CBSE Test Paper-03

Class - 12 Physics (Atoms)

1. Emission line spectrum of atoms contains
    a. Only a few colors in the form of isolated sharp parallel lines generally produced by heated gases
    b. All colors of visible light without sharp boundaries
    c. All colors in the form of isolated sharp parallel lines generally produced by heated resistances
    d. Only a few colors in the form of isolated sharp parallel lines generally produced by chilled gases
2. Find the kinetic, potential, and total energies of the hydrogen atom in the first excited level, and find the wavelength of the photon emitted in a transition from that level to the ground level.
    a. K_2 = 3.90 eV, U_2 = -6.80 eV and E_2 = -3.40 eV. For the ground level (n = 1) E_1 = -13.6 eV, λ = 142 nm
    b. K_2 = 3.80 eV, U_2 =-6.80 eV and E_2 = -3.40 eV. For the ground level (n = 1) E_1 = -13.6 eV, λ = 132 nm
    c. K_2 = 3.40 eV, U_2 =-6.80 eV and E_2 = -3.40 eV. For the ground level (n = 1) E_1 = -13.6 eV, λ = 122 nm
    d. K_2 = 3.60 eV, U_2 = -6.80 eV and E_2 = -3.40 eV. For the ground level (n = 1) E_1 = -13.6 eV, λ = 129 nm
3. A triply ionized beryllium ion Be^{3+}, (a beryllium atom with three electrons removed), behaves very much like a hydrogen atom except that the nuclear charge is four times as great. For a given value of n, how does the radius of an orbit in Be^{3+} compare to that for hydrogen?
    a. $\frac{1}{7}$th of hydrogen radius
    b. $\frac{1}{4}$th of hydrogen radius
    c. $\frac{1}{6}$th of hydrogen radius
    d. $\frac{1}{5}$th of hydrogen radius
4. The energy of the electron revolving in the orbit of Bohr radius is
    a. 13.6 MeV

b. -13.6 eV
c. -13.6 MeV
d. 13.6 eV

5. Which of these statements about Bohr model is correct?
   a. Bohr model is pure quantum mechanical theory
   b. Bohr model is based classical electromagnetic theory
   c. Bohr model combines classical and early quantum concepts
   d. Bohr model postulates wavy paths around the nucleus

6. What is the maximum number of spectral lines emitted by a hydrogen atom when it is in the third excited state?

7. State Bohr's quantisation condition for defining stationary orbits.

8. Write the expression for Bohr's radius in hydrogen atom.

9. In the Rutherford scattering experiment, the distance of closest approach for an a-particle is d_0 . If a-particle is replaced by a proton, then how much kinetic energy in comparison to α-particle will be required to have the same distance of closest approach d_0?

10. In the ground state of hydrogen atom, its Bohr radius is given as 5.3×10^{-11} m.The atom is excited such that the radius becomes 21.2×10^{-11} m. Find (i) the value of the principal quantum number and (ii) the total energy of the atom in this excited state.

11. Using Bohr's postulates of the atomic model, derive the expression for radius of nth electron orbit. Hence, obtain the expression for Bohr's radius.

12. State any two postulates of Bohr's theory of hydrogen atom. What is the maximum possible number of spectral lines observed when the hydrogen atom is in its second excited state? Justify your answer.
    Calculate the ratio of the maximum and minimum wavelengths of the radiations emitted in this process.

13. The energy of the electron, the hydrogen atom, is known to be expressible in the form $E_n = \frac{-13.6eV}{n^2}$ (n = 1, 2, 3, . . .)

Use this expression to show that the

i. Electron in the hydrogen atom can not have an energy of -2V.
ii. Spacing between the lines (consecutive energy levels) within the given set of the observed hydrogen atom spectrum decreases as n increases.

14. Prove that the ionization energy of hydrogen atom is 13.6 eV.

15. Using Bohr's formula for energy quantization determine:
    i. the longest wavelength in the Lyman series of hydrogen atom spectrum.
    ii. the excitation energy of the n =3 level of He^{+} atom
    iii. the ionization potential of the ground state of Li^{++} atom.

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Answers

1. a. Only a few colors in the form of isolated sharp parallel lines generally produced by heated gases

   Explanation: Since the energy states of an element are definite, the atoms of that element emit light of some particular wavelength only. Every element has different atomic spectrum and so this emission spectrum can be used to find the material composition.

