Chapter 3

Transient Analysis (AC and DC)

LEARNING OBJECTIVES

After reading this chapter, you will be able to understand:

- Classification of transients
- Singularity functions
- Step response of an *RC* circuit
- Transient response
- Steady-state response
- Higher-order circuits
- AC transients
- Sinusoidal steady-state analysis
- Phasor
- Inductor
- Sinusoidal steady state analysis of *RLC* circuits
- AC transients
- Sinusoidal steady-state analysis
- Sinusoidal steady state analysis of *RLC* circuits

Introduction

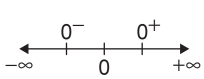
Whenever a circuit is switched from one condition to another, either by a change in the applied source or a change in the circuit elements, there is a transitional period during which the branch currents and element voltages change from their initial values to new ones. This period is called the transient period. After the transient period has passed, the circuit is said to be in steady state.

Transient in the system is because of the presence of energy storing elements (i.e., *L* and *C*).

Since the energy stored in a memory element cannot change instantaneously, i.e., within zero time.

The network consists of only resistances, no transients in the system at the time of switching. Since the resistor can accommodate any amount of voltage and currents.

The equivalent form of the elements in terms of the initial condition of the elements.

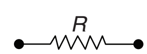
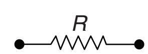
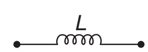
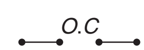
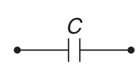
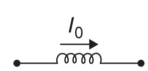
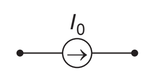
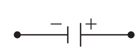
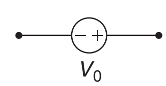


1. $Z_L = sL\ \Omega$

2. $Z_C = \dfrac{1}{sC}\ \Omega$

At $t = 0^+ \Rightarrow f = \infty \Rightarrow Z_L = \infty \Rightarrow L \Rightarrow 0.C$

$Z_C = \dfrac{1}{2\pi fc} = 0\ \Omega \Rightarrow C \Rightarrow$ Short circuit

S. No	Element (Initial Condition)	Equivalent Circuit at $t = 0^+$
1.		
2.		
3.		
4.		
5.	 $V_0 = \dfrac{q_0}{C}$	

A long time after the switching action ($t \to \infty$) is the steady state. In *S.S*, the inductor behaviours is a short circuit and capacitor behaviours is an open circuit.

$t \to \infty \Rightarrow f = 0.$

$Z_L = sL\ \Omega \Rightarrow Z_L = 0\ \Omega \Rightarrow S.C$

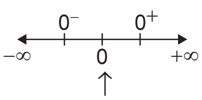
$Z_C = \dfrac{1}{sC}\ \Omega \Rightarrow Z_C = \infty \Rightarrow$ Open circuit.

* The equivalent form of the elements in terms of the final condition of the element.

S.No	Element (and Initial Condition)	Equivalent Circuit at $t = \infty$. (Steady State Values)
1.	R	R
2.	L	S.C
3.	C	O.C
4.	I_0	S.C, I_0
5.	C, $V_0 = \frac{q_0}{C}$	V_0, O.C

Notes:

1. Inductor current at $t = 0^-$ and $t = 0^+$ instants



Current flowing through an inductor is given as

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} V_L(t)\cdot dt$$

$$= \frac{1}{L}\int_{-\infty}^{0^-} V_L(t)dt + \frac{1}{L}\int_{0^-}^{0^+} V_L(t)dt$$

$$i_L(0^+) = i_L(0^-) + 0$$

$$\therefore \quad i_L(0^+) = i_L(0^-)$$

Therefore, inductor current cannot change instantaneously.

2. Capacitor voltage at $t = 0^-$ and at $t = 0^+$ instants

$$V_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C \cdot dt$$

$$= V_C(0^-) + \frac{1}{C}\int_{0^-}^{t} i_C(t)\cdot dt$$

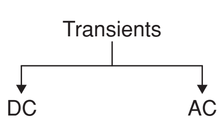
At $t = 0^+$

$$V_C(0^+) = V_C(0^-) + \frac{1}{C}\int_{0^-}^{0^+} i_C(t)\cdot dt$$

$$V_C(0^+) = V_C(0^-)\ \text{V}$$

Classification of Transients

The transient effects are more for DC as compared to AC and the transient free condition is possible to only for AC excitations.



DC Transients

Source free circuits

1. RL Circuit $\rightarrow$ initial current through the inductor ($L \rightarrow I_0$)
2. RC Circuit $\rightarrow$ initial voltage (V_0) across the capacitor ($C \rightarrow V_0$)
3. RLC Circuit $\rightarrow$ initial current through inductor or voltage across capacitor.

Source free RC circuits: A source free RC circuit occurs when its DC source is suddenly disconnected. The energy already stored in the capacitor is released to the resistors.

Consider a series combination of a resistor and an initially charged capacitor as shown in figure 1.

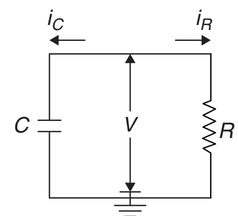


Figure 1 A source-free RC circuit.

The capacitor is initially charged, we can assume that at time $t = 0$, the initial voltage is

$$V(0) = V_0$$

Applying KCL at the node of circuit in figure 1

$$i_C + i_R = 0$$

$$C\cdot\frac{dV}{dt} + \frac{V}{R} = 0 \quad \Rightarrow \quad \frac{dV}{dt} + \frac{1}{RC}\cdot V = 0$$

$$\frac{dV}{V} = -\frac{1}{RC}\cdot dt$$

Integrating both sides, we get

$$\ln\ V = \frac{-t}{RC} + \ln\ A$$

where A is the integration constant. Thus,

$$\ln\frac{V}{A} = \frac{-t}{RC} \quad \Rightarrow \quad \frac{V}{A} = e^{\frac{-t}{RC}}$$

$\therefore \quad V(t) = A\cdot e^{-t/RC}$

But from initial conditions at $t = 0$

$$V(0) = A\cdot 1\ A = V(0) = V_0$$

$$\boxed{\therefore \quad V(t) = V_0 \cdot e^{\frac{-t}{RC}}\text{V}}$$

i.e., it is called the natural response of the circuit. The natural response of a circuit refers to the behaviour of the circuit itself, with no external sources of excitation.

The natural response is illustrated graphically in figure 2.

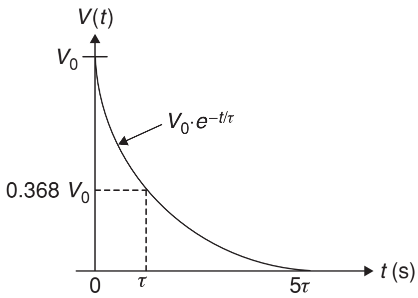


Figure 2 The voltage response of the *RC* circuit.

Time constant: The time constant of a circuit is the time required for the response to decay to a factor of $1/e$ or 36.8% of its initial value.

At $t = \tau$

$$V_0 \cdot e^{-\tau/RC} = V_0 \cdot e^{-1} = 0.368\ V_0$$

$\therefore \quad \tau = RC$

$$\boxed{\therefore \quad V(t) = V_0 \cdot e^{\frac{-t}{RC}} = V_0 \cdot e^{\frac{-t}{\tau}}\ \text{V}}$$

Note: In finding the time constant $\tau = RC$, R is obtained the thevenin's equivalent resistance at the terminals of the capacitor

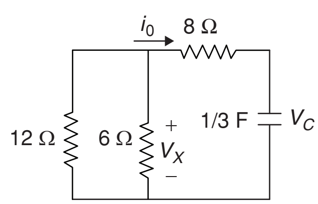
$\therefore \quad \tau = R_{eq} \cdot C = R_{th} \cdot C$

$$i_R(t) = \frac{V(t)}{R} = \frac{V_0}{R} \cdot e^{\frac{-t}{\tau}}\ \text{A}$$

Power dissipated in the resistor (R) is

$$P(t) = V \cdot i_R = \frac{V_0^2}{R} \cdot e^{\frac{-2t}{\tau}}$$

Examples 1: The circuit shown in the below figure. Let $V_C(0) = 15$ V



The value of $V_C(t)$, at $t = 2$ s

(A) 12 V (B) 13.62 V

(C) 2.04 V (D) 9.09 V

Solution: (D)

We know for source free *RC* circuits

$V_c(t) = V_0 \cdot e^{-t/RC}$

$\tau = R_{eq} \cdot C \quad \Rightarrow \quad R_{eq} = 8 + (6 \| 12) \quad \Rightarrow \quad 12\ \Omega$

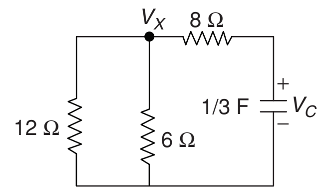
$\tau = 12 \times 1/3 = 4$ s

$V_c(t) = 15 \cdot e^{-t/4}$ V

at $t = 2$ s

$V_c(2) = 15 \cdot e^{-2/4} \quad \Rightarrow \quad 15 \cdot e^{-0.5} = 9.09$ V

Examples 2: The value of V_x for $t \geq 0$



(A) $V_x = 5 \cdot e^{-4t}$ V (B) $V_x = 12 \cdot e^{-0.25t}$ V

(C) $V_x = 5 \cdot e^{-0.25t}$ V (D) $V_x = 15 \cdot e^{-0.71t}$ V

Solution: (C)

Applying KCL at node V_x.

$$\frac{V_x}{12} + \frac{V_x}{6} + \frac{V_x - V_c}{8} = 0$$

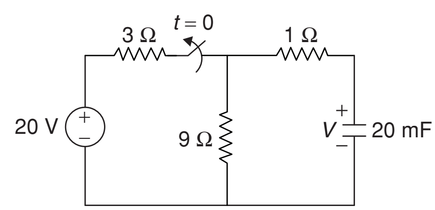
$2V_x + 4V_x + 3(V_x - V_c) = 0$

$9V_x = 3V_c$

$V_x = 1/3\ V_c \quad \Rightarrow \quad V_x = 1/3 \times 15 \cdot e^{-t/4}$ V.

$V_x = 5 \cdot e^{-0.25t}$ V.

Examples 3: The switch in the circuit in the below figure has been closed for a long time, and it is opened at $t = 0$



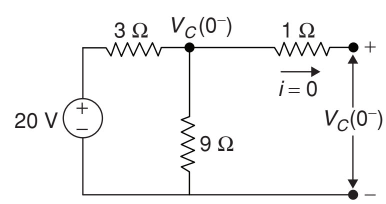
(i) The $V_C(t)$ for $t \geq 0$ is

(A) $V_C(t) = 15 \cdot e^{-t}$ V (B) $V_C(t) = 12 \cdot e^{-0.2t}$ V

(C) $V_C(t) = 15 \cdot e^{-5t}$ V (D) $V_C(t) = 12 \cdot e^{-5t}$ V

Solution: (C)

For $t < 0$: The switch is closed, the capacitor is open circuit to DC.



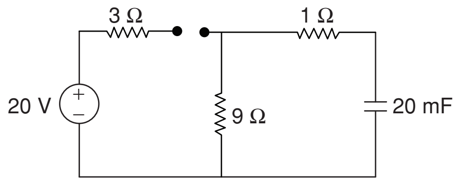
Circuit is in *S* and *S*.

$$\therefore \quad V_c(0^-) = 20 \times \frac{9}{9+3} = 15 \text{ V for } t < 0.$$

Since the voltage across capacitor does not change instantaneously,

$V_c(0^-) = V_c(0^+) = 15$ V

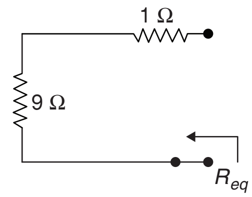
For $t \geq 0$. The switch is opened. The circuit shown in the following figure.



$\therefore \quad V_C(t) = V_0 \cdot e^{-t/}\tau$

$\tau = R_{eq} \cdot c.$

$R_{eq} \Rightarrow$



$R_{eq} = (9 + 1) = 10\ \Omega$

$\tau = 10 \times 20 \times 10^{-3}\ \tau = 0.2$ s.

Thus, the voltage across the capacitor for $t \geq 0$ is

$V(t) = V_0 \cdot e^{-t/\tau}$

$= 15 \cdot e^{-t/0.2}$ V

$\therefore V(t) = 15 \cdot e^{-5t}$ V

(ii) The initial energy stored in the capacitor is

(A) 4.5 J (B) 0 J

(C) 2.25 J (D) 4 J

Solution: (C)

$$W_c(0) = \frac{1}{2}CV_c^2(0) = \frac{1}{2}CV_o^2$$

$= 1/2 \times 20 \times 10^{-3} \times (15)^2 = 2.25$ J

The source free RL circuits: Consider the series connection of a RL circuit as shown in figure.3.

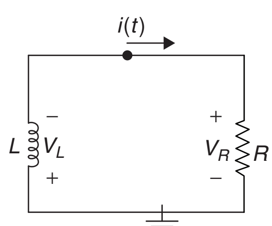


Figure 3 A source free *RL* circuit.

At $t = 0$, we assume that the inductor has on initial current I_0.

$\therefore i(0) = I_0$

Applying KVL around the loop in figure 3.

$$V_L + V_R = 0$$

$$L \cdot \frac{di}{dt} + iR = 0$$

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

$$\text{Let } \frac{d}{dt} = D$$

$$\left(D + \frac{R}{L}\right)i = 0$$

$D = -R/L.$

So $i(t) = A \cdot e^{-Dt} \quad \Rightarrow \quad i(t) = A \cdot e^{-Rt/L}$

$\therefore i_L(t) = A \cdot e^{-t/(L/R)}$

at $t = 0 \quad \Rightarrow \quad i_L(0) = I_0$

$i_L(0) = I_0 = A \cdot e^{-0} \quad \Rightarrow \quad A = I_0$

$$\boxed{i_L(t) = I_0 \cdot e^{-\frac{Rt}{L}} \text{A}}$$

$\therefore \quad \tau = \frac{L}{R}$ s

$\tau = \frac{L_{eq}}{R_{eq}}$ s

$\therefore i(t) = I_0.e^{-t/\tau}$ A

This shows that the natural response of the *RL* circuit is an exponential decay of the initial current.

The current response shown in figure 4.

