CHAPTER

9

Term-II

DIFFERENTIAL EQUATIONS

Syllabus

- *Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree type:* $\frac{dy}{dx} + f(y/x)$. *Solutions of linear differential equations of the type:*

  $\frac{dy}{dx} + py = q$, *where p and q are the functions of x or constants*

STAND ALONE MCQs

(1 Mark each)

Q. 1. The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{dy}{dx}\right) \text{ is}$$

(A) 1 **(B)** 2

(C) 3 **(D)** not defined

Ans. Option (D) is correct.

Explanation: The degree of above differential equation is not defined because when we expand $\sin\left(\frac{dy}{dx}\right)$ we get an infinite series in the increasing powers of $\frac{dy}{dx}$. Therefore its degree is not defined.

Q. 2. The order and degree of the differential equation

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/4} + x^{1/5} = 0 \text{ respectively, are}$$

(A) 2 and 4 **(B)** 2 and 2

(C) 2 and 3 **(D)** 3 and 3

Ans. Option (A) is correct.

Explanation :

Given that, $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/4} = -x^{1/5}$

$\Rightarrow \quad \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/4} = -x^{1/5}$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)^{1/4} = -\left(x^{1/5} + \frac{d^2y}{dx^2}\right)$

On squaring both sides, we get

$$\left(\frac{dy}{dx}\right)^{1/2} = \left(x^{1/5} + \frac{d^2y}{dx^2}\right)^2$$

Again, on squaring both sides, we have

$$\frac{dy}{dx} = \left(x^{1/5} + \frac{d^2y}{dx^2}\right)^4$$

Order = 2, degree = 4

Q. 3. If $y = e^{-x}(A\cos x + B\sin x)$, then y is a solution of

(A) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$ **(B)** $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

(C) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$ **(D)** $\frac{d^2y}{dx^2} + 2y = 0$

Ans. Option (C) is correct.

Explanation:
Given that, $y = e^{-x}$ ($A\cos x + B\sin x$)
On differentiating both sides w.r.t., x we get

$$\frac{dy}{dx} = -e^{-x}(A\cos x + B\sin x) + e^{-x}(-A\sin x + B\cos x)$$

$$\frac{dy}{dx} = -y + e^{-x}(-A\sin x + B\cos x)$$

Again, differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{-dy}{dx} + e^{-x}(-A\cos x - B\sin x) - e^{-x}(-A\sin x + B\cos x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{dy}{dx} - y\left[\frac{dy}{dx} + y\right]$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{dy}{dx} - y - \frac{dy}{dx} - y$$

$$\Rightarrow \frac{d^2y}{dx^2} = -2\frac{dy}{dx} - 2y$$

$$\Rightarrow \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

Q. 4. The solution of differential equation $xdy - ydx = 0$ represents
(A) a rectangular hyperbola
(B) parabola whose vertex is at origin
(C) straight line passing through origin
(D) a circle whose centre is at origin

Ans. Option (C) is correct.

Explanation: Given that,

$$xdy - ydx = 0$$

$$\Rightarrow xdy = ydx$$

$$\Rightarrow \frac{dy}{y} = \frac{dx}{x}$$

On integrating both sides, we get

$$\log y = \log x + \log C$$

$$\Rightarrow \log y = \log Cx$$

$$\Rightarrow y = Cx$$

which is a straight line passing through the origin.

Q. 5. The integrating factor of differential equation $\cos x\frac{dy}{dx} + y\sin x = 1$ is
(A) $\cos x$ **(B)** $\tan x$
(C) $\sec x$ **(D)** $\sin x$

Ans. Option (C) is correct.

Explanation: Given that,

$$\cos x\frac{dy}{dx} + y\sin x = 1$$

$$\Rightarrow \frac{dy}{dx} + y\tan x = \sec x$$

Here, $P = \tan x$ and $Q = \sec x$

$$\text{IF} = e^{\int P dx} = e^{\int \tan x dx} = e^{\ln \sec x}$$

$$\therefore \text{IF} = \sec x$$

Q. 6. Family $y = Ax + A^3$ of curves is represented by the differential equation of degree :
(A) 1 **(B)** 2
(C) 3 **(D)** 4

Ans. Option (A) is correct.

Explanation:
Given that, $y = Ax + A^3$

$$\Rightarrow \frac{dy}{dx} = A$$

[We can differentiate above equation only once because it has only one arbitrary constant.]
$\therefore$ Degree = 1

Q. 7. Which of the following is a second-order differential equation?
(A) $(y')^2 + x = y^2$ **(B)** $y'y'' + y = \sin x$
(C) $y''' + (y'')^2 + y = 0$ **(D)** $y' = y^2$

Ans. Option (B) is correct.

