

9.5 Stellar evolution

Evolutionary timescales

Free-fall timescale ^a	$\tau_{\text{ff}} = \left(\frac{3\pi}{32G\rho_0} \right)^{1/2}$	(9.53)	τ_{ff} free-fall timescale G constant of gravitation ρ_0 initial mass density
Kelvin–Helmholtz timescale	$\tau_{\text{KH}} = \frac{-U_{\text{g}}}{L}$	(9.54)	τ_{KH} Kelvin–Helmholtz timescale U_{g} gravitational potential energy
	$\simeq \frac{GM^2}{R_0L}$	(9.55)	M body's mass R_0 body's initial radius L body's luminosity

^aFor the gravitational collapse of a uniform sphere.

Star formation

Jeans length ^a	$\lambda_{\text{J}} = \left(\frac{\pi}{G\rho} \frac{dp}{d\rho} \right)^{1/2}$	(9.56)	λ_{J} Jeans length G constant of gravitation ρ cloud mass density p pressure
Jeans mass	$M_{\text{J}} = \frac{\pi}{6} \rho \lambda_{\text{J}}^3$	(9.57)	M_{J} (spherical) Jeans mass
Eddington limiting luminosity ^b	$L_{\text{E}} = \frac{4\pi GM_{\text{p}}c}{\sigma_{\text{T}}}$	(9.58)	L_{E} Eddington luminosity M stellar mass $M_{\odot}$ solar mass
	$\simeq 1.26 \times 10^{31} \frac{M}{M_{\odot}} \text{ W}$	(9.59)	m_{p} proton mass c speed of light σ_{T} Thomson cross section

^aNote that $(dp/d\rho)^{1/2}$ is the sound speed in the cloud.

^bAssuming the opacity is mostly from Thomson scattering.

Stellar theory^a

Conservation of mass	$\frac{dM_r}{dr} = 4\pi\rho r^2$	(9.60)	r radial distance M_r mass interior to r ρ mass density
Hydrostatic equilibrium	$\frac{dp}{dr} = \frac{-G\rho M_r}{r^2}$	(9.61)	p pressure G constant of gravitation
Energy release	$\frac{dL_r}{dr} = 4\pi\rho r^2\epsilon$	(9.62)	L_r luminosity interior to r ϵ power generated per unit mass
Radiative transport	$\frac{dT}{dr} = \frac{-3}{16\sigma} \frac{\langle\kappa\rangle\rho}{T^3} \frac{L_r}{4\pi r^2}$	(9.63)	T temperature σ Stefan–Boltzmann constant $\langle\kappa\rangle$ mean opacity
Convective transport	$\frac{dT}{dr} = \frac{\gamma-1}{\gamma} \frac{T}{p} \frac{dp}{dr}$	(9.64)	γ ratio of heat capacities, c_p/c_V

^aFor stars in static equilibrium with adiabatic convection. Note that ρ is a function of r . κ and ϵ are functions of temperature and composition.

Stellar fusion processes^a

PP I chain	PP II chain	PP III chain
$p^+ + p^+ \rightarrow {}^2_1\text{H} + e^+ + \nu_e$ ${}^2_1\text{H} + p^+ \rightarrow {}^3_2\text{He} + \gamma$ ${}^3_2\text{He} + {}^3_2\text{He} \rightarrow {}^4_2\text{He} + 2p^+$	$p^+ + p^+ \rightarrow {}^2_1\text{H} + e^+ + \nu_e$ ${}^2_1\text{H} + p^+ \rightarrow {}^3_2\text{He} + \gamma$ ${}^3_2\text{He} + {}^4_2\text{He} \rightarrow {}^7_4\text{Be} + \gamma$ ${}^7_4\text{Be} + e^- \rightarrow {}^7_3\text{Li} + \nu_e$ ${}^7_3\text{Li} + p^+ \rightarrow 2{}^4_2\text{He}$	$p^+ + p^+ \rightarrow {}^2_1\text{H} + e^+ + \nu_e$ ${}^2_1\text{H} + p^+ \rightarrow {}^3_2\text{He} + \gamma$ ${}^3_2\text{He} + {}^4_2\text{He} \rightarrow {}^7_4\text{Be} + \gamma$ ${}^7_4\text{Be} + p^+ \rightarrow {}^8_5\text{B} + \gamma$