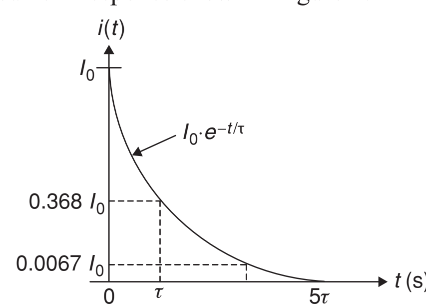
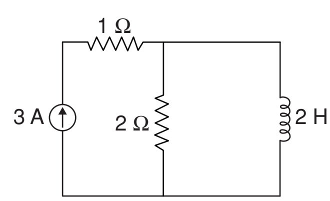


Figure 4 The current response of the *RL* circuit

Note: When $\quad \tau = RC = \frac{L}{R}$

$$R^2 = \frac{L}{C} \quad \Rightarrow \quad R = \sqrt{\frac{L}{C}}\ \Omega$$

Examples 4: The time constant of the given circuit τ is



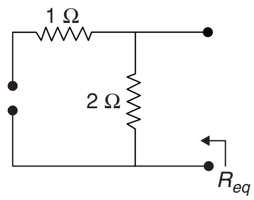
(A) 1 s (B) 1.5 s

(C) $\frac{2}{3}$ s (D) 4 s

Solution: (A)

$$\tau = \frac{L_{eq}}{R_{eq}}.$$

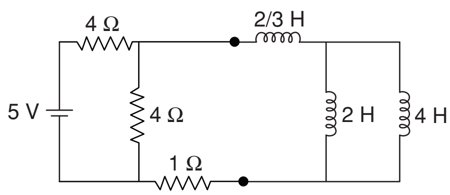
$$R_{eq} \Rightarrow R_{th}$$



$$\Rightarrow \quad R_{eq} = 2\ \Omega.$$

$$\tau = \frac{2}{2} = 1\text{ s}$$

Examples 5:



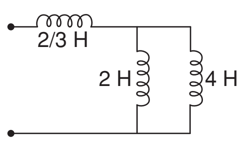
The time constant τ is

(A) 1 s (B) 1.5 s

(C) $\frac{2}{3}$ s (D) 4 s

Solution: (C)

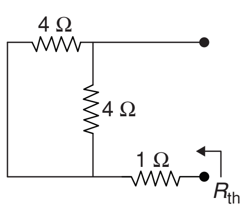
$$\tau = \frac{L_{eq}}{R_{eq}}\text{ s}$$



$$L_{eq} = \frac{2}{3} + (2 \,||\, 4)$$

$$= \frac{2}{3} + \frac{4}{3} = 2\text{ H.}$$

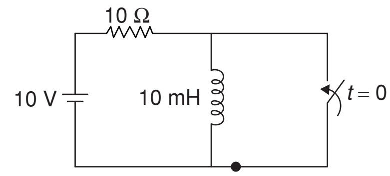
$$R_{eq} \quad \Rightarrow \quad R_{th}$$



$$R_{th} = 1 + (4||4) = 3\ \Omega$$

$$\tau = \frac{2}{3}\text{ s}$$

Examples 6: The circuit shown in the figure is in a steady state. When the switch is closed at $t = 0$. The current through the inductor at $t = 0^+$ is



(A) 0 A (B) 0.5 A

(C) 1 A (D) 2 A

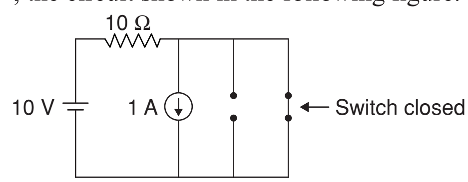
Solution: For $t < 0$:

at $t = 0^-$; the switch is opened.

In steady state. L behaves as a short circuit.

$$i_L(0^-) = \frac{10}{10} = 1\text{ A.}$$

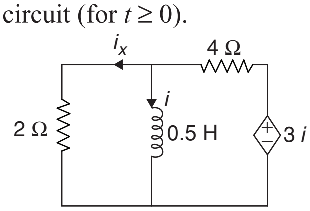
at $t = 0^+$; the circuit shown in the following figure.



Therefore, inductor does not allow sudden change in current

$\therefore\ i_L(0^+) = i_L(0^-) = 1$ A.

Examples 7: Assume that $i(0)$ = 10 A. Calculate the $i(t)$ and $i_x(t)$ in the circuit (for $t \geq 0$).



(A) $i(t) = 10 \cdot e^{-2t/3}$ A; $i_x(t) = -5/3 \cdot e^{-2t/3}$ A

(B) $i(t) = 10 \cdot e^{-3t/8}$ A; $i_x(t) = 5/3\ e^{-3t/8}$ A

(C) $i(t) = 10 \cdot e^{-4t}$ A; $i_x(t) = -20 \cdot e^{-3t/8}$ A

(D) None of the above

Solution: (A)

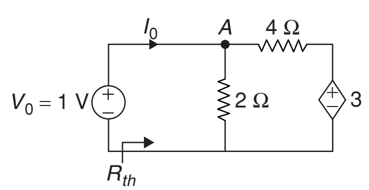
$$i(t) = I_0 \cdot e^{\frac{-t}{\tau}}$$

Given $I_0 = 10$ A

$$\tau = \frac{L_{eq}}{R_{eq}}$$

$$R_{eq} = R_{th}:$$

The equivalent resistance is the same as the thevenin's resistance at the inductor terminals.



Here $I_0 = -i$

Applying KCL at node A.

$$\frac{V_0}{2} - I_0 + \frac{V_0 - 3i}{4} = 0$$

$$2V_0 - 4I_0 + V_0 + 3I_0 = 0$$
$$3V_0 = I_0$$

$$\frac{V_0}{I_0} = R_{th} = \frac{1}{3}\,\Omega$$

$$\tau = \frac{L_{eq}}{R_{th}} = \frac{0.5}{\left(\frac{1}{3}\right)} = 1.5\text{ s}$$

Therefore, thus, the current through the inductor is $i(t)$

$$i(t) = 10 \cdot e^{-t/1.5}\text{ A}$$

$$i(t) = 10 \cdot e^{-2t/3}\text{ A for } t \geq 0.$$

$$V_L(t) = L \cdot \frac{di(t)}{dt}$$

$$= 0.5 \times 10 \frac{d}{dt} e^{\frac{-2t}{3}}$$

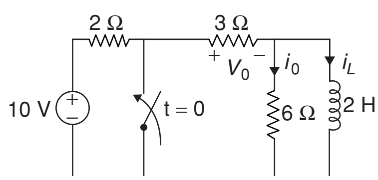
$$= 5 \times \left(\frac{-2}{3}\right) \cdot e^{\frac{-2t}{3}} V$$

$$= \frac{-10}{3} \cdot e^{\frac{-2t}{3}} V.$$

Voltage across inductor equal to voltage across 2 Ω resistors.

$$\therefore \quad i_x(t) = \frac{V_L(t)}{2} = \frac{-5}{3} \cdot e^{\frac{-2t}{3}}\text{ A. for } t > 0.$$

Examples 8: In the circuit shown in figure assuming that the switch was open for along time.



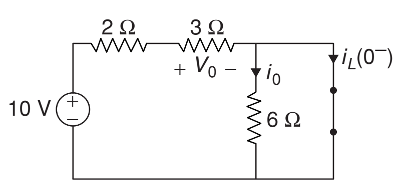
Find i_0, V_0, and i_L for all times $(-\infty \leq t \leq \infty)$.

Solution: $t < 0$:

for $t < 0$ the switch is open.

At $t = 0^-$

the inductor acts like a short circuit to DC



The current through 6 Ω $\Rightarrow$ $i_0 = 0$ A.

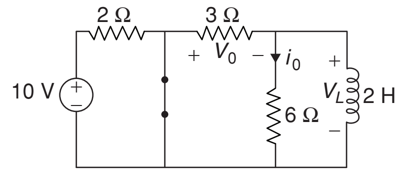
$$V_{6\Omega} = 0\text{ V} \quad \Rightarrow \quad i_0 = 0\text{ A}$$

$$i_{L(0^-)} = \frac{10}{5} = 2\text{ A.}$$

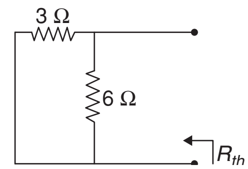
$$V_0 = 3 \times 2 = 6\text{ V.}$$

(i) for $t > 0$: The switch is closed

$i_L(t) = I_0 \cdot e^{-t/\tau}$ A.



find R_{th}:



$$R_{th} = 6 \,\|\, 3 \quad \Rightarrow \quad \frac{6 \times 3}{9}$$

$$R_{th} = 2\ \Omega.$$

$$\tau = \frac{L}{R_{th}} = \frac{2}{2} = 1\text{ s}$$

thus $i_L(t) = i(0^+) \cdot e^{\frac{-t}{1}}$

$= 2.e^{-t}$A. for $t > 0$.

(ii) $i_0(t) = \dfrac{V_L(t)}{6}$

$$\Rightarrow \quad V_L(t) = L \cdot \frac{di(t)}{dt} = 2 \times \frac{d}{dt}(2 \times e^{-t})$$

$$= -4.e^{-t}\text{V.}$$

$$i_0(t) = \frac{-2}{3} \cdot e^{-t}\text{A. for } t > 0.$$

(iii) $V_0 = -V_L$

$\Rightarrow V_0 = 4.e^{-t}$ V for $t > 0$

thus for all time

$$i_0(t) = \begin{cases} 0A : t < 0 \\ \dfrac{-2}{3} \cdot e^{-t} A, t \geq 0 \end{cases}$$

$$V_0(t) = \begin{cases} 6\text{ V}, t < 0 \\ 4 \cdot e^{-t} V, t > 0. \end{cases}$$

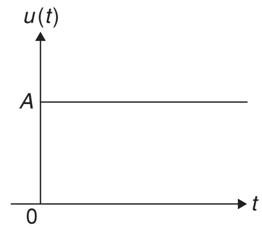
$$i(t) = \begin{cases} 2\ A; t < 0 \\ 2 \cdot e^{-t} A, t \geq 0 \end{cases}$$

Singularity Functions

A basic understanding of singularity functions will help us make sense of the response of first order circuits to a sudden application of an independent DC voltage or current source.

The most widely used singularity functions in circuit analysis are the step ramp and impulse functions.

Step function



The mathematical terms:

$$u(t) = \begin{cases} 0 & t < 0 \\ A; & t > 0 \end{cases}$$

The unit step function is undefined at $t = 0$

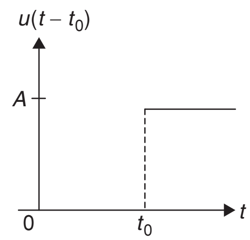


Figure 5 The step function delayed by t_0 units.

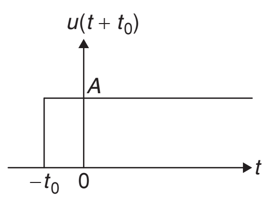
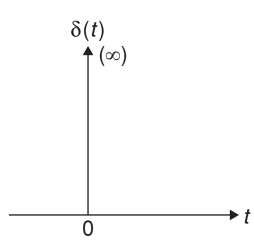


Figure 6 The step function advanced by t_0

Impulse function $\delta(t)$

The derivative of the step function $u(t)$ is the impulse function $\delta(t)$.

$$\delta(t) = \frac{d}{dt}u(t) = \begin{cases} \text{undefined; } (\infty) : \text{for } t = 0 \\ 0\text{; else where} \end{cases}$$



1. $\int_{0-}^{0+} \delta(t)dt = 1$

2. $\int_{t1}^{t2} f(t)\cdot\delta(t-t_0)dt = f(t_0); t_1 < t_0 < t_2 = 0$; else where

Ramp function

Integrating the unit step function $u(t)$ results in the unit ramp function $r(t)$.

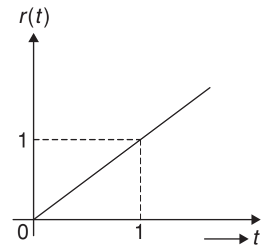


Figure 7 The unit ramp function

$$r(t) = \int_{-\infty}^{t} u(t)\cdot dt = t\cdot u(t)$$

$$r(t) = \begin{cases} 0 & t<0 \\ \text{At} & t \geq 0 \end{cases}$$

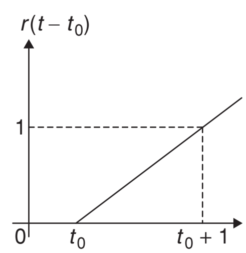


Figure 8 Unit ramp function delayed by t_0.

$$r(t-t_0) = \begin{cases} 0 & t < t_0 \\ t-t_0 & t \geq t_0 \end{cases}$$

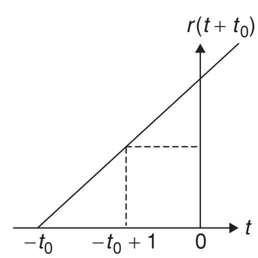


Figure 9 The unit ramp function advanced by to

$$r(t+t_0) = \begin{cases} 0 & t < -t_0 \\ t+t_0 & t \geq -t_0 \end{cases}$$

Step Response of an RC Circuit

When the DC source of an *RC* circuit is applied, the voltage or current source can be modelled as a step function, and the response is known as a step response.

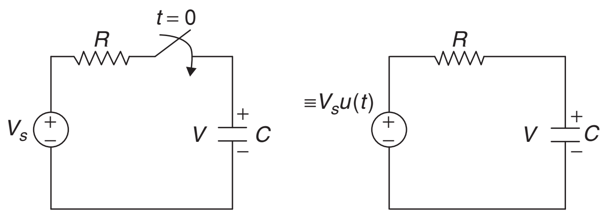


Figure 10 An *RC* circuit with voltage source.

We assume an initial voltage V_0 on the capacitor, since the voltage of the capacitor does not change instantaneously.

$$V(0^-) = V(0^+) = V_0$$

Applying KCL for $t \geq 0$.

$$C \cdot \frac{dV}{dt} + \frac{V - V_S}{R} = 0.$$

$$\frac{dV}{dt} + \frac{V}{RC} = \frac{V_S}{RC} \tag{1}$$

$$\frac{dV}{dt} = -\frac{(V - V_S)}{RC}$$

$$\frac{dV}{V - V_s} = -\frac{1}{RC} \cdot dt$$

Integrating both sides and introducing the initial conditions.

$$\ln t[V - V_S]_{V_0}^{V(t)} = \left[-\frac{t}{RC}\right]_0^t$$

$$\ln t[V(t) - V_s] - \ln t[V_0 - V_s] = \frac{-t}{RC} + 0$$

$$\ln t\left[\frac{v(t) - v_s}{v_0 - v_s}\right] = \frac{-t}{RC} \tag{2}$$

$$\frac{V(t) - V_s}{V_0 - V_s} = e^{\frac{-t}{RC}}$$

We know $\tau = RC$

$$V(t) - V_s = (V_0 - V_s) \cdot e^{-t/\tau}$$

$$V(t) = V_S + (V_0 - V_S) \cdot e^{-t/\tau}$$

for $t > 0$.

Thus

$$V(t) = \begin{cases} V_0; t \leq 0 \\ V_S + (V_0 - V_s) \cdot e^{\frac{-t}{\tau}} \;; t > 0. \end{cases}$$

This is known as the complete response of the *RC* circuit.

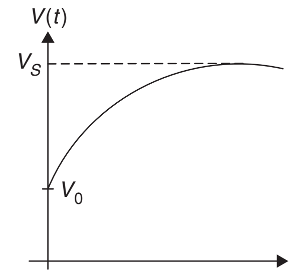


Figure 11 Response of an *RC* circuit With initially charged capacitor.

$$i(t) = C \cdot \frac{dV(t)}{dt}$$

$$i(t) = \frac{V_s}{R} \cdot e^{\frac{-t}{\tau}}; t > 0.$$

with initial values zero.

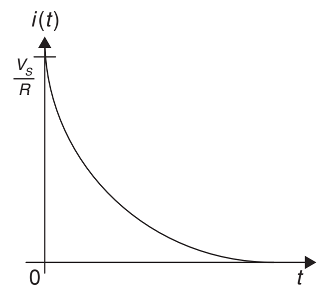


Figure 12 Current response.

Complete response = natural response + forced response.

$\therefore\ V = V_n + V_f$

where $V_n = V_0 \cdot e^{\frac{-t}{\tau}}$

$V_f = V_S(1 - e^{\frac{-t}{\tau}})$

$V_n \Rightarrow$ transient response
$V_f \Rightarrow$ steady state response

Transient response: The transient response is the circuit's temporary response that will dies out with time.

Steady-state response: The steady-state response is the behaviour of the circuit a long time after an external excitation is applied.

Therefore, complete response may be written as

$$\Rightarrow \quad V(t) = V(\infty) + [V(0) - V(\infty)].e^{-t/\tau}\,\text{V}$$

Where $V(0) = V_0 \Rightarrow$ The initial capacitor voltage

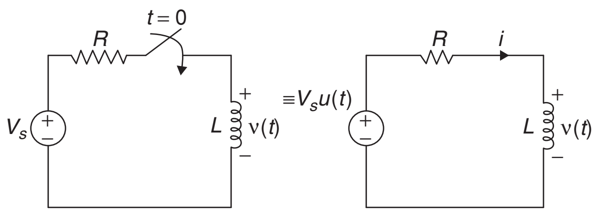
$V(\infty) \Rightarrow$ final capacitor voltage (or) *S.S.* value

$\tau \Rightarrow$ time constant

Note: If the switch changes position at time $t = t_0$ instead of $t = 0$, there is a time delay in response. So that above equation becomes

$$\boxed{V(t) = V(\infty) + [V(t_0) - V(\infty)].e^{-(t - t0)/\tau}\ \text{Volts.}}$$

Step Response of an *RL* Circuit



An RL circuit with a step input voltage

Let the response be the sum of the transient response and the steady state response.

$$i = i_{tr} + i_s \tag{3}$$

We know that i_{tr} is always a decaying exponential.

$$\text{i.e., } i_{tr} = A\cdot e^{\frac{-t}{\tau}},\ \tau = \frac{L}{R}$$

The steady-state is the value of current along time after the switch is closed. At that time, the inductor becomes a short circuit, and the voltage across it is zero. The entire source voltage V_S appears across R.

