Hydraulics Test 3

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. In a fluid the velocity field is given by
$$V = (3x + 2y)\, i + (2z + 3x^2)\, j + (2t - 3z)\, k$$
The velocity at point (1, 1, 1) at time 2 second is
(A) 6.82 units (B) 7.14 units
(C) 7.93 units (D) 8.26 units

Statement for Linked Answer Questions 2 and 3:
The velocity along the centre line of a nozzleof length ℓ is given by,
$$V = 2\,t\left(1 - \frac{x}{2\ell}\right)^2$$
where V = velocity in m/s, t = time in seconds,
x = distance from inlet of the nozzle

2. Convective acceleration when x = 1 m and ℓ = 1.5 m and t = 5 seconds is
(A) –19.75 m/s^2 (B) 19.75 m/s^2
(C) 26.35 m/s^2 (D) –26.35 m/s^2

3. Total acceleration for the same conditions as above is
(A) 20.64 m/s^2 (B) –20.64 m/s^2
(C) –18.86 m/s^2 (D) 18.86 m/s^2

4. Velocity distribution in the boundary layer of fluid flow over a surface is given by
$$\frac{u}{U} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\frac{y^2}{\delta^2}$$
When δ = the boundary layer thickness
U = maximum velocity of fluid
u = fluid velocity at y
The ratio of displacement thickness to boundary layer thickness is
(A) $\frac{5}{12}$ (B) $\frac{7}{12}$
(C) $\frac{5}{7}$ (D) $\frac{3}{7}$

5. A Kaplan turbine develops 20000 kW at an average head of 35 m. Assuming a speed ratio of 2, flow ratio of 0.6 and overall efficiency of 90% and taking boss to runner diameter ratio as 0.35, speed of the turbine is
(A) 365 rpm (B) 388 rpm
(C) 409 rpm (D) 418 rpm

6. A liquid is flowing between two parallel plates. One plate is moving relative to other with a velocity of 4 m/s in the negative direction. If pressure gradient $\frac{\partial p}{\partial x} = -100 \times 10^6\ \text{N/m}^3$, viscosity μ = 0.4 poise and distance between the plates is 1 m, discharge per meter width is
(A) 182.5 L/s (B) 190.2 L/s
(C) 206.8 L/s (D) 242.4 L/s

7. A moving vane with velocity 20 m/s having an inlet angle zero degree and an outlet angle 25° receives water from a jet at a velocity of 48 m/s. Assuming mass flow rate of 1kg/s, the force acting on the vanes and its inclination is.
(A) 62.16 N, 15°
(B) 58.72N, 13.6°
(C) 54.68 N, 12.5°
(D) 50.25N, 10.4°

8. In the model study of a motor boat in a lake, 1 : 25 scale model is used. It is assumed that viscous resistance due to water and air is negligible compared to the wave resistance. If the speed of the model is 2 m/s, determine the speed of the prototype for dynamically similar conditions
(A) 8 m/s (B) 10 m/s
(C) 12 m/s (D) 15 m/s

9. A pipe line through which oil flows has a sudden expansion in it such that maximum pressure rise occurs. Energy loss in the sudden expansion in metres of oil is given by (V_1 = velocity before expansion)
(A) $\frac{V_1^2}{4g}$ (B) $\frac{V_1^2}{6g}$
(C) $\frac{V_1^2}{8g}$ (D) $\frac{V_1^2}{2\sqrt{2}g}$

10. Prandtl's velocity distribution for a boundary layer is given by
(A) $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$
(B) $\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$
(C) $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^3 + \left(\frac{y}{\delta}\right)^4$
(D) $\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$

11. Specific speed of a hydraulic pump is the speed of geometrically similar pump working against unit head and
(A) consuming unit power
(B) having unit velocity of flow
(C) having unit radial velocity
(D) delivering unit quantity of water

