Next, cut a triangle A′ D′ E′ congruent to triangle ADE on a separate sheet with the help of a tracing paper and place Δ A′D′E′ in such a way that A′D′ coincides with BC as shown in Fig 11.11.Note that there are two parallelograms ABCD and EE′CD on the same base DC and between the same parallels AE′ and DC. What can you say about their areas?

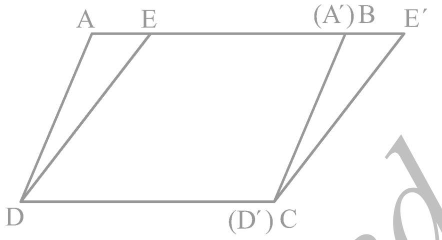

Fig. 11.11

As Δ ADE ≅ Δ A′D′E′

Therefore ar (ADE) = ar (A′D′E′)

Also ar (ABCD) = ar (ADE) + ar (EBCD)

= ar (A′D′E′) + ar (EBCD)

= ar (EE′CD)

So, the two parallelograms are equal in area.

Let us now try to prove this relation between the two such parallelograms.

Theorem 11.1 : *Parallelograms on the same base and between the same parallels are equal in area.*

Proof : Two parallelograms ABCD and EFCD, on the same base DC and between the same parallels AF and DC are given (see Fig.11.12).

We need to prove that ar (ABCD) = ar (EFCD).

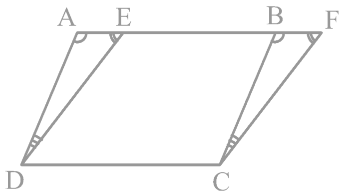

Fig. 11.12

In Δ ADE and Δ BCF,

∠ DAE = ∠ CBF (Corresponding angles from AD || BC and transversal AF) (1)

∠ AED = ∠ BFC (Corresponding angles from ED || FC and transversal AF) (2)

Therefore, ∠ ADE = ∠ BCF (Angle sum property of a triangle) (3)

Also, AD = BC (Opposite sides of the parallelogram ABCD) (4)

So, Δ ADE ≅ Δ BCF [By ASA rule, using (1), (3), and (4)]

Therefore, ar (ADE) = ar (BCF) (Congruent figures have equal areas) (5)

Now, ar (ABCD) = ar (ADE) + ar (EDCB)

= ar (BCF) + ar (EDCB) [From(5)]

= ar (EFCD)

So, parallelograms ABCD and EFCD are equal in area.

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Let us now take some examples to illustrate the use of the above theorem.

Example 1 : In Fig. 11.13, ABCD is a parallelogram and EFCD is a rectangle.

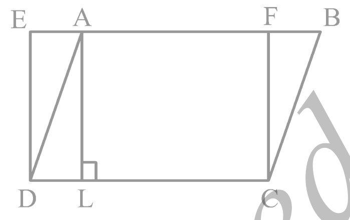

Fig. 11.13

Also, AL ⊥ DC. Prove that

(i) ar (ABCD) = ar (EFCD)

(ii) ar (ABCD) = DC × AL

Solution : (i) As a rectangle is also a parallelogram,

therefore, ar (ABCD) = ar (EFCD) (Theorem 11.1)

(ii) From above result,

ar (ABCD) = DC × FC (Area of the rectangle = length × breadth) (1)

As AL ⊥ DC, therefore, AFCL is also a rectangle

So, AL = FC (2)

Therefore, ar (ABCD) = DC × AL [From (1) and (2)]

Can you see from the Result (ii) above *that area of a parallelogram is the product of its any side and the coresponding altitude*. Do you remember that you have studied this formula for area of a parallelogram in Class VII. On the basis of this formula, Theorem 11.1 can be rewritten as *parallelograms on the same base or equal bases and between the same parallels are equal in area*.

Can you write the converse of the above statement? It is as follows: *Parallelograms on the same base* (*or equal bases*) *and having equal areas lie between the same parallels.* Is the converse true? Prove the converse using the formula for area of the parallelogram.

Example 2 : If a triangle and a parallelogram are on the same base and between the same parallels, then prove that the area of the triangle is equal to half the area of the parallelogram.

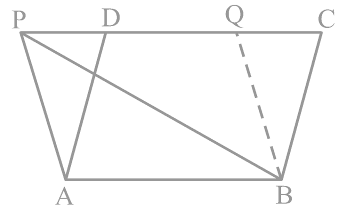

Fig. 11.14

Solution : Let Δ ABP and parallelogram ABCD be on the same base AB and between the same parallels AB and PC (see Fig. 11.14).

You wish to prove that ar (PAB) = $\frac{1}{2}$ ar (ABCD)

Draw BQ || AP to obtain another parallelogram ABQP. Now parallelograms ABQP and ABCD are on the same base AB and between the same parallels AB and PC.

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Therefore, ar (ABQP) = ar (ABCD) (By Theorem 9.1) (1)

But Δ PAB $\cong$ Δ BQP (Diagonal PB divides parallelogram ABQP into two congruent triangles.)

So, ar (PAB) = ar (BQP) (2)

Therefore, ar (PAB) = $\frac{1}{2}$ ar (ABQP) [From (2)] (3)

This gives ar (PAB) = $\frac{1}{2}$ ar (ABCD) [From (1) and (3)]

EXERCISE 11.2

1. In Fig. 9.15, ABCD is a parallelogram, AE $\perp$ DC and CF $\perp$ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.

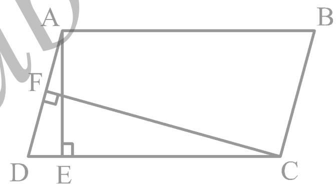

Fig. 11.15

2. If E,F,G and H are respectively the mid-points of the sides of a parallelogram ABCD, show that ar (EFGH) = $\frac{1}{2}$ ar (ABCD).
3. P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar (APB) = ar (BQC).
4. In Fig. 11.16, P is a point in the interior of a parallelogram ABCD. Show that

   (i) ar (APB) + ar (PCD) = $\frac{1}{2}$ ar (ABCD)

   (ii) ar (APD) + ar (PBC) = ar (APB) + ar (PCD)

   [*Hint* : Through P, draw a line parallel to AB.]

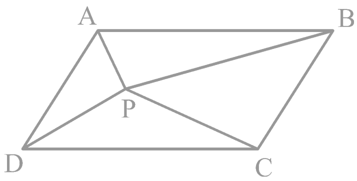

Fig. 11.16

5. In Fig. 11.17, PQRS and ABRS are parallelograms and X is any point on side BR. Show that

   (i) ar (PQRS) = ar (ABRS)

   (ii) ar (AX S) = $\frac{1}{2}$ ar (PQRS)

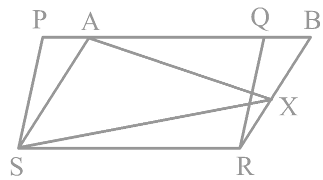

Fig. 11.17

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6. A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the fields is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?

11.4 Triangles on the same Base and between the same Parallels

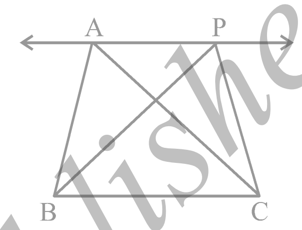

Fig. 11.18

Let us look at Fig. 11.18. In it, you have two triangles ABC and PBC on the same base BC and between the same parallels BC and AP. What can you say about the areas of such triangles? To answer this question, you may perform the activity of drawing several pairs of triangles on the same base and between the same parallels on the graph sheet and find their areas by the method of counting the squares. Each time, you will find that the areas of the two triangles are (approximately) equal. This activity can be performed using a geoboard also. You will again find that the two areas are (approximately) equal.

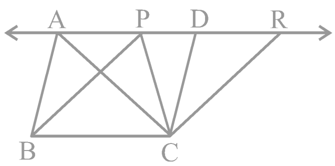

Fig. 11.19

To obtain a logical answer to the above question, you may proceed as follows:

In Fig. 11.18, draw CD || BA and CR || BP such that D and R lie on line AP(see Fig.11.19).

From this, you obtain two parallelograms PBCR and ABCD on the same base BC and between the same parallels BC and AR.

Therefore, ar (ABCD) = ar (PBCR) (Why?)

Now Δ ABC $\cong$ Δ CDA and Δ PBC $\cong$ Δ CRP (Why?)

So, ar (ABC) = $\frac{1}{2}$ ar (ABCD) and ar (PBC) = $\frac{1}{2}$ ar (PBCR) (Why?)

Therefore, ar (ABC) = ar (PBC)

In this way, you have arrived at the following theorem:

Theorem 11.2 : *Two triangles on the same base (or equal bases) and between the same parallels are equal in area.*

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Now, suppose ABCD is a parallelogram whose one of the diagonals is AC (see Fig. 11.20). Let AN $\perp$ DC. Note that

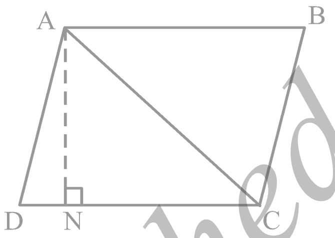

Fig. 11.20

$$\Delta\, ADC \cong \Delta\, CBA \qquad \text{(Why?)}$$

So, $\quad ar\,(ADC) = ar\,(CBA) \qquad \text{(Why?)}$

Therefore, $\quad ar\,(ADC) = \frac{1}{2}\, ar\,(ABCD)$

$$= \frac{1}{2}\,(DC \times AN) \qquad \text{(Why?)}$$

So, $\quad$ area of $\Delta\, ADC = \frac{1}{2} \times$ base DC $\times$ corresponding altitude AN

In other words, *area of a triangle is half the product of its base* (*or any side*) *and the corresponding altitude* (*or height*). Do you remember that you have learnt this formula for area of a triangle in Class VII ? From this formula, you can see that *two triangles with same base* (*or equal bases*) *and equal areas will have equal corresponding altitudes.*

For having equal corresponding altitudes, the triangles must lie between the same parallels. From this, you arrive at the following converse of Theorem 11.2 .

Theorem 11.3 : *Two triangles having the same base* (*or equal bases*) *and equal areas lie between the same parallels.*

Let us now take some examples to illustrate the use of the above results.

Example 3 : Show that a median of a triangle divides it into two triangles of equal areas.

Solution : Let ABC be a triangle and let AD be one of its medians (see Fig. 11.21). You wish to show that

$$ar\,(ABD) = ar\,(ACD).$$

Since the formula for area involves altitude, let us draw AN $\perp$ BC.

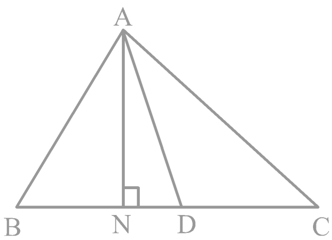

Fig. 11.21

Now $\quad ar(ABD) = \frac{1}{2} \times$ base $\times$ altitude (of Δ ABD)

$$= \frac{1}{2} \times BD \times AN$$

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$= \frac{1}{2} \times CD \times AN$ (As BD = CD)

$= \frac{1}{2} \times$ base $\times$ altitude (of Δ ACD)

= ar(ACD)

Example 4 : In Fig. 11.22, ABCD is a quadrilateral and BE || AC and also BE meets DC produced at E. Show that area of Δ ADE is equal to the area of the quadrilateral ABCD.

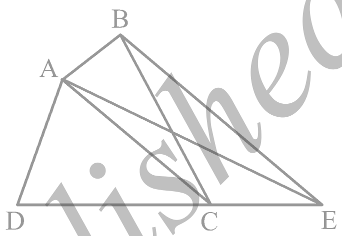

Fig. 11.22

Solution : Observe the figure carefully .

Δ BAC and Δ EAC lie on the same base AC and between the same parallels AC and BE.