Explanation: The second-order differential equation is $y'y'' + y = \sin x$.

Q. 8. The integrating factor of differential equation $(1-x^2)\frac{dy}{dx} - xy = 1$ is
(A) $-x$ **(B)** $\frac{x}{1+x^2}$
(C) $\sqrt{1-x^2}$ **(D)** $\frac{1}{2}\log(1-x^2)$

Ans. Option (C) is correct.

Explanation: Given that,

$$(1-x^2)\frac{dy}{dx} - xy = 1$$

$$\Rightarrow \frac{dy}{dx} - \frac{x}{1-x^2}y = \frac{1}{1-x^2}$$

which is a linear differential equation.

$$\text{IF} = e^{-\int \frac{x}{1-x^2}dx}$$

Put $1 - x^2 = t$

$$\Rightarrow -2xdx = dt$$

$$\Rightarrow xdx = -\frac{dt}{2}$$

Now, $\text{IF} = e^{\frac{1}{2}\int \frac{dt}{t}}$

$$= e^{\frac{1}{2}\log t}$$
$$= e^{\frac{1}{2}\log(1-x^2)}$$
$$= \sqrt{1-x^2}$$

Q. 9. The degree of differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3+6y^5=0$ is

(A) 1 **(B)** 2

(C) 3 **(D)** 5

Ans. Option (A) is correct.

Explanation:

$$\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3+6y^5 = 0$$

We know that, the degree of a differential equation is exponent of highest order derivative.

$\therefore$ Degree = 1

Q. 10. The integrating factor of differential equation $\frac{dy}{dx}+y\tan x-\sec x=0$ is

(A) $\cos x$ **(B)** $\sec x$

(C) $e^{\cos x}$ **(D)** $e^{\sec x}$

Ans. Option (B) is correct.

Explanation: Given that,

$$\frac{dy}{dx}+y\tan x-\sec x = 0$$

Here, $P = \tan x$, $Q = \sec x$

$$\text{IF} = e^{\int P dx} = e^{\int \tan x dx}$$
$$= e^{(\log \sec x)}$$
$$= \sec x$$

Q. 11. $y = ae^{mx}+be^{-mx}$ satisfies which of the following differential equation

(A) $\frac{dy}{dx}+my=0$ **(B)** $\frac{dy}{dx}-my=0$

(C) $\frac{d^2y}{dx^2}-m^2y=0$ **(D)** $\frac{d^2y}{dx^2}+m^2y=0$

Ans. Option (C) is correct.

Explanation: Given that,

$$y = ae^{mx}+be^{-mx}$$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = mae^{mx}-bme^{-mx}$$

Again, differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = m^2ae^{mx}-bm^2e^{-mx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = m^2(ae^{mn}+be^{-mn})$$

$$\Rightarrow \frac{d^2y}{dx^2} = m^2y$$

$$\Rightarrow \frac{d^2y}{dx^2}-m^2y = 0$$

Q. 12. The solution of $x\frac{dy}{dx}+y=e^x$ is

(A) $y=\frac{e^x}{x}+\frac{k}{x}$ **(B)** $y = xe^x + Cx$

(C) $y = xe^x + k$ **(D)** $x=\frac{e^y}{y}+\frac{k}{y}$

Ans. Option (A) is correct.

Explanation: Given that,

$$x\frac{dy}{dx}+y = e^x$$

$$\Rightarrow \frac{dy}{dx}+\frac{y}{x} = \frac{e^x}{x}$$

which is a linear differential equation.

$$\therefore \text{IF} = e^{\int\frac{1}{x}dx}$$
$$= e^{(\log x)}$$
$$= x$$

The general solution is

$$y\cdot x = \int\left(\frac{e^x}{x}\cdot x\right)dx$$

$$\Rightarrow y\cdot x = \int e^x dx$$

$$\Rightarrow y\cdot x = e^x+k$$

$$\Rightarrow y = \frac{e^x}{x}+\frac{k}{x}$$

Q. 13. The general solution of $\frac{dy}{dx}=2xe^{x^2-y}$ is

(A) $e^{x^2-y}=C$ **(B)** $e^{-y}+e^{x^2}=C$

(C) $e^y=e^{x^2}+C$ **(D)** $e^{x^2+y}=C$

Ans. Option (C) is correct.