12. Match List – I with List – II and select correct answer using the codes given

	List – I		List – II
a.	Pelton turbine (single jet)	1.	Mediumdischarge, low head
b.	Francis turbine	2.	High discharge, low head
c.	Kaplan turbine	3.	medium head
		4.	Low discharge high head

	a b c		a b c
(A)	1 2 3	(B)	4 3 2
(C)	2 3 4	(D)	3 4 1

13. Efficiency of the turbine is least under part load condition in the case of *a*
(A) Pelton turbine
(B) Francis turbine
(C) Kaplan turbine
(D) Propeller turbine

14. Cavitation in a hydraulic turbine is most likely to occur at the turbine
(A) entry
(B) exit
(C) stator exit
(D) rotor exit

15. A hydraulic turbine develops 1000 kW power for a head of 40 m. If the turbine is to work under a head of 20 m the power developed in kW is
(A) 250 (B) 500
(C) $\frac{500}{\sqrt{8}}$ (D) $\frac{100}{\sqrt{8}}$

16. A water turbine delivering 16 MW power is to be tested with the help of a geometrically similar model of 1 : 4 scale. If the speed of the model is same as that of the prototype then determine, power developed by the model, assuming same efficiencies for model and prototype
(A) $\frac{800}{32}$ kW (B) $\frac{1000}{128}$ kW
(C) $\frac{1000}{64}$ kW (D) $\frac{500}{64}$ kW

17. A jet of water of cross sectional area '*a*' strikes on a series of flat plates mounted on a wheel. If the jet velocity is *v* and velocity of the plates is *u*, the force exerted on plate is
(A) $\rho a(v-u)^2$ (B) $\rho a v^2(v-u)$
(C) $\rho a(v-u)^2 u$ (D) $\rho a v(v-u)$

18. A jet issuing from nozzle with diameter of 180 mm has a velocity of 90 m/s and strikes the pelton wheel and gives a shaft power of 7500 kW. If the coefficient of velocity of nozzle is 0.98, the overall efficiency of the turbine is
(A) 80% (B) 77.65%
(C) 90% (D) 78.83%

19. In two pelton wheels, the first wheel deflects the water through 160° and the other through 170°. The ratio of the maximum efficiencies of the first and second wheels is (all other operating parameters being same for both)
(A) 1.023
(B) 0.966
(C) 1.035
(D) 0.977

Common Data for Questions 20 and 21:
A pelton wheel develops 5520 kW under a head of 240 m at an overall efficiency of 80% when revolving at a speed of 200 rpm.

20. The unit discharge is
(A) 0.172 (B) 0.189
(C) 0.135 (D) 0.212

21. The unit power is
(A) 1.645 (B) 1.581
(C) 1.485 (D) 1.321

Common Data for Questions 22 and 22:
A 10 cm diameter water jet having a velocity of 45 m/s strikes a flat plate, the normal of which is inclined at 45° to the axis of the jet.

22. What will be the normal force on the plate if the plate is stationary?
(A) 5.62 kN
(B) 11.25 kN
(C) 15.9 kN
(D) Data insufficient

23. What will be the normal force on the plate if the plate is moving towards the jet with a velocity of 15 m/s
(A) 5 kN
(B) 10 kN
(C) 20 kN
(D) 30 kN

Common Data for Questions 24 and 25:
A pipe line 1200 m long supplies water to 3 single jet pelton wheels. The head above the nozzle is 360 m. The velocity coefficient for the nozzle is 0.98 and coefficient of friction for the pipeline is 0.02. The turbine efficiency based on the head at the nozzle is 0.85. The specific speed of each turbine is 15.3 and the head loss due to friction in the pipeline is 12 m of water. If the operating speed of each turbine is 560 rpm.