Therefore, ar(BAC) = ar(EAC) (By Theorem 11.2)

So, ar(BAC) + ar(ADC) = ar(EAC) + ar(ADC) (Adding same areas on both sides)

or ar(ABCD) = ar(ADE)

EXERCISE 11.3

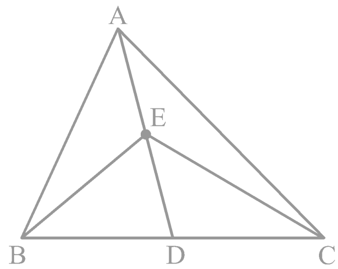

Fig. 11.23

1. In Fig.11.23, E is any point on median AD of a Δ ABC. Show that ar (ABE) = ar (ACE).
2. In a triangle ABC, E is the mid-point of median AD. Show that ar (BED) = $\frac{1}{4}$ ar(ABC) .
3. Show that the diagonals of a parallelogram divide it into four triangles of equal area.
4. In Fig. 11.24, ABC and ABD are two triangles on the same base AB. If line- segment CD is bisected by AB at O, show that ar(ABC) = ar (ABD).

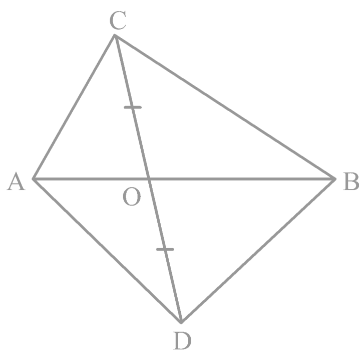

Fig. 11.24

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5. D, E and F are respectively the mid-points of the sides BC, CA and AB of a Δ ABC. Show that

(i) BDEF is a parallelogram. (ii) ar (DEF) = $\frac{1}{4}$ ar (ABC)

(iii) ar (BDEF) = $\frac{1}{2}$ ar (ABC)

6. In Fig. 11.25, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, then show that:

(i) ar (DOC) = ar (AOB)

(ii) ar (DCB) = ar (ACB)

(iii) DA || CB or ABCD is a parallelogram.

[*Hint* : From D and B, draw perpendiculars to AC.]

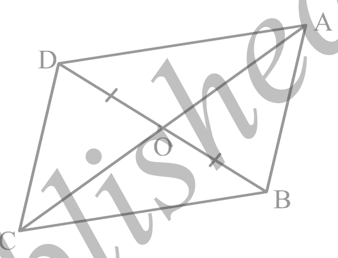

Fig. 11.25

7. D and E are points on sides AB and AC respectively of Δ ABC such that ar (DBC) = ar (EBC). Prove that DE || BC.

8. XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and F respectively, show that

ar (ABE) = ar (ACF)

9. The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see Fig. 11.26). Show that ar (ABCD) = ar (PBQR).

[*Hint* : Join AC and PQ. Now compare ar (ACQ) and ar (APQ).]

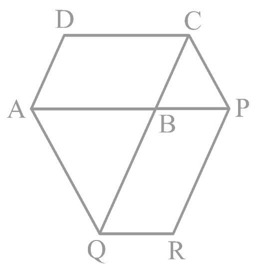

Fig. 11.26

10. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar (AOD) = ar (BOC).

11. In Fig. 11.27, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that

(i) ar (ACB) = ar (ACF)

(ii) ar (AEDF) = ar (ABCDE)

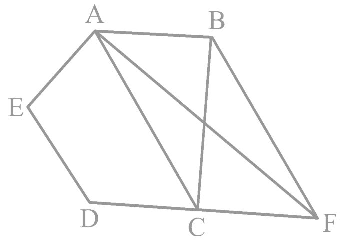

Fig. 11.27

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12. A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

13. ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar (ADX) = ar (ACY).

[*Hint* : Join CX.]

14. In Fig.11.28, AP || BQ || CR. Prove that ar (AQC) = ar (PBR).

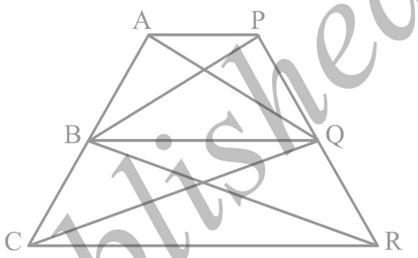

Fig. 11.28

15. Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (AOD) = ar (BOC). Prove that ABCD is a trapezium.

16. In Fig.11.29, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.

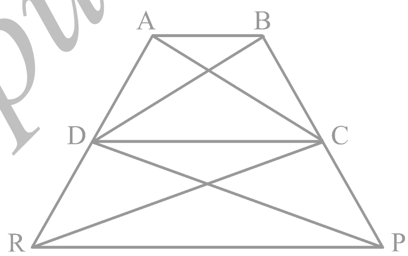

Fig. 11.29

EXERCISE 11.4 (Optional)*

1. Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.

2. In Fig. 11.30, D and E are two points on BC such that BD = DE = EC. Show that ar (ABD) = ar (ADE) = ar (AEC).

Can you now answer the question that you have left in the 'Introduction' of this chapter, whether the field of *Budhia* has been actually divided into three parts of equal area?

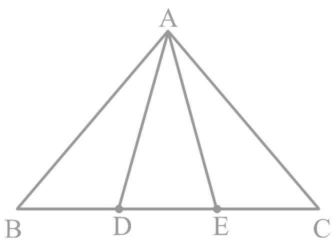

Fig. 11.30

*These exercises are not from examination point of view.

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[**Remark:** Note that by taking BD = DE = EC, the triangle ABC is divided into three triangles ABD, ADE and AEC of equal areas. In the same way, by dividing BC into *n* equal parts and joining the points of division so obtained to the opposite vertex of BC, you can divide ΔABC into *n* triangles of equal areas.]

3. In Fig. 11.31, ABCD, DCFE and ABFE are parallelograms. Show that ar (ADE) = ar (BCF).

4. In Fig. 11.32, ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersect DC at P, show that ar (BPC) = ar (DPQ).

[*Hint* : Join AC.]

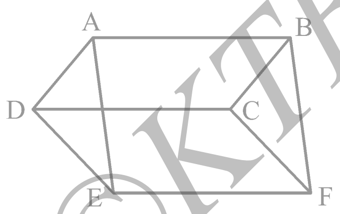

Fig. 11.31

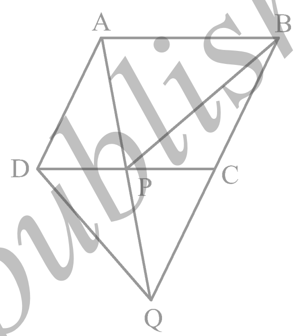

Fig. 11.32

5. In Fig.11.33, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, show that

(i) ar (BDE) = $\frac{1}{4}$ ar (ABC)

(ii) ar (BDE) = $\frac{1}{2}$ ar (BAE)

(iii) ar (ABC) = 2 ar (BEC)

(iv) ar (BFE) = ar (AFD)

(v) ar (BFE) = 2 ar (FED)

(vi) ar (FED) = $\frac{1}{8}$ ar (AFC)

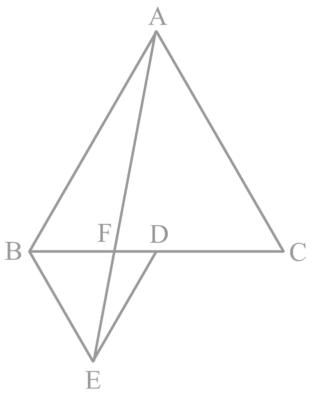

Fig. 11.33

[*Hint* : Join EC and AD. Show that BE || AC and DE || AB, etc.]

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6. Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that

ar (APB) × ar (CPD) = ar (APD) × ar (BPC).

[*Hint* : From A and C, draw perpendiculars to BD.]

7. P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP, show that

(i) ar (PRQ) = $\frac{1}{2}$ ar (ARC) (ii) ar (RQC) = $\frac{3}{8}$ ar (ABC)

(iii) ar (PBQ) = ar (ARC)

8. In Fig. 11.34, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX ⊥ DE meets BC at Y. Show that:

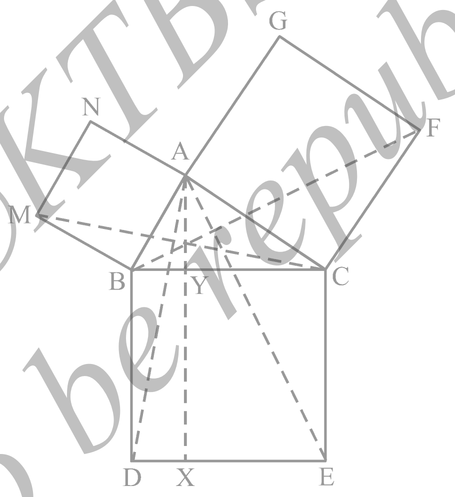

Fig. 11.34

(i) Δ MBC ≅ Δ ABD (ii) ar (BYXD) = 2 ar (MBC)

(iii) ar (BYXD) = ar (ABMN) (iv) Δ FCB ≅ Δ ACE

(v) ar (CYXE) = 2 ar (FCB) (vi) ar (CYXE) = ar (ACFG)

(vii) ar (BCED) = ar (ABMN) + ar (ACFG)

Note : Result (vii) is the famous *Theorem of Pythagoras*. You shall learn a simpler proof of this theorem in Class X.

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11.5 Summary

In this chapter, you have studied the following points :

1. Area of a figure is a number (in some unit) associated with the part of the plane enclosed by that figure.
2. Two congruent figures have equal areas but the converse need not be true.
3. If a planar region formed by a figure T is made up of two non-overlapping planar regions formed by figures P and Q, then ar (T) = ar (P) + ar (Q), where ar (X) denotes the area of figure X.
4. Two figures are said to be on the same base and between the same parallels, if they have a common base (side) and the vertices, (or the vertex) opposite to the common base of each figure lie on a line parallel to the base.
5. Parallelograms on the same base (or equal bases) and between the same parallels are equal in area.
6. Area of a parallelogram is the product of its base and the corresponding altitude.
7. Parallelograms on the same base (or equal bases) and having equal areas lie between the same parallels.
8. If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.
9. Triangles on the same base (or equal bases) and between the same parallels are equal in area.
10. Area of a triangle is half the product of its base and the corresponding altitude.
11. Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.
12. A median of a triangle divides it into two triangles of equal areas.

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CHAPTER 12

CIRCLES

12.1 Introduction

You may have come across many objects in daily life, which are round in shape, such as wheels of a vehicle, bangles, dials of many clocks, coins of denominations 50 p, Re 1 and Rs 5, key rings, buttons of shirts, etc. (see Fig.12.1). In a clock, you might have observed that the second's hand goes round the dial of the clock rapidly and its tip moves in a round path. This path traced by the tip of the second's hand is called a *circle*. In this chapter, you will study about circles, other related terms and some properties of a circle.

Fig. 12.1

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12.2 Circles and Its Related Terms: A Review

Take a compass and fix a pencil in it. Put its pointed leg on a point on a sheet of a paper. Open the other leg to some distance. Keeping the pointed leg on the same point, rotate the other leg through one revolution. What is the closed figure traced by the pencil on paper? As you know, it is a circle (see Fig.12.2). How did you get a circle? You kept one point fixed (A in Fig.12.2) and drew all the points that were at a fixed distance from A. This gives us the following definition:

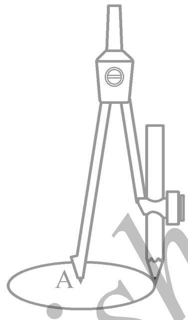

Fig. 12.2

The collection of all the points in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle.

The fixed point is called the *centre* of the circle and the fixed distance is called the *radius* of the circle. In Fig.12.3, O is the centre and the length OP is the radius of the circle.

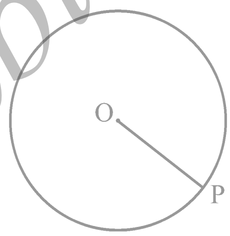

Fig. 12.3

Remark : Note that the line segment joining the centre and any point on the circle is also called a *radius* of the circle. That is, 'radius' is used in two senses-in the sense of a line segment and also in the sense of its length.