Thus, the steady state response is

$$i_{ss} = \frac{V_s}{R} A.$$

$$i = A\cdot e^{\frac{-t}{\tau}} + \frac{V_s}{R} \tag{4}$$

We know $i(0^-) = i(0^-) = I_0$

Thus at $t = 0$, Equation (4) becomes

$$i(0) = A + \frac{V_s}{R} = I_0$$

$$A = I_0 - \frac{V_s}{R}$$

Sub A in Equation (2), we get

$$i(t) = \left(I_0 - \frac{V_s}{R}\right)\cdot e^{\frac{-t}{\tau}} + \frac{V_s}{R}$$

This is the comp ute response of the *RL* circuit

$$\boxed{i(t) = i(\infty) + [i(0) - i(\infty)]\cdot e^{\frac{-t}{\tau}} A.}$$

Note: If the switching takes place at time $t = t_0$ instead of $t = 0$,

$$i(t) = i(\infty) + [i(t_0) - i(\infty)]\cdot e^{\frac{-(t\ \ t_0)}{\tau}}$$

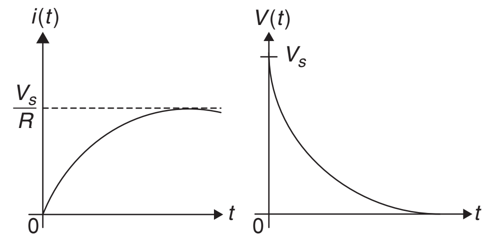


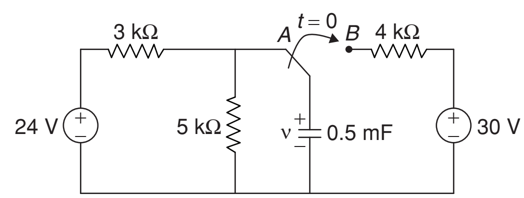
Figure 13 Step response of an RL circuit with no initial inductor current.

Note: With sources, elements behaviour at

$$t = 0^+ \text{ and } t \to \infty$$

1. At $t = 0^+ \Rightarrow$ L = open circuit
   C = short circuit
2. At $t \to \infty \Rightarrow f = 0$
   $\Rightarrow X_L = j\omega L = 0\ \Omega \quad \Rightarrow$ short circuit
   $X_C = \frac{1}{j\omega C} = \infty\Omega \quad \Rightarrow$ open circuit

Examples 9: The switch in figure has been in position A for a long time. At $t = 0$, the switch moves to B. determine $V(t)$ for $t > 0$ and calculate its value at $t = 2$ s.



(A) $V_C(2) = 24.5$ V (B) $V_C(2) = 5.5$ V
(C) $V_C(2) = -14.5$ V (D) $V_C(2) = 18.4$ V

Solution: (A)

For $t < 0$, the switch is at position A.

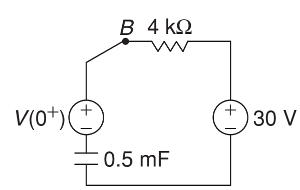
at $t = 0^-$: $\to$ capacitor acts like open circuit

$$v(0^-) = 5\ \text{k}\Omega \times \frac{24\ \text{V}}{8\ \text{k}\Omega} = 15\ \text{V}.$$

we know $v(0^+) = v(0^-) = 15$ V.

(capacitor voltages can't change instantaneously)

For $t > 0$, the switch is a position B



$\tau = RC = 4 \times 10^3 \times 0.5 \times 10^{-3} = 2$ s.

at $t \to \infty$

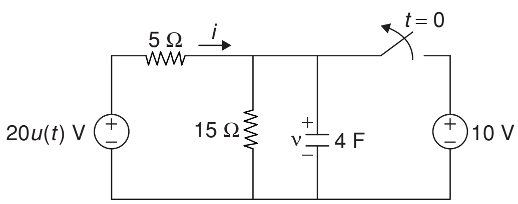
Capacitor acts like an open circuit to dc at steady state.
Thus, $V(\infty) = 30$ V

$$V_C(t) = V(\infty) + [V(0^+) - V_C(\infty)] \cdot e^{-t/\tau}\ V$$
$$= 30 + (15 - 30) \cdot e^{-t/2}\ \text{s.}$$
$$= 30 - 15 \cdot e^{-0.5t}\ \text{s.}$$

at $t = 2$ s

$$V_C(2) = 30 - 15 \cdot e^{-1}$$
$$= 30 - \frac{15}{e} = 24.5\ \text{V}$$

Examples 10: The switch has been closed for a long time and is opened at $t = 0$



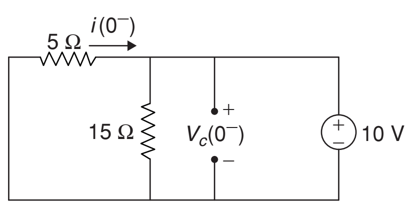
The value of $V_C(t)$ for $t > 0$ would be

(A) $V_C(t) = 15 - 5 \cdot e^{-t/15}$ V
(B) $V_C(t) = 5 - 15 \cdot e^{-t/15}$ V
(C) $V_C(t) = 15 + 5 \cdot e^{-0.25t}$ V
(D) $V_C(t) = 15 + 10 \cdot e^{-t/15}$ V

Solution: (A)
For $t < 0$:
at $t = 0^-$, switch is closed. And $C \Rightarrow$ open circuit

$$u(t) = \begin{cases} 0; & t < 0 \\ 1; & t > 0. \end{cases}$$

The shown in below at $t < 0$.

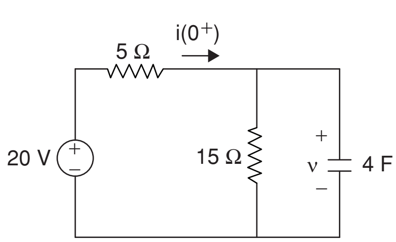


circuit is in steady state from the above circuit.

$$i(0^-) = \frac{10}{5} = 2\ \text{A.}$$

$$V_C(0^-) = 10\ \text{V.} = V_C(0^+)$$

for $t > 0$: Switch opened.
at $t = 0^+$



$$\begin{cases} \tau = R_{eq} \cdot c \\ R_{eq} = \dfrac{5\times 15}{5+15} = \dfrac{5\times 15}{20} = \dfrac{15}{4}\ \Omega \\ \tau = \dfrac{15}{4}\times 4 = 5\ \text{s} \end{cases}$$

$C \rightarrow$ Short circuit

$$i(0^+) = \frac{20}{5} = 4\ \text{A.}$$

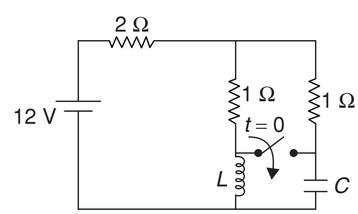
at $t \rightarrow \infty$; C open circuit.

$$V_C(\infty) = V_{15\,\Omega} = 20 \times \frac{15}{15+5} = 15\ \text{V}$$

We know total response $V_C(t)$ is

$$V_C(t) = V_C(\infty) + [V_C(0^+) - V_C(\infty)] \cdot e^{\frac{-t}{\tau}}$$
$$= 15 + [10 - 15] \cdot e^{-t/15}$$
$$V_C(t) = 15 - 5\, e^{-t/15}\ \text{V}$$

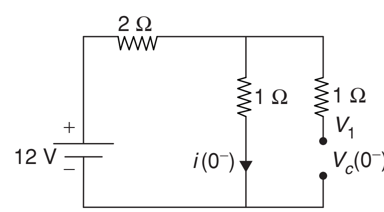
Examples 11: In the circuit shown below, the switch is closed at $t = 0$. What is the Initial value of the current through the capacitor?



(A) 0.8 A
(B) 1.6 A
(C) 2.4 A
(D) 3.2 A

Solution:
For $t < 0$
At $t = 0^-$; The switch is opened
$L \Rightarrow$ Short circuit
$C \Rightarrow$ Open circuit

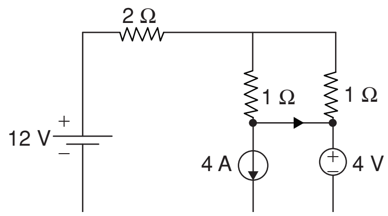
The equivalent circuit shown below:

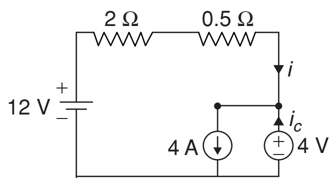


$$i(0^-) = \frac{12}{3} = 4\ \text{A.} = i_L(0^+)$$

$$V_C(0^-) = 1 \times 4 = 4\ \text{V} = V_C(0^+)$$

at $t = 0^+$, the equivalent circuit is shown below:

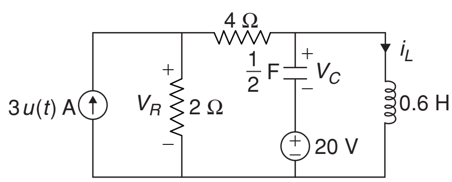




$$i = \frac{12-4}{2.5} = 3.2 \text{ A}$$

$$i + i_C = 4 \quad \Rightarrow \quad i_C = 0.8 \text{ A.}$$

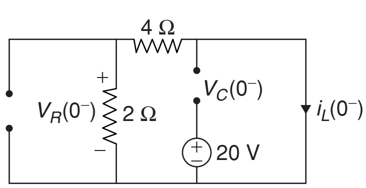
Common Data for Questions 12 to 14:



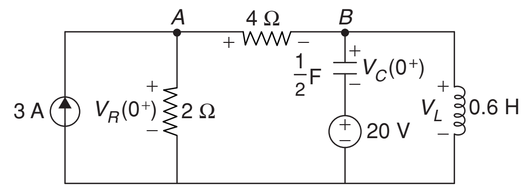
Examples 12: The values of $i_L(0^+)$, $V_C(0^+)$, and $V_R(0^+)$ would be.

(A) $i_L(0^+) = 0$ A; $V_C(0^+) = 20$ V; and $V_R(0^+) = 4$ V
(B) $i_L(0^+) = 5$ A; $V_C(0^+) = -15$ V; and $V_R(0^+) = 4$ V
(C) $i_L(0^+) = 3$ A; $V_C(0^+) = -20$ V; and $V_R(0^+) = 4$ V
(D) $i_L(0^+) = 0$ A; $V_C(0^+) = -20$ V; and $V_R(0^+) = +4$ V

Solution: (D)
For $t < 0$, $3.\, u(t) = 0$; at $t = 0^-$
Since the circuit is in steady state.
∴ L ⇒ short circuit
C ⇒ open circuit



from this figure:
$i_L(0^-) = 0$; $V_R(0^-) = 0$; and $V_C(0^-) = -20$ V
for $t > 0$:
the circuit shown in below at $t = 0^+$
∴ L ⇒ open circuit
C ⇒ short circuit



The inductor current and capacitor voltages cannot change instantaneously
$i_L(0^+) = 0$; $i_L(0^-) = 0$ and $V_C(0^+) = V_C(0^-) = -20$ V
apply KCL at node A.

$$3 = \frac{V_R(0^+)}{2} + \frac{V_0(0^+)}{4}$$

Applying KVL to the middle Loop.

$$V_R(0^+) - V_0(0^+) - V_c(0^+) - 20 = 0$$

$V_C(0^+) = -20$ V
∴ $V_R(0^+) = V_0(0^+)$.
$3\, V_R(0^+) = 12$
⇒ $V_R(0^+) = 4$ V
∴ $V_R(0^+) = 4$ V, $V_C(0^+) = -20$ V. $I_L(0^+) = 0$ A.

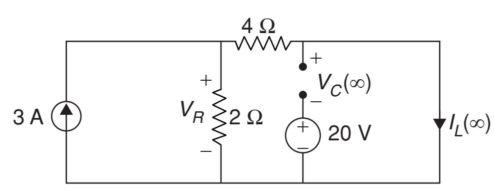
Examples 13: The steady state values of the I_L V_C, and V_R would be

(A) $V_R(\infty) = 4$ V, $V_C(\infty) = -20$ V, $I_L(\infty) = 0$ A
(B) $V_R(\infty) = 4$ V, $V_C(\infty) = -20$ V, $I_L(\infty) = 1$ A
(C) $V_R(\infty) = -4$ V, $V_C(\infty) = 20$ V, $I_L(\infty) = 4$ A
(D) None of the above

Solution: (B)

As $t \to \infty$. The circuit reaches steady-state.
In steady state
∴ L ⇒ short circuit
C ⇒ open circuit

The circuit becomes.



$$i_L(\infty) = 2 \times \frac{3}{6} = 1 \text{ A.}$$

$$V_C(\infty) = -20 \text{ V.}$$

$$V_R(\infty) = 2 \times 2 = 4 \text{ V.}$$

∴ $V_R(\infty) = 4$ V, $V_C(\infty) = -20$ V and $i_L(\infty) = 1$ A.

Examples 14: The values of $\frac{di_L(0^+)}{dt}$ and $\frac{dV_C(0^+)}{dt}$ would be

(A) 0 A/s, 2 V/s
(B) 0 A/s, 0.5 V/s
(C) 2 A/s, 2 V/s
(D) −2 A/s, −2 V/s

Solution: (A)
for $t > 0$:
at $t = 0^+$.

$$V_L(0^+) = L \cdot \frac{di_L(0^+)}{dt} \Rightarrow \frac{di_L(0^+)}{dt} = \frac{V_L(0^+)}{L}$$

But $V_L(0^+) = 0 \text{ V} \Rightarrow \frac{di_L(0^+)}{dt} = 0 \frac{\text{A}}{\text{s}}$

$$i_C = C \cdot \frac{dV_c}{dt} \Rightarrow \frac{dV_C(0^+)}{dt} = \frac{i_C(0^+)}{C}$$

$$\frac{dV_C(0^+)}{dt} = \frac{i_C(0^+)}{C}$$

Apply KCL at node B,

$$\frac{V_0(0^+)}{4} = i_C(0^+) + i_L(0^+)$$

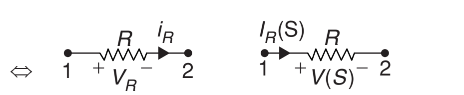
$$i_C(0^+) = \frac{4}{4} = 1 \text{ A}$$

$$\frac{dV_C(0^+)}{dt} = \frac{1}{\frac{1}{2}} = 2 \text{ V/s}$$

Equivalent Circuits for *R*, *L*, and *C* in *S* Domain
Laplace Transform

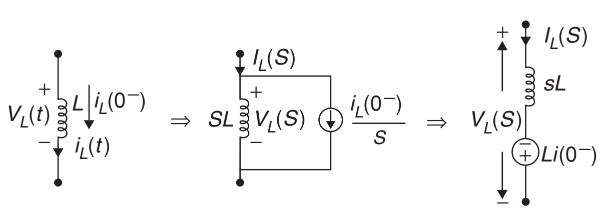
$$i(t) \leftrightarrow I(s)$$
$$V(t) \leftrightarrow V(s)$$
$$R \leftrightarrow R$$
$$L \leftrightarrow sL\ \Omega$$
$$C \leftrightarrow \frac{1}{sC}\ \Omega$$

(i) *R*: Resistor (*R*):

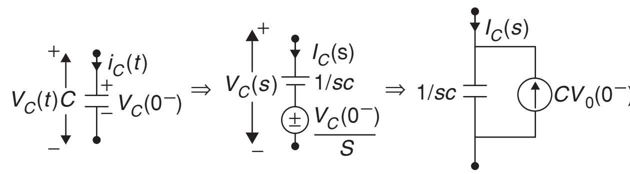


1. Time domain   2. Frequency domain.

(ii) Inductor (*L*):



(iii) Capacitor (*C*):

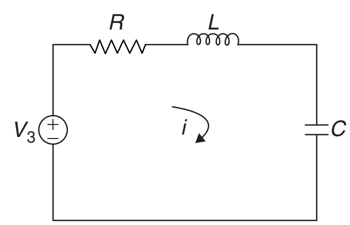


1. Time domain
2. s-domain
3. s-domain Voltage source current source.

Higher–order Circuits

When two or more energy storage elements are present, the network equations will result in second order differential equations.