Explanation : Given that,

$$\frac{dy}{dx} = 2xe^{x^2-y}$$
$$= 2xe^{x^2}\cdot e^{-y}$$

$$\Rightarrow e^y\frac{dy}{dx} = 2xe^{x^2}$$

$$\Rightarrow e^y dy = 2xe^{x^2}dx$$

On integrating both sides, we get

$$\int e^y dy = 2\int xe^{x^2}dx$$

Put $x^2 = t$ in RHS integral, we get

$$2xdx = dt$$

$$\int e^y dy = \int e^t dt$$

$$\Rightarrow \quad e^y = e^t + C$$

$$\Rightarrow \quad e^y = e^{x^2} + C$$

Q. 14. The solution of equation $(2y - 1)dx - (2x + 3)dy = 0$ is

(A) $\frac{2x-1}{2y+3} = k$ (B) $\frac{2y+1}{2x-3} = k$

(C) $\frac{2x+3}{2y-1} = k$ (D) $\frac{2x-1}{2y-1} = k$

Ans. Option (C) is correct.

Explanation: Given that,

$$(2y-1)dx - (2x+3)dy = 0$$

$$\Rightarrow \quad (2y-1)dx = (2x+3)dy$$

$$\Rightarrow \quad \frac{dx}{2x+3} = \frac{dy}{2y-1}$$

On integrating both sides, we get

$$\frac{1}{2}\log(2x+3) = \frac{1}{2}\log(2y-1) + \log C$$

$$\Rightarrow \quad \frac{1}{2}[\log(2x+3) - \log(2y-1)] = \log C$$

$$\Rightarrow \quad \frac{1}{2}\log\left(\frac{2x+3}{2y-1}\right) = \log C$$

$$\Rightarrow \quad \left(\frac{2x+3}{2y-1}\right)^{1/2} = C$$

$$\Rightarrow \quad \frac{2x+3}{2y-1} = C^2$$

$$\Rightarrow \quad \frac{2x+3}{2y-1} = k,$$

where, $k = C^2$

Q. 15. The solution of

$\frac{dy}{dx} + y = e^{-x}$, $y(0) = 0$ is

(A) $y = e^{-x}(x - 1)$ (B) $y = xe^x$

(C) $y = xe^{-x} + 1$ (D) $y = xe^{-x}$

Ans. Option (D) is correct.

Explanation: Given that,

$$\frac{dy}{dx} + y = e^{-x}$$

which is a linear differential equation.

Here, $P = 1$ and $Q = e^{-x}$

$$\text{IF} = e^{\int dx} = e^x$$

The general solution is

$$y \cdot e^x = \int e^{-x} \cdot e^x dx + C$$

$$\Rightarrow \quad ye^x - \int dx + C$$

$$\Rightarrow \quad ye^x = x + C \quad \text{...(i)}$$

When $x = 0$ and $y = 0$ then, $0 = 0 + C \Rightarrow C = 0$

eqn. (i) becomes $y \cdot e^x = x \Rightarrow y = xe^{-x}$

Q. 16. The general solution of $\frac{dy}{dx} + y\tan x = \sec x$ is

(A) $y\sec x = \tan x + C$

(B) $y\tan x = \sec x + C$

(C) $\tan x = y\tan x + C$

(D) $x\sec x = \tan y + C$

Ans. Option (A) is correct.

Explanation: Given differential equation is

$$\frac{dy}{dx} + y\tan x = \sec x$$

which is a linear differential equation

Here, $P = \tan x$, $Q = \sec x$,

$$\therefore \quad \text{IF} = e^{\int \tan x dx} = e^{\log|\sec x|} = \sec x$$

The general solution is

$$y \cdot \sec x = \int \sec x \cdot \sec x + C$$

$$\Rightarrow \quad y \cdot \sec x = \int \sec^2 x \, dx + C$$

$$\Rightarrow \quad y \cdot \sec x = \tan x + C$$

Q. 17. The general solution of differential equation $(e^x + 1)ydy = (y+1)e^x dx$ is

(A) $(y + 1) = k(e^x + 1)$

(B) $y + 1 = e^x + 1 + k$

(C) $y = \log\{k(y+1)(e^x + 1)\}$

(D) $y = \log \frac{x+1}{y+1} + k$

Ans. Option (C) is correct.

Explanation: Given differential equation

$$(e^x + 1)ydy = (y+1)e^x dx$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{e^x(1+y)}{(e^x+1)y}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{(e^x+1)y}{e^x(1+y)}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{e^x y}{e^x(1+y)} + \frac{y}{e^x(1+y)}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{y}{1+y} + \frac{y}{(1+y)e^x}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{y}{1+y}\left(1 + \frac{1}{e^x}\right)$$

$\Rightarrow \quad \frac{dx}{dy} = \frac{y}{1+y}\left(\frac{e^x+1}{e^x}\right)$

$\Rightarrow \quad \left(\frac{y}{1+y}\right)dy = \left(\frac{e^x}{e^x+1}\right)dx$