You are already familiar with some of the following concepts from Class VI. We are just recalling them.

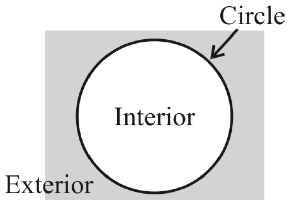

Fig. 12.4

A circle divides the plane on which it lies into three parts. They are: (i) inside the circle, which is also called the *interior* of the circle; (ii) the *circle* and (iii) outside the circle, which is also called the *exterior* of the circle (see Fig.12.4). The circle and its interior make up the *circular region*.

If you take two points P and Q on a circle, then the line segment PQ is called a *chord* of the circle (see Fig. 12.5). The chord, which passes through the centre of the circle, is called a *diameter* of the circle. As in the case of radius, the word 'diameter' is also used in two senses, that is, as a line segment and also as its length. Do you find any other chord of the circle longer than a diameter? No, you see that a *diameter is the longest chord and all diameters have the same length, which is equal to two*

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times the radius. In Fig.12.5, AOB is a diameter of the circle. How many diameters does a circle have? Draw a circle and see how many diameters you can find.

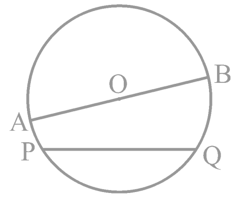

Fig. 12.5

A piece of a circle between two points is called an *arc*. Look at the pieces of the circle between two points P and Q in Fig.12.6. You find that there are two pieces, one longer and the other smaller (see Fig.12.7). The longer one is called the *major arc* PQ and the shorter one is called the *minor arc* PQ. The minor arc PQ is also denoted by $\overset{\frown}{PQ}$ and the major arc PQ by $\overset{\frown}{PRQ}$, where R is some point on the arc between P and Q. Unless otherwise stated, arc PQ or $\overset{\frown}{PQ}$ stands for minor arc PQ. When P and Q are ends of a diameter, then both arcs are equal and each is called a *semicircle*.

Fig. 12.6

The length of the complete circle is called its *circumference*. The region between a chord and either of its arcs is called a *segment* of the circular region or simply a *segment* of the circle. You will find that there are two types of segments also, which are the *major segment* and the *minor segment* (see Fig. 12.8). The region between an arc and the two radii, joining the centre to the end points of the arc is called a *sector*. Like segments, you find that the minor arc corresponds to the *minor sector* and the major arc corresponds to the *major sector*. In Fig. 12.9, the region OPQ is the minor sector and remaining part of the circular region is the major sector. When two arcs are equal, that is, each is a semicircle, then both segments and both sectors become the same and each is known as a *semicircular region*.

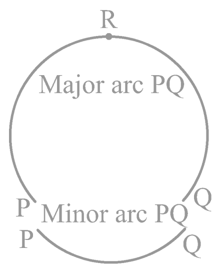

Fig. 12.7

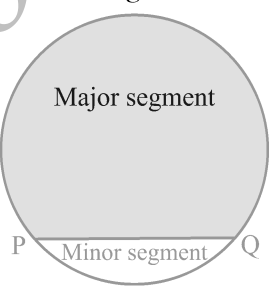

Fig. 12.8

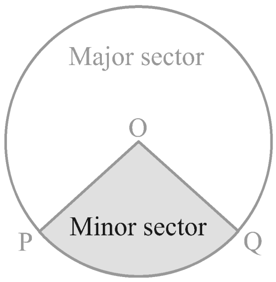

Fig. 12.9

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EXERCISE 12.1

1. Fill in the blanks:

(i) The centre of a circle lies in ____________ of the circle. (exterior/ interior)

(ii) A point, whose distance from the centre of a circle is greater than its radius lies in ____________ of the circle. (exterior/ interior)

(iii) The longest chord of a circle is a ____________ of the circle.

(iv) An arc is a ____________ when its ends are the ends of a diameter.

(v) Segment of a circle is the region between an arc and ____________ of the circle.

(vi) A circle divides the plane, on which it lies, in ____________ parts.

2. Write True or False: Give reasons for your answers.

(i) Line segment joining the centre to any point on the circle is a radius of the circle.

(ii) A circle has only finite number of equal chords.

(iii) If a circle is divided into three equal arcs, each is a major arc.

(iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.

(v) Sector is the region between the chord and its corresponding arc.

(vi) A circle is a plane figure.

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12.3 Angle Subtended by a Chord at a Point

Take a line segment PQ and a point R not on the line containing PQ. Join PR and QR (see Fig. 12.10). Then $\angle$ PRQ is called the angle subtended by the line segment PQ at the point R. What are angles POQ, PRQ and PSQ called in Fig. 12.11? $\angle$ POQ is the angle subtended by the chord PQ at the centre O, $\angle$ PRQ and $\angle$ PSQ are respectively the angles subtended by PQ at points R and S on the major and minor arcs PQ.

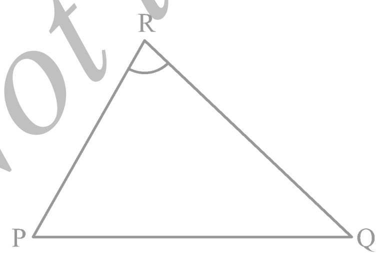

Fig. 12.10

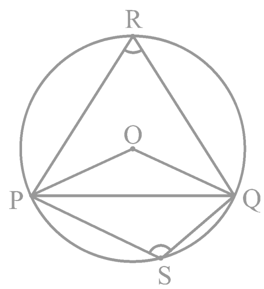

Fig. 12.11

Let us examine the relationship between the size of the chord and the angle subtended by it at the centre. You may see by drawing different chords of a circle and angles subtended by them at the centre that the longer is the chord, the bigger will be the angle subtended by it at the centre. What will happen if you take two equal chords of a circle? Will the angles subtended at the centre be the same or not?

Draw two or more equal chords of a circle and measure the angles subtended by them at the centre (see Fig.12.12). You will find that the angles subtended by them at the centre are equal. Let us give a proof of this fact.

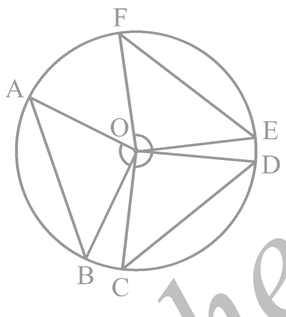

Fig. 12.12

Theorem 12.1 : *Equal chords of a circle subtend equal angles at the centre.*

Proof : You are given two equal chords AB and CD of a circle with centre O (see Fig.12.13). You want to prove that ∠ AOB = ∠ COD.

In triangles AOB and COD,

OA = OC (Radii of a circle)

OB = OD (Radii of a circle)

AB = CD (Given)

Therefore, Δ AOB ≅ Δ COD (SSS rule)

This gives ∠ AOB = ∠ COD

(Corresponding parts of congruent triangles)

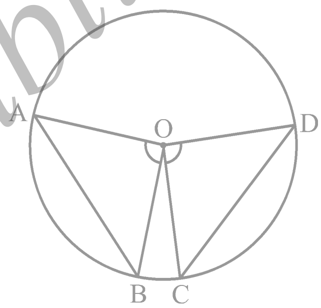

Fig. 12.13

Remark : For convenience, the abbreviation CPCT will be used in place of 'Corresponding parts of congruent triangles', because we use this very frequently as you will see.

Now if two chords of a circle subtend equal angles at the centre, what can you say about the chords? Are they equal or not? Let us examine this by the following activity:

Take a tracing paper and trace a circle on it. Cut it along the circle to get a disc. At its centre O, draw an angle AOB where A, B are points on the circle. Make another angle POQ at the centre equal to ∠AOB. Cut the disc along AB and PQ (see Fig. 12.14). You will get two segments ACB and PRQ of the circle. If you put one on the other, what do you observe? They cover each other, i.e., they are congruent. So AB = PQ.

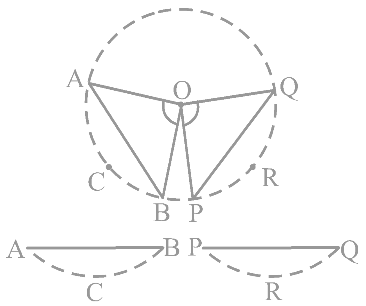

Fig. 12.14

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Though you have seen it for this particular case, try it out for other equal angles too. The chords will all turn out to be equal because of the following theorem:

Theorem 12.2 : *If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.*

The above theorem is the converse of the Theorem 12.1. Note that in Fig. 12.13, if you take $\angle$ AOB = $\angle$ COD, then

Δ AOB $\cong$ Δ COD (Why?)

Can you now see that AB = CD?

EXERCISE 12.2

1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.
2. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

12.4 Perpendicular from the Centre to a Chord

Activity : Draw a circle on a tracing paper. Let O be its centre. Draw a chord AB. Fold the paper along a line through O so that a portion of the chord falls on the other. Let the crease cut AB at the point M. Then, $\angle$ OMA = $\angle$ OMB = 90° or OM is perpendicular to AB. Does the point B coincide with A (see Fig.12.15)?

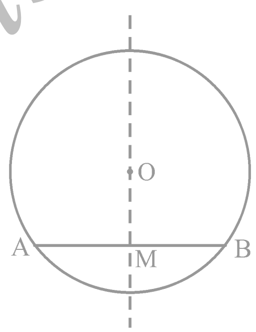

Fig. 12.15

Yes it will. So MA = MB.

Give a proof yourself by joining OA and OB and proving the right triangles OMA and OMB to be congruent. This example is a particular instance of the following result:

Theorem 12.3 : *The perpendicular from the centre of a circle to a chord bisects the chord.*

What is the converse of this theorem? To write this, first let us be clear what is assumed in Theorem 12.3 and what is proved. Given that the perpendicular from the centre of a circle to a chord is drawn and to prove that it bisects the chord. Thus in the converse, what the hypothesis is 'if a line from the centre bisects a chord of a circle' and what is to be proved is 'the line is perpendicular to the chord'. So the converse is:

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Theorem 12.4 : *The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.*

Is this true? Try it for few cases and see. You will see that it is true for these cases. See if it is true, in general, by doing the following exercise. We will write the stages and you give the reasons.

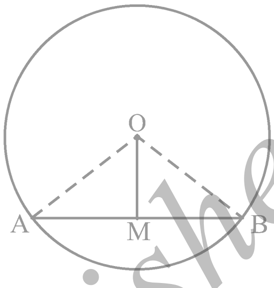

Fig. 12.16

Let AB be a chord of a circle with centre O and O is joined to the mid-point M of AB. You have to prove that OM ⊥ AB. Join OA and OB (see Fig. 12.16). In triangles OAM and OBM,

OA = OB (Why ?)

AM = BM (Why ?)

OM = OM (Common)

Therefore, $\Delta OAM \cong \Delta OBM$ (How ?)

This gives $\angle OMA = \angle OMB = 90°$ (Why ?)

12.5 Circle through Three Points

You have learnt in Chapter 6, that two points are sufficient to determine a line. That is, there is one and only one line passing through two points. A natural question arises. How many points are sufficient to determine a circle?

Take a point P. How many circles can be drawn through this point? You see that there may be as many circles as you like passing through this point [see Fig. 12.17(i)]. Now take two points P and Q. You again see that there may be an infinite number of circles passing through P and Q [see Fig.12.17(ii)]. What will happen when you take three points A, B and C? Can you draw a circle passing through three collinear points?

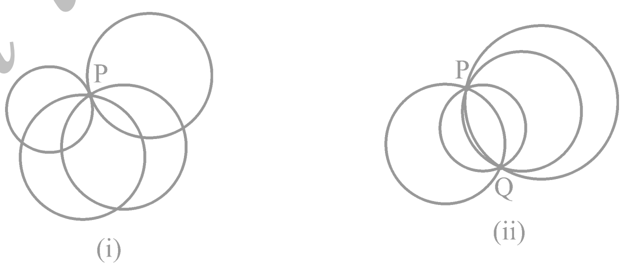

Fig. 12. 17

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No. If the points lie on a line, then the third point will lie inside or outside the circle passing through two points (see Fig 12.18).

