CBSE Test Paper 03

Chapter 13 Surface Area and Volume

1. A rectangular piece of paper is 44 cm long and 18 cm wide. If a cylinder is formed by rolling the paper along its length, then the radius of the base of the cylinder is **(1)**

   a. 14 cm
   b. 21 cm
   c. 7 cm
   d. 22 cm

2. A frustum of a cone is of height 12cm with radii of its circular ends as 2cm and 4cm. The volume of the frustum is **(1)**

   a. $352cm^3$
   b. None of these
   c. $354cm^3$
   d. $350cm^3$

3. A cone is cut through a plane parallel to its base and the small cone is (upper part) removed. The part that is left over is called **(1)**

   a. cylinder
   b. cone
   c. sphere
   d. frustum of a cone

4. The volume of the cuboid whose length, breadth and height is 12cm, 8cm and 6cm is **(1)**

   a. 568 cu.cm
   b. 576 cu.cm
   c. 570 cu.cm
   d. 576 sq.cm

5. A hollow sphere of external and internal diameters 8cm and 4cm respectively is

melted into a cone of base diameter 8cm. The height of the cone is **(1)**

a. 14cm
b. 13cm
c. 12cm
d. 15cm

6. Given that 1 cubic cm of marble weighs 25 g. The weight of a marble block 28 cm in width and 5 cm thick is 112 kg. Find the length of the marble block. **(1)**
7. Find the area of a triangle having base 25 cm and height 10.8 cm. **(1)**
8. The largest sphere is carved out of a cube of side 10.5 cm. Find the volume of the sphere. **(1)**
9. Find the ratio of the volume of a cube to the volume of a sphere, if sphere fits completely inside the cube. **(1)**
10. Find the volume of a frustum of base radii 7 cm and 3.5 cm and height 2 cm. **(1)**
11. 50 circular plates each of radius 7 cm and thickness 5 mm are placed one above another to form a solid right circular cylinder. Find the total surface area of the cylinder so formed. **(2)**
12. A hemispherical depression is cut out from one face of a cubical block of side 7 cm, such that the diameter of the hemisphere is equal to the edge of the cube. Find the surface area of the remaining solid. **(2)**
13. A spherical ball of lead 3 cm in diameter is melted and recast into three spherical balls. The diameters of two of these balls are 1 cm and 1.5 cm. Find the diameter of the third ball. **(2)**
14. The height of a solid cylinder is 15 cm. and the diameter of its base is 7 cm. Two equal conical holes each of radius 3 cm, and height 4 cm are cut off. Find the volume of the remaining solid. **(3)**
15. The diameter of a sphere is 42 cm. It is melted and drawn into a cylindrical wire of diameter 2.8 cm. Find the length of the wire. **(3)**

16. A vessel is in the form of a hemispherical bowl surmounted by a hollow cylinder of same diameter. The diameter of the hemispherical bowl is 14 cm and the total height of the vessel is 13 cm. Find the total surface area of the vessel. **(3)** $\left[\text{Use } \pi = \frac{22}{7}\right]$

17. The diameter of a copper sphere is 18 cm. It is melted and drawn into a long wire of uniform cross section. If the length of the wire is 108 m, find its diameter. **(3)**

18. An iron pillar has some part in the form of a right circular cylinder and remaining in the form of a right circular cone. The radius of base of each of cone and cylinder is 8 cm. The cylindrical part is 240 cm high and the conical part is 36 cm high. Find the weight of the pillar, if one cubic cm of iron weighs 10 g. **(4)**

19. A friction clutch is in the form of a frustum of a cone, the diameter of the ends being 32 cm and 20 cm and length 8 cm. Find its bearing surface and volume. **(4)**

20. Water flows at the rate of 10 m per min. through a cylindrical pipe 5 mm in diameter. How long would it take to fill a conical vessel whose diameter at the base is 40 cm and depth 24 cm? **(4)**

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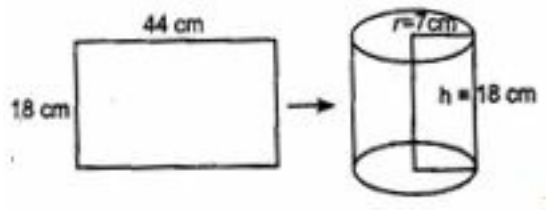
Solution

1. c. 7 cm

Explanation: 

Explanation here:: Let be$r$ the radius of the cylinder.