On integrating both sides, we get

$$\int \frac{y}{1+y}dy = \int \frac{e^x}{1+e^x}dx$$

$\Rightarrow \quad \int \frac{1+y-1}{1+y}dy = \int \frac{e^x}{1+e^x}dx$

$\Rightarrow \quad \int 1dy - \int \frac{1}{1+y}dy = \int \frac{e^x}{1+e^x}dx$

$\Rightarrow \quad y - \log|(1+y) = \log|(1+e^x) + \log k$

$\Rightarrow \quad y = \log(1+y) + \log(1+e^x) + \log(k)$

$\Rightarrow \quad y = \log\{k(1+y)(1+e^x)\}$

Q. 18. The solution of differential equation $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$ is

(A) $y(1+x^2) = C + \tan^{-1} x$

(B) $\frac{y}{1+x^2} = C + \tan^{-1} x$

(C) $y\log(1+x^2) = C + \tan^{-1} x$

(D) $y(1+x^2) = C + \sin^{-1} x$

Ans. Option (A) is correct.

Explanation: Given that,

$$\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$$

Here, $\quad P = \frac{2x}{1+x^2}$

and $\quad Q = \frac{1}{(1+x^2)^2}$

which is a linear differential equation.

$\therefore \quad \text{IF} = e^{\int \frac{2x}{1+x^2}dx}$

Put $\quad 1+x^2 = t$

$\Rightarrow \quad 2xdx = dt$

$\therefore \quad \text{IF} = e^{\int \frac{dt}{t}}$

$= e^{\log t}$

$= e^{\log(1+x^2)}$

$= 1+x^2$

The general solution is

$$y \cdot (1+x^2) = \int (1+x^2)\frac{1}{(1+x^2)^2} + C$$

$\Rightarrow \quad y(1+x^2) = \int \frac{1}{1+x^2}dx + C$

$\Rightarrow \quad y(1+x^2) = \tan^{-1} x + C$

Q. 19. The order of the differential equation

$$2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0 \text{ is}$$

(A) 2 **(B)** 1

(C) 0 **(D)** not defined

Ans. Option (A) is correct.

Explanation :

$$2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$$

The highest order derivative present in the given differential equation is $\frac{d^2y}{dx^2}$. Therefore, its order is two.

Q. 20. The numbers of arbitrary constants in the general solution of a differential equation of fourth order are :

(A) 0 **(B)** 2

(C) 3 **(D)** 4

Ans. Option (D) is correct.

Explanation: We know that the number of constants in the general solution of a differential equation of order n is equal to its order.

Therefore, the number of constants in the general equation of fourth-order differential equation is four.

Q. 21. The numbers of arbitrary constants in the particular solution of a differential equation of third order are :

(A) 3 **(B)** 2

(C) 1 **(D)** 0

Ans. Option (D) is correct.

Explanation : In the particular solution of a differential equation, there are no arbitrary constants.

Q. 22. Which of the following differential equations has $y = x$ as one of its particular solution?

(A) $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = x$ **(B)** $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = x$

(C) $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0$ **(D)** $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy - 0$

Ans. Option (C) is correct.

Explanation: The given equation of curve is $y=x$

Differentiating with respect to x, we get :

$$\frac{dy}{dx} = 1 \quad ...(i)$$

Again, differentiating with respect to x, we get :

$$\frac{d^2y}{dx^2} = 0 \quad ...(ii)$$

Now, on substituting the values of y, $\frac{d^2y}{dx^2}$, and $\frac{dy}{dx}$ from equation (i) and (ii) in each of the given alternatives, we find that only the differential equation given in alternative C is correct.

$$\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0 - x^2 \cdot 1 + x \cdot x$$
$$= -x^2 + x^2$$
$$= 0$$

Q. 23. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

(A) $e^x + e^{-y} = C$ (B) $e^x + e^y = C$

(C) $e^{-x} + e^y = C$ (D) $e^{-x} + e^{-y} = C$

Ans. Option (A) is correct.

Explanation:

$$\frac{dy}{dx} = e^{x+y}$$
$$= e^x \cdot e^y$$
$$\Rightarrow \frac{dy}{e^y} = e^x dx$$
$$\Rightarrow e^{-y}dy = e^x dx$$

Integrating both sides, we get:

$$\int e^{-y}dy = \int e^x dx$$
$$\Rightarrow -e^{-y} = e^x + k$$
$$\Rightarrow e^x + e^{-y} = -k$$
$$\Rightarrow e^x + e^{-y} = C \quad \text{(where, } C = -k\text{)}$$

Q. 24. The Integrating Factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is

(A) e^{-x} (B) e^{-y}

(C) $\frac{1}{x}$ (D) x

Ans. Option (C) is correct.