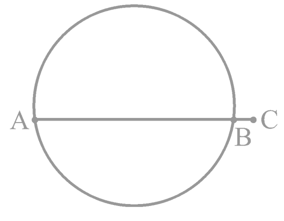

Fig. 12.18

So, let us take three points A, B and C, which are not on the same line, or in other words, they are not collinear [see Fig. 12.19(i)]. Draw perpendicular bisectors of AB and BC say, PQ and RS respectively. Let these perpendicular bisectors intersect at one point O. (Note that PQ and RS will intersect because they are not parallel) [see Fig. 12.19(ii)].

Fig. 12.19

Now O lies on the perpendicular bisector PQ of AB, you have OA = OB, as every point on the perpendicular bisector of a line segment is equidistant from its end points, proved in Chapter 7.

Similarly, as O lies on the perpendicular bisector RS of BC, you get

$$OB = OC$$

So OA = OB = OC, which means that the points A, B and C are at equal distances from the point O. So if you draw a circle with centre O and radius OA, it will also pass through B and C. This shows that there is a circle passing through the three points A, B and C. You know that two lines (perpendicular bisectors) can intersect at only one point, so you can draw only one circle with radius OA. In other words, there is a unique circle passing through A, B and C. You have now proved the following theorem:

Theorem 12.5 : *There is one and only one circle passing through three given non-collinear points.*

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Remark : If ABC is a triangle, then by Theorem 12.5, there is a unique circle passing through the three vertices A, B and C of the triangle. This circle is called the *circumcircle* of the Δ ABC. Its centre and radius are called respectively the *circumcentre* and the *circumradius* of the triangle.

Example 1 : Given an arc of a circle, complete the circle.

Solution : Let arc PQ of a circle be given. We have to complete the circle, which means that we have to find its centre and radius. Take a point R on the arc. Join PR and RQ. Use the construction that has been used in proving Theorem 12.5, to find the centre and radius.

Taking the centre and the radius so obtained, we can complete the circle (see Fig. 12.20).

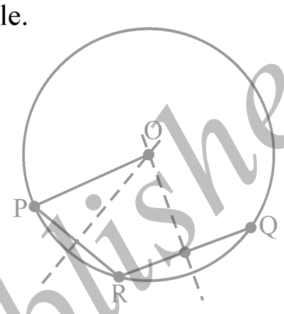

Fig. 12.20

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EXERCISE 12.3

1. Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?
2. Suppose you are given a circle. Give a construction to find its centre.
3. If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.

12.6 Equal Chords and Their Distances from the Centre

Let AB be a line and P be a point. Since there are infinite numbers of points on a line, if you join these points to P, you will get infinitely many line segments PL_1, PL_2, PM, PL_3, PL_4, etc. Which of these is the distance of AB from P? You may think a while and get the answer. Out of these line segments, the perpendicular from P to AB, namely PM in Fig. 12.21, will be the least. In Mathematics, we define this least length PM to be **the distance of AB from P**. So you may say that:

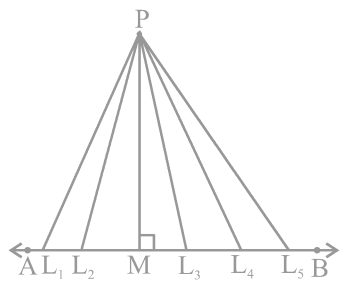

Fig. 12.21

The length of the perpendicular from a point to a line is the distance of the line from the point.

Note that if the point lies on the line, the distance of the line from the point is zero.

A circle can have infinitely many chords. You may observe by drawing chords of a circle that longer chord is nearer to the centre than the smaller chord. You may observe it by drawing several chords of a circle of different lengths and measuring their distances from the centre. What is the distance of the diameter, which is the longest chord from the centre? Since the centre lies on it, the distance is zero. Do you think that there is some relationship between the length of chords and their distances from the centre? Let us see if this is so.

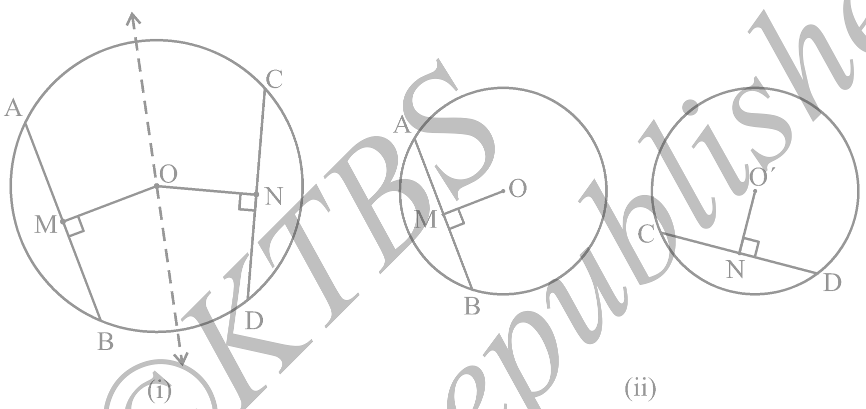

Fig. 12.22

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Activity : Draw a circle of any radius on a tracing paper. Draw two equal chords AB and CD of it and also the perpendiculars OM and ON on them from the centre O. Fold the figure so that D falls on B and C falls on A [see Fig.12.22 (i)]. You may observe that O lies on the crease and N falls on M. Therefore, OM = ON. Repeat the activity by drawing congruent circles with centres O and O′ and taking equal chords AB and CD one on each. Draw perpendiculars OM and O′N on them [see Fig. 12.22(ii)]. Cut one circular disc and put it on the other so that AB coincides with CD. Then you will find that O coincides with O′ and M coincides with N. In this way you verified the following:

Theorem 12.6 : *Equal chords of a circle* (*or of congruent circles*) *are equidistant from the centre* (*or centres*)*.*

Next, it will be seen whether the converse of this theorem is true or not. For this, draw a circle with centre O. From the centre O, draw two line segments OL and OM of equal length and lying inside the circle [see Fig. 12.23(i)]. Then draw chords PQ and RS of the circle perpendicular to OL and OM respectively [see Fig 12.23(ii)]. Measure the lengths of PQ and RS. Are these different? No, both are equal. Repeat the activity for more equal line segments and drawing the chords perpendicular to

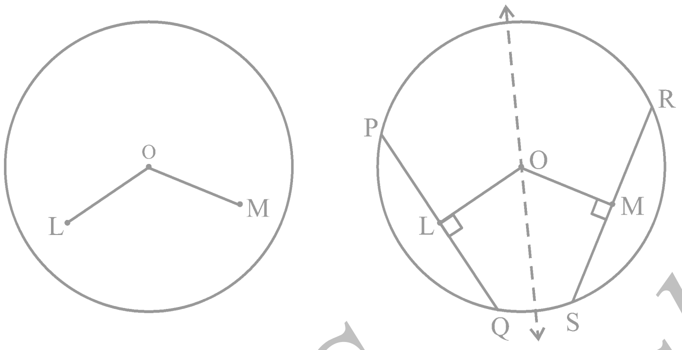

Fig. 12.23

them. This verifies the converse of the Theorem 12.6 which is stated as follows:

Theorem 12.7 : *Chords equidistant from the centre of a circle are equal in length.*

We now take an example to illustrate the use of the above results:

Example 2 : If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection, prove that the chords are equal.

Solution : Given that AB and CD are two chords of a circle, with centre O intersecting at a point E. PQ is a diameter through E, such that $\angle$ AEQ = $\angle$ DEQ (see Fig.12.24). You have to prove that AB = CD. Draw perpendiculars OL and OM on chords AB and CD, respectively. Now

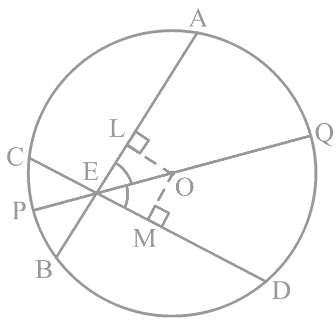

Fig. 12.24

$\angle$ LOE = 180° – 90° – $\angle$ LEO = 90° – $\angle$ LEO
(Angle sum property of a triangle)

= 90° – $\angle$ AEQ = 90° – $\angle$ DEQ

= 90° – $\angle$ MEO = $\angle$ MOE

In triangles OLE and OME,

	$\angle$ LEO = $\angle$ MEO	(Why ?)
	$\angle$ LOE = $\angle$ MOE	(Proved above)
	EO = EO	(Common)
Therefore,	Δ OLE ≅ Δ OME	(Why ?)
This gives	OL = OM	(CPCT)
So,	AB = CD	(Why ?)

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EXERCISE 12.4

1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.
2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.
3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.
4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see Fig. 12.25).
5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip?

Fig. 12.25

6. A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

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12.7 Angle Subtended by an Arc of a Circle

You have seen that the end points of a chord other than diameter of a circle cuts it into two arcs – one major and other minor. If you take two equal chords, what can you say about the size of arcs? Is one arc made by first chord equal to the corresponding arc made by another chord? In fact, they are more than just equal in length. They are congruent in the sense that if one arc is put on the other, without bending or twisting, one superimposes the other completely.

You can verify this fact by cutting the arc, corresponding to the chord CD from the circle along CD and put it on the corresponding arc made by equal chord AB. You will find that the arc CD superimpose the arc AB completely (see Fig. 12.26). This shows that equal chords make congruent arcs and conversely congruent arcs make equal chords of a circle. You can state it as follows:

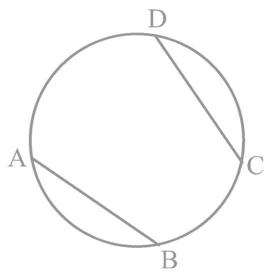

Fig. 12.26

If two chords of a circle are equal, then their corresponding arcs are congruent and conversely, if two arcs are congruent, then their corresponding chords are equal.

Also the angle subtended by an arc at the centre is defined to be angle subtended by the corresponding chord at the centre in the sense that the minor arc subtends the angle and the major arc subtends the reflex angle. Therefore, in Fig 12.27, the angle subtended by the minor arc PQ at O is ∠POQ and the angle subtended by the major arc PQ at O is reflex angle POQ.

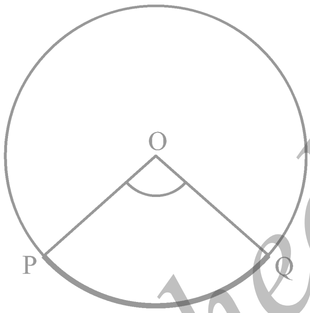

Fig. 12.27

In view of the property above and Theorem 12.1, the following result is true:

Congruent arcs (or equal arcs) of a circle subtend equal angles at the centre.

Therefore, the angle subtended by a chord of a circle at its centre is equal to the angle subtended by the corresponding (minor) arc at the centre. The following theorem gives the relationship between the angles subtended by an arc at the centre and at a point on the circle.

Theorem 12.8 : *The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.*

Proof : Given an arc PQ of a circle subtending angles POQ at the centre O and PAQ at a point A on the remaining part of the circle. We need to prove that ∠ POQ = 2 ∠ PAQ.