24. The total power developed by the turbine in kW
(A) 6069 (B) 5059
(C) 7079 (D) 4079

25. The diameter of the nozzle in mm
(A) 45.7 (B) 95.6
(C) 76.2 (D) 81.9

Answer Keys

1. B	**2.** A	**3.** C	**4.** A	**5.** C	**6.** C	**7.** C	**8.** B	**9.** C	**10.** B
11. D	**12.** B	**13.** D	**14.** D	**15.** D	**16.** C	**17.** D	**18.** B	**19.** D	**20.** B
21. C	**22.** B	**23.** C	**24.** B	**25.** B					

Hints and Explanations

1. The velocity components are

$u = 3x + 2y$

$v = 2z + 3x^2$

$w = 2t - 3z$

Substituting,

$x = 1, y = 1, z = 1$ and $t = 2$

$u = 3 + 2 = 5$

$v = 2 + 3 = 5$

$w = 2 \times 2 - 3 = 1$

$$V = \sqrt{u^2+v^2+w^2} = \sqrt{5^2+5^2+1^2} = \sqrt{51}$$

$= 7.14$ units. Choice (B)

2. Convective acceleration

$$= v\frac{\partial v}{\partial x} = 2t\left(1-\frac{x}{2\ell}\right)^2 \times 2t \times 2\left(1-\frac{x}{2\ell}\right)\left(-\frac{1}{2\ell}\right)$$

$$= -\frac{4t^2}{\ell}\left(1-\frac{x}{2\ell}\right)^3$$

Given that $\ell = 1.5$ m

$x = 1$ and $t = 5$ seconds

$$\text{convective acceleration} = \frac{-4\times(5)^2}{1.5}\left(1-\frac{1}{2\times1.5}\right)^3$$

$= -19.75$ m/s^2. Choice (A)

3. Total acceleration

= Convective acceleration + local acceleration

$$\text{local acceleration} = \frac{\partial v}{\partial t} = \left(1-\frac{x}{2\ell}\right)^2 \times 2$$

$$= \left(1-\frac{1}{2\times1.5}\right)^2 \times 2$$

$= 0.889$ m/s^2

Total acceleration $= -19.75 + 0.889 = -18.86$ m/s^2. Choice (C)

4. $$\frac{u}{U} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\frac{y^2}{\delta^2}$$

Displacement thickness

$$\delta^* = \int_0^\delta \left(1-\frac{u}{U}\right)dy = \int_0^\delta 1-\frac{3}{2}\frac{y}{\delta}+\frac{1}{2}\frac{y^2}{\delta^2}$$

$$= \left[y - \frac{3}{2}\frac{y^2}{2\delta} + \frac{1}{2}\cdot\frac{y^3}{3\delta^2}\right]_0^\delta = \delta - \frac{3}{4}\delta + \frac{\delta}{6} = \frac{5}{12}\delta$$

$$\frac{\delta^*}{\delta} = \frac{5}{12}\frac{\delta}{\delta} = \frac{5}{12}.$$ Choice (A)

5. $u_1 = k_u\sqrt{2gH} = 2\sqrt{2\times9.81\times35} = 52.4$ m/s

$v_{f1} = k_f\sqrt{2gH} = 0.6\sqrt{2\times9.81\times35} = 15.7$ m/s

$$\frac{\text{shaft power}}{\text{wQH}} = \text{overall } \eta$$

i.e., $$\frac{20000\times10^3}{9810\times Q\times35} = 0.9$$

$Q = 64.72$ m^3/s

But $Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right)\times v_{f1}$ or

$$64.72 = \frac{\pi}{4}D_o^2\left(1-0.35^2\right)\times15.7$$

$D_o = 2.45$ m

$u_1 = \frac{\pi D_o N}{60}$ where N = speed of turbine

i.e., $$52.4 = \frac{\pi\times2.45\times N}{60}$$

$N = 409.2$ rpm. Choice (C)