2. c. K_2 = 3.40 eV, U_2 =-6.80 eV and E_2 = -3.40 eV. For the ground level (n = 1) E_1 = -13.6 eV, λ = 122 nm

   Explanation: Total energy = KE + PE

   In 1st orbit, n = 1

   -13.6 = -2 K.E + K.E = -K.E

   K.E = 13.6 eV

   P.E = -2 K.E = -27.2 eV

   In 2nd orbit, n = 2

   Total energy = -13.6 / 2^2 eV

   K.E = - Total energy = 3.4 eV

   P.E = -2 K.E = -6.8 eV

   Wavelength is found from Lyman series:

   $\frac{1}{\lambda} = R(\frac{1}{1^2} - \frac{1}{2^2})$ = 122 nm

3. b. $\frac{1}{4}$th of hydrogen radius

   Explanation: $r_n = \frac{n^2 h^2 \varepsilon_0}{\pi m e^2}$ for hydrogen atom

   $r_{n-ion} = \frac{n^2 h^2 \varepsilon_0}{\pi m Z e^2}$ for ion

   Z =4, so radius of Be^{3+} is four times smaller than hydrogen atom

4. b. -13.6 eV

   Explanation: Atomic energies are expressed in electron volts (eV) rather than joules (J).

   $E_n = -\frac{13.6}{n^2} eV$

The lowest state of the atom is that of the lowest energy, ie with the electron revolving in the smallest radius, n = 1, which is the Bohr radius a_0.

when n = 1, E_n = -13.6 eV

5. c. Bohr model combines classical and early quantum concepts

Explanation: Bohr combined classical and early quantum concepts and gave his theory in form of three postulates.

i. An electron in an atom could revolve in certain stable orbits without emitting radiant energy.
ii. Electron revolves around the nucleus in those orbits whose angular momentum is an integral multiple of $\frac{h}{2\pi}$ where h is the Planck's constant.
iii. An electron might make a transition from one orbit to another of lower energy by emitting energy equal to the energy difference between the intial and final states. Frequency of emitted photon is $h\nu = E_i - E_f$

6. Number of spectral lines obtained due to transition of electron from n=4 to n = 1 is given by formula $N = \frac{n(n-1)}{2}$

$N = \frac{(4)(4-1)}{2} = 6.$

7. According to Bohr, an electron can revolve only in certain discrete, non-radiating orbits for which the total angular momentum of the revolving electron is an integral multiple of $\frac{h}{2\pi}$ i.e,

$mvr = nh/2\pi$ where n = 1, 2, 3 ,4..........

h is Planck's constant.

8. Expression for Bohr's radius in hydrogen atom

$r_n = \frac{n^2h^2}{4\pi^2mkZe^2} \Rightarrow r_1 = \frac{n^2h^2}{4\pi^2mke^2}$

where, n = principal quantum number,

m = mass of electron

$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 N - m^2/c^2$

Z =atomic number of atom = 1 and

h =Planck's constant

9. Distance of closest approach

$d_0 = \frac{2kZe^2}{K.E} = \frac{4kZe^2}{mv^2}$

kinetic energy $\propto$ Z (atomic number)

$$\Rightarrow \quad \frac{K_{\text{proton}}}{K_\alpha} = \frac{Z_{\text{proton}}}{Z_\alpha} = \frac{1}{2} \Rightarrow K_{\text{proton}} : K_\alpha = 1 : 2$$

10. i. According to the question,

   Given, $r_1 = 5.3 \times 10^{-11}$ m and $r_2 = 21.2 \times 10^{-11}$ m

   $n_1 = 1$

   We know that, $r \propto n^2$

   $$\frac{r_1}{r_2} = \frac{n_1^2}{n_2^2} \Rightarrow \frac{1}{n_2^2} = \frac{5.3 \times 10^{-11}}{21.2 \times 10^{-11}}$$

   $$\Rightarrow \quad n_2^2 = 4 \Rightarrow n_2 = 2$$

   ii. We know that,

   $$E = \frac{-13.6}{n^2} = \frac{-13.6}{4} = -3.4\text{eV}$$

11. In atomic physics, the Rutherford–Bohr model presented by Niels Bohr and Ernest Rutherford in 1913, is a system consisting of a small, dense nucleus surrounded by revolving electrons —similar to the structure of the Solar System but with attraction provided by electrostatic forces rather than gravity. A hydrogen like atom consists of a tiny positively charged nucleus and an electron revolving in a stable circular orbit around the nucleus

   Let e, m and v be respectively the charge, mass and velocityof the electron and r the radius of the orbit.

   The positive charge on the nucleus is Ze, where Z is the atomic number (in case of hydrogen atom, Z = 1). As, the centripetal force is provided by the electrostatic force of attraction, we have

   $$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(Ze) \times e}{r^2} \text{ or } mv^2 = \frac{Ze^2}{4\pi\varepsilon_0 r} \text{............(i)}$$

   From the first postulate of Bohr's atomic model, the angular momentum of the electron is

   $$mvr = n\frac{h}{2\pi} \text{.............(ii)}$$

   where, n (= I, 2, 3, ...) is **principal quantum number**.

   From Eqs. (i) and (ii), we get

   $$r = n^2 \frac{h^2\varepsilon_0}{\pi m Z e^2} \text{ ....(iii)}$$

   This is the equation for the radii of the permitted orbits.

   According to this equation, $r_n \propto n^2$

   Since, n = 1, 2. 3, ... it follows that the radii of the permitted orbits increase in the ratio

1: 4: 9: 16:.... from the first orbit.

Bohr's Radius The radius of the first orbit (n = 1) of hydrogen atom (Z = 1) will be

$r = h^2\varepsilon_0/\pi me^2$

This is called Bohr's radius and its value is 0.53. Since $r_n \propto n^2$, the radius of the second orbit of hydrogen atom will be (4$\times$0.53)$\overset{o}{A}$ and that of the third orbit (9 $\times$ 0.53) $\overset{o}{A}$ and can be extended for other orbits according to formula.

12. Bohr's Postulates
    i. Every atom consists of small and massive central core, known as nucleus around which electron revolve and necessary centripetal force prevailed by electrostatic force of attraction between positively charged nucleus and negatively charged electrons.
    ii. The electrons are revolved around the nucleus in only those circular orbits which satisfy the quantum condition that the angular momentum of electrons is equal to integral multiple of $\frac{h}{2\pi}$ where, h is Planck's constant.
    $mvr = \frac{nh}{2\pi}$
    where, n = 1, 2, 3,...
    In second excited state i.e., n =3, two spectral lines namely Lyman series and Balmer series can be obtained corresponding to transition of electron from n = 3 to n = 1 and n = 3 to n = 2, respectively.
    For Lyman series, n = 3 to n = 1, for minimum wavelength
    $\frac{1}{\lambda_{\text{man}}} = R\left[\frac{1}{1^2} - \frac{1}{3^2}\right] = \frac{8R}{9}$ ....(i)
    For Balmer series(maximum wavelength),
    $\frac{1}{\lambda_{\text{max}}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$
    $= \left(\frac{9-4}{36}\right) R = \frac{5R}{36}$ ....(ii)
    On dividing Eq. (i) by Eq. (ii), we get
    $\frac{\lambda_{\text{max}}}{\lambda_{\text{man}}} = \frac{8R/9}{5R/36} = \frac{8R}{9} \times \frac{36}{5R} = \frac{32}{5}$
    $\Rightarrow \lambda_{\text{max}} : \lambda_{\text{man}} = 32 : 5$
13. As, $E_n = -\frac{13.6eV}{n^2}$

    Putting n = 1, 2, 3, . . ., n we get

    $E_1 = \frac{-13.6}{12} = -13.6eV$

    $E_2 = \frac{-13.6}{2^2} = -\frac{13.6}{4} = 3.4\,eV$

$E_3 = \frac{-13.6}{3^2} = -\frac{13.6}{9} = -1.51eV$

$E_4 = \frac{-13.6}{4^2} = \frac{-13.6}{16} = -.85V \, E_n = \frac{-13.6}{\infty^2} = 0eV$

i. Hence, it can be observed that the electron in the hydrogen atom can not have an energy of -2V.

ii. As n increases, energies of the excited states come closer and closer together. Therefore, as n increases, E_n becomes less negative until at $n = \infty$, i.e. E_n = 0.

14. We know that

$E = -\frac{2\pi^2 mK^2 Z^2 e^4}{n^2 h^2}$(n = 1, Z = 1)

$W = k^2 \frac{2\pi^2 me^4}{h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Ionization energy is the energy required to remove an electron from ground state to infinity.

Here, n_1 = 1, n_2 = ∞

$\therefore W = k^2 \frac{2\pi^2 me^4}{h^2}\left(\frac{1}{1} - \frac{1}{\infty}\right)$

$= k^2 \frac{2\pi^2 me^4}{h^2}$

or $W = \frac{(9\times10^9)^2 \times 2(3.142)^2 \times 9\times10^{-31} \times (1.6\times10^{-19})^4}{(6.63\times10^{-34})^2}$

$= 21.45 \times 10^{-19} J = \frac{21.45\times10^{-19}}{1.6\times10^{-19}} eV$ = 13.6 eV

15. i. Wavelengths of radiation of the Lyman series are given by

$\lambda_n = \frac{64\pi^3 \varepsilon_0^2 h^3 c}{me^4}\left(\frac{n^2}{n^2 - 1^2}\right)$

n = 2 corresponds to the longest wavelength

= 1225 Å

ii. The energy required to excite the electron from the ground state (n = 1) to the n = 3 state is

$\mathrm{E}_3 - E_1 = \frac{mZ^2 e^4}{32\pi^2 \varepsilon_0^2 h^2}\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$

= 48.1 eV

where Z = 2

iii. Ionization energy is given by

$E_\infty - E_1 = \frac{mZ^2 e^4}{32\pi^2 \varepsilon_0^2 h^2}$ (with Z = 3) = 122 eV

Thus, ionization potential is 122 V.