Series RLC circuits



Apply KVL

$$R_i + L \cdot \frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i \cdot dt = V_s. \quad (5)$$

To eliminate the integral, differentiate with respect to t.

$$R \cdot \frac{di}{dt} + L \cdot \frac{d^2 i}{dt^2} + \frac{i}{C} = 0$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \cdot \frac{di}{dt} + \frac{i}{LC} = 0. \quad (6)$$

Apply Laplace transform to Equation (6)

$$\Rightarrow \quad s^2 + \frac{R}{L} \cdot s + \frac{1}{LC} = 0.$$

Characteristic equation of the differential equation.

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \quad (7)$$

Second order characteristic equation
From Equation (7)

$$\omega_n = \frac{1}{\sqrt{LC}} \text{ and } 2\xi\omega_n = \frac{R}{L}$$

$$\xi\omega_n = \frac{R}{2L}$$

$$\tau = \frac{1}{\xi\omega_n} = 2\frac{L}{R}$$

$$\boxed{\xi = \frac{R}{2}\cdot\sqrt{\frac{C}{L}}}$$

The roots of the characteristic equation are

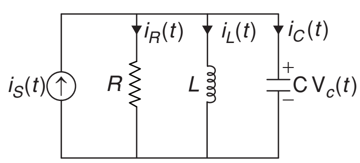
$$s = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Let $\alpha = \frac{R}{2L} = \frac{1}{\tau}$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_n^2}$$

1. If $\alpha > \omega_n$ (or) $\xi > 1$, we have over damped oscillations.
2. $\xi = 1$ or $\alpha = \omega_n$, $\Rightarrow$ Critically damped
3. $0 < \xi < 1$ or $\alpha < \omega_n$, we have under damped oscillators.

Parallel RLC Circuit



$$i_S(t) = i_R(t) + i_L(t) + i_C(t)$$

Let us assume that the voltage across the capacitor is $V_C(t)$

$$\therefore \quad \frac{V_C(t)}{R} + \frac{1}{L}\int_0^t V_C(t)dt + C\cdot\frac{dV_C(t)}{dt} = i_s(t)$$

simplifying the above equation, it becomes

$$\frac{d^2V_C(t)}{dt} + \frac{1}{RC}\cdot\frac{dv_c(t)}{dt} + \frac{1}{LC}\cdot V_C(t) = 0$$

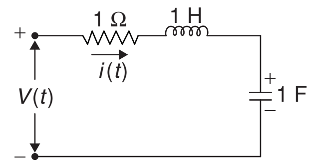
In S-domain

$$s^2V_C(s) + \frac{1}{RC}\cdot sV_c(0) + \frac{1}{LC}\cdot V_c(s) = 02$$

$$s^2 + \frac{1}{RC}\cdot s + \frac{1}{LC} = 0.$$

$$\boxed{\xi = \frac{1}{2R}\cdot\sqrt{\frac{L}{C}};}\ \omega_n = \frac{1}{\sqrt{LC}}.$$

Example 15: The circuit shown in figure has initial current $i_L(0^-) = 1$ A. Through the inductor and an initial voltage $V_C(0^-) = -1$ V across the capacitor for input $V(t) = u(t)$, the Laplace transform of the current $i(t)$ for $t \geq 0$ is



(A) $\frac{s}{s^2+s+1}$ (B) $\frac{s+2}{s^2+s+1}$

(C) $\frac{s-1}{s^2+s+1}$ (D) $\frac{s-1}{s^2+s+1}$

Solution: (B)

Apply KVL, the loop equation is

$$V(t) = R\cdot i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt.$$

but $V(t) = u(t)$, $i_L(0^-) = 1$ A and $V_C(0^-) = -1$ V

Take LPF both sides.

$$L[sI(s) - I(0^-)] + RI(s) + \frac{1}{sC}\cdot I(S) + \frac{V_C(0^-)}{s} = \frac{1}{s}$$

Sub $R = 1\ \Omega$, and $L = 1$ H

$C = 1$ F

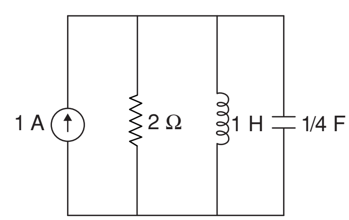
$$sI(s) - 1 + I(s) + \frac{1}{s}I(S) \quad \frac{1}{s} = \frac{1}{s}$$

$$I(s)\left[s + \frac{1}{s} + 1\right] = 1 + \frac{2}{s}$$

$$I(s)\ [s^2 + s + 1] = s + 2$$

$$I(s) = \frac{s+2}{s^2+s+1}$$

Example 16: The circuit is



(A) Critically damped (B) Undamped

(C) Under damped (D) Over damped

Solution: (D)

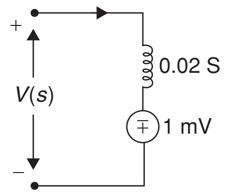
For RLC parallel circuit

$$\xi = \frac{1}{2R}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{2\times 2}\times\sqrt{\frac{4}{1}};\ \xi = \frac{1}{2}.$$

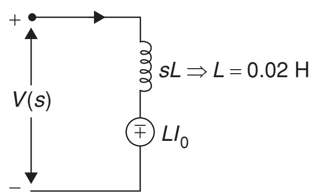
$\therefore\ \xi < 1 \Rightarrow$ under damped system.

Example 17: The value of Initial current is



(A) 0.5 A (B) 0.05 A
(C) 0.01 A (D) 0.2 μA.

Solution: (B)

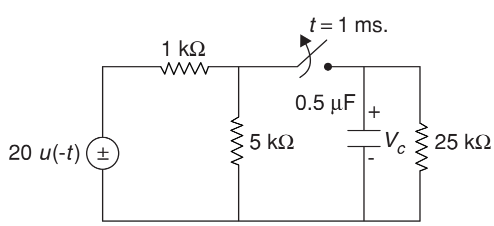


$$L.\,I_0 = 1 \times 10^{-3}$$

$$I_0 = \frac{1\times10^{-3}}{0.02} = 0.05 \text{ A}$$

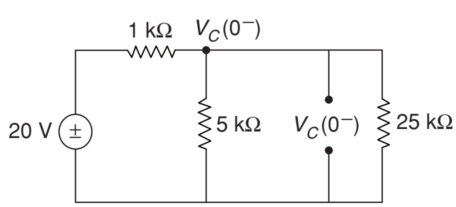
$$I_0 = 50 \text{ mA}$$

Example 18: In the following circuit, the 20 V source has been applied for a long time. The switch is opened at $t = 1$ m s.



At $t = 3$ ms the value of $V_C(t)$ is
(A) 1.314 V (B) 1.128 V
(C) 16.13 V (D) None of the above.

Solution: (B)
For $t < 0$:
at $t = 0^-$ switch was closed
Capacitor behaves like an open circuit.



$$\frac{V_C(0^-)-20}{1\text{ k}\Omega} + \frac{V_C(0^-)}{5\text{ k}\Omega} + \frac{V_C(0^-)}{25\text{ k}\Omega} = 0$$

$$25\,(V_C(0^-) - 20) + 5\,V_C(0^-) + V_C(0^-) = 0$$

$$31\,V_C(0^-) = 25 \times 20$$

$$\Rightarrow \quad V_C(0^-) = \frac{25\times20}{31} = 16.13 \text{ V}$$

$$R_{eq} = (1\text{ k}\Omega \parallel 5\text{ k}\Omega \parallel 25\text{ k}\Omega)$$

$$\frac{1}{R_{eq}} = \frac{1}{1\text{ k}\Omega} + \frac{1}{5\text{ k}\Omega} + \frac{1}{25\text{ k}\Omega}$$

$$= \frac{25+5+1}{25\text{ k}\Omega}$$

$$R_{eq} = \frac{25}{31}\text{ k}\Omega = 806\ \Omega$$

$$\tau = R_{eq} \times C = 806 \times 0.5 \times 10^{-6}$$

$$\tau = 0.4 \text{ ms.}$$

$$V_C(t) = V_0.e^{-t/\tau} = 16.13 \times e^{-2500t}\text{ V}$$

$$V_C(1\text{ ms}) = 16.13 \times e^{-2500\times1\times10^{-3}}\text{ V}$$

$$= 16.13.\ e^{-2.5} = 1.324 \text{ V for } 0 < t < 1 \text{ ms}$$

$$V_C(t) = 1.324 \times e^{\frac{-(t-1m)}{25\times0.5\times10^{-3}}}\text{ V}$$

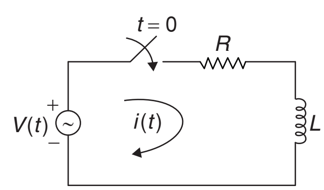
$$V_C(3\text{ m}) = 1.324 \times e^{-80\times2\times10^{-3}}$$

$$= 1.128 \text{ V}$$

AC Transients

Transient Response with Sinusoidal Excitation

Series R – L Circuit



Let $V(t) = V_m \cdot \cos(\omega t + \phi)$ V.
for $t > 0$: Switch is closed.
Apply KVL

$$R\cdot i + L\cdot\frac{di}{dt} = V_m\cdot\cos(\omega t+\phi)$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V_m}{L}\cdot\cos(\omega t+\phi) \qquad (8)$$

Let $$\frac{d}{dt} = D.$$

$$\left(D+\frac{R}{L}\right)i = \frac{V_m}{L}\cdot\cos(\omega t+\phi) \qquad (9)$$

The complete solution = complement function + particular solution.

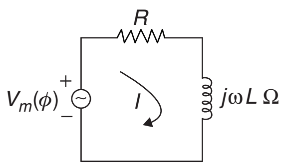
That is, C.S. = C.F. + P.I.

The complementary function of Equation (2) is

$$i_{tr}(t) = A \cdot e^{-\frac{R}{L}t} \text{ A}$$

Next, we are to obtain the particular solution of current $i(t)$: $\Rightarrow i_{ss}(t)$

Transform the above network into phasor domain.



Network is phasor domain

Apply KVL to above circuit

$$V_m\angle\phi - R.I - j\omega LI = 0.$$

$$I = \frac{V_m\angle\phi}{R + j\omega L}$$

$$\Rightarrow \quad I = \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \cdot \angle\phi - \tan^{-1}\frac{\omega L}{R}$$

I in time domain $i_{ss}(t)$.

$$i_{ss}(t) = \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \cdot \cos\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R}\right) \text{A.}$$

$$i(t) = i_{tr}(t) + i_{ss}(t)$$

$$= \text{Transient} + \text{Steady state response}$$

$$= A\cdot e^{-\frac{R}{L}t} + \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \cdot \cos\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R}\right) \text{A}$$

at $t = 0^- \Rightarrow i(0^-) = 0 = i(0^+)$.

$$A + \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \cdot \cos\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R}\right) = 0$$

$$A = \frac{-V_m}{\sqrt{R^2+(\omega L)^2}} \cdot \cos\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R}\right) \text{A.}$$

$\Rightarrow$ As $A << 1$, the transient effects are less for AC at $t = 0$.

Transient free condition

That is, $A = 0$.

$$\Rightarrow \quad \cos\left(\phi - \tan^{-1}\frac{\omega L}{R}\right)$$

$$\phi - \tan^{-1}\frac{\omega L}{R} = \frac{\pi}{2}$$

$$\phi = \frac{\pi}{2} + \tan^{-1}\frac{\omega L}{R}; \text{ at } t = 0.$$

Case 1: If the switch is closed at $t = t_0$. Then the condition for transient free response is

$$\omega t_0 + \phi = \frac{\pi}{2} + \tan^{-1}\frac{\omega L}{R}$$

Note: If the excitation is $V(t) = V_m \cdot \sin(\omega t + \phi)$.

Then the condition for transient free response.

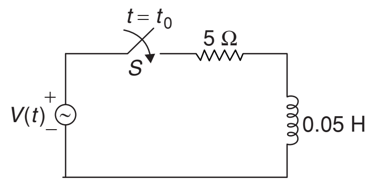
$\Rightarrow \quad \omega t_0 + \phi = \tan^{-1}\omega\tau$ at $t = t_0$

$\therefore\ i(t) = i_{ss}(t)$

Summary:

S.No	Excitation	Circuit	Transient free condition (at $t = t_0$)
(i)	$V(t) = V_m \sin(\omega t + \varphi)$	Series *RL* (or)	$\omega t_0 + \phi = \tan^{-1}\frac{\omega L}{R}$ $= \tan^{-1}\omega\tau.$
		Series *RC*	$\omega t_0 + \phi = \tan^{-1}\omega RC$ $= \tan^{-1}\omega\tau.$
(ii)	$V(t) = V_m \cos(\omega t + \varphi)$	Series *RL* (or) Series *RC*	$\omega t_0 + \phi = \tan^{-1}\omega t + \frac{\pi}{2}$ where $t = \frac{L}{R}$ for *RL* $\tau = RC$ for *RC*.
(iii)	$i(t) = I_m \sin(\omega t + \varphi)$	Parallel *RL* (or) Parallel *RC*	$\omega t_0 + \phi = \tan^{-1}\omega\tau$ where $\tau = RC$ for *RC* circuits. $\tau = \frac{L}{R}$ for *RL* circuits.
(iv)	$i(t) = I_m \cos(\omega t + \varphi)$	Parallel *RL* (or) Parallel *RC*	$\tau = \frac{L}{R}$ for *RL* circuits, *RC* for *RC* circuits $\omega t_0 + \phi = \tan^{-1}\omega t + \frac{\pi}{2}$

Example 19:



If $V(t) = 5\cos\left(100\pi t + \frac{\pi}{4}\right)$ V, the value of t_0 which results in a transient free response.

Solution: We know for *RL* series circuits transient free condition.
Given input co sinusoidal

$$\therefore \quad \omega t_0 + \phi = \tan^{-1}\frac{\omega L}{R} + \frac{\pi}{2}$$

from the given data.

$$\omega = 100\pi \text{ and } \phi = \frac{\pi}{4}$$

$$100\pi t + \frac{\pi}{4} = \tan^{-1}\left(\frac{100\pi \times 0.05}{5}\right) + \frac{\pi}{2}$$

$$100\pi t = 117.34$$

$$t = 0.3735 \text{ s}$$

Sinusoidal Steady-State Analysis

A sinusoidal forcing function produces both a transient response and steady state response.

The transient response dies out with time so that only steady state response remains. When the transient response has become negligibly small compared to steady-state response, we say that the circuit is operating at sinusoidal steady state.

Sinusoids

Let us consider a general expression for the sinusoidal.

$$V(t) = V_m \cdot \sin(\omega t + \varphi)$$

where

$V_m \Rightarrow$ amplitude
$\omega \Rightarrow$ angular frequency
$\omega t + \varphi \Rightarrow$ argument of the sinusoidal
$\varphi \Rightarrow$ phase

A sinusoid can be expressed in either sine or cosine form.

