CLASS: XI CHAPTER: CONIC SECTIONS

EXERCISE: 11.3

In each of the exercises 1 to 9, find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and length of latus rectum of the ellipse.

Q No 1 $\frac{x^2}{36}+\frac{y^2}{16}=1$

Sol The equation of ellipse is $\frac{x^2}{36}+\frac{y^2}{16}=1$.

$\therefore \quad a^2=36 \; ; \; b^2=16$

$\Rightarrow \quad a=6 \; ; \; b=4.$

Now $c=\sqrt{a^2-b^2}=\sqrt{36-16}=\sqrt{20}=2\sqrt{5}$

$\therefore$ Foci are $(\pm c, 0)$ ie $(2\sqrt{5},0)$ and $(-2\sqrt{5},0)$.

Vertices are $(\pm a, 0)$ ie $(6,0)$ and $(-6,0)$

Length of Major axis $=2a=2\times6=12.$

Length of minor axis $=2b=2\times4=8$

Eccentricity $e=\frac{c}{a}=\frac{2\sqrt{5}}{6}=\frac{\sqrt{5}}{3}$

Length of Latus Rectum $=\frac{2b^2}{a}=\frac{2(4)^2}{6}=\frac{2\times16}{6}=\frac{16}{3}.$

Q No 2 $\frac{x^2}{4}+\frac{y^2}{25}=1.$

Sol. The equation of ellipse is $\frac{x^2}{4}+\frac{y^2}{25}=1$

Comparing with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

$a^2=25 \; ; \; b^2=4$

$\Rightarrow \quad a=5 \; ; \; b=2.$

$\therefore \quad c=\sqrt{a^2-b^2}=\sqrt{(5)^2-(2)^2}=\sqrt{25-4}=\sqrt{21}$

$\therefore$ Foci are $(0,\pm c)$ ie $(0,\sqrt{21}), (0,-\sqrt{21})$

Vertices are $(0,\pm a)$ ie $(0,\pm5)$

Length of Major axis $=2a=2\times5=10$

Length of Minor axis $=2b=2\times2=4$

Eccentricity $e=\frac{c}{a}=\frac{\sqrt{21}}{5}$, Length of Latus Rectum $=\frac{2b^2}{a}=\frac{2\times4}{5}=\frac{8}{5}$

Q.No.3 $\frac{x^2}{16}+\frac{y^2}{9}=1$

Sol. The equation of ellipse is $\frac{x^2}{16}+\frac{y^2}{9}=1$.

Here $a^2=16$ and $b^2=9$

$\Rightarrow a=4$ and $b=3$

$\therefore c=\sqrt{a^2-b^2}=\sqrt{16-9}=\sqrt{7}$

$\therefore$ Foci are $(\pm c, 0)$ ie $(\pm\sqrt{7}, 0)$

Vertices are $(\pm a, 0)$ ie $(\pm 4, 0)$

Length of Major Axis $=2a=2\times 4=8$

Length of Minor Axis $=2b=2\times 3=6$

Eccentricity $e=\frac{c}{a}=\frac{\sqrt{7}}{4}$

Length of Latus Rectum $=\frac{2b^2}{a}=\frac{2\times 9}{4}=\frac{9}{2}$

Q.No.4. $\frac{x^2}{25}+\frac{y^2}{100}=1$

Sol Equation of Ellipse is $\frac{x^2}{25}+\frac{y^2}{100}=1$

Here $a^2=100$ and $b^2=25$

$\Rightarrow a=10$ and $b=5$

$\therefore c=\sqrt{a^2-b^2}=\sqrt{100-25}=\sqrt{75}=5\sqrt{3}$.

$\therefore$ Foci are $(0,\pm c)=(0,\pm 5\sqrt{3})$

Vertices are $(0,\pm a)=(0,\pm 10)$

Length of Major Axis $=2a=2\times 10=20$

Length of Minor Axis $:2b=2\times 5=10$

Eccentricity $e=\frac{c}{a}=\frac{5\sqrt{3}}{10}=\frac{\sqrt{3}}{2}$

Length of Latus Rectum $=\frac{2b^2}{a}=\frac{2\times 25}{10}=5$.

Q.No 5. $\frac{x^2}{49}+\frac{y^2}{36}=1$

Sol: Equation of Ellipse is $\frac{x^2}{49}+\frac{y^2}{36}=1$.