Given: Circumference of cylinder = 44 cm

$\Rightarrow 2\pi r = 44$

$\Rightarrow 2 \times \frac{22}{7} \times r = 44$

$\Rightarrow r = 7$ r =7 cm

2. a. $352cm^3$

Explanation: Given: r_1 = 4cm, r_2 = 2 cm and h = 12 cm

$\therefore$ Volume of Frustum = $\frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

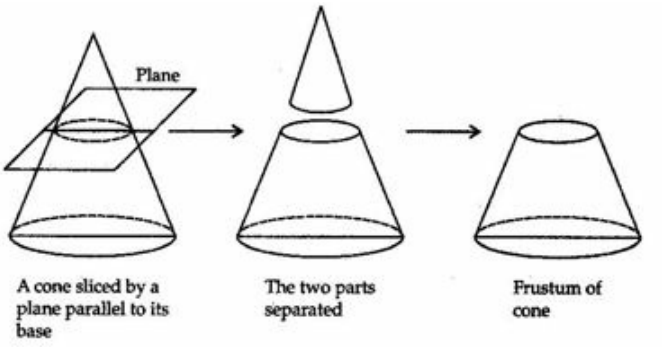
= $\frac{1}{3} \times \frac{22}{7} \times 12\left(4^2 + 2^2 + 4 \times 2\right)$

= $\frac{88}{7}(16 + 4 + 8)$

= $\frac{88}{7} \times 28 = 352$ cm^3

3. d. frustum of a cone

Explanation: A cone is cut through a plane parallel to its base and the small cone is (upper part) removed. The part that is left over is called Frustum of a cone.



4. b. 576 cu.cm

Explanation: Volume of cuboid = Length $\times$ Breadth $\times$Height

$\Rightarrow$ V = 12 $\times$ 8 $\times$ 6 = 576 cu. cm

5. a. 14cm

   Explanation: Given: External and internal diameter of hollow sphere = 8 cm and 4 cm respectively.

   Then External Radius = $(r_1) = \frac{8}{2}$ = 4 cm and Internal Radius = $(r_2) = \frac{4}{2}$ = 2 cm

   And Diameter of base of cone = 8 cm,

   then Radius of base of cone (r) = $\frac{8}{2}$ = 4 cm

   According to question,

   Volume of hollow sphere = Volume of Cone

   $\Rightarrow \frac{4}{3}\pi\left(r_1^3 - r_2^3\right) = \frac{1}{3}\pi r^2 h$

   $\Rightarrow 4 \times \left(4^3 - 3^3\right) = (4)^2 \times h$

   $\Rightarrow h = \frac{4(64-8)}{16}$

   $\Rightarrow h = \frac{4\times 56}{16} = 14$ cm

6. Total weight = 112 kg

   Volume = $\frac{112\times 1000}{25}$ = 4480 cm^3

   Volume = $l \times b \times h$

   $\Rightarrow \quad 4480 = l \times 28 \times 5$

   $\Rightarrow \quad l = 32\text{cm}$

7. Area of the given triangle $= \left(\frac{1}{2} \times \text{ base } \times \text{ height }\right)$

   $= \left(\frac{1}{2} \times 25 \times 10.8\right) \text{cm}^2 = 135\text{cm}^2$

8. Side of cube = 10.5 cm

   The largest sphere is carved out from this cube

   Then, the diameter of sphere = 10.5 cm

   $\therefore$ radius of sphere = $\frac{10.5}{2}$ cm

   $\therefore$ Volume of sphere = $\frac{4}{3}\pi r^3$

   = $\frac{4}{3} \times \frac{22}{7} \times \frac{10.5}{2} \times \frac{10.5}{2} \times \frac{10.5}{2}$

   = 606.375 cm^3

9. Since the sphere fits inside the cube, the diameter of the sphere is equal to the side of

the cube.