6. $U = -3$ m/s; $\frac{dp}{dx} = -100 \times 10^6$ N/m^3

$\mu = 0.4$ poise $= 0.04$ Ns/m^2

$b = 1$ mm

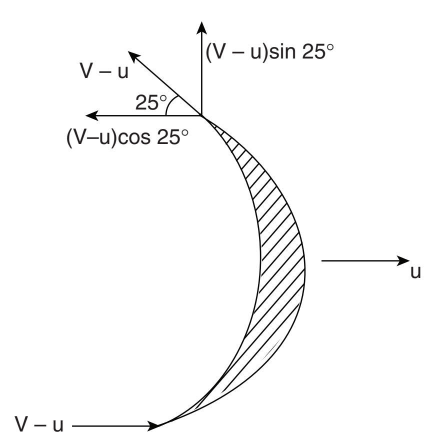
Discharge per unit width is given by,

$$q = U\frac{b}{2} - \frac{b^3}{12\mu}\cdot\frac{\partial p}{\partial x}$$

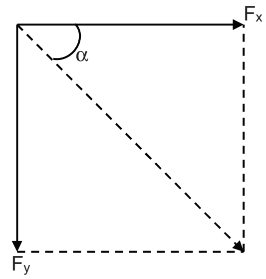
$$= (-3)\times\frac{0.001}{2} - \frac{(0.001)}{12\times0.04}\times(-100\times106)$$

$= 0.2068$ m^3/s $= 206.8$ L/s Choice (C)

7.



$u = 20$ m/s
$v_{1x} = (v - u) = 48 - 20 = 28$ m/s
$v_{1y} = 0$
$v_{2x} = -(v - u)\cos 25 = -28\cos 25 = -25.38$ m/s
$v_{2y} = (v - u)\sin 25 = 28\sin 25 = 11.83$ m/s
$F_x = 1_x [v_{1x} - v_{2x}] = 28 - (-25.38) = 53.38$ m/s
$F_y = 1\,x\,[v_{1y} - v_{2y}] = 0 - 11.83 = -11.83$ m/s



Resultant force

$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{53.38^2 + (-11.83)^2} = 54.68$ N

$\tan\alpha = \frac{F_y}{F_x} = \frac{11.83}{53.38} = 0.2216$

$\alpha = 12.5°$. Choice (C)

8. Since wave resistance is dominant, dynamic similarity will be attained when Froude number is same for model and prototype

i.e., $\left(\frac{V}{\sqrt{Lg}}\right)_m = \left(\frac{V}{\sqrt{Lg}}\right)_p \Rightarrow \frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}} = \sqrt{25} = 5$

$\Rightarrow V_p = 2 \times 5 = 10$ m/s Choice (B)

9. Maximum pressure rise occurs when $\frac{D_1}{D_2} = \frac{1}{\sqrt{2}}$ or

$D_2 = \sqrt{2}\,D_1$

$\Rightarrow A_2 = 2A_1 \because A \propto D^2$

$\Rightarrow V_2 = \frac{V_1}{2}$

$\because A_1V_1 = A_2V_2 = Q$

Energy (head) loss in sudden expansion

$$= \frac{(V_1 - V_2)^2}{2g} = \frac{\left(V_1 - \frac{V_1}{2}\right)^2}{2g}$$

$$= \frac{V_1^2}{2g}\left(1 - \frac{1}{2}\right)^2 = \frac{V_1^2}{2g} \times \frac{1}{4} = \frac{V_1^2}{8g} \text{ m of oil}$$

Choice (C)

11. Specific speed of hydraulic pump $N_s = \frac{N\sqrt{Q}}{H^{\frac{3}{4}}}$

It is the speed when working against unit head and delivering unit quantity of water. Choice (D)

13. Blades of a propeller turbine are fixed and not adjustable under part load condition. So under part load, water enters with shock and eddies are formed which reduces its efficiency. Choice (D)

15. $P_1 = 1000$ kW, $H_1 = 40$ m
$H_2 = 20$ m
For both conditions, unit power is same

i.e., $P_u = \frac{P_1}{(H_1)^{\frac{3}{2}}} = \frac{P_2}{(H_2)^{\frac{3}{2}}} \Rightarrow P_2 = P_1\left(\frac{H_2}{H_1}\right)^{\frac{3}{2}}$

$$= 1000 \times \left(\frac{20}{40}\right)^{\frac{3}{2}} = 1000 \times \left(\frac{1}{2}\right)^{\frac{3}{2}}$$

$$= \frac{1000}{\sqrt{8}} \text{ kW}$$ Choice (D)