Explanation: The given differential equation is:

$$x\frac{dy}{dx} - y = 2x^2$$
$$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = 2x$$

This is a linear differential equation of the form:

$$\frac{dy}{dx} + py = Q$$

(where $p = -\frac{1}{x}$ and $Q = 2x$)

The integrating factor (IF) is given by the relation,

$$\text{IF} = \int p dx$$
$$\therefore \text{IF} = e^{\int -\frac{1}{x}dx}$$
$$= e^{-\log x}$$
$$= e^{\log(x^{-1})}$$
$$= x^{-1}$$
$$= \frac{1}{x}$$

? ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Q. 1. Assertion (A): The order of the differential equation given by $\frac{dy}{dx} + 4y = \sin x$ is 1.

Reason (R): Since the order of a differential equation is defined as the order of the highest derivative occurring in the differential equation, *i.e.*, for nth derivative $\frac{d^ny}{d^xn}$ if $n = 1$. then it's order = 1.

Given differential equation contains only $\frac{dy}{dx}$ derivative with variables and constants.

Ans. Option (A) is correct.

Explanation: Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Q. 2. Assertion (A): The degree of the differential equation given by $\frac{dy}{dx} = \frac{x^4 - y^4}{(x^2 + y^2)xy}$ is 1.

Reason (R): The degree of a differential equation is the degree of the highest order derivative when differential coefficients are free from radicals and fraction.

The given differential equation has first order derivative which is free from radical and fraction with power = 1, thus it has a degree of 1.

Ans. Option (A) is correct.

> *Explanation:* Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Q. 3. Assertion (A): Solution of the differential equation $\frac{dy}{dx}=e^{3x-2y}+x^2e^{-2y}$ is $\frac{e^{2y}}{3}=\frac{e^{3x}}{3}+\frac{x^2}{2}+C$

Reason (R):

$$\frac{dy}{dx}=e^{3x-2y}+x^2e^{-2y}$$
$$\frac{dy}{dx}=e^{-2y}(e^{3x}+x^2)$$

separating the variables

$$e^{2y}dy=(e^{3x}+x^2)dx \qquad \text{[integrating]}$$
$$\int e^{2y}dy=\int(e^{3x}+x^2)dx$$
$$\frac{e^{2y}}{2}=\frac{e^{3x}}{3}+\frac{x^3}{3}+C.$$

Ans. Option (D) is correct.

> *Explanation:* Assertion (A) is wrong. The correct solution is given in Reason (R).

Q. 4. Assertion (A): The solution of differential equation $\frac{dy}{dx}=\frac{y}{x}+\tan\frac{y}{x}$ is $\cos\left(\frac{y}{x}\right)=xc$

Reason (R): $\frac{dy}{dx}=\frac{y}{x}+\tan\frac{y}{x}$ we can clearly see that it is an homogeneous equation substituting

$$y=vx$$
$$\Rightarrow \quad \frac{dy}{dx}=v+x\frac{dv}{dx}$$
$$\Rightarrow \quad v+x\frac{dv}{dx}=v+\tan v$$

separating the variables and integrating we get

$$\int\frac{1}{\tan v}dv=\int\frac{1}{x}dx$$
$$\log(\sin v)=\log x+\log C$$
$$\sin(v)=xC$$
$$\Rightarrow \quad \sin\left(\frac{y}{x}\right)=xC$$

is the solution where, C is constant.

Ans. Option (D) is correct.

> *Explanation:* Assertion (A) is wrong. The correct solution is given in Reason (R).

Q. 5. Assertion (A): The order and degree of the differential equation $\sqrt{\frac{d^2y}{dx^2}}=\sqrt{\frac{dy}{dx}+5}$ are 2 and 1 respectively

Reason (R): The differential equation

$$\left(\frac{dx}{dy}\right)^3+2y^{1/2}=x$$

is of order 1 and degree 3.

Ans. Option (B) is correct.

> *Explanation:* Squaring both sides of the given differential equation,
>
> $$\left(\sqrt{\frac{d^2y}{dx^2}}\right)^2=\left(\sqrt{\frac{dy}{dx}+5}\right)^2$$
> $$\Rightarrow \quad \frac{d^2y}{dx^2}=\frac{dy}{dx}+5$$
>
> The highest order is 2 and its power is 1
>
> ∴ Order is 2, degree is 1
>
> Hence, Assertion (A) is true.
>
> The equation given in reason (R) is,
>
> $$\left(\frac{1}{\frac{dy}{dx}}\right)^3+2\sqrt{y}=x$$
> $$\Rightarrow \quad \frac{1+2\sqrt{y}\left(\frac{dy}{dx}\right)^3}{\left(\frac{dy}{dx}\right)^3}=x$$
> $$\Rightarrow \quad 1+2\sqrt{y}\left(\frac{dy}{dx}\right)^3=x\left(\frac{dy}{dx}\right)^3$$
>
> Highest order is 1 and its power is 3
>
> ∴ Order is 1 and degree is 3.
>
> Hence, reason (R) is also true.