This is achieved by using the following trigonometric identities:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B.$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B.$$

Note:

$$\sin(\omega t \pm 180°) = -\sin \omega t$$

$$\cos(\omega t \pm 180°) = -\cos \omega t$$

$$\sin(\omega t \pm 90°) = \pm \cos \omega t$$

$$\cos(\omega t \pm 90°) = \pm \sin \omega t.$$

Examples 20: Given the sinusoidal signal

$$V(t) = 10\sin\left(4\pi t + \frac{\pi}{6}\right).$$

Calculate its power and period.
(A) 100 W, 2 s
(B) 50 W, 0.5 s
(C) 10 W, 0.5 s
(D) 0 W, 2 s

Solution: (B)
for sinusoidal signals

$$\text{power} = \frac{V_m^2}{2R} (\because \; V_m = \text{maximum voltage})$$

but R is not given
So Let $R = 1\ \Omega$

$$P = \frac{V_m^2}{2} = \frac{(10)^2}{2} = 50 \text{ W}$$

$$\omega = 2\pi f = 4\pi$$

$$T = \frac{2\pi}{4\pi} = \frac{1}{2} \quad \Rightarrow \quad 0.5 \text{ s}$$

Example 21: Calculate the phase angle between $V_1 = -10 \cos(\omega t + 40°)$ and

$$V_2 = 8 \sin(\omega t - 20°).$$

(A) 30° (B) 60° (C) −60° (D) 20°

Solution: In order to compare V_1 and V_2, we must express them in same form.

$V_1 = -10 \cos(\omega t + 40°)$
$= +10 \sin(\varpi t + 40° - 90°).$
$= 10 \sin(\omega t - 50°).$
$V_2 = 12 \sin(\omega t - 20°)$
V_2 Leads V_1 by 30°.

Phasor

A phasor is a complex number that represents the amplitude and phase of a sinusoid.

$Z = x + jy \Rightarrow$ rectangular form

$Z = r\angle\varphi \Rightarrow$ polar form

$Z = r.\, e^{j\varphi} \Rightarrow$ exponential form

Euler's identity

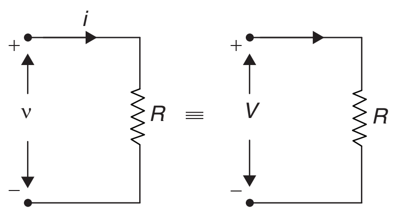
$$e^{\pm j\phi} = \cos\phi \pm j\sin\phi$$

Table 1 *Sinusoidal phasor transformation:*

S.No	Time domain	Phasor domain
(i)	(i) $V_m \cos(\omega t + \varphi)$	$V_m \angle\varphi$
(ii)	(ii) $V_m \sin(\omega t + \varphi)$	$V_m \angle\varphi - 90°$
(iii)	(iii) $I_m \cos(\omega t + \varphi)$	$I_m \angle\varphi$
(iv)	(iv) $I_m \sin(\omega t + \varphi)$	$I_m \angle\varphi - 90°$
(v)	(v) $\frac{dv}{dt} \Leftrightarrow$	$j\omega v$
(vi)	(vi) $\int v.dt \Leftrightarrow$	$\frac{V}{j\omega}$

Phasor relationships for circuit elements:

Resistor



(i) Time domain (ii) Frequency domain

If the current through a resistor R is

$$i = I_m \cos(\omega t + \varphi).$$

The voltage across it is given by

$$V = iR = R.I_m \cos(\omega t + \varphi)$$

The phasor form of this voltage is

$$V = R \times I_m \angle\varphi$$

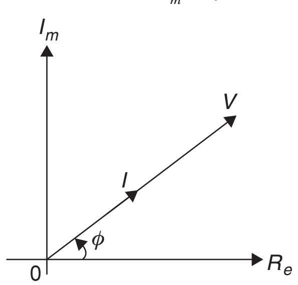
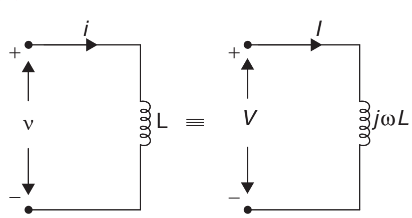


Figure 14 Phasor diagram for the resistor.

Inductor (L)



Time domain	Frequency domain
$T - D$	$F - D$
$V = L \cdot \frac{di}{dt}$	$V = j\omega L \cdot I$

The voltage across the inductor is

$$V = L \cdot \frac{di}{dt} = -\omega L \cdot I_m \sin(\omega t + \phi)$$

$$V = \omega L.I_m \cos(\omega t + \varphi + 90°)$$

which transform to the phasor

$$V = \omega L \cdot I_m.e^{j(\varphi + 90°)} = \omega L \cdot I_m.\angle\varphi + 90°$$

$$\text{but } I_m \cdot \angle\varphi = I \text{ and } e^{j90°} = j.$$

$$\therefore V = j\omega L.I.$$

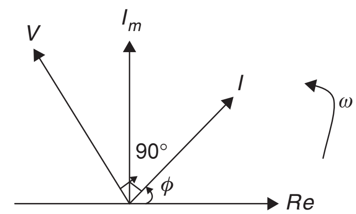
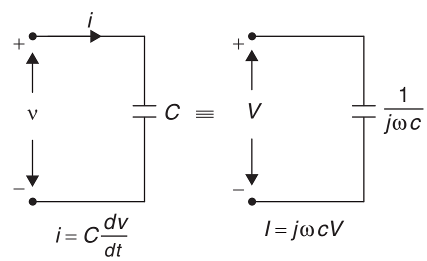


Figure 15 Phasor diagram for the inductor *I* lags V. (or) V leads *I*.

Capacitor (C)



For the capacitor C, assume the voltage across it is $V = V_m \cos(\omega t + \varphi)$.

$$i = C \cdot \frac{dV}{dt} \Leftrightarrow I = j\omega CV$$

$$T - D \qquad F - D$$

$$\therefore \quad V = \frac{I}{j\omega c}$$

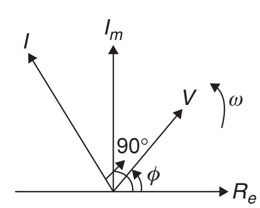


Figure 16 Phasor diagram for the capacitor. *I* leads *V*. or *V* lags *I*.

Note: Summary of $V - I$ relationships

Elements	$T - D$	$F - D$
R	$v = Ri$	$V = R.I$
L	$v = L \cdot di/dt$	$V = j\omega LI$
C	$I = C\, dv/dt$	$I = I/j\omega C$

Examples 22: The voltage $V = 5\cos(40t + 60°)$, is applied to a 0.5 H inductor. The steady state current through the inductor is

(A) $i(t) = 4\sin(40t + 60°)$ A
(B) $i(t) = 2\cos(40t + 30°)$ A
(C) $i(t) = 0.25\cos(40t - 30°)$ A
(D) $i(t) = 0.25\cos(\omega t + 60°)$ A

Solution: (C)
For the inductor, $V = j\omega L\, I$
form the given data.
$\omega = 40$ rad/s.
$L = 1/2\ H$. and $V = 5\ \angle 60°$ V.

$$I = \frac{V}{j\omega L} = \frac{5\angle 60°}{j \times 40 \times 0.5}$$

$$= \frac{1}{4}\angle 60° - 90° = 0.25\angle -30°.$$

$\therefore \quad i(t) = 0.25.\ \cos(40t - 30°)$ A.

Example 23: If voltage $V = 4\sin(50t + 30°)$ is applied to a 100 μF, capacitor. The steady state current through the capacitor is

(A) $i(t) = 20\sin(50t + 60°)$ mA
(B) $i(t) = 20\cos(50t + 30°)$ mA
(C) $i(t) = 20\cos(50t - 30°)$ mA
(D) $i(t) = 20\cos(50t - 60°)$ mA

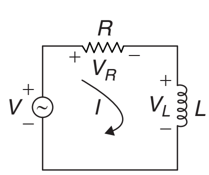
Solution: (B)
Given V = $4\sin(50t + 30°)$

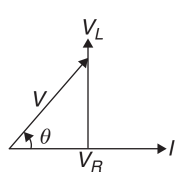
$$i = C \cdot \frac{dV}{dt} \Leftrightarrow V = \frac{I}{j\omega C};\ V = 4\angle \quad 60°$$

$I = j\omega V.C \quad \Rightarrow \quad j \times 50 \times 100 \times 10^{-6} \times 4\ \angle -60°.$
$I = 20\cos(50t - 60° + 90°)$ mA
$i(t) = 20\cos(50t + 30°)$ mA.

Sinusoidal Steady-State Analysis of RLC Circuits

RL Series Circuit





$$V = V_R + V_L$$
$$= I \cdot R + j\omega L \cdot I$$

$$V = \sqrt{V_R^2 + V_L^2}$$

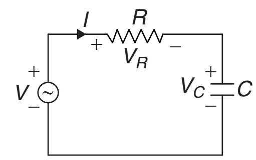
$$\theta = \tan^{-1}\left[\frac{V_L}{V_R}\right]$$

where $V_R = I \cdot R$

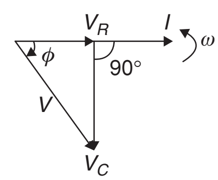
Power factor $= \cos\theta = \dfrac{V_R}{V}$

$$V_L = I \cdot X_L \angle\, 90°\text{Lag.}$$

Series RC Circuit



$$V = V_R + V_C$$
$$V_R = I \cdot R\ ;\ V_C = I \cdot Z_C = IX_C\angle -90°.$$

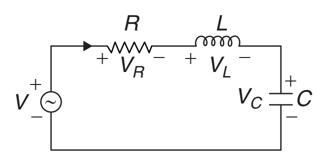


Phasor diagram

$$V = \sqrt{V_R^2 + V_C^2}$$

$$pf = \cos\phi = \frac{V_R}{V}\ \text{Lead}$$

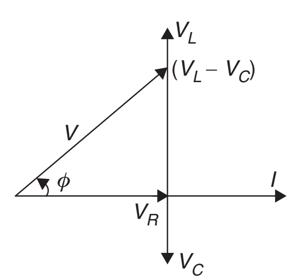
Series RLC Circuit



$$V = V_R + V_L + V_C$$
$$= I \cdot R + j\omega L \cdot I + \frac{1}{j\omega c} \cdot I$$

$$= I.R + I.\ X_L\angle 90° + I.\ X_C\angle -90°.$$

1. $V_L > V_C$:



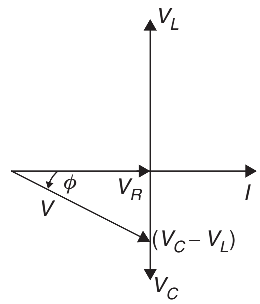
from the above phasor diagram

$$|V| = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$\phi = \tan^{-1}\left[\frac{V_L - V_C}{V_R}\right] \Rightarrow$ Impedance or Admittance angle

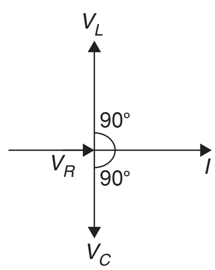
Power factor $= pf = \cos\phi = \frac{V_R}{V}$; Lag

2. If $V_L < V_C$:



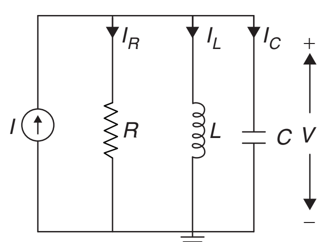
$$\cos\phi = \frac{V_R}{V}; \text{ Lead}$$

3. If $V_L = V_C$
   In this case



$\cos\phi = \frac{V_R}{V} = 1 \Rightarrow$ unity power factor

Parallel RLC Circuit

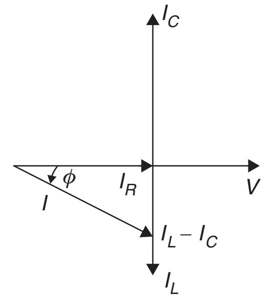


$$I = I_R + I_L + I_C$$

$$I = \frac{V}{R} + \frac{V}{j\omega L} + j\omega c \cdot \text{V}$$

$$\Rightarrow \quad I = \frac{V}{R} + \frac{V}{X_L}\angle -90° + \frac{V}{X_C}\angle 90°.$$

1. $I_L > I_C$

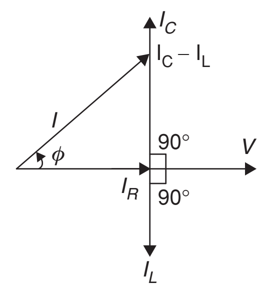


from the above phasor diagram

$$|I| = \sqrt{I_R^2 + (I_L - I_C)^2}$$

$$pf = \cos\phi = \frac{I_R}{I}; \text{ Lag}$$

2. $I_L < I_C$

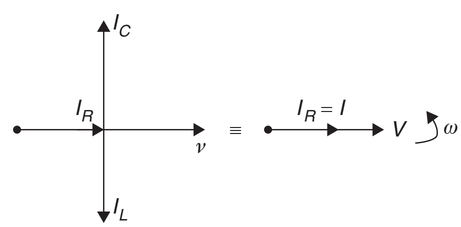


$$|I| = \sqrt{I_R^2 + (I_L - I_C)^2}$$

$\phi = \tan^{-1}\left[\frac{I_L - I_C}{I_R}\right] \Rightarrow$ Impedance (or) admittance angle

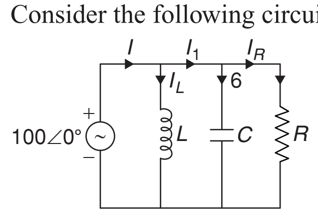
$$\cos\phi = pf = \frac{I_R}{I} \text{ Lead}$$

3. $I_L = I_C$
   $|I| = I_{R.}$



$\cos\phi = \frac{I_R}{I} = 1 \Rightarrow$ unity power factor.

Examples 24: Consider the following circuit:



If $|I_1| = 10$ A and $|I| = 12$ A, then the values of I_L and I_R would be
(A) $I_L = -14.94$ A, $I_R = 8$ A
(B) $I_L = 2$ A, $I_R = 4$ A
(C) $I_L = 14.94$ A, $I_R = 8$ A
(D) None of the above

Solution: (A)

$$I = \sqrt{I_R^2 + (I_L - I_C)^2}$$

$$I_1 = \sqrt{I_R^2 + I_C^2}$$

$$100 = I_R^2 + 36$$

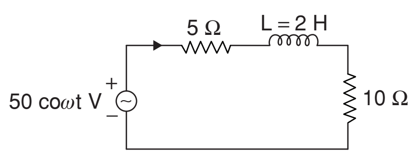
$$I_R^2 = 64 \quad \Rightarrow \quad I_R = 8 \text{ A}$$

$$12 = \sqrt{8^2 + (I_L - 6)^2}$$

$$(I_L - 6)^2 = 80$$

$$I_L = 6 + 8.944 = 14.94 \text{ A}$$

Example 25:



If the power dissipated in the 5 Ω resistor is 15 W, then the power factor of the circuit is
(A) $p.f = 0.68$
(B) $p.f = 0.854$
(C) $p.f = 0.52$
(D) None of the above.

Solution: (C)
For *RL* series circuit

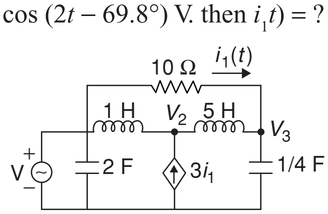
$$pf = \cos\phi = \frac{V_R}{V}$$

$P = V \cdot I = I^2 \cdot R$
$I^2 \times 5 = 15 \quad \Rightarrow \quad I^2 = 3$
$I = 1.732$ A
$\therefore \quad V_R = 15 \times 1.732 \approx 26$
$|V| = 50 \angle 0°$.

$$\cos\phi = \frac{26}{50} = 0.52 \quad \Rightarrow \text{ power factor}$$

Impedance angle $\varphi = 58.66°$

Example 26: In the circuit shown below it is known that
$V_2(t) = 0.757 \cos(2t + 66.7°)$ V,
$V_3(t) = 0.606 \cos(2t - 69.8°)$ V. then $i_1 t) = ?$



(A) $0.194 \cos(2t + 35.73°)$ A
(B) $0.318 \cos(2t + 177°)$ A
(C) $0.196 \cos(2t - 35.6°)$ A
(D) $0.318 (2t - 177°)$ A

Solution: (A)
Apply KCL at node V_3.