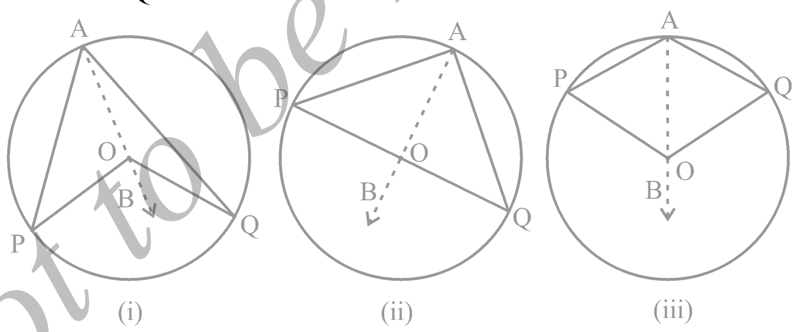

Fig. 12.28

Consider the three different cases as given in Fig. 12.28. In (i), arc PQ is minor; in (ii), arc PQ is a semicircle and in (iii), arc PQ is major.

Let us begin by joining AO and extending it to a point B.

In all the cases,

$$\angle BOQ = \angle OAQ + \angle AQO$$

because an exterior angle of a triangle is equal to the sum of the two interior opposite angles.

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Also in Δ OAQ,

$$OA = OQ \quad \text{(Radii of a circle)}$$

Therefore, $\angle OAQ = \angle OQA$ (Theorem 5.5)

This gives $\angle BOQ = 2 \angle OAQ$ (1)

Similarly, $\angle BOP = 2 \angle OAP$ (2)

From (1) and (2), $\angle BOP + \angle BOQ = 2(\angle OAP + \angle OAQ)$

This is the same as $\angle POQ = 2 \angle PAQ$ (3)

For the case (iii), where PQ is the major arc, (3) is replaced by

$$\text{reflex angle } POQ = 2 \angle PAQ$$

Remark : Suppose we join points P and Q and form a chord PQ in the above figures. Then $\angle PAQ$ is also called the angle formed in the segment PAQP.

In Theorem 12.8, A can be any point on the remaining part of the circle. So if you take any other point C on the remaining part of the circle (see Fig. 12.29), you have

$$\angle POQ = 2 \angle PCQ = 2 \angle PAQ$$

Therefore, $\angle PCQ = \angle PAQ$.

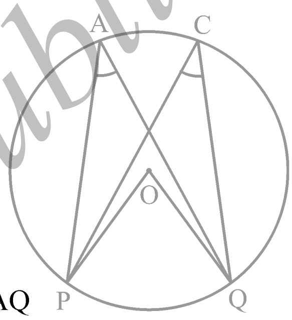

Fig. 12.29

This proves the following:

Theorem 12.9 : *Angles in the same segment of a circle are equal.*

Again let us discuss the case (ii) of Theorem 12.8 separately. Here $\angle PAQ$ is an angle in the segment, which is a semicircle. Also, $\angle PAQ = \frac{1}{2} \angle POQ = \frac{1}{2} \times 180° = 90°$.
If you take any other point C on the semicircle, again you get that

$$\angle PCQ = 90°$$

Therefore, you find another property of the circle as:

Angle in a semicircle is a right angle.

The converse of Theorem 12.9 is also true. It can be stated as:

Theorem 12.10 : *If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle (i.e. they are concyclic).*

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You can see the truth of this result as follows:

In Fig. 12.30, AB is a line segment, which subtends equal angles at two points C and D. That is

$$\angle ACB = \angle ADB$$

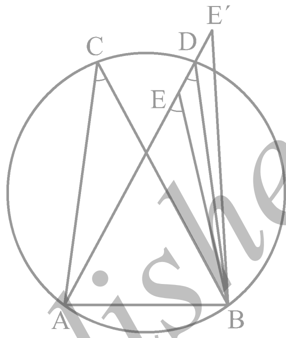

Fig. 12.30

To show that the points A, B, C and D lie on a circle let us draw a circle through the points A, C and B. Suppose it does not pass through the point D. Then it will intersect AD (or extended AD) at a point, say E (or E′).

If points A, C, E and B lie on a circle,

$\angle ACB = \angle AEB$ (Why?)

But it is given that $\angle ACB = \angle ADB$.

Therefore, $\angle AEB = \angle ADB$.

This is not possible unless E coincides with D. (Why?)

Similarly, E′ should also coincide with D.

12.8 Cyclic Quadrilaterals

A quadrilateral ABCD is called *cyclic* if all the four vertices of it lie on a circle (see Fig 12.31). You will find a peculiar property in such quadrilaterals. Draw several cyclic quadrilaterals of different sides and name each of these as ABCD. (This can be done by drawing several circles of different radii and taking four points on each of them.) Measure the opposite angles and write your observations in the following table.

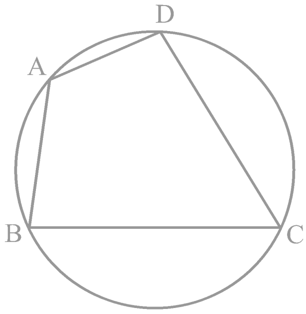

Fig. 12.31

S.No. of Quadrilateral	$\angle A$	$\angle B$	$\angle C$	$\angle D$	$\angle A + \angle C$	$\angle B + \angle D$
1.						
2.						
3.						
4.						
5.						
6.						

What do you infer from the table?

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You find that $\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$, neglecting the error in measurements. This verifies the following:

Theorem 12.11 : *The sum of either pair of opposite angles of a cyclic quadrilateral is* 180°.

In fact, the converse of this theorem, which is stated below is also true.

Theorem 12.12 : *If the sum of a pair of opposite angles of a quadrilateral is* 180°, *the quadrilateral is cyclic.*

You can see the truth of this theorem by following a method similar to the method adopted for Theorem 12.10.

Example 3 : In Fig. 12.32, AB is a diameter of the circle, CD is a chord equal to the radius of the circle. AC and BD when extended intersect at a point E. Prove that $\angle AEB = 60°$.

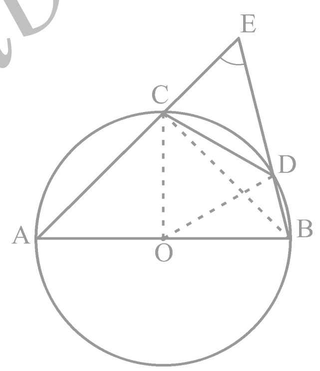

Fig. 12.32

Solution : Join OC, OD and BC.

Triangle ODC is equilateral (Why?)

Therefore, $\angle COD = 60°$

Now, $\angle CBD = \frac{1}{2} \angle COD$ (Theorem 10.8)

This gives $\angle CBD = 30°$

Again, $\angle ACB = 90°$ (Why ?)

So, $\angle BCE = 180° - \angle ACB = 90°$

Which gives $\angle CEB = 90° - 30° = 60°$, i.e., $\angle AEB = 60°$

Example 4 : In Fig 12.33, ABCD is a cyclic quadrilateral in which AC and BD are its diagonals. If $\angle DBC = 55°$ and $\angle BAC = 45°$, find $\angle BCD$.

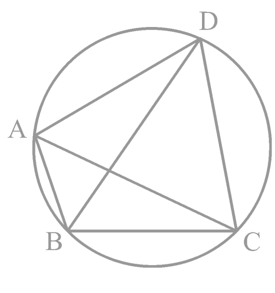

Fig. 12.33

Solution : $\angle CAD = \angle DBC = 55°$

(Angles in the same segment)

Therefore, $\angle DAB = \angle CAD + \angle BAC$

$= 55° + 45° = 100°$

But $\angle DAB + \angle BCD = 180°$

(Opposite angles of a cyclic quadrilateral)

So, $\angle BCD = 180° - 100° = 80°$

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Example 5 : Two circles intersect at two points A and B. AD and AC are diameters to the two circles (see Fig.12.34). Prove that B lies on the line segment DC.

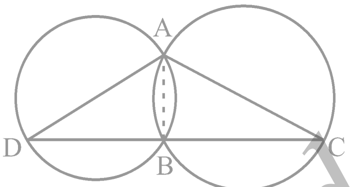

Fig. 12.34

Solution : Join AB.

$\angle$ ABD = 90° (Angle in a semicircle)

$\angle$ ABC = 90° (Angle in a semicircle)

So, $\angle$ ABD + $\angle$ ABC = 90° + 90° = 180°

Therefore, DBC is a line. That is B lies on the line segment DC.

Example 6 : Prove that the quadrilateral formed (if possible) by the internal angle bisectors of any quadrilateral is cyclic.

Solution : In Fig. 12.35, ABCD is a quadrilateral in which the angle bisectors AH, BF, CF and DH of internal angles A, B, C and D respectively form a quadrilateral EFGH.

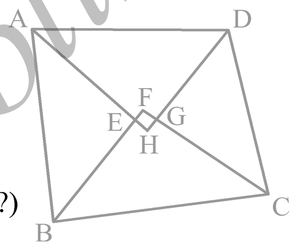

Fig. 12.35

Now, $\angle$ FEH = $\angle$ AEB = 180° – $\angle$ EAB – $\angle$ EBA (Why ?)

$$= 180° - \frac{1}{2}(\angle A + \angle B)$$

and $\angle$ FGH = $\angle$ CGD = 180° – $\angle$ GCD – $\angle$ GDC (Why ?)

$$= 180° - \frac{1}{2}(\angle C + \angle D)$$

Therefore, $\angle FEH + \angle FGH = 180° - \frac{1}{2}(\angle A + \angle B) + 180° - \frac{1}{2}(\angle C + \angle D)$

$$= 360° - \frac{1}{2}(\angle A + \angle B + \angle C + \angle D) = 360° - \frac{1}{2} \times 360°$$

$$= 360° - 180° = 180°$$

Therefore, by Theorem 12.12, the quadrilateral EFGH is cyclic.

EXERCISE 12.5

1. In Fig. 12.36, A,B and C are three points on a circle with centre O such that $\angle$ BOC = 30° and $\angle$ AOB = 60°. If D is a point on the circle other than the arc ABC, find $\angle$ADC.

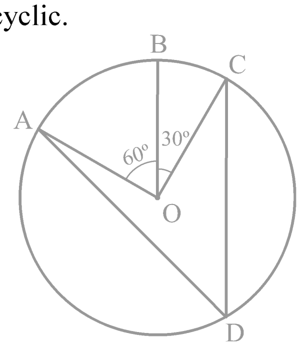

Fig. 12.36

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2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

3. In Fig. 12.37, ∠ PQR = 100°, where P, Q and R are points on a circle with centre O. Find ∠ OPR.

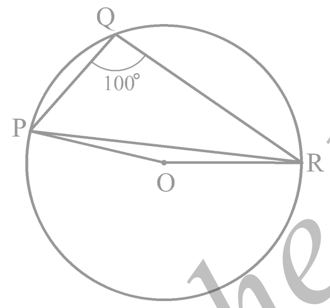

Fig. 12.37

4. In Fig. 12.38, ∠ ABC = 69°, ∠ ACB = 31°, find ∠ BDC.

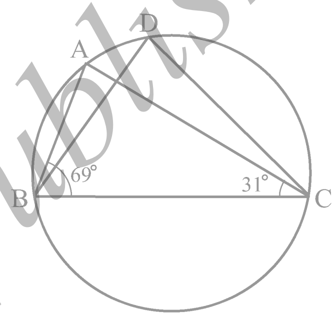

Fig. 12.38

5. In Fig. 12.39, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠ BEC = 130° and ∠ ECD = 20°. Find ∠ BAC.

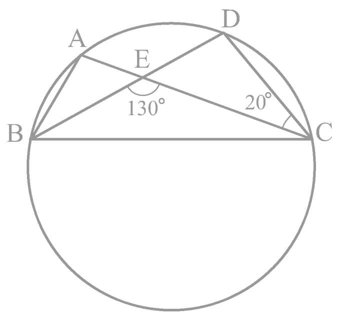

Fig. 12.39

6. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If ∠ DBC = 70°, ∠ BAC is 30°, find ∠ BCD. Further, if AB = BC, find ∠ ECD.

7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

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9. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Fig. 12.40). Prove that $\angle ACP = \angle QCD$.