16. Given- $P_p = 10$ MW

$\frac{D_p}{D_m} = \frac{1}{4}$

$\frac{N_p}{N_m} = 1$

$\eta_p = \eta_m$

Power coefficient $\frac{P}{N^3D^5}$ is

same in both cases

i.e., $\frac{P_p}{N_p^3 D_p^5} = \frac{P_m}{N_m^3\, D_m^5}$

$\therefore\ P_m = P_p \times \left(\frac{N_m}{Np}\right)^3 \times \left(\frac{D_m}{D_p}^5\right)$

$$= 16 \times 1 \times \left(\frac{1}{4}\right)^5 = \frac{1}{4^3} = \frac{1}{64}\text{MW}$$

$$= \frac{1000}{64}\text{ kW}$$ Choice (C)

17. Force exerted
= mass/second × change in velocity of the jet
$= \rho av(v - u)$ Choice (D)

18. $V = C_v\sqrt{2gH}$

$90 = 0.98\sqrt{2 \times 9.81 \times H}$

$H = 429.87$ m

$\eta_0 = \frac{P}{wQH} = \frac{7500 \times 10^3}{9810 \times 2.29 \times 429.87} = 0.7766$

$Q = \frac{\pi}{4}\, d^2 \times V$

$= \frac{\pi}{4}(0.18)^2 \times 90 = 2.29$ m³/s

$\eta_0 = 0.7766 \times 100 = 77.66$ % Choice (B)

19. Maximum efficiency is expressed as $\eta_H = \dfrac{1+\cos\phi}{2}$

$\phi_1 = 180 - 160 = 20°$
$\phi_2 = 180 - 170 = 10°$

$$\eta_{H1} = \frac{1+\cos 20°}{2}$$

$$\eta_{H2} = \frac{1+\cos 10°}{2}$$

$$\frac{\eta_{H1}}{\eta_{H1}} = \frac{(1+\cos 20°)}{2} \times \frac{2}{1+\cos 10°}$$

$$= \frac{1.9397}{1.9848} = 0.977$$ Choice (D)

20. $\eta_0 = \dfrac{P}{wQH}$

$$0.8 = \frac{5520 \times 10^3}{9810 \times Q \times 240}$$

$Q = 2.931\ m^3/sec$
Unit Discharge

$$Q_u = \frac{Q}{\sqrt{H}} = \frac{2.931}{\sqrt{240}} = 0.189$$ Choice (B)

21. Unit power $Pu = \dfrac{P}{H^{\frac{3}{2}}} = \dfrac{5520}{(240)^{\frac{3}{2}}} = 1.485$ Choice (C)

22. $F_n = \rho A V^2 \sin\theta$
$V = 45$ m/s
$\theta = 45° = 1000 \times 7.8544 \times 10^{-3} \times (45)^2 \sin 45°$
$= 11246$ N

$$F_n = \frac{11246}{1000} = 11.25 kN$$ Choice (B)

23. Relative velocity $= v + u = 45 + 15 = 60$ m/s
$F_n = \rho A\ (v+u)^2 \sin\theta$
$= 1000 \times 7.854 \times 10^{-3} (60)^2 \sin 45°$
$= 19993$ N

$$F_n = \frac{19993}{1000} = 20 \text{ kN}$$ Choice (C)

24. Net head available at the base of the nozzle
$= 360 - 12 = 348$m

$$N_s = \frac{N\sqrt{p}}{H^{\frac{5}{4}}}$$

$$15.33 = \frac{560\sqrt{p}}{348^{\frac{5}{4}}}$$

$P = 1686.37$ kW
Total power $= 1686.37 \times 3 = 5059.11$ kW Choice (B)

25. $\eta_0 = \dfrac{P}{wQH}$

$$0.85 = \frac{1686.37 \times 10^3}{9810 \times Q\ 348}$$

$Q = 0.581\ m^3/sec$
Total discharge $= 3 \times 0.581$
$Q = 1.743\ m^3/s$
$V = k_v\sqrt{2gh} = 0.98\sqrt{2 \times 9.81 \times 348}$
$= 80.98$ m/s
Thus, if the diameter of the nozzle is '*d*' then

$$0.581 = \frac{\pi}{4} d^2 \times 80.98$$

$D = 0.0956$ m $= 95.6$ mm Choice (B)