices as:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
c. Assertion (A) is true but Reason (R) is false.
d. Assertion (A) is false but Reason (R) is true.

Q 3. Assertion (A): The side of an equilateral triangle is 6 cm, then the height of the triangle is 9 cm.

Reason (R): The height of an equilateral triangle is $\frac{\sqrt{3}}{2}a$.

Q 4. Assertion (A): The sides of a triangle are in the ratio of 25 : 14 : 12 and its perimeter is 510 cm. Then the area of the triangle is 4449.08 cm^2.

(Use $\sqrt{391} = 19.77$)

Reason (R): The semi-perimeter of a triangle with sides a, b and c is $\frac{(a+b+c)}{2}$.

Fill in the Blanks

Q 5. If height of a triangle is doubled and base is tripled then its area become .... times.

Q 6. Area of a triangle with perimeter 42 cm and length of two sides is 18 cm and 10 cm is given by ....

True/False

Q 7. Heron's formula cannot be use to calculate the area of special type of triangle.

Q 8. The length of longest altitude is the pendicular distance from the opposite vertex to the smallest side of a triangle.

Case Study Based Question

Q 9. To, spread awareness about traffic signal, Central Board of Secondary Education (CBSE) decided to conduct an activity of making a traffic signal board and should take a photo of themselves telling about that particular traffic signal. One of the student decided to make a triangular sign board showing road work ahead. The dimensions of the sign board are 11 cm, 12 cm and 13 cm.

On the basis of the above information, solve the following questions:

(i) If dimension of sign board are given, then which formula is used for finding the covered area?

(ii) Find the smallest altitude of a triangle.

OR

If cost of painting is ₹ 2 per cm^2, then find the total cost of painting the sign board in one side.

(Use $\sqrt{105} = 10.25$)

(iii) What the area of sign board?

Very Short Answer Type Questions

Q 10. If the perimeter of an equilateral triangle is 60 m, then find the area of triangle.

Q 11. The base and hypotenuse of a right triangle are respectively 5 cm and 13 cm long. Find its area.

Short Answer Type-I Questions

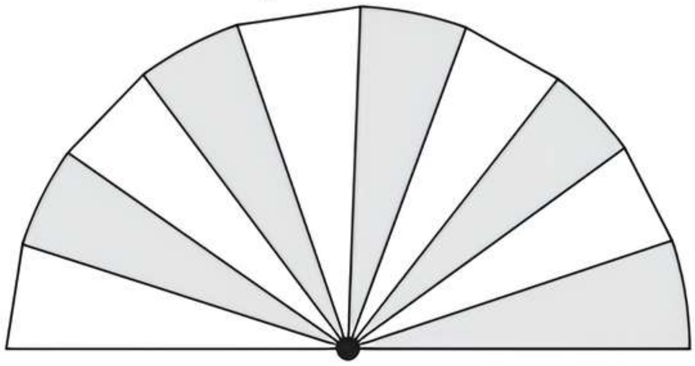
Q 12. The perimeter of a triangular field is 450 m and its sides are in the ratio 13 : 12 : 5. Find the area of triangle.

Q 13. The lengths of the sides of a triangle are 5cm, 12cm and 13cm. Find the length of perpendicular from the opposite vertex to the side whose length is 13 cm.

Short Answer Type-II Questions

Q 14. A triangle has sides 35cm, 54cm and 61cm long. Find its area. Also, find the smallest of its altitudes.

Q 15. A hand fan is made by stitching 10 equal size triangular strips of two different types of paper as shown in figure.



The dimensions of equal strips are 25 cm, 25 cm and 14 cm. Find the area of each type of paper needed to make the hand fan.

Long Answer Type Question

Q 16. The perimeter of an isosceles triangle is 42cm and its base is $\left(\frac{3}{2}\right)$ times each of the equal sides.

Find the length of each side of the triangle, area of the triangle and the height of the base side of the triangle.