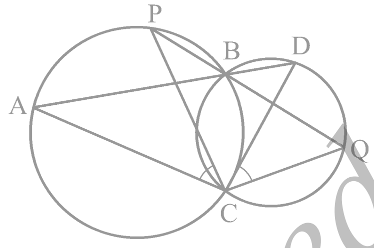

Fig. 12.40

10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

11. ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

12. Prove that a cyclic parallelogram is a rectangle.

EXERCISE 12.6 (Optional)*

1. Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.

2. Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

3. The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre?

4. Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that $\angle ABC$ is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

5. Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.

6. ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that AE = AD.

7. AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters, (ii) ABCD is a rectangle.

8. Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are $90° - \frac{1}{2}A$, $90° - \frac{1}{2}B$ and $90° - \frac{1}{2}C$.

*These exercises are not from examination point of view.

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9. Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

10. In any triangle ABC, if the angle bisector of $\angle$A and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.

12.9 Summary

In this chapter, you have studied the following points:

1. A circle is the collection of all points in a plane, which are equidistant from a fixed point in the plane.

2. Equal chords of a circle (or of congruent circles) subtend equal angles at the centre.

3. If the angles subtended by two chords of a circle (or of congruent circles) at the centre (corresponding centres) are equal, the chords are equal.

4. The perpendicular from the centre of a circle to a chord bisects the chord.

5. The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

6. There is one and only one circle passing through three non-collinear points.

7. Equal chords of a circle (or of congruent circles) are equidistant from the centre (or corresponding centres).

8. Chords equidistant from the centre (or corresponding centres) of a circle (or of congruent circles) are equal.

9. If two arcs of a circle are congruent, then their corresponding chords are equal and conversely if two chords of a circle are equal, then their corresponding arcs (minor, major) are congruent.

10. Congruent arcs of a circle subtend equal angles at the centre.

11. The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

12. Angles in the same segment of a circle are equal.

13. Angle in a semicircle is a right angle.

14. If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle.

15. The sum of either pair of opposite angles of a cyclic quadrilateral is 180^0.

16. If sum of a pair of opposite angles of a quadrilateral is 180^0, the quadrilateral is cyclic.

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CHAPTER 13

SURFACE AREAS AND VOLUMES

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13.1 Introduction

Wherever we look, usually we see solids. So far, in all our study, we have been dealing with figures that can be easily drawn on our notebooks or blackboards. These are called *plane figures*. We have understood what rectangles, squares and circles are, what we mean by their perimeters and areas, and how we can find them. We have learnt these in earlier classes. It would be interesting to see what happens if we cut out many of these plane figures of the same shape and size from cardboard sheet and stack them up in a vertical pile. By this process, we shall obtain some *solid figures* (briefly called *solids*) such as a cuboid, a cylinder, etc. In the earlier classes, you have also learnt to find the surface areas and volumes of cuboids, cubes and cylinders. We shall now learn to find the surface areas and volumes of cuboids and cylinders in details and extend this study to some other solids such as cones and spheres.

13.2 Surface Area of a Cuboid and a Cube

Have you looked at a bundle of many sheets of paper? How does it look? Does it look like what you see in Fig. 13.1?

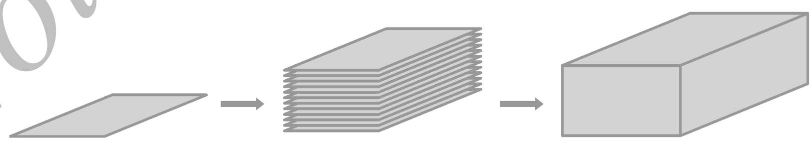
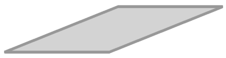


Fig. 13.1

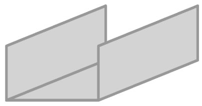
That makes up a cuboid. How much of brown paper would you need, if you want to cover this cuboid? Let us see:

First we would need a rectangular piece to cover the bottom of the bundle. That would be as shown in Fig. 13.2 (a)



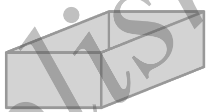
(a)

Then we would need two long rectangular pieces to cover the two side ends. Now, it would look like Fig. 13.2 (b).



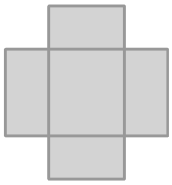
(b)

Now to cover the front and back ends, we would need two more rectangular pieces of a different size. With them, we would now have a figure as shown in Fig. 13.2(c).



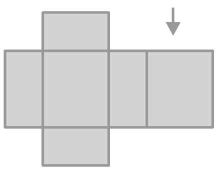
(c)

This figure, when opened out, would look like Fig. 13.2 (d).



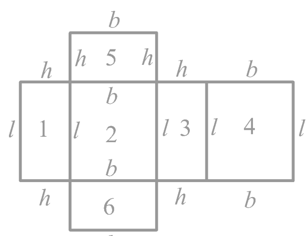
(d)

Finally, to cover the top of the bundle, we would require another rectangular piece exactly like the one at the bottom, which if we attach on the right side, it would look like Fig. 13.2(e).



(e)

So we have used six rectangular pieces to cover the complete outer surface of the cuboid.



(f)

Fig. 13.2

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This shows us that the outer surface of a cuboid is made up of six rectangles (in fact, rectangular regions, called the faces of the cuboid), whose areas can be found by multiplying length by breadth for each of them separately and then adding the six areas together.

Now, if we take the length of the cuboid as l, breadth as b and the height as h, then the figure with these dimensions would be like the shape you see in Fig. 13.2(f).

So, the sum of the areas of the six rectangles is:

Area of rectangle 1 $(= l \times h)$

$+$

Area of rectangle 2 $(= l \times b)$

$+$

Area of rectangle 3 $(= l \times h)$

$+$

Area of rectangle 4 $(= l \times b)$

$+$

Area of rectangle 5 $(= b \times h)$

$+$

Area of rectangle 6 $(= b \times h)$

$= 2(l \times b) + 2(b \times h) + 2(l \times h)$

$= 2(lb + bh + hl)$

This gives us:

Surface Area of a Cuboid = 2(*lb* + *bh* + *hl*)

where l, b and h are respectively the three edges of the cuboid.

Note : The unit of area is taken as the square unit, because we measure the magnitude of a region by filling it with squares of side of unit length.

For example, if we have a cuboid whose length, breadth and height are 15 cm, 10 cm and 20 cm respectively, then its surface area would be:

$$2[(15 \times 10) + (10 \times 20) + (20 \times 15)] \text{ cm}^2$$
$$= 2(150 + 200 + 300) \text{ cm}^2$$
$$= 2 \times 650 \text{ cm}^2$$
$$= 1300 \text{ cm}^2$$

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Recall that a cuboid, whose length, breadth and height are all equal, is called a *cube*. If each edge of the cube is a, then the surface area of this cube would be

$$2(a \times a + a \times a + a \times a), \text{ i.e., } 6a^2 \text{ (see Fig. 13.3), giving us}$$

Surface Area of a Cube = $6a^2$

where a is the edge of the cube.

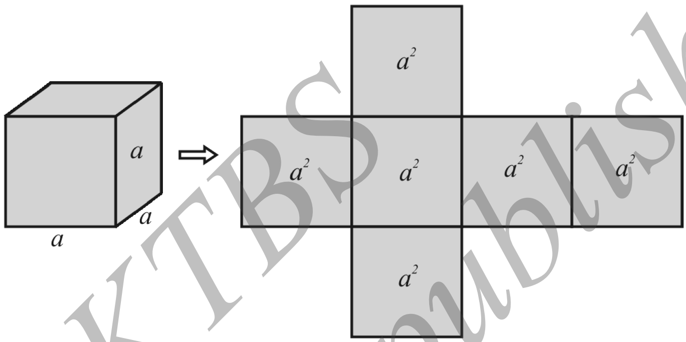

Fig. 13.3

Suppose, out of the six faces of a cuboid, we only find the area of the four faces, leaving the bottom and top faces. In such a case, the area of these four faces is called the **lateral surface area** of the cuboid. So, *lateral surface area of a cuboid of length l, breadth b and height h is equal to* $2lh + 2bh$ *or* $2(l + b)h$. Similarly, *lateral surface area of a cube of side a is equal to* $4a^2$.

Keeping in view of the above, the surface area of a cuboid (or a cube) is sometimes also referred to as the **total surface area**. Let us now solve some examples.

Example 1 : Mary wants to decorate her Christmas tree. She wants to place the tree on a wooden box covered with coloured paper with picture of Santa Claus on it (see Fig. 13.4). She must know the exact quantity of paper to buy for this purpose. If the box has length, breadth and height as 80 cm, 40 cm and 20 cm respectively how many square sheets of paper of side 40 cm would she require?

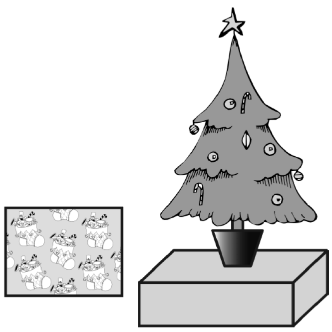

Fig. 13.4

Solution : Since Mary wants to paste the paper on the outer surface of the box; the quantity of paper required would be equal to the surface area of the box which is of the shape of a cuboid. The dimensions of the box are:

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Length =80 cm, Breadth = 40 cm, Height = 20 cm.

The surface area of the box = $2(lb + bh + hl)$

$= 2[(80 \times 40) + (40 \times 20) + (20 \times 80)]$ cm^2

$= 2[3200 + 800 + 1600]$ cm^2

$= 2 \times 5600$ cm^2 = 11200 cm^2

The area of each sheet of the paper = 40×40 cm^2

= 1600 cm^2

Therefore, number of sheets required $= \dfrac{\text{surface area of box}}{\text{area of one sheet of paper}}$

$= \dfrac{11200}{1600} = 7$

So, she would require 7 sheets.

Example 2 : Hameed has built a cubical water tank with lid for his house, with each outer edge 1.5 m long. He gets the outer surface of the tank excluding the base, covered with square tiles of side 25 cm (see Fig. 13.5). Find how much he would spend for the tiles, if the cost of the tiles is ₹ 360 per dozen.

Solution : Since Hameed is getting the five outer faces of the tank covered with tiles, he would need to know the surface area of the tank, to decide on the number of tiles required.

Edge of the cubical tank = 1.5 m = 150 cm (= a)

So, surface area of the tank = $5 \times 150 \times 150$ cm^2

Area of each square tile = side × side = 25×25 cm^2

So, the number of tiles required $= \dfrac{\text{surface area of the tank}}{\text{area of each tile}}$

$= \dfrac{5 \times 150 \times 150}{25 \times 25} = 180$

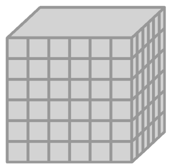

Fig. 13.5

Cost of 1 dozen tiles, i.e., cost of 12 tiles = ₹ 360

Therefore, cost of one tile = ₹ $\dfrac{360}{12}$ = ₹ 30

So, the cost of 180 tiles = 180 × ₹ 30 = ₹ 5400

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EXERCISE 13.1

1. A plastic box 1.5 m long, 1.25 m wide and 65 cm deep is to be made. It is opened at the top. Ignoring the thickness of the plastic sheet, determine:
   (i) The area of the sheet required for making the box.
   (ii) The cost of sheet for it, if a sheet measuring $1m^2$ costs Rs 20.
2. The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of ₹ 7.50 per m^2.
3. The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of ₹ 10 per m^2 is ₹ 15000, find the height of the hall.
   [**Hint :** Area of the four walls = Lateral surface area.]
4. The paint in a certain container is sufficient to paint an area equal to 9.375 m^2. How many bricks of dimensions 22.5 cm × 10 cm × 7.5 cm can be painted out of this container?
5. A cubical box has each edge 10 cm and another cuboidal box is 12.5 cm long, 10 cm wide and 8 cm high.
   (i) Which box has the greater lateral surface area and by how much?
   (ii) Which box has the smaller total surface area and by how much?
6. A small indoor greenhouse (herbarium) is made entirely of glass panes (including base) held together with tape. It is 30 cm long, 25 cm wide and 25 cm high.
   (i) What is the area of the glass?
   (ii) How much of tape is needed for all the 12 edges?
7. Shanti Sweets Stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm × 20 cm × 5 cm and the smaller of dimensions 15 cm × 12 cm × 5 cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is ₹ 4 for 1000 cm^2, find the cost of cardboard required for supplying 250 boxes of each kind.
8. Parveen wanted to make a temporary shelter for her car, by making a box-like structure with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small, and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5 m, with base dimensions 4 m × 3 m?