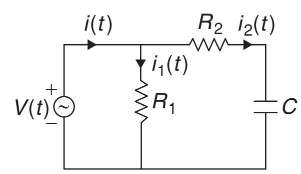
$$I_1 = \frac{V_3}{1}\left(\frac{j}{2}\right) + \frac{V_3 - V_2}{j \times 2 \times 5}$$

$$I_1 = j\,0.1\, V_2 + j\,0.4 \times V_3$$
$$= 0.1 \times \angle 90° \times 0.757 \angle 66.7° + 0.4 \angle 90° \times 0.606 \angle -69.8°$$
$$= 0.0757 \angle 156.7° + 0.2424 \angle 20.2°.$$
$$I_1 = 0.1945 \angle 35.736° \text{A}$$

Locus Diagrams

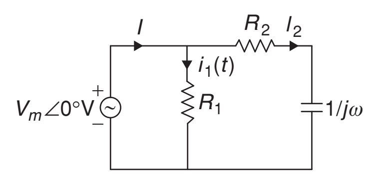
The locus diagram or circle diagram is the graphical representation of the electrical circuit. The frequency response of a circuit has been exhibited by drawing separately the angle and magnitude of a network function against variable parameter (e.g., ω, L, R, C)

Examples 27:



Consider $V(t) = V_m \cos \omega t$ V, if frequency "ω" of the source is varying from 0 to ∞. Draw the locus of the current phasor I_2 ________

Solution: Transform the given network into phasor domain



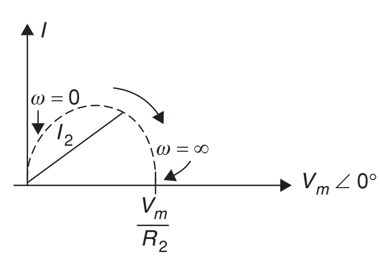
$$I = I_1 + I_2$$

$$I_1 = \frac{V_m \angle 0°}{R_1};\; I_2 = \frac{V_m \angle 0°}{R_2 + \dfrac{1}{j\omega c}}$$

If $\omega = 0$, $I_2 = 0$ [∵ capacitor open circuit]

$$\omega = \infty,\; I_2 = \frac{V_m \angle 0°}{R_2}$$

Locus of I_2:



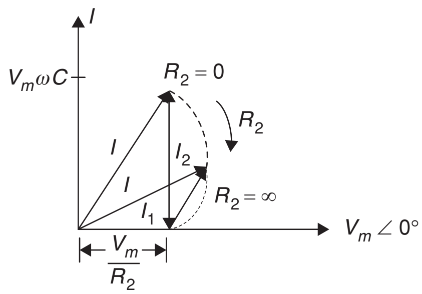
Case 2: In the above circuit instead of 'ω' If R_2 is varying from 0 to ∞.

If $R_2 = 0$; $I_2 = j\,\omega.C\,V_m = \omega \cdot CV_m < 90°$

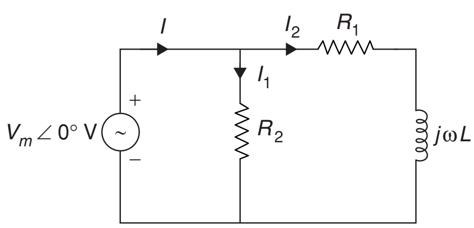
$R_2 = \infty$; $I_2 = 0$,

$$I_1 = \frac{V_m\angle 0°}{R_1}$$

Locus diagram of *I*:



Example 28:



If R_1 is varied from '0' to '∞', draw the locus diagram of *I*.

Solution: $I = I_1 + I_2$

$$I = \frac{V_m\angle 0°}{R_2} + \frac{V_m}{R_1 + j\omega L}$$

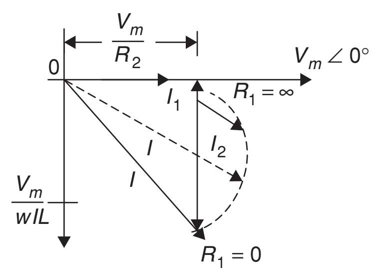
If $R_1 = 0$

$$I = \frac{V_m}{R_2} + \frac{V_m\angle -90°}{\omega L}$$

If $R_1 = \infty$

$$I = \frac{V_m\angle 0°}{R_2} + 0$$

$$\therefore \quad I = \frac{V_m\angle 0°}{R_2}$$



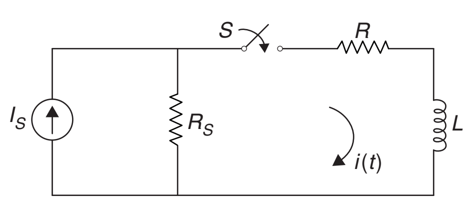
Locus diagram of *I*

Exercises

Practice Problems I

Directions for questions 1 to 32: Select the correct alternative from the given choices.

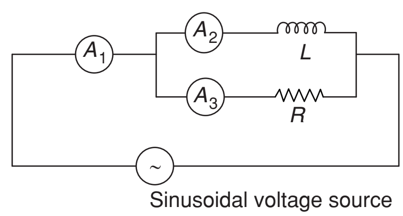
1. In the following circuit, the switch *S* is closed at $t = 0$. The rate of change of current $\frac{di}{dt}(0+)$ is given by



(A) 0 (B) $\frac{R_s L_s}{L}$

(C) $\frac{(R+R_s)I_s}{L}$ (D) ∞

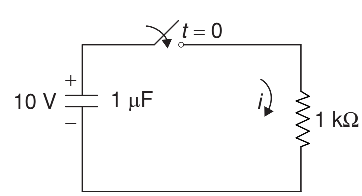
2. In the given figure, A_1, A_2, and A_3 are ideal ammeters. If A_2 and A_3 read 3 *A* and 4 *A*, respectively, then A_1 should read



(A) 1 A (B) 5 A

(C) 7 A (D) None of these

3.



The current in the circuit when the switch is closed at $t = 0$

(A) 10 e^{-100t} (B) 0.01 e^{-1000t}

(C) 0.1 e^{-1000t} (D) 10 $e^{-0.1t}$

4. An input voltage $V(t) = 10\sqrt{2}\cos(t + 10°) + 10\sqrt{5}\cos(2t + 10°)$V is applied to a series combination of resistance $R = 1\ \Omega$ and an inductance $L = 1$ H. The resulting steady state current $i(t)$ in ampere is

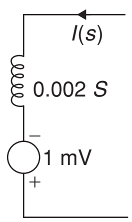
(A) $10\cos(t + 55°) + 10\cos(2t + 10° + \tan^{-1}2)$

(B) $10\cos(t + 55°) + 10\sqrt{\frac{3}{2}}\cos(2t + 55°)$

(C) $10\cos(t - 35°) + 10\cos(2t + 10° - \tan^{-1}2)$

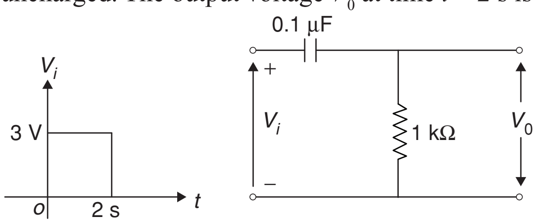
(D) $10\cos(t - 35°) + 10\sqrt{\frac{3}{2}}\cos(2t - 35°)$

5. A$_2$ mH inductor with some initial current can be represented as shown in the figure, where S is the Laplace transform variable. The value of initial current is



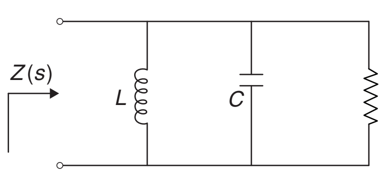
(A) 0.5 A (B) 2.0 A
(C) 1.0 A (D) 0 A

6. A square pulse of 3 V amplitude is applied to $C - R$ circuit shown in the figure. The capacitor is initially uncharged. The output voltage V_0 at time $t = 2$ s is



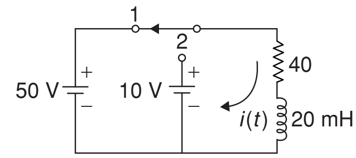
(A) 3 V (B) − 3 V
(C) 0 (D) − 4 V

7. The driving point impedance of the following network is given by $Z(s) = \frac{0.2s}{s^2 + 0.1s + 2}$. The component values are



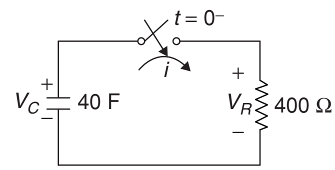
(A) $L = 5$ H, $R = 0.5\ \Omega$, $C = 0.1$ F
(B) $L = 0.1$ H, $R = 0.5\ \Omega$, $C = 5$ F
(C) $L = 5$ H, $R = 2\ \Omega$, $C = 0.1$ F
(D) $L = 0.1$ H, $R = 2\ \Omega$, $C = 5$ F

8. The switch has been in position 1 for a long time, it is moved to position 2 at $t = 0$. The expression for $i(t)$ for $t > 0$ is



(A) $0.25\ e^{-2000t}$ (B) $0.25 + e^{-2000t}$
(C) $0.5\ e^{-2000t}$ (D) $0.5 + e^{-2000t}$

9.



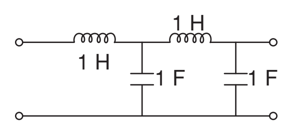
At $t = 0^-$, just before the switch is closed, $V_C = 100$ V. The current $i(t)$ for $t > 0$ is

(A) $100\ e^{-62.5t}$ (B) $50\ e^{-160t}$
(C) $0.25\ e^{-62.5t}$ (D) $50\ e^{-62.5t}$

10. A series RL circuit with $R = 5\ \Omega$ and $L = 2$ mH, has an applied voltage V = 150 sin5000 t, the resulting current i is

(A) 13.4 sin5000t
(B) 10 cos(5000t – 36.4°)
(C) 13.4 cos(5000t – 63.4°)
(D) 13.4 sin(5000t – 63.4°)

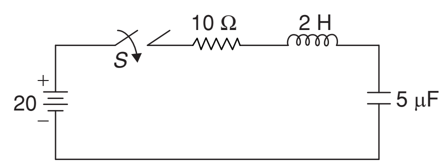
11. Driving point impedance of the network shown in the figure is



(A) $\frac{s^4 + 3s^2 + 1}{s^3 + 2s}$ (B) $\frac{s^2 + 1}{s(s^2 + 2)}$

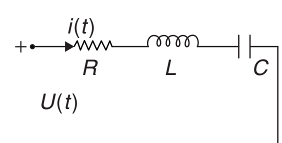
(C) $\frac{s^4 + 3s^3 + 2s^2 + 1}{s^3 + 2s}$ (D) $\frac{s^2 + 1}{s + 1}$

12. Initially, the circuit shown in the given figure was relaxed. If the switch is closed at $t = 0$ the values of $i(0^+)$, $\frac{di(0^+)}{dt}$, $\frac{d^2i(0^+)}{dt^2}$ will, respectively, be ______.



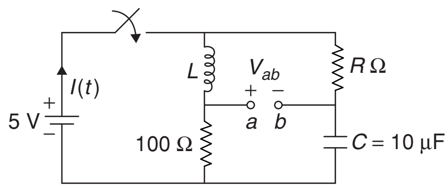
(A) 0,10 and 100 (B) 0,10 and 50
(C) 0, 10 and –100 (D) 0,10 and –50

13. Transient response of the following circuit ______.



(A) Rises exponentially
(B) Decays exponentially
(C) Oscillatory and the oscillations die down with time
(D) Will have sustained oscillations

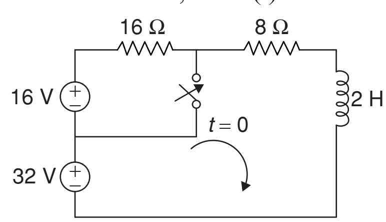
14. In the circuit shown below
$i_L(0^-) = 0$, $V_c(0^-) = 0$
Switch S is closed at $t = 0$ $i(0^+) = 20$ mA, and $V_{ab}=0$ for $t \geq 0$



The value of R is ______.

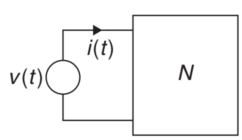
(A) $\frac{1}{4}$ kΩ (B) 250 Ω
(C) 350 Ω (D) 100 Ω

15. For the circuit shown in the following figure. If the switch is closed at $t = 0$; then $i(t)$ for $t \geq 0$ will be



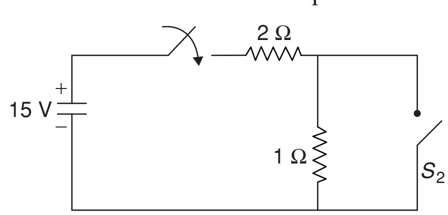
(A) $4 + 2e^{-4t}$ (B) $4 - 2e^{-4t}$
(C) $4 + 2e^{4t}$ (D) $4 - 2^{4t}$

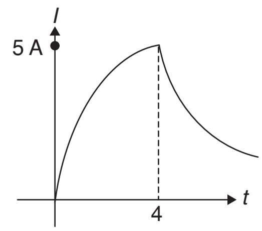
16. The network shown in the figure consists of only two elements. The response for unit step excitation is $i(t) = e^{-5t}$ then the elements are

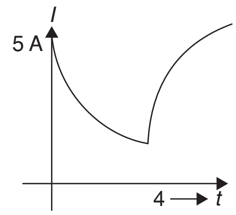


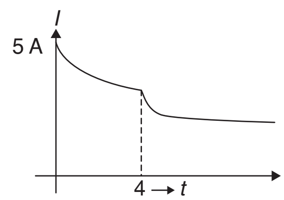
(A) $R = 1$ Ω, $L = 5$ H in series
(B) $R = 1$ Ω, $C = \frac{1}{5}$ F in series
(C) $R = 1$ Ω, $L = 5$ H in parallel
(D) $R = 1$ Ω, $C = \frac{1}{5}$ F in parallel

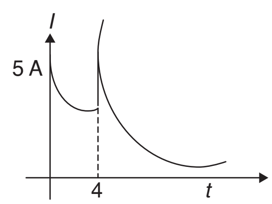
17. The capacitor in the circuit is initially charged to 15 V with S_1 and S_2 open. S_1 is closed at $t = 0$ while S_2 is closed at $t = 4$ s. The wave form of the capacitor current is



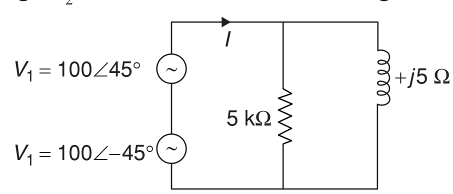
(A)


(B)


(C)


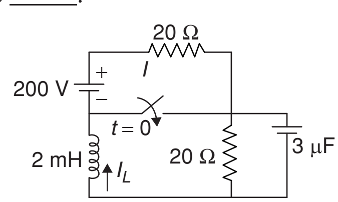
(D)


18. The phase angle of the current I with respect to the voltage V_2 in the circuit shown in the figure is



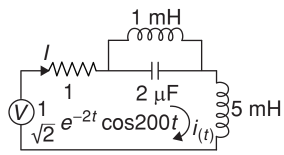
(A) 0° (B) –45°
(C) +45° (D) +90°

19. In the circuit of figure the switch S has been opened for along time. It is closed at $t = 0$. The values of $V_L(0^+)$ and $I_L(0^+)$ are ______.



(A) 200 V, –5 A
(B) 0 V, 5 A
(C) 100 V, 5 A
(D) 100 V, –5 A

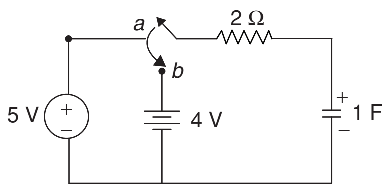
20. Obtain the value of current $i(t)$ in the given circuit at steady state.