Q. 6. Assertion (A): The differential equation formed by eliminating *a* and *b* from $y=ae^x+be^{-x}$ is $\frac{d^2y}{dx^2}-y=0$

Reason (R):

$$y=ae^x+be^{-x} \qquad ...(i)$$

Differentiating w.r.t. 'x'

$$\frac{dy}{dx}=ae^x-be^{-x}$$

Differentiating again w.r.t. 'x'

$$\frac{d^2y}{dx^2}=ae^x+be^{-x} \qquad ...(ii)$$

Subtracting eqn. (i) from eqn. (ii)

$$\frac{d^2y}{dx^2}-y=ae^x+be^{-x}-ae^x-be^{-x}$$
$$=0$$

Ans. Option (B) is correct.

> *Explanation:* Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

CASE-BASED MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

A Veterinary doctor was examining a sick cat brought by a pet lover. When it was brought to the hospital, it was already dead. The pet lover wanted to find its time of death. He took the temperature of the cat at 11.30 pm which was 94.6°F. He took the temperature again after one hour; the temperature was lower than the first observation. It was 93.4°F. The room in which the cat was put is always at 70°F. The normal temperature of the cat is taken as 98.6°F when it was alive. The doctor estimated the time of death using Newton law of cooling which is governed by the differential equation: $\frac{dT}{dt} \propto (T-70)$, where 70°F is the room temperature and T is the temperature of the object at time t.

Substituting the two different observations of T and t made, in the solution of the differential equation $\frac{dT}{dt} = k(T-70)$ where k is a constant of proportion, time of death is calculated. **[CBSE QB-2021]**

Q. 1. What will be the degree of the above given differential equation.

(A) 2 **(B)** 1

(C) 0 **(D)** 3

Ans. Option (B) is correct.

Q. 2. Which method of solving a differential equation helped in calculation of the time of death?

(A) Variable separable method

(B) Solving Homogeneous differential equation

(C) Solving Linear differential equation

(D) all of the above

Ans. Option (A) is correct.

Q. 3. If the temperature was measured 2 hours after 11.30 pm, what will be the change in time of death?

(A) No change

(B) Death time increased

(C) Death time decreased

(D) Death time always constant

Ans. Option (A) is correct.

Q. 4. The solution of the differential equation $\frac{dT}{dt} = k(T-70)$ is given by,

(A) $\log|T-70| = kt + C$

(B) $\log|T-70| = \log|kt| + C$

(C) $T-70 = kt + C$

(D) $T-70 = kt\,C$

Ans. Option (A) is correct.

Q. 5. If $t = 0$ when T is 72, then the value of C is

(A) –2 **(B)** 0

(C) 2 **(D)** log 2

Ans. Option (D) is correct.

II. Read the following text and answer the following questions on the basis of the same:

Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution to the differential equation $\frac{dy}{dx} = k(50-y)$ where x denotes the number of weeks and y the number of children who have been given the drops. **[CBSE QB-2021]**

Q. 1. State the order of the above given differential equation.

(A) 2 **(B)** 1

(C) 0 **(D)** Can't define

Ans. Option (B) is correct.

Q. 2. Which method of solving a differential equation can be used to solve $\frac{dy}{dx} = k(50-y)$?

(A) Variable separable method

(B) Solving Homogeneous differential equation

(C) Solving Linear differential equation

(D) all of the above

Ans. Option (A) is correct.

Q. 3. The solution of the differential equation $\frac{dy}{dx} = k(50-y)$ is given by,

(A) $\log|50-y| = kx + C$

(B) $-\log|50-y| = kx + C$

(C) $\log|50-y| = \log|kx| + C$

(D) $50-y = kx + C$

Ans. Option (B) is correct.

Explanation:

$$\frac{dy}{dx} = k(50-y)$$

$$\int \frac{dy}{50-y} = \int K dx$$

$$-\log|50-y| = Kx + C$$

Q. 4. The value of C in the particular solution given that $y(0) = 0$ and $k = 0.049$ is

(A) $\log 50$ (B) $\log \frac{1}{50}$

(C) 50 (D) –50

Ans. Option (B) is correct.