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13.3 Surface Area of a Right Circular Cylinder

If we take a number of circular sheets of paper and stack them up as we stacked up rectangular sheets earlier, what would we get (see Fig. 13.6)?

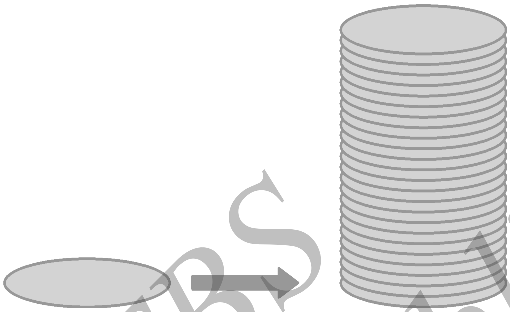

Fig. 13.6

Here, if the stack is kept vertically up, we get what is called a *right circular cylinder*, since it has been kept at right angles to the base, and the base is circular. Let us see what kind of cylinder is *not* a right circular cylinder.

In Fig 13.7 (a), you see a cylinder, which is certainly circular, but it is not at right angles to the base. So, we can *not* say this a *right* circular cylinder.

Of course, if we have a cylinder with a non circular base, as you see in Fig. 13.7 (b), then we also cannot call it a right circular cylinder.

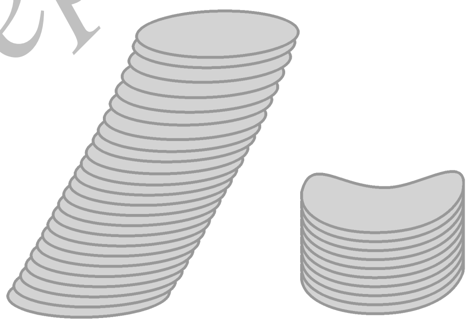

Fig. 13.7

Remark : Here, we will be dealing with only right circular cylinders. So, unless stated otherwise, the word cylinder would mean a right circular cylinder.

Now, if a cylinder is to be covered with coloured paper, how will we do it with the minimum amount of paper? First take a rectangular sheet of paper, whose length is just enough to go round the cylinder and whose breadth is equal to the height of the cylinder as shown in Fig. 13.8.

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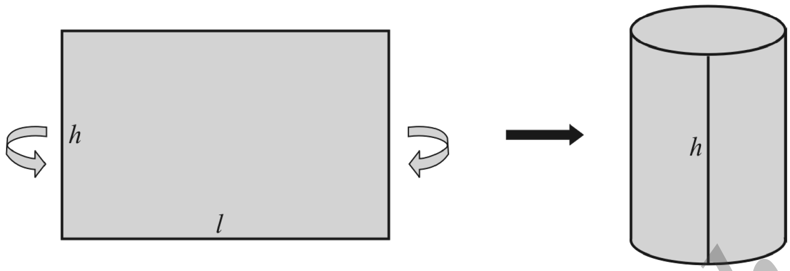

Fig. 13.8

The area of the sheet gives us the curved surface area of the cylinder. Note that the length of the sheet is equal to the circumference of the circular base which is equal to $2\pi r$.

So, curved surface area of the cylinder

= area of the rectangular sheet = length × breadth

= perimeter of the base of the cylinder × h

= $2\pi r \times h$

Therefore, **Curved Surface Area of a Cylinder = $2\pi rh$**

where r is the radius of the base of the cylinder and h is the height of the cylinder.

Remark : In the case of a cylinder, unless stated otherwise, 'radius of a cylinder' shall mean' base radius of the cylinder'.

If the top and the bottom of the cylinder are also to be covered, then we need two circles (infact, circular regions) to do that, each of radius r, and thus having an area of πr^2 each (see Fig. 13.9), giving us the total surface area as $2\pi rh + 2\pi r^2 = 2\pi r(r + h)$.

So, **Total Surface Area of a Cylinder = $2\pi r(r + h)$**

where h is the height of the cylinder and r its radius.

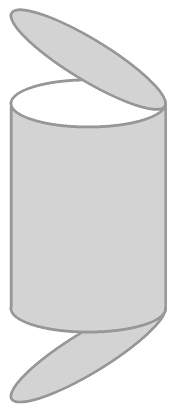

Fig. 13.9

Remark : You may recall from Chapter 1 that π is an irrational number. So, the value

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of π is a non-terminating, non-repeating decimal. But when we use its value in our calculations, we usually take its value as approximately equal to $\frac{22}{7}$ or 3.14.

Example 3 : Savitri had to make a model of a cylindrical kaleidoscope for her science project. She wanted to use chart paper to make the curved surface of the kaleidoscope. (see Fig 13.10). What would be the area of chart paper required by her, if she wanted to make a kaleidoscope of length 25 cm with a 3.5 cm radius? You may take $\pi = \frac{22}{7}$.

Solution : Radius of the base of the cylindrical kaleidoscope (r) = 3.5 cm.

Height (length) of kaleidoscope (h) = 25 cm.

Area of chart paper required = curved surface area of the kaleidoscope

$$= 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 3.5 \times 25 \text{ cm}^2$$

$$= 550 \text{ cm}^2$$

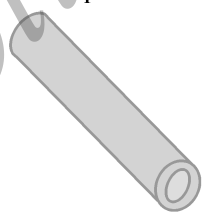

Fig. 13.10

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EXERCISE 13.2

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

1. The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the base of the cylinder.
2. It is required to make a closed cylindrical tank of height 1 m and base diameter 140 cm from a metal sheet. How many square metres of the sheet are required for the same?
3. A metal pipe is 77 cm long. The inner diameter of a cross section is 4 cm, the outer diameter being 4.4 cm (see Fig. 13.11). Find its
   (i) inner curved surface area,
   (ii) outer curved surface area,
   (iii) total surface area.

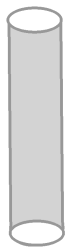

Fig. 13.11

4. The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground in m^2.

5. A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of ₹ 12.50 per m^2.

6. Curved surface area of a right circular cylinder is 4.4 m^2. If the radius of the base of the cylinder is 0.7 m, find its height.

7. The inner diameter of a circular well is 3.5 m. It is 10 m deep. Find

(i) its inner curved surface area,

(ii) the cost of plastering this curved surface at the rate of ₹ 40 per m^2.

8. In a hot water heating system, there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system.

9. Find

(i) the lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in diameter and 4.5 m high.

(ii) how much steel was actually used, if $\frac{1}{12}$ of the steel actually used was wasted in making the tank.

10. In Fig. 13.12, you see the frame of a lampshade. It is to be covered with a decorative cloth. The frame has a base diameter of 20 cm and height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required for covering the lampshade.

Fig. 13.12

11. The students of a Vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder with a base, using cardboard. Each penholder was to be of radius 3 cm and height 10.5 cm. The Vidyalaya was to supply the competitors with cardboard. If there were 35 competitors, how much cardboard was required to be bought for the competition?

13.4 Surface Area of a Right Circular Cone

So far, we have been generating solids by stacking up congruent figures. Incidentally, such figures are called *prisms*. Now let us look at another kind of solid which is not a prism (These kinds of solids are called *pyramids.*). Let us see how we can generate them.

Activity : Cut out a right-angled triangle ABC right angled at B. Paste a long thick string along one of the perpendicular sides say AB of the triangle [see Fig. 13.13(a)]. Hold the string with your hands on either sides of the triangle and rotate the triangle

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about the string a number of times. What happens? Do you recognize the shape that the triangle is forming as it rotates around the string [see Fig. 13.13(b)]? Does it remind you of the time you had eaten an ice-cream heaped into a container of that shape [see Fig. 13.13 (c) and (d)]?

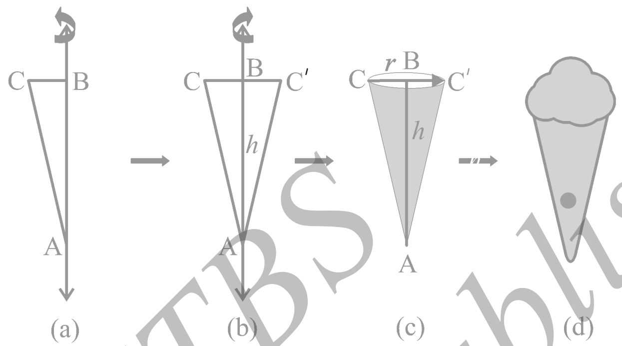

Fig. 13.13

This is called a *right circular cone*. In Fig. 13.13(c) of the right circular cone, the point A is called the vertex, AB is called the height, BC is called the *radius* and AC is called the slant height of the cone. Here B will be the centre of circular base of the cone. The height, radius and slant height of the cone are usually denoted by *h*, *r* and *l* respectively. Once again, let us see what kind of cone we can *not* call a right circular cone. Here, you are (see Fig. 13.14)! What you see in these figures are not right circular cones; because in (a), the line joining its vertex to the centre of its base is not at right angle to the base, and in (b) the base is not circular.

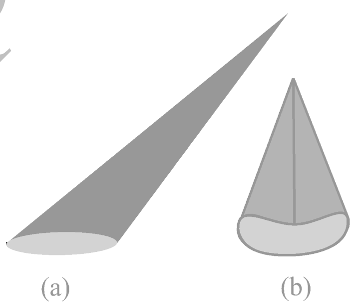

Fig. 13.14

As in the case of cylinder, since we will be studying only about right circular cones, remember that by 'cone' in this chapter, we shall mean a 'right circular cone.'

Activity : (i) Cut out a neatly made paper cone that does not have any overlapped paper, straight along its side, and opening it out, to see the shape of paper that forms the surface of the cone. (The line along which you cut the cone is the *slant height* of the cone which is represented by *l*). It looks like a part of a round cake.

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(ii) If you now bring the sides marked A and B at the tips together, you can see that the curved portion of Fig. 13.15 (c) will form the circular base of the cone.

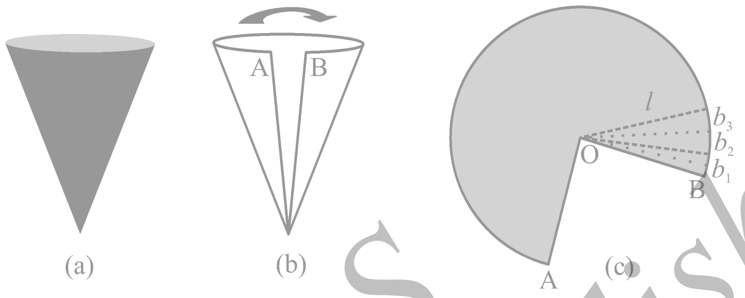

Fig. 13.15

(iii) If the paper like the one in Fig. 13.15 (c) is now cut into hundreds of little pieces, along the lines drawn from the point O, each cut portion is almost a small triangle, whose height is the slant height l of the cone.

(iv) Now the area of each triangle = $\frac{1}{2}$ × base of each triangle × l.

So, area of the entire piece of paper

$$= \text{sum of the areas of all the triangles}$$

$$= \frac{1}{2}b_1 l + \frac{1}{2}b_2 l + \frac{1}{2}b_3 l + \cdots = \frac{1}{2}l\left(b_1 + b_2 + b_3 + \cdots\right)$$

$$= \frac{1}{2} \times l \times \text{length of entire curved boundary of Fig. 13.15(c)}$$

(as $b_1 + b_2 + b_3 + \ldots$ makes up the curved portion of the figure)

But the curved portion of the figure makes up the perimeter of the base of the cone and the circumference of the base of the cone = $2\pi r$, where r is the base radius of the cone.