(A) $\frac{1}{\sqrt{2}}\cos(200t+45)$ (B) $\frac{1}{\sqrt{2}}$

(C) 0 (D) ∞

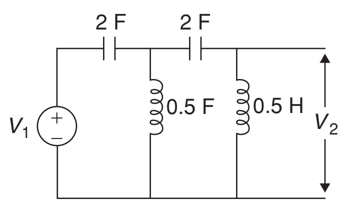
21. For the network shown, the switch is at position 'a' initially. At steady state, the switch is thrown to position 'b' Now $i(0^-)=2$ A, $V_c=2$ V are the initial conditions. Find the circuit current



(A) e^{-3t} (B) $-3e^{\frac{-3}{2^t}}$

(C) $3e^{\frac{-3}{2^t}}$ (D) $3e^{-2t}$

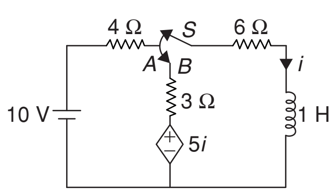
22. Obtain the driving point impedance of the network given in the diagram.



(A) $\frac{s^3+s^2+1}{(s+1)s}$ (B) $\frac{s^4+s^2+s}{1+2s^2}$

(C) $\frac{s^4+s^3+2s^2+1}{(s+1)s}$ (D) $\frac{s^4+s^2+1}{(1+2s^2)2s}$

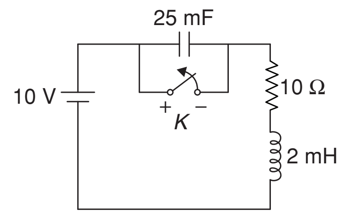
23. For the given circuit switch S is at position 'A' when $t<0$. At $t=0$ the switch is thrown to position B. What will the value of current 'i' in the circuit at the instant $t=4$ s.



(A) 0.1 μA (B) .01 mA

(C) 10 μA (D) 200 mA

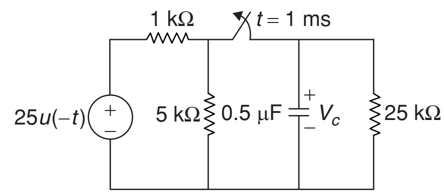
24. In the given network, the switch K is closed for a long time and circuit is in steady state. Now at $t=0$ the switch is opened. Find $V_c(0^+)$ and $i(0^+)$ in the circuit.



(A) 1 A, 0 V (B) 0 A, 5 V

(C) 0, α (D) 0, 0

25. In the following circuit the 25 V source has been applied for a long time the switch is opened at $t=1$ ms

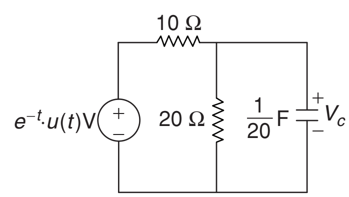


At $t=5$ m s the value of V_c is

(A) 1.23 V (B) 20.16 V

(C) 1.69 V (D) −1.23 V

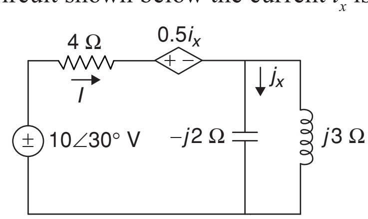
26. In the following circuit capacitor is initially uncharged. At $t=0^+$ the value of $\frac{dV_c}{dt}$ and $\frac{d^2V_c}{dt^2}$



(A) 0 V/s, 8 V/s² (B) −2 V/s, 8 V/s²

(C) 2 V/s, −8 V/s (D) None of these

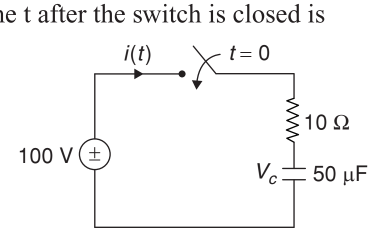
27. In the circuit shown below the current i_x is



(A) 3.94 ∠ 46.28° A (B) 4.62 ∠ 97.38° A

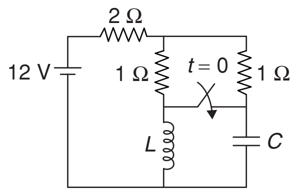
(C) 7.42 ∠ 92.49° A (D) 6.78 ∠ 49.27° A

28. In the circuit shown below. The initial charge on the capacitor is 2.5 mC, with the voltage polarity as indicated. The switch is closed at time $t=0$. The current $i(t)$ at a time t after the switch is closed is



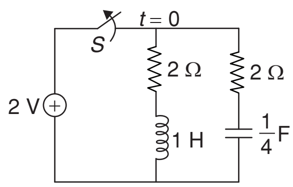
(A) $i(t) = 15 \exp(-2 \times 10^3 t)$ A
(B) $i(t) = 5 \exp(-2 \times 10^3 t)$ A
(C) $i(t) = 10 \exp(-2 \times 10^3 t)$ A
(D) $i(t) = -5 \exp(-2 \times 10^3 t)$ A

29. In the circuit shown below the switch is closed at $t = 0$. What is the initial value of the current through the capacitor?



(A) 0.8 A (B) 2.4 A
(C) 1.6 A (D) 3.2 A

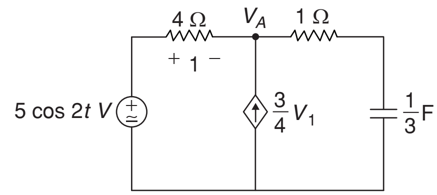
30.



The time constant of the circuit after the switch is opened would be

(A) 2 s (B) 0.5 s
(C) 1 s (D) None of these

31. The power factor seen by the voltage source is



(A) 0.8 (lagging) (B) 0.8 (leading)
(C) 36.9 (Lag) (D) − 36.9 (Leading)

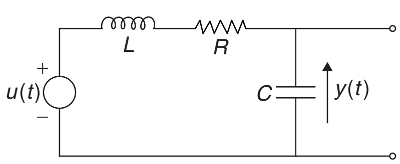
32. An input voltage
$V(t) = 10\sqrt{2}\cos(t + 10°) + 10\sqrt{5}\cdot\cos(2t + 10°)$ V is applied to a series combination of resistance $R = 1$ Ω and an inductance $L = 1$ H. the resulting steady state current $i(t)$ is

(A) $10\cos(t + 55°) + 10\cos(2t + 10° + \tan^{-1}2)$ A
(B) $10\cos(t + 55°) + 10\sqrt{\frac{3}{2}}\cdot\cos(2t + 55°)$ A
(C) $10\cos(t - 35°) + 10.\cos(2t + 10° - \tan^{-1}2)$ A
(D) $\frac{10\sqrt{3}}{2}\angle 90°$ A

Practice Problems 2

Directions for questions 1 to 24: Select the correct alternative from the given choices.

1. The condition on R, L and C such that the step response $y(t)$ in the figure has no oscillations, is



(A) $R \geq \frac{1}{2}\sqrt{\frac{L}{C}}$ (B) $R \geq \sqrt{\frac{L}{C}}$

(C) $R \geq 2\sqrt{\frac{L}{C}}$ (D) $R = \frac{1}{\sqrt{LC}}$

2. A series circuit consists of two elements has the following current and applied voltage $i = 4\cos(2000t + 11.32°)$ A
$\upsilon = 200\sin(2000t + 50°)$ V. The circuit elements are
(A) Resistance and capacitance
(B) Capacitance and inductance
(C) Inductance and resistance
(D) Both resistances

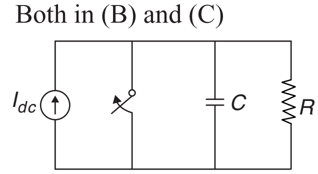
3. Transient current of an RLC circuit is oscillatory when

(A) $R = 2\sqrt{\frac{L}{C}}$ (B) $R = 0$

(C) $R > 2\sqrt{\frac{L}{C}}$ (D) $R < 2\sqrt{\frac{L}{C}}$

4. The transient response occurs
(A) Only in resistive circuits
(B) Only in inductive circuits
(C) Only in capacitive circuits
(D) Both in (B) and (C)

5.

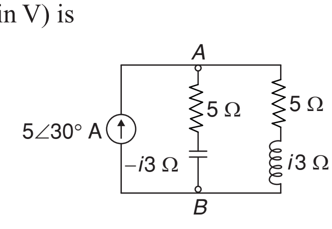


The initial voltage across the capacitor when the switch S is opened at $t = 0$

(A) zero (B) $C\cdot\frac{I_{dc}}{s}$

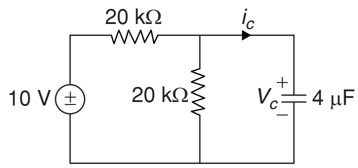
(C) $\frac{1}{Cs}I_{dc}$ (D) $Cs\,I_{dc}$

6. In the AC network shown in the figure, the phasor voltage V_{AB} (in V) is



(A) 0 (B) $5\angle 30°$
(C) $12.5\angle 30°$ (D) $17\angle 30°$

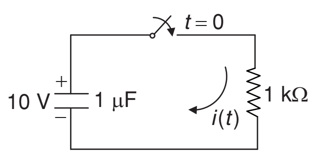
7. In the circuit shown, V_C is 0 V at $t = 0$ V at $t = 0$ s. For $t > 0$, the capacitor current $i_C(t)$, where 't' is in seconds, is given by



(A) 0.50 exp (– 25t) mA
(B) 0.25 exp (– 25t) mA
(C) 0.50 exp (–12.5t) mA
(D) 0.25 exp (– 6.25t) mA

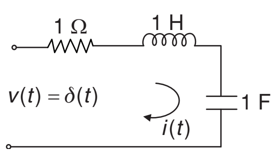
8. A series RL circuit, with $R = 10\ \Omega$ and $L = 1$ H, has a 100 V source applied at $t = 0$. The current for $t > 0$ is
(A) $10\ e^{-10t}$
(B) $10\ (1 - e^{-10t})$
(C) $100\ e^{-100t}$
(D) $100\ (1 - e^{-100t})$

9. The current in the circuit when the switch is closed at $t = 0$ is



(A) $10\ e^{-100t}$ (B) $0.01\ e^{-1000t}$
(C) $0.1\ e^{-1000t}$ (D) $10\ e^{0.1t}$

10.

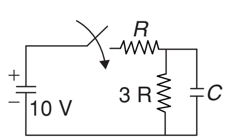


The circuit shown in the figure is initially relaxed. The Laplace transform of the current $i(t)$ is

(A) $\dfrac{s+1}{s^2+2s+1}$ (B) $\dfrac{s+1}{s^2+s+1}$

(C) $\dfrac{s}{s^2+s+1}$ (D) $\dfrac{s}{s^2+2s+1}$

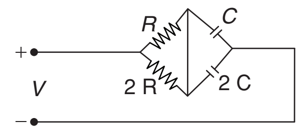
11. The time constant of the network shown in the figure is



(A) 4 RC (B) $\dfrac{3}{4}RC$

(C) 3 RC (D) RC

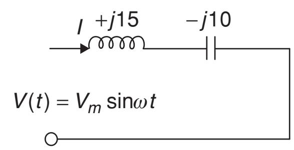
12. The time constant of the network shown in the figure is ______.



(A) $\dfrac{2}{3}RC$ (B) $\dfrac{3}{2}RC$

(C) 2 RC (D) RC

13. The network shown in the figure draws a current of "I"

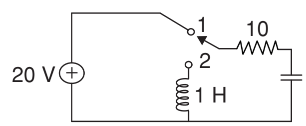


If the supply frequency is doubled then the current drawn by the circuit is ______.

(A) $\dfrac{I}{2}$ (B) 2 I

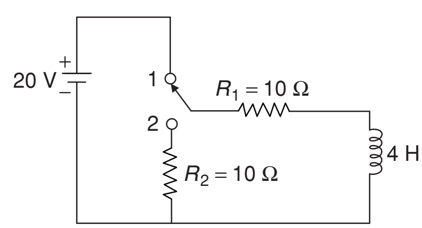
(C) $\dfrac{I}{5}$ (D) $\dfrac{2I}{5}$

14. In the circuit shown in the figure, the switch is thrown from position 1 to 2 at $t = 0$, after being at position 1 for a long time. The value of $\dfrac{d^2i(0^+)}{dt^2}$ is ______.



(A) 200 (B) – 200
(C) – 100 (D) 100

Common Data for Questions 15 and 16: The circuit shown in the figure is initially under a steady state condition.



The switch is moved from position 1 to position 2 at $t = 0$.

15. The current through inductor immediately after switching is ______.

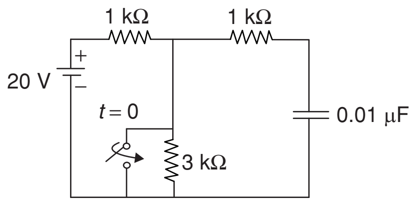
(A) 2 A (B) $\dfrac{1}{2}$ A

(C) 1 A (D) 5 A

16. The expression for current $i(t)$ is ______.
(A) e^{-5t} (B) $2e^{-5t}$

(C) $\dfrac{2}{5}e^{-5t}$ (D) $5e^{-2t}$

17. The switch in the circuit shown in the figure closes at $t = 0$. Find current i_c for all times.



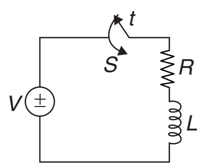
(A) $5\times10^{-3}\times e^{-105t}$ A

(B) $15\times10^{-2}\times e^{-105t}$ A

(C) $15\times10^{-3}\times e^{-105t}$ A

(D) $10\times10^{-3}\times e^{-105t}$ A

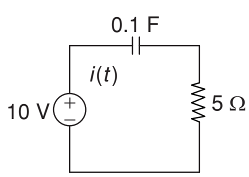
18. For the given circuit, the current passing through inductor 'L' at the instant $t = 0^+$ is



(A) $\frac{V}{R}$ (B) Infinity

(C) $\frac{V}{R + jwL}$ (D) 0

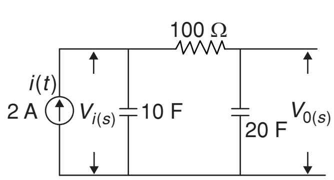
19. Determine the current i for $t \geq 0$, if $V_c(0) = 1$ V for the circuit shown



(A) $\frac{0.9e^{-5t}}{2}$ (B) $1.8\, e^{-2t}$

(C) $5e^{-2t}$ (D) 3.6

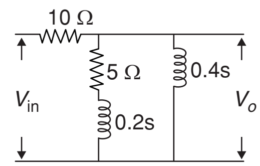
20. Find the transfer function of the given system $\left(\frac{V_0(s)}{V_1(s)}\right)$



(A) $\frac{1}{2000s+1}$ (B) $\frac{10s}{2000s+1}$

(C) $\frac{100}{1000s+1}$ (D) $\frac{1}{1000s+1}$

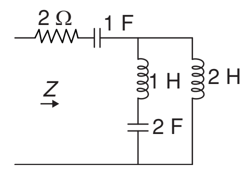
21. Obtain the transfer function of the following system:



(A) $\frac{0.4s}{0.2s+1s}$ (B) $\frac{s(2+0.4s)}{0.4s^2+8s+100}$

(C) $\frac{s(2+.08s)}{0.08s^2+8s+50}$ (D) $\frac{2+.04s}{s^2+8s+50}$

22. Find the driving point admittance of the network given below.



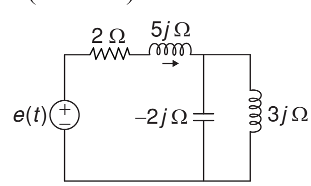
(A) $\frac{3s^3+2s}{2s^4+6s^3+7s^2+4s+2}$