Let the side of the cube =2r = diameter of the sphere

So, the radius of the sphere = r

Now, Volume of the cube: Volume of the sphere

= (side of the cube)^3 : ($\frac{4}{3}\pi(radius)^3$)

$= (2r)^3 : \frac{4}{3}\pi r^3$

$= 8r^3 : \frac{4}{3}\pi r^3$

$= 24 : 4\pi$

$= 6 : \pi$

10. R = 7 cm r = 3.5 cm h = 2 cm

Volume of frustum = $\frac{\pi h}{3}\left(\mathrm{R}^2 + r^2 + \mathrm{R}r\right)$

$= \frac{1}{3} \times 3.14 \times 2\left[7^2 + 3.5^2 + 7 \times 3.5\right]$ cm^3

$= \frac{1}{3} \times 3.14 \times 2[49 + 12.25 + 7 \times 3.5]$cm^3

$= \frac{6.28 \times 85.75}{3}$cm^3 = 179.5 cm^3

11. Clearly, we have

radius of the cylinder so formed, r = 7 cm, and

height of the cylinder so formed, h = (50 $\times$ 5) mm

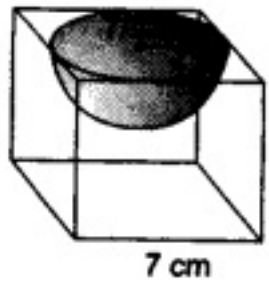
= 250 mm = 25 cm.

$\therefore$ total surface area of the cylinder so formed

$= 2\pi r(h + r) = \left[2 \times \frac{22}{7} \times 7 \times (25 + 7)\right]$ cm^2

= (2 $\times$ 22 $\times$ 32) cm^2 = 1408 cm^2.

12. Edge of the cube, a = 7 cm.



Radius of the hemisphere, $r = \frac{7}{2}$cm.

Surface area of remaining solid

= total surface area of the cube - area of the top of hemispherical part + curved

surface area of the hemisphere

$= 6a^2 - \pi r^2 + 2\pi r^2 = 6a^2 + \pi r^2$

$= \left(6 \times 7 \times 7 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right) \text{cm}^2$

= (294 + 38.5)cm^2 = 332.5 cm^2.

13. Radius of the ball = $\frac{3}{2}\text{cm} = 1.5\text{cm}$

Volume of the bigger ball = $\frac{4}{3}\pi(1.5)^3\text{cm}^3 \ldots . (i)$

Radii of two smaller balls are $\frac{1}{2}\text{cm and } \frac{1.5}{2}\text{cm}$

∴ total volume of the two smaller balls

$= \left[\frac{4}{3}\pi(0.5)^3 + \frac{4}{3}\pi(0.75)^3\right] \text{cm}^3$

$= \frac{4}{3}\pi\left[(0.5)^3 + (0.75)^3\right] \text{cm}^3 --(\text{i})$

Let the radius of the third ball be r cm.

∴Volume of the third ball = $\frac{4}{3}\pi r^3 cm^3 \ldots (iii)$

From (i), (ii) and (iii), we have

$\frac{4}{3}\pi(1.5)^3 = \frac{4}{3}\pi\left[(0.5)^3 + (0.75)^3\right] + \frac{4}{3}\pi r^3 \Rightarrow$(1.5)^3 = (0.5)^3 + (0.75)^3 + r^3

$\Rightarrow$r^3 = (1.5)^3 - (0.5)^3 - (0.75)^3 = r^3 = 3.375 - 0.125 - 0.421875

$\Rightarrow r^3 = 2.828125 \Rightarrow r = 1.41$

∴ diameter of the third ball = $2 \times 1.41\text{cm} = 2.82\text{cm}$

14. Given that diameter of cylinder = 7cm

Radius of cylinder (R) = $\frac{7}{2}$ cm

Also, Height of cylinder(H) = 15 cm

Let radius of conical hole = r = 3 cm

Height of conical hole = h = 4 cm

∴ Volume of remaining solid = volume of cylinder - 2 $\times$volume of conical hole

$= \pi R^2 H - 2 \times \frac{1}{3}\pi r^2 h$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 15 - 2 \times \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 4$