Here $a^2=49$ and $b^2=36$

$\Rightarrow a=7$ and $b=6$

$\therefore c=\sqrt{a^2-b^2}=\sqrt{49-36}=\sqrt{13}$

$\therefore$ Foci are $(\pm c, 0)$ ie $(\pm\sqrt{13}, 0)$

Vertices are $(\pm a, 0)$ ie $(\pm 7, 0)$

Length of Major Axis $= 2a = 2\times7 = 14$.

Length of Minor Axis $= 2b = 2\times6 = 12$.

Eccentsicity $e = \frac{c}{a} = \frac{\sqrt{13}}{7}$

Length of Latus Rectum $= \frac{2b^2}{a} = \frac{2\times36}{7} = \frac{72}{7}$

Q No. 6 $\frac{x^2}{100} + \frac{y^2}{400} = 1$

Sol: The equation of ellipse is $\frac{x^2}{100} + \frac{y^2}{400} = 1$

$\therefore$ Here $a^2 = 400$, $b^2 = 100$.

$\Rightarrow a = 20$, $b = 10$

$\therefore c = \sqrt{a^2 - b^2} = \sqrt{400-100} = \sqrt{300} = \sqrt{3\times100} = 10\sqrt{3}$

$\therefore$ Foci are $(0, \pm c)$. ie $(0, \pm 10\sqrt{3})$

Vertices are $(0, \pm a)$ ie $(0, \pm 20)$

Length of major axis $= 2a = 2\times20 = 40$.

Length of minor axis $= 2b = 2\times10 = 20$

Eccentsicity $e = \frac{c}{a} = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2}$

Length of Latus Rectum $= \frac{2b^2}{a} = \frac{2\times100}{20} = 10$

Q No. 7: $36x^2 + 4y^2 = 144$

Sol. The equation of ellipse is $36x^2 + 4y^2 = 144$

or $\frac{36x^2}{144} + \frac{4y^2}{144} = 1$ or $\frac{x^2}{4} + \frac{y^2}{36} = 1$.

Here $a^2 = 36$ and $b^2 = 4$

$\Rightarrow a = 6$ and $b = 2$.

$\therefore c = \sqrt{a^2 - b^2} = \sqrt{36-4} = \sqrt{32} = \sqrt{16\times2} = 4\sqrt{2}$

$\therefore$ Foci are $(0, \pm c)$ ie $(0, \pm 4\sqrt{2})$

Vertices are $(0, \pm a)$ ie $(0, \pm 6)$

Length of major axis $= 2a = 2\times6 = 12$.

Length of minor axis $= 2b = 2\times2 = 4$.

Eccentricity $e = \frac{c}{a} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$

Length of Latus Rectum $= \frac{2b^2}{a} = \frac{2\times4}{6} = \frac{4}{3}$

Q No. 8 $16x^2+y^2=16$

Sol. The equation of ellipse is $16x^2+y^2=16$

or $\frac{x^2}{1}+\frac{y^2}{16}=1$

$\therefore$ $a^2=16$, $b^2=1$.

$\therefore$ $a=4$; $b=1$.

$\therefore$ $c=\sqrt{a^2-b^2}=\sqrt{16-1}=\sqrt{15}$

Foci are $(0, \pm c)$ i.e. $(0, \pm\sqrt{15})$

Vertices are $(0, \pm a)$ ie $(0, \pm 4)$

Length of major axis $= 2a = 2\times4 = 8$

Length of minor axis $= 2b = 2\times1 = 2$

Eccentricity $e = \frac{c}{a} = \frac{\sqrt{15}}{4}$.

Length of Latus Rectum $= \frac{2b^2}{a} = \frac{2\times1}{4} = \frac{1}{2}$.

Q No. 9 $4x^2+9y^2=36$

Sol: The equation of ellipse is $4x^2+9y^2=36$

or $\frac{x^2}{9}+\frac{y^2}{4}=1$.

$\therefore$ $a^2=9$ and $b^2=4$.

$\Rightarrow$ $a=3$ and $b=2$.

$\therefore$ $c=\sqrt{a^2-b^2}=\sqrt{9-4}=\sqrt{5}$

Foci are $(\pm c, 0)$ ie $(\pm\sqrt{5}, 0)$

Vertices are $(\pm a, 0)$ ie $(\pm 3, 0)$

Length of Major axis $= 2a = 2\times3 = 6$.