(B) $\frac{3s^2+2}{2s^4+5s^2+4s+2}$

(C) $\frac{(3s^2+2)s}{2s^4+7s^3+6s^2+4s}$

(D) $\frac{5s^3+2s}{s^4+7s^3+6s^2+4s+2}$

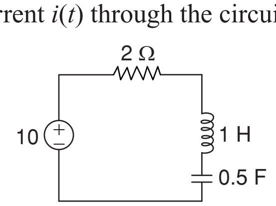
23. Find the voltage V_{ab} across the impedance of $(2 + 5j)$ Ω in the network.
The supply voltage.
$e(t) = 10 \sin(2\pi t + 45)$



(A) $20\angle 44°$ (B) $24\angle{-40°}$

(C) $4.48\angle{-108°}$ (D) $2.25\angle 63.43°$

24. Find the current $i(t)$ through the circuit given



(A) $10e^{-t}\cos t$ (B) $\frac{10}{\sqrt{2}}e^{t}\cos\sqrt{2}t$

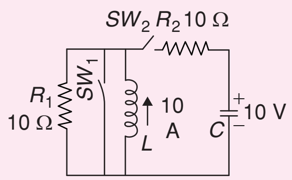
(C) $10e^{-t}\sin t$ (D) $\frac{10}{\sqrt{2}}e^{t}\sin\sqrt{2}t$

PREVIOUS YEARS' QUESTIONS

1. An ideal capacitor is charged to a voltage V_0 and connected at $t = 0$ across an ideal inductor L. (The circuit now consists of a capacitor and inductor alone). If we let $\omega_0 = \frac{1}{\sqrt{LC}}$, the voltage across the capacitor at time $t > 0$ is given by **[2006]**

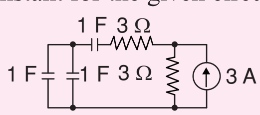
(A) V_0 (B) $V_0 \cos(\omega_0 t)$
(C) $V_0 \sin(\omega_0 t)$ (D) $V_0 e^{-\omega_0 t} \cos(\omega_0 t)$

2. In the circuit shown in figure switch SW_1 is initially CLOSED and SW_2 is OPEN. The inductor L carries a current of 10 A and the capacitor is charged to 10 V with polarities as indicated. SW_2 is initially CLOSED at $t = 0$ –and SW_1 is OPENED at $t = 0$. The current through C and the voltage across L at $t = 0+$ is **[2007]**



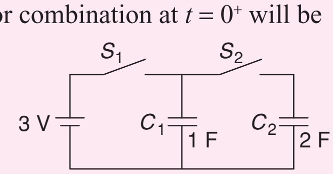
(A) 55 A, 4.5 V (B) 5.5 A, 45 V
(C) 45 A, 5.5 V (D) 4.5 A, 55 V

3. The time constant for the given circuit will be **[2008]**



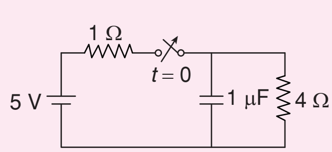
(A) $\frac{1}{9}$ s (B) $\frac{1}{4}$ s
(C) 4 s (D) 9 s

4. In the figure shown, all elements used are ideal. For time $t < 0$, S_1 remained closed and S_2 open. At $t = 0$, S_1 is opened and S_2 is closed. If the voltage V_{c2} across the capacitor C_2 at $t = 0$ is zero, the voltage across the capacitor combination at $t = 0^+$ will be **[2009]**



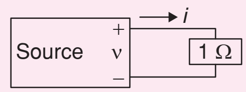
(A) 1 V (B) 2 V
(C) 1.5 V (D) 3 V

5. The switch in the circuit has been closed for a long time. It is opened at $t = 0$. At $t = 0^+$, the current through the 1 μF capacitor is **[2010]**



(A) 0 A (B) 1 A
(C) 1.25 A (D) 5 A

6. As shown in the figure, a 1 Ω resistance is connected across a source that has a load line $v + i = 100$. The current through the resistance is **[2010]**

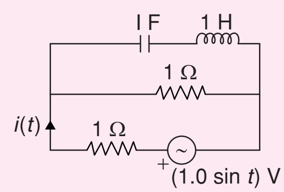


(A) 25 A (B) 50 A
(C) 100 A (D) 200 A

7. The voltage applied to a circuit is $100\sqrt{2}\cos(100\pi)$ V and the circuit draws a current of $10\sqrt{2}\sin(100\pi + \pi/4)$ amperes. Taking the voltage as the reference phasor, the phasor representation of the current in amperes is **[2011]**

(A) $10\sqrt{2}\angle\frac{-\pi}{4}$ (B) $10\sqrt{2}\angle\frac{-\pi}{4}$
(C) $10\angle+\pi/4$ (D) $10\sqrt{2}\angle+\frac{\pi}{4}$

8. The rms value of the current $i(t)$ in the circuit shown below is **[2011]**



(A) $\frac{1}{2}$ A (B) $\frac{1}{\sqrt{2}}$
(C) 1 A (D) $\sqrt{2}$ A

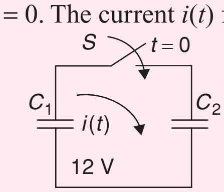
9. The current I_C in the figure above is **[2011]**

(A) $j\,2$ A (B) $-j\frac{1}{\sqrt{2}}$ A
(C) $+j\frac{1}{\sqrt{2}}$ A (D) $+j2$ A

10. The power dissipated in the resistor R is **[2011]**

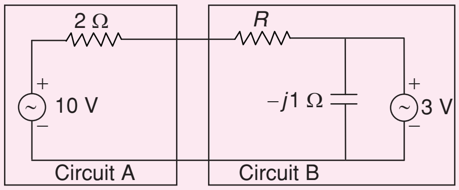
(A) 0.5 W (B) 1 W
(C) $\sqrt{2}$ W (D) 2 W

11. In the following figure, C_1 and C_2 are ideal capacitors. C_1 has been charged to 12 V before the ideal switch S is closed at $t = 0$. The current $i(t)$ for all t is **[2012]**



(A) Zero
(B) A step function
(C) An exponentially decaying function
(D) An impulse function

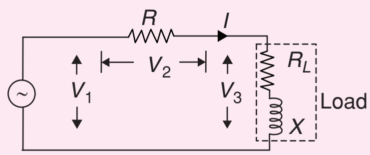
12. Assuming both the voltage sources are in phase, the value of R for which maximum power is transferred from circuit A to circuit B is **[2012]**



(A) 0.8 Ω (B) 1.4 Ω
(C) 2 Ω (D) 2.8 Ω

Common Data for Questions 13 and 14: In the circuit shown, the three voltmeter readings are V_1 = 220 V, V_2 = 122 V, V_3 = 136 V.

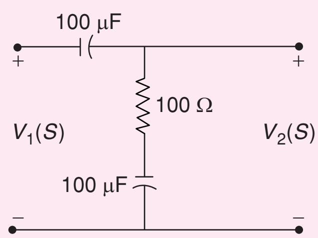
13. The power factor of the load is **[2012]**



(A) 0.45 (B) 0.50
(C) 0.55 (D) 0.60

14. If R_L = 5 Ω, the approximate power consumption in the load is **[2012]**
(A) 700 W (B) 750 W
(C) 800 W (D) 850 W

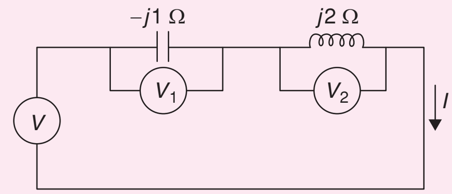
15. The transfer function $\frac{V_2(s)}{V_1(s)}$ of the circuit shown below is **[2013]**



(A) $\frac{0.5s+1}{s+1}$ (B) $\frac{3s+6}{s+2}$
(C) $\frac{s+2}{s+1}$ (D) $\frac{s+1}{s+2}$

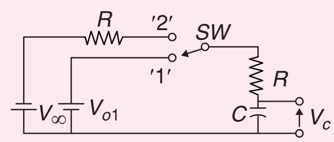
16. A single phase load is supplied by a single-phase voltage source. If the current flowing from the load to the source is 10∠–150° A and if the voltage at the load terminals is 100∠60° V, then the **[2013]**
(A) Load absorbs real power and delivers reactive power.
(B) Load absorbs real power and absorbs reactive power.
(C) Load delivers real power and delivers reactive power.
(D) Load delivers real power and absorbs reactive power.

17. Three moving iron type voltmeters are connected as shown below. Voltmeter readings are V, V_1, and V_2, as indicated. The correct relation among the voltmeter readings is **[2013]**



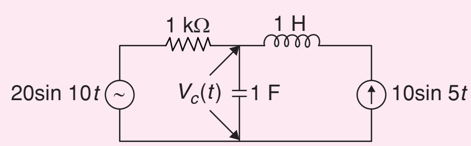
(A) $V = \frac{V_1}{\sqrt{2}} + \frac{V_2}{\sqrt{2}}$
(B) $V = V_1 + V_2$
(C) $V = V_1 V_2$
(D) $V = V_2 - V_1$

18. The switch SW shown in the circuit is kept at position '1' for a long duration. At t = 0+, the switch is moved to position '2'. Assuming $|V_{o2}| > |V_{o1}|$, the voltage $v_c(t)$ across the capacitor is **[2014]**



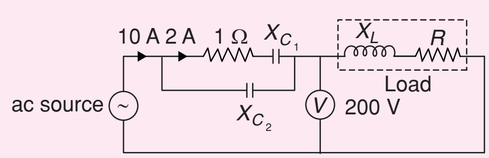
(A) $V_c(t) = -V_{o2}(1 - e^{-t/RC}) - V_{o1}$
(B) $V_c(t) = V_{o2}(1 - e^{-t/RC}) + V_{o1}$
(C) $V_c(t) = -(V_{o2} + V_{o1})(1 - e^{-t/RC}) - V_{o1}$
(D) $V_c(t) = (V_{o2} - V_{o1})(1 - e^{-t/RC}) + V_{o1}$

19. The voltage across the capacitor, as shown in the figure, is expressed as
$v_c(t) = A_1 \sin(\omega_1 t - \theta_1) + A_2 \sin(\omega_2 t - \theta_2)$ **[2014]**



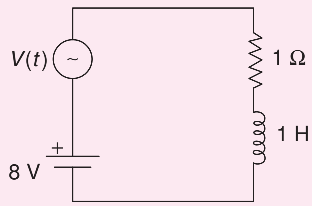
The values of A_1 and A_2, respectively, are
(A) 2.0 and 1.98
(B) 2.0 and 4.20
(C) 2.5 and 3.50
(D) 5.0 and 6.40

20. The total power dissipated in the circuit, shown in the figure, is 1 kW. **[2014]**

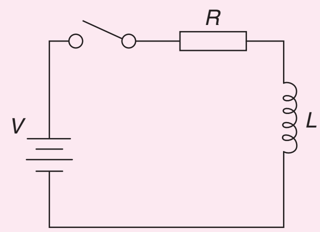


The voltmeter, across the load, reads 200 V. The value of X_L is________.

21. The circuit shown in the figure has two sources connected in series. The instantaneous voltage of the AC source (in Volts) is given by $v(t) = 12 \sin t$. If the circuit is in steady state, then the rms value of the current (in Ampere) flowing in the circuit is _______. **[2015]**



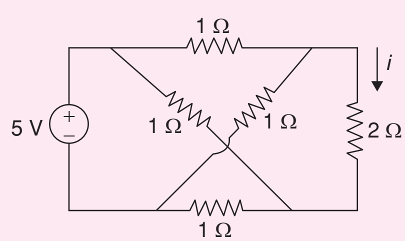
22. A series RL circuit is excited at $t = 0$ by closing a switch as shown in the figure. Assuming zero initial conditions, the value of $\frac{d^2t}{dt^2}$ at $t = 0^+$ is **[2015]**



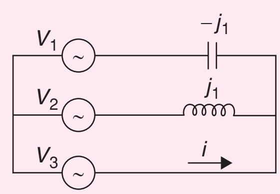
(A) $\frac{V}{L}$ (B) $\frac{-V}{R}$

(C) 0 (D) $\frac{-RV}{L^2}$

23. The current i(Ampere) in the 2 | resistor of the given network is ______. **[2015]**

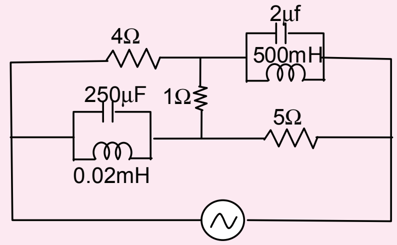


24. In the given network $V_1 = 100\ \angle 0°$ V, $V_2 = 100\ \angle -120°$ V, $V_3 = 100\ \angle +120°$ V. The phasor current i (in Ampere) is **[2015]**

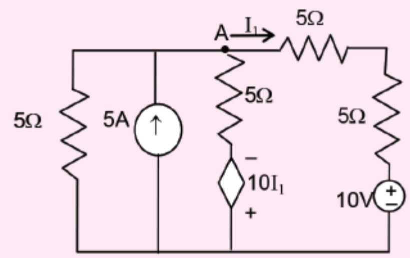


(A) 173.2 ∠–60° (B) 173.2 ∠120°
(C) 100.0 ∠–60° (D) 100.0 ∠120°

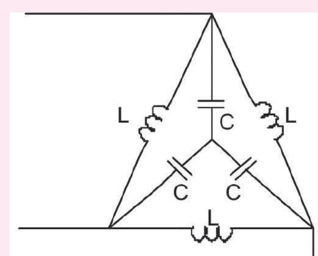
25. In the circuit shown below, the supply voltage is 10 sin(1000) volts. The peak value of the steady state current through the 1Ω resistor, in amperes, is ______. **[2016]**



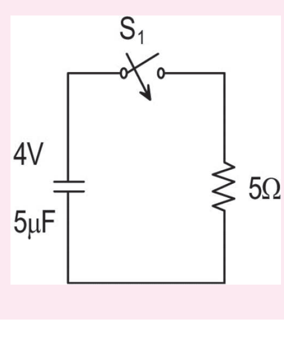
26. The circuit below is excited by a sinusoidal source. The value of R, in Ω, for which the admittance of the circuit becomes a pure conductance at all frequencies is_______. **[2016]**



27. In the balanced 3-phase, 50Hz, circuit shown below, the value of inductance (L) is 10mH. The value of the capacitance (C) for which all the line currents are zero, in millifarads, is ________. **[2016]**



28. In the circuit shown below, the initial capacitor voltage is 4V. switch S_1 is closed at $t = 0$. The charge (in µC) lost by the capacitor form $t = 25$µS to $t = 100$µS is ________. **[2016]**



ANSWER KEYS

EXERCISES

Practice Problems 1

1. B	**2.** B	**3.** B	**4.** C	**5.** A	**6.** C	**7.** D	**8.** B	**9.** C	**10.** D
11. A	**12.** D	**13.** C	**14.** B	**15.** B	**16.** B	**17.** D	**18.** A	**19.** C	**20.** C
21. B	**22.** D	**23.** A	**24.** A	**25.** A	**26.** C	**27.** B	**28.** A	**29.** A	**30.** B
31. B	**32.** C								

Practice Problems 2

1. C	**2.** D	**3.** D	**4.** D	**5.** A	**6.** D	**7.** A	**8.** B	**9.** B	**10.** C
11. B	**12.** C	**13.** C	**14.** B	**15.** A	**16.** B	**17.** C	**18.** D	**19.** B	**20.** A
21. C	**22.** A	**23.** B	**24.** C						

Previous Years' Questions

1. B	**2.** D	**3.** C	**4.** A	**5.** B	**6.** B	**7.** B	**8.** B	**9.** D	**10.** B
11. D	**12.** A	**13.** A	**14.** B	**15.** D	**16.** B	**17.** D	**18.** D	**19.** A	
20. 17.34	**21.** 9.99 A	**22.** D	**23.** 0 A	**24.** B	**25.** 1	**26.** 14.14	**27.** 3	**28.** 6.99	