$= \frac{22}{7}\left[\frac{7}{2} \times \frac{7}{2} \times 15 - 2 \times \frac{1}{3} \times 3 \times 3 \times 4\right]$

$= \frac{22}{7}[183.75 - 24]$

$= \frac{22}{7} \times 159.75$

= 502.1cm^3

Therefore, the volume of remaining solid is 502.1 cm^3

15. Diameter of sphere = 42 cm

    Radius of sphere = $\left(\frac{42}{2}\right)\text{cm} = 21\text{cm}$

    Volume of sphere = $\frac{4}{3}\pi r^3 = \left(\frac{4}{3} \times \pi \times 21 \times 21 \times 21\right)\text{cm}^3$

    Diameter of cylindrical wire = 2.8 cm

    Radius of cylindrical wire = $\left(\frac{2.8}{2}\right)\text{cm} = 1.4\text{cm}$

    Volume of cylindrical wire = $\pi r^2 h = (\pi \times 1.4 \times 1.4 \times h)\text{cm}^3$

    = (1.96πh)cm^3

    Volume of cylindrical wire = volume of sphere

    $\therefore 1.96\pi h = \frac{4}{3} \times \pi \times 21 \times 21 \times 21$

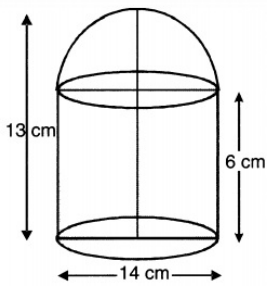
    $h = \left(\frac{4}{3} \times \pi \times 21 \times 21 \times 21 \times \frac{1}{1.96} \times \frac{1}{\pi}\right)\text{cm}$

    h = 6300

    $h\left(\frac{6300}{100}\right)m = 63\text{m}$

    Hence length of the wire is 63 m.

16. Radius of hemispherical portion = radius of cylindrical portion, $r = \frac{14}{2} = 7cm$

    Height of cylinder, $h = 13 - 7 = 6cm$

    Total Surface area of vessel = Curved surface area of hemisphere + Curved surface area of cylinder

    $= 2\pi r^2 + 2\pi rh$

    $= 2\pi r(r + h)$

    $= 2 \times \frac{22}{7} \times 7(7 + 6)$

    $= 44 \times 13$

    $= 572\text{cm}^2$

17. Diameter of copper sphere = 18 cm

    Therefore, Radius of copper sphere = $\left(\frac{18}{2}\right)cm = 9\ cm$

    Volume of sphere $= \left(\frac{4}{3} \times \pi \times r^3\right)\text{cm}^3$

$= \left(\frac{4}{3}\pi \times 9 \times 9 \times 9\right) \text{cm}^3 = 972\pi \text{cm}^3$

Length of wire = 108m = 10800 cm

Let the radius of wire be r cm

$Volume\ of\ wire = \left(\pi r^2 l\right) cm^3 = \left(\pi r^2 \times 10800\right) \text{cm}$

But the volume of wire = Volume of sphere

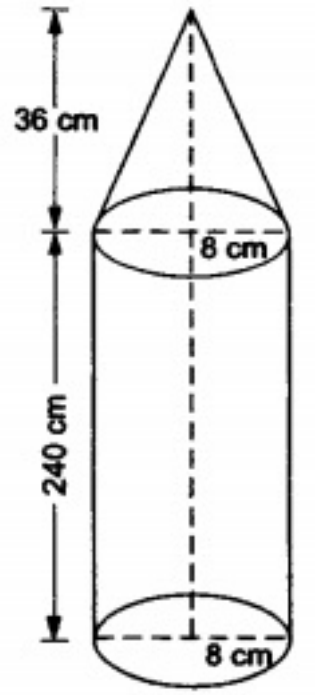
$\Rightarrow \pi r^2 \times 10800 = 972\pi$

$r^2 = \frac{972\pi}{10800\pi} = 0.09 \text{cm}^2$

$r = \sqrt{0.09} \text{cm} = 0.3$

Hence, the diameter = 2r = 2(0.3) cm = 0.6 cm.

18. Let us suppose that r denotes the radius of the cylinder = 8 cm.



Suppose R denotes the radius of the cone = 8 cm.