Length of Minor axis $= 2b = 2\times2 = 4$

Eccentricity $e = \frac{c}{a} = \frac{\sqrt{5}}{3}$.

Length of Latus Rectum $= \frac{2b^2}{a} = \frac{2\times4}{3} = \frac{8}{3}$

In each of the following Exercises 10 to 20 find the equation for the ellipse that satisfies the given conditio

Q No 10 Vertices $(\pm 5, 0)$, Foci $(\pm 4, 0)$

Sol. Since the vertices are on x-axis

$\therefore$ Eqn of ellipse is of the form.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \ldots\ldots (1)$$

Vertices are $(\pm 5, 0)$

$\therefore \quad a = 5 \quad \Rightarrow a^2 = 25$

Foci are $(\pm 4, 0)$

$\therefore \quad c = 4.$

Now $c^2 = a^2 - b^2 \Rightarrow 16 = 25 - b^2 \quad \rightarrow b^2 = 9$

Putting values of a^2 and b^2 in (1)

$$\frac{x^2}{25} + \frac{y^2}{9} = 1.$$

Q No 11. Vertices $(0, \pm 13)$, foci $(0, \pm 5)$

Since the vertices are on y-axis

$\therefore$ Equation ellipse is of the form.

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1. \quad \ldots (1)$$

Vertices are $(0, \pm 13)$.

$\therefore \quad a = 13 \quad \Rightarrow a^2 = 169$

Foci are $(0, \pm 5)$

$\therefore \quad c = 5$

Now $c^2 = a^2 - b^2 \Rightarrow 25 = 169 - b^2 \Rightarrow b^2 = 144$

$\therefore$ from (1) $\quad \dfrac{x^2}{144} + \dfrac{y^2}{169} = 1$, which is required eqn.

Q No 12. Vertices $(\pm 6, 0)$, foci $(\pm 4, 0)$

Sol. Since the vertices are on x-axis

$\therefore$ Equation of ellipse is of form $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \quad \ldots (1)$

Vertices are $(\pm 6, 0)$

$\therefore a = 6 \Rightarrow a^2 = 36$

Foci are $(\pm 4, 0) \Rightarrow c = 4$

Now $c^2 = a^2 - b^2 \Rightarrow 16 = 36 - b^2 \Rightarrow b^2 = 20$

$\therefore$ from (1) $\frac{x^2}{36} + \frac{y^2}{20} = 1$, which is required eqn.

Q No. 13: Ends of major axis $(\pm 3, 0)$, ends of minor axis $(0, \pm 2)$

Sol: Since the ends of major axis are on x-axis

$\therefore$ Equation of ellipse is of form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \cdots (1)$

Ends of major axis are $(\pm 3, 0) \Rightarrow a = 3 \Rightarrow a^2 = 9$

Ends of minor axis are $(0, \pm 2) \Rightarrow b = 2 \Rightarrow b^2 = 4$

$\therefore$ from (1) $\frac{x^2}{9} + \frac{y^2}{4} = 1$. which is required eqn.

Q No 14. Ends of major axis $(0, \pm\sqrt{5})$, ends of minor axis $(\pm 1, 0)$

Sol: Since ends of major axis are on y-axis

$\therefore$ Eqn. of ellipse is of form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1. \quad \cdots (1)$

Ends of major axis are $(0, \pm\sqrt{5}) \Rightarrow a = \sqrt{5} \Rightarrow a^2 = 5$

Ends of minor axis are $(\pm 1, 0) \Rightarrow b = 1 \Rightarrow b^2 = 1$.

$\therefore$ from (1) $\frac{x^2}{1} + \frac{y^2}{5} = 1$ which is required eqn.

Q No 15. Length of major axis 26, foci $(\pm 5, 0)$

Sol: Since foci are on x-axis

$\therefore$ Eqn of ellipse is of form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \cdots (1)$

Length of major axis is $2a = 26 \Rightarrow a = 13 \Rightarrow a^2 = 169$

Foci are $(\pm 5, 0) \Rightarrow c = 5$

Now $c^2 = a^2 - b^2 \Rightarrow 25 = 169 - b^2 \Rightarrow b^2 = 144$.

