



Algebraic Identity

Objective

To verify the identity $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$, for simple cases using a set of unit cubes.

Pre-requisite knowledge

- 1. Express the volume of an object as the number of unit cubes in it.
- Knowledge of the identity

 (a b) (a² + ab + b²) = (a b) a² + (a b) (ab) + (a b) b²

Material Required

27 unit cubes made of wood (dimension is 1 unit × 1 unit × 1 unit).

Procedure

For representing $a^3 - b^3$

- Take a = 3 and make a cube of dimension 3 × 3 × 3 using 27 unit cubes. [Fig 22 (a)]
- Take b = 1 and remove a cube of dimension 1 × 1 × 1 from a³. [Fig 22 (b) and Fig 22 (c)]

For representing $(a - b) (a^2 + ab + b^2)$

- We will use the identity (a b) (a² + ab + b²) = (a b) a² + (a b) (ab) + (a b) b². For the given values of a and b, the dimensions of cuboids formed in right hand side of the identity are as follows
 - $(a b) a^2 = 2 \times 3 \times 3$ cubic units
 - $(a b) (ab) = 2 \times 3 \times 1$ cubic units
 - $(a b) b^2 = 2 \times 1 \times 1$ cubic units
- 2. Take the shape obtained by removing b³ from a³ and show that it splits into three cuboids of dimensions obtained in the above step. [Fig 22 (d)]

Observations

- 1. No. of unit cubes in $a^3 = 27$
- 2. No. of unit cubes in $b^3 = 1$
- 3. No. of unit cubes in $a^3 b^3 = 26$
- 4. No. of unit cubes in $(a b) a^2 = 18$
- 5. No. of unit cubes in (a b) (ab) = 6
- 6. No. of unit cubes in $(a b) b^2 = 2$
- 7. No. of unit cubes in $(a b) a^2 + (a b) (ab) + (a b) b^2 = 18 + 6 + 2 = 26$

8. It is verified that, $a^3 - b^3 = (a - b) a^2 + (a - b) ab + (a - b) b^2$ = $(a - b) (a^2 + ab + b^2)$

Learning Outcomes

- 1. The students obtain the skill of making cuboids of various dimensions using unit cubes.
- 2. The students obtain the skill of adding and subtracting the volume of cuboids.
- 3. Showing the volume of a cube as the sum of cuboids helps them to get a geometric feeling of volume.

Remark

- 1. Teachers can choose any value of a and b and verify the result.
- 2. Teachers should ensure that the students calculate the dimensions of cuboids formed on the right hand side for representing $(a b) (a^2 + ab + b^2)$.
- 3. The colour difference of the cubes is to suggest transformations (for activity 22-25).