Explanation:

Given, $y(0) = 0$ and $k = 0.049$

We have, $-\log|50-y| = Kx + C$

$$\log|50-y| = -Kx - C$$

$$\log|50-0| = 0 - C$$

$$[\because x = 0, K = 0.049, y(0) = 0]$$

$$\log 50 = -C$$

$$C = \log\frac{1}{50}$$

Q. 5. Which of the following solutions may be used to find the number of children who have been given the polio drops?

(A) $y = 50 - e^{kx}$ (B) $y = 50 - e^{-kx}$

(C) $y = 50(1 - e^{-kx})$ (D) $y = 50(e^{-kx} - 1)$

Ans. Option (C) is correct.

Explanation: We have

$$-\log|50-y| - Kx + C$$

$$-\log|50-y| = Kx + \log\frac{1}{50}$$

$$\log\frac{50-y}{50} = -Kx$$

$$\frac{50-y}{50} = e^{-Kx}$$

$$50 - y = 50e^{-Kx}$$

$$y = 50 - 50e^{-Kx}$$

$$y = 50(1-e^{-Kx})$$

III. Read the following text and answer the following questions on the basis of the same:

The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours.

Q. 1. $\int \frac{1}{Kx} dx =$ _______.

(A) $\log|x| + C$ (B) $\log|Kx| + C$

(C) $\frac{1}{K}\log|x| + C$ (D) $\frac{-1}{Kx^2} + C$

Ans. Option (C) is correct.

Explanation:

$$\int \frac{1}{Kx} dx = \frac{1}{K}\int \frac{1}{x} dx$$

$$= \frac{1}{K}\log|x| + C$$

Q. 2. If 'N' is the number of bacteria, the corresponding differential equation is _______.

(A) $\frac{dN}{dt} = Kt$ (B) $\frac{dN}{dt} = KN$

(C) $\frac{dK}{dt} = N$ (D) $\frac{dK}{dN} = t$

Ans. Option (B) is correct.

Explanation: Given that N is the number of bacteria.

$$\frac{dN}{dt} \propto N$$

$$\Rightarrow \quad \frac{dN}{dt} = KN$$

Q. 3. The general solution is _______.

(A) $\log|N| = Kt + C$ (B) $\log|Nt| = K + C$

(C) $\log|N| = t$ (D) $\log|Kt| = N + C$

Ans. Option (A) is correct.

Explanation:

$$\int \frac{dN}{N} = K\int dt$$

$$\log|N| = Kt + C \quad \text{...(i)}$$

Q. 4. If N_0 is the initial count of bacteria, after 10 hours the count is _______.

(A) $\frac{1}{5}\log 3$ (B) $3\log N_0$

(C) $9N_0$ (D) $2N_0$

Ans. Option (C) is correct.

Explanation: Given when $t = 0, N = N_0$.

From (i), $\log|N_0| = C$

$\therefore$ (i) $\rightarrow$ $\log|N| = Kt + \log|N_0|$

$$\Rightarrow \log\left|\frac{N}{N_0}\right| = Kt \quad ...(ii)$$

Given when $t = 5, N = 3N_0$.

From (ii), $\log|3| = 5K$

$$\Rightarrow K = \frac{1}{5}\log 3$$

$\therefore$ The particular solution is

$$\log\left|\frac{N}{N_0}\right| = \frac{t}{5}\log 3 \quad ...(3)$$

When $t = 10$,

$$\log\left|\frac{N}{N_0}\right| = 2\log 3 = \log 9$$

$$\frac{N}{N_0} = 9$$

$$\Rightarrow N = 9N_0$$

Q. 5. The bacteria becomes 10 times in _______ hours.

(A) $5\log 7$ **(B)** $\frac{5\log 10}{\log 3}$

(C) $\frac{5}{\log 3}$ **(D)** $\log\left(\frac{10^5}{3}\right)$

Ans. Option (B) is correct.

Explanation:

Given $N = 10N_0$,

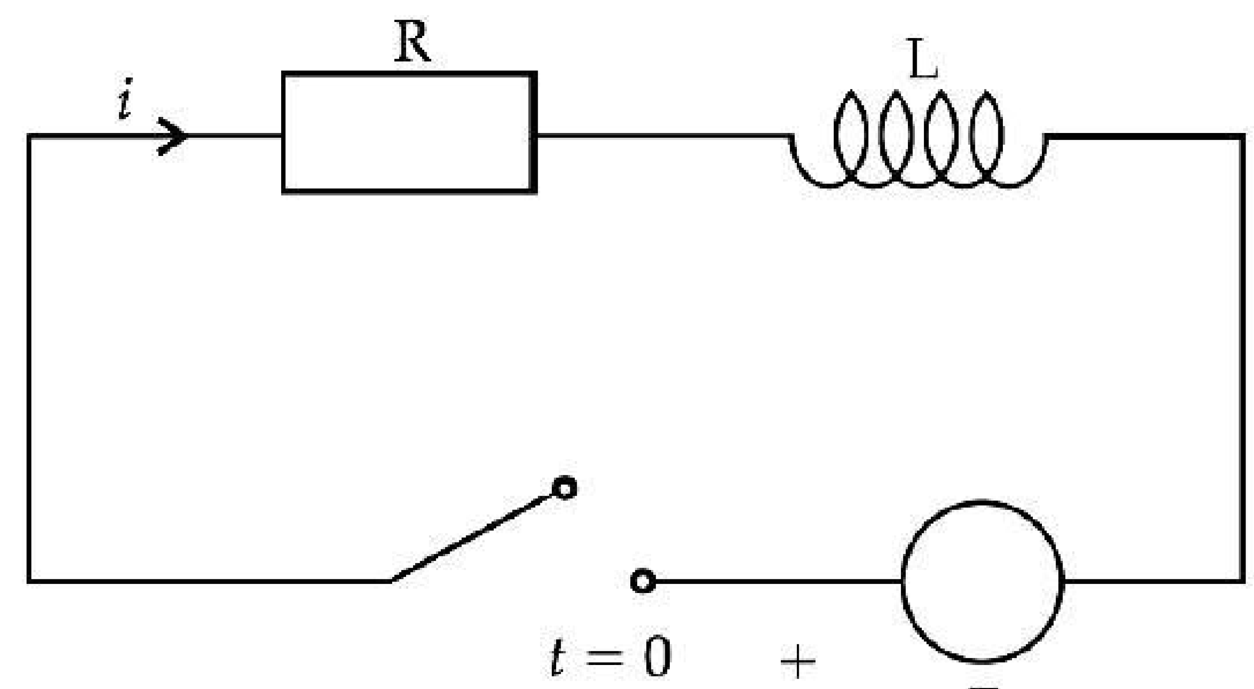
$$\log\left|\frac{N}{N_0}\right| = \frac{t}{5}\log 3$$

$$\text{(iii)} \Rightarrow \log 10 = \frac{t}{5}\log 3$$

$$t = \frac{5\log 10}{\log 3}$$

IV. Read the following text and answer the following questions on the basis of the same:

A differential equation of the form $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x alone, is called a first order linear differential equation. It has many applications in physics including RL circuits.



Switch on

Consider the linear differential equation

$$x\frac{dy}{dx} + 2y = x^2$$

Q. 1. $\int Q\,dx =$ _______.

(A) $\log|x| + C$ **(B)** $2x + C$

(C) $\log x^2 + C$ **(D)** $\frac{x^2}{2} + C$

Ans. Option (D) is correct.

Explanation: The given differential equation can be expressed as

$$\frac{dy}{dx} + \frac{2y}{x} = x$$

$$P = \frac{2}{x},\ Q = x$$

$$\int Q\,dx = \int x\,dx$$

$$= \frac{x^2}{2} + C$$

Q. 2. The value of $\int P\,dx =$ _______.

(A) $\frac{x^3}{3} + C$ **(B)** $\log x^2 + C$

(C) $\log|x| + C$ **(D)** $\frac{x^2}{2} + C$

Ans. Option (B) is correct.

Explanation:

$$\int P\,dx = \int \frac{2}{x}dx$$

$$= 2\log|x| + C$$

$$= \log x^2 + C$$

Q. 3. The integrating factor is _______.

(A) $\log|x|$ **(B)** $2x$

(C) x^2 **(D)** $e^{\log x}$

Ans. Option (C) is correct.

Explanation:

$$\text{IF} = e^{\int P\,dx}$$

$$= e^{\log x^2}$$

$$= x^2$$

Q. 4. The general solution is _______.

(A) $y = \frac{x^2}{4} + C$ **(B)** $yx^2 = \frac{x^4}{4} + C$

(C) $xy^2 = \frac{x^3}{3} + C$ **(D)** $y^2 = \frac{x^2}{3} + C$

Ans. Option (B) is correct.

Explanation: The general solution is

$$y(x^2) = \int (x^2 \times x)dx$$

$$x^2y = \frac{x^4}{4} + C$$

Q. 5. If $y(1) = 0$, then $y(2) =$ _______.

(A) 0

(B) 1

(C) $\frac{15}{4}$

(D) $\frac{15}{16}$

Ans. Option (D) is correct.

Explanation:

Given $y(1) = 0$

$$\Rightarrow \quad 0 = \frac{1}{4} + C$$

$$\Rightarrow \quad C = \frac{-1}{4}$$

The particular solution is

$$yx^2 = \frac{x^4 - 1}{4}$$

When $x = 2$,

$$4y = \frac{15}{4}$$

$$\Rightarrow \quad y = \frac{15}{16}$$