$\therefore$ from (1) $\frac{x^2}{169} + \frac{y^2}{144} = 1$

Q No 16. Length of minor axis 16, foci $(0, \pm 6)$

Sol Since foci are on y-axis

$\therefore$ Eqn of ellipse is of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ ...(1)

Now Length of minor axis i.e $2b = 16 \Rightarrow b = 8 \Rightarrow b^2 = 64$.

foci are $(0, \pm 6) \Rightarrow c = 6 \Rightarrow c^2 = 36$

$\therefore$ $c^2 = a^2 - b^2 \Rightarrow a^2 = c^2 + b^2 = 36 + 64 = 100$

$\therefore$ From (1) $\frac{x^2}{64} + \frac{y^2}{100} = 1$. which is reqd eqn.

Q No 17. Foci $(\pm 3, 0)$, $a = 4$.

Sol. Since foci are on x-axis

$\therefore$ Eqn. of ellipse is of form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(1)

Now $a = 4 \Rightarrow a^2 = 16$.

foci are $(\pm 3, 0) \Rightarrow c = 3 \Rightarrow c^2 = 9$

Now $c^2 = a^2 - b^2 \Rightarrow 9 = 16 - b^2 \Rightarrow b^2 = 7$.

$\therefore$ from (1) $\frac{x^2}{16} + \frac{y^2}{7} = 1$ which is reqd. eqn.

Q No 18. $b = 3$, $c = 4$, centre at origin, foci on x-axis

Sol. Since foci are on x-axis and centre at origin

$\therefore$ Eqn. of ellipse is of form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(1)

Now $b = 3 \Rightarrow b^2 = 9$

$c = 4 \Rightarrow c^2 = 16$

$\therefore$ $c^2 = a^2 - b^2 \Rightarrow 16 = a^2 - 9 \Rightarrow a^2 = 25$.

$\therefore$ from (1) Eqn of ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1$

Q No. 19 Centre at $(0,0)$ major axis on y-axis and passes through the points $(3,2)$ and $(1,6)$

Sol Since major axis is on y-axis and centre at origin $(0,0)$

$\therefore$ Eqn of ellipse is of form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ ...(1)

Since it passes through points (3,2) and (1,6)

$\therefore \quad \frac{9}{b^2} + \frac{4}{a^2} = 1 \quad \cdots (2) \qquad$ and $\qquad \frac{1}{b^2} + \frac{36}{a^2} = 1 \cdots (3)$

Multiplying (2) by 9, we get

$$\frac{81}{b^2} + \frac{36}{a^2} = 9 \quad \cdots (4)$$

Subtracting (3) from (4) we get $\frac{80}{b^2} = 8 \Rightarrow b^2 = \frac{80}{8} = 10$

From (2) $\quad \frac{9}{10} + \frac{4}{a^2} = 1 \Rightarrow \frac{4}{a^2} = \frac{1}{10} \Rightarrow a^2 = 40.$

$\therefore$ Eqn of ellipse is from (1) $\quad \frac{x^2}{10} + \frac{y^2}{40} = 1.$

Q No. 20. Major axis on x-axis and passes through the points (4,3) and (6,2)

Sol. Since major axis is on x-axis.

$\therefore$ Eqn of ellipse is of form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \cdots (1)$

Since it passes through (4,3) and (6,2)

$$\therefore \quad \frac{16}{a^2} + \frac{9}{b^2} = 1 \quad \cdots (2)$$

and $$\frac{36}{a^2} + \frac{4}{b^2} = 1 \quad \cdots (3)$$

Multiplying (2) by 4 and (3) by 9 we get

$$\frac{64}{a^2} + \frac{36}{b^2} = 4 \quad \cdots (4)$$

$$\frac{324}{a^2} + \frac{36}{b^2} = 9 \quad \cdots (5)$$

Subtracting (4) from (5) we get

$$\frac{260}{a^2} = 5 \Rightarrow a^2 = \frac{260}{5} = 52.$$

Putting $a^2 = 52$ in (2) we get

$$\frac{16}{52} + \frac{9}{b^2} = 1 \quad \text{or} \quad \frac{9}{b^2} = 1 - \frac{16}{52} \quad \text{or} \quad \frac{9}{b^2} = 1 - \frac{4}{13}$$

$$\text{or} \quad \frac{9}{b^2} = \frac{9}{13} \Rightarrow b^2 = 13$$

$\therefore$ From (1) $\quad \frac{x^2}{52} + \frac{y^2}{13} = 1 \quad$ or $\quad x^2 + 4y^2 = 52$

which is required equation of ellipse.