So, **Curved Surface Area of a Cone =** $\mathbf{\frac{1}{2} \times \mathit{l} \times 2\pi \mathit{r} = \pi \mathit{rl}}$

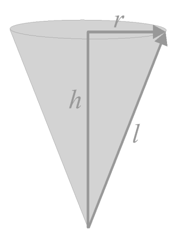

Fig. 13.16

where r is its base radius and l its slant height.

Note that $l^2 = r^2 + h^2$ (as can be seen from Fig. 13.16), by applying Pythagoras Theorem. Here h is the *height* of the cone.

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Therefore, $l = \sqrt{r^2 + h^2}$

Now if the base of the cone is to be closed, then a circular piece of paper of radius r is also required whose area is πr^2.

So, **Total Surface Area of a Cone = $\pi rl + \pi r^2 = \pi r(l + r)$**

Example 4 : Find the curved surface area of a right circular cone whose slant height is 10 cm and base radius is 7 cm.

Solution : Curved surface area = πrl

$$= \frac{22}{7} \times 7 \times 10 \text{ cm}^2$$

$$= 220 \text{ cm}^2$$

Example 5 : The height of a cone is 16 cm and its base radius is 12 cm. Find the curved surface area and the total surface area of the cone (Use $\pi = 3.14$).

Solution : Here, $h = 16$ cm and $r = 12$ cm.

So, from $l^2 = h^2 + r^2$, we have

$$l = \sqrt{16^2 + 12^2} \text{ cm} = 20 \text{ cm}$$

So, curved surface area = πrl

$$= 3.14 \times 12 \times 20 \text{ cm}^2$$

$$= 753.6 \text{ cm}^2$$

Further, total surface area = $\pi rl + \pi r^2$

$$= (753.6 + 3.14 \times 12 \times 12) \text{ cm}^2$$

$$= (753.6 + 452.16) \text{ cm}^2$$

$$= 1205.76 \text{ cm}^2$$

Example 6 : A corn cob (see Fig. 13.17), shaped somewhat like a cone, has the radius of its broadest end as 2.1 cm and length (height) as 20 cm. If each 1 cm^2 of the surface of the cob carries an average of four grains, find how many grains you would find on the entire cob.

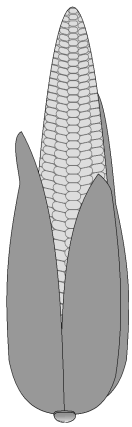

Fig. 13.17

Solution : Since the grains of corn are found only on the curved surface of the corn cob, we would need to know the curved surface area of the corn cob to find the total number of grains on it. In this question, we are given the height of the cone, so we need to find its slant height.

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Here, $l = \sqrt{r^2 + h^2} = \sqrt{(2.1)^2 + 20^2}$ cm

$= \sqrt{404.41}$ cm = 20.11 cm

Therefore, the curved surface area of the corn cob = πrl

$= \frac{22}{7} \times 2.1 \times 20.11 \text{ cm}^2 = 132.726 \text{ cm}^2 = 132.73 \text{ cm}^2$ (approx.)

Number of grains of corn on 1 cm^2 of the surface of the corn cob = 4

Therefore, number of grains on the entire curved surface of the cob

= 132.73 × 4 = 530.92 = 531 (approx.)

So, there would be approximately 531 grains of corn on the cob.

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EXERCISE 13.3

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

1. Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area.
2. Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.
3. Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find (i) radius of the base and (ii) total surface area of the cone.
4. A conical tent is 10 m high and the radius of its base is 24 m. Find

   (i) slant height of the tent.

   (ii) cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is ₹ 70.
5. What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm (Use $\pi = 3.14$).
6. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of ₹ 210 per 100 m^2.
7. A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.
8. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is ₹ 12 per m^2, what will be the cost of painting all these cones? (Use $\pi = 3.14$ and take $\sqrt{1.04} = 1.02$)

13.5 Surface Area of a Sphere

What is a sphere? Is it the same as a circle? Can you draw a circle on a paper? Yes, you can, because a circle is a plane closed figure whose every point lies at a constant distance (called **radius**) from a fixed point, which is called the **centre** of the circle. Now if you paste a string along a diameter of a circular disc and rotate it as you had rotated the triangle in the previous section, you see a new solid (see Fig 13.18). What does it resemble? A ball? Yes. It is called a **sphere**.

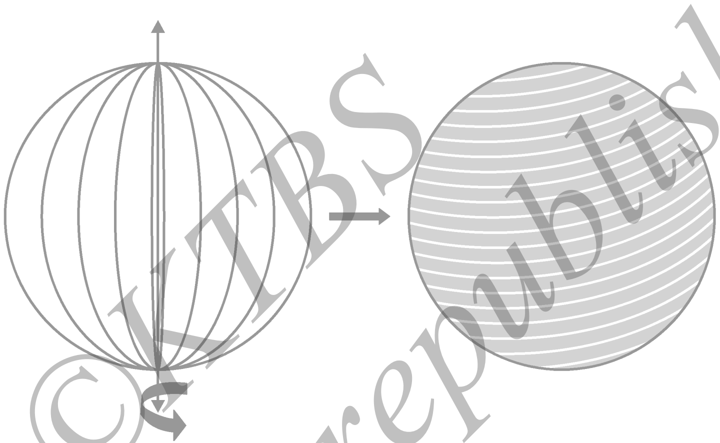

Fig. 13.18

Can you guess what happens to the centre of the circle, when it forms a sphere on rotation? Of course, it becomes the centre of the sphere. So, *a sphere is a three dimensional figure* (*solid figure*), *which is made up of all points in the space, which lie at a constant distance called the radius, from a fixed point called the centre of the sphere.*

Note : A sphere is like the surface of a ball. The word *solid sphere* is used for the solid whose surface is a sphere.

Activity : Have you ever played with a top or have you at least watched someone play with one? You must be aware of how a string is wound around it. Now, let us take a rubber ball and drive a nail into it. Taking support of the nail, let us wind a string around the ball. When you have reached the 'fullest' part of the ball, use pins to keep the string in place, and continue to wind the string around the remaining part of the ball, till you have completely covered the ball [see Fig. 13.19(a)]. Mark the starting and finishing points on the string, and slowly unwind the string from the surface of the ball.

Now, ask your teacher to help you in measuring the diameter of the ball, from which you easily get its radius. Then on a sheet of paper, draw four circles with radius equal

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to the radius of the ball. Start filling the circles one by one, with the string you had wound around the ball [see Fig. 13.19(b)].

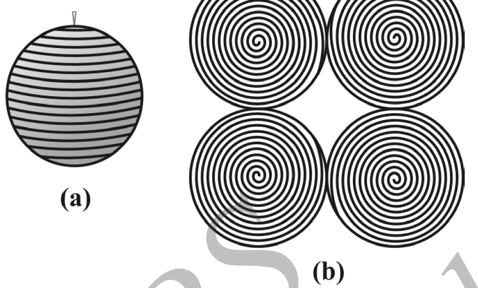

Fig. 13.19

What have you achieved in all this?

The string, which had completely covered the surface area of the sphere, has been used to completely fill the regions of four circles, all of the same radius as of the sphere.

So, what does that mean? This suggests that the surface area of a sphere of radius r

$$= 4 \text{ times the area of a circle of radius } r = 4 \times (\pi r^2)$$

So, **Surface Area of a Sphere = $4 \pi r^2$**

where r is the radius of the sphere.

How many faces do you see in the surface of a sphere? There is only one, which is curved.

Now, let us take a solid sphere, and slice it exactly 'through the middle' with a plane that passes through its centre. What happens to the sphere?

Yes, it gets divided into two equal parts (see Fig. 13.20)! What will each half be called? It is called a **hemisphere**. (Because 'hemi' also means 'half')

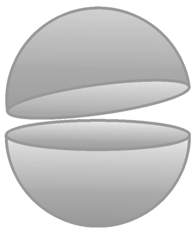

Fig. 13.20

And what about the surface of a hemisphere? How many faces does it have?

Two! There is a curved face and a flat face (base).

The curved surface area of a hemisphere is half the surface area of the sphere, which is $\frac{1}{2}$ of $4\pi r^2$.

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Therefore, **Curved Surface Area of a Hemisphere = $2\pi r^2$**

where r is the radius of the sphere of which the hemisphere is a part.

Now taking the two faces of a hemisphere, its surface area $2\pi r^2 + \pi r^2$

So, **Total Surface Area of a Hemisphere = $3\pi r^2$**

Example 7 : Find the surface area of a sphere of radius 7 cm.

Solution : The surface area of a sphere of radius 7 cm would be

$$4\pi r^2 = 4 \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2 = 616 \text{ cm}^2$$

Example 8 : Find (i) the curved surface area and (ii) the total surface area of a hemisphere of radius 21 cm.

Solution : The curved surface area of a hemisphere of radius 21 cm would be

$$= 2\pi r^2 = 2 \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2 = 2772 \text{ cm}^2$$

(ii) the total surface area of the hemisphere would be

$$3\pi r^2 = 3 \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2 = 4158 \text{ cm}^2$$

Example 9 : The hollow sphere, in which the circus motorcyclist performs his stunts, has a diameter of 7 m. Find the area available to the motorcyclist for riding.

Solution : Diameter of the sphere = 7 m. Therefore, radius is 3.5 m. So, the riding space available for the motorcyclist is the surface area of the 'sphere' which is given by

$$4\pi r^2 = 4 \times \frac{22}{7} \times 3.5 \times 3.5 \text{ m}^2$$

$$= 154 \text{ m}^2$$

Example 10 : A hemispherical dome of a building needs to be painted (see Fig. 13.21). If the circumference of the base of the dome is 17.6 m, find the cost of painting it, given the cost of painting is ₹ 5 per 100 cm^2.

Solution : Since only the rounded surface of the dome is to be painted, we would need to find the curved surface area of the hemisphere to know the extent of painting that needs to be done. Now, circumference of the dome = 17.6 m. Therefore, $17.6 = 2\pi r$.

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So, the radius of the dome = $17.6 \times \frac{7}{2 \times 22}$ m = 2.8 m

The curved surface area of the dome = $2\pi r^2$

$$= 2 \times \frac{22}{7} \times 2.8 \times 2.8 \text{ m}^2$$

$$= 49.28 \text{ m}^2$$

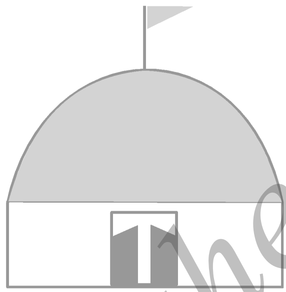

Fig. 13.21

Now, cost of painting 100 cm^2 is ₹ 5.

So, cost of painting 1 m^2 = ₹ 500

Therefore, cost of painting the whole dome

= ₹ 500 × 49.28

= ₹ 24640

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EXERCISE 13.4

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

1. Find the surface area of a sphere of radius:
   (i) 10.5 cm (ii) 5.6 cm (iii) 14 cm
2. Find the surface area of a sphere of diameter:
   (i) 14 cm (ii) 21 cm (iii) 3.5 m
3. Find the total surface area of a hemisphere of radius 10 cm. (Use $\pi = 3.14$)
4. The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.
5. A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin-plating it on the inside at the rate of ₹ 16 per 100 cm^2.
6. Find the radius of a sphere whose surface area is 154 cm^2.
7. The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.
8. A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.
9. A right circular cylinder just encloses a sphere of radius r (see Fig. 13.22). Find
   (i) surface area of the sphere,
   (ii) curved surface area of the cylinder,
   (iii) ratio of the areas obtained in (i) and (ii).

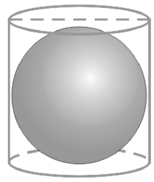

Fig. 13.22