Let h be the height of the cylinder = 240cm.

Suppose H is the height of the cone = 36 cm.

Total volume of the iron = volume of the cylinder + volume of the cone

$= \pi r^2 h + \frac{1}{3}\pi R^2 H = \pi r^2 \left(h + \frac{1}{3} H\right)$ [as r=R= 8cm each]

$= \left[\frac{22}{7} \times 8 \times 8 \times \left(240 + \frac{1}{3} \times 36\right)\right] \text{cm}^3$

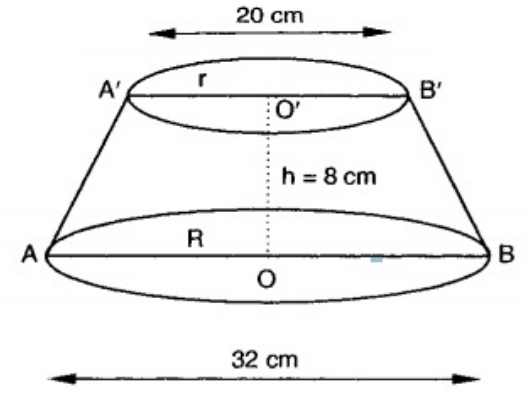
= 50688 cm^3

$\therefore$ Weight of the pilllar = volume in cm^3 $\times$ weight per cm^3

$= \left(\frac{50688 \times 10}{1000}\right) \text{kg} = 506.88 \text{kg}$

Therefore, the weight of the pillar is 506.88 kg.

19. Let ABB 'A' be the friction clutch of slant height l cm.



We have,

R = 16 cm, r = 10 cm and h = 8 cm

$\therefore \quad l^2 = h^2 + (R - r)^2$

$= (8)^2 + (16 - 10)^2$

= 64 + $(6)^2$

$\Rightarrow \quad l^2 = 64 + 36 \Rightarrow l = 10\text{cm}$

Bearing surface of the clutch = Lateral surface of the frustum

$\Rightarrow \quad S = \pi (R + r) l$

$= \frac{22}{7} \times (16 + 10) \times 10\text{cm}^2$

$= 817.14\text{cm}^2$

$V = \frac{1}{3}\pi h \left(R^2 + Rr + r^2\right)$

$\Rightarrow \quad V = \frac{1}{3} \times \frac{22}{7} \times 8 \times \left(16^2 + 16 \times 10 + 10^2\right) \text{cm}^3$

$\Rightarrow \quad V = \frac{1}{3} \times \frac{22}{7} \times 8 \times (256 + 160 + 100)\text{cm}^3 = \frac{176}{21} \times 516\text{cm}^3$ = 4324.57 cm^3

20. Radius of the pipe is $\frac{5}{2}\text{mm} = \frac{5}{2} \times \frac{1}{10}$ cm = $\frac{1}{4}$ cm

Speed of water = $\frac{10 \text{ m}}{\text{min}} = \frac{1000 \text{ cm}}{\text{min}}$

⇒ v = $\frac{10}{60}$ m/s = $\frac{1}{6}$ m/s

∴ Volume of flowing water = Volume of cone

⇒ Area of base × height (dist.) = $\frac{1}{3}\pi R^2 H$

⇒ A × vt = $\frac{1}{3}\pi R^2 H$ [as distance=speed ×time]

⇒ πr^2.vt= $\frac{1}{3}$ $\pi R^2 H$

$\Rightarrow r^2.vt= \frac{1}{3} R^2H$

$\Rightarrow \frac{1}{400} \times \frac{1}{400} \times \frac{1}{6}t = \frac{1}{3} \times 0.2 \times 0.2 \times 0.24$

$\Rightarrow t = \frac{2\times2\times24\times400\times400}{3\times10000}$

$\Rightarrow$ t = 4 × 24 × 4 × 4 × 2 sec

$= \frac{4\times24\times4\times4\times2}{60}\text{min} = \frac{512}{10}$ = 51.2 min

= 51 min + 0.2 min = 51 min + 0.2 × 60sec

$\Rightarrow$ t = 51 min and 12 sec.

Hence, conical tank will fill in 51 min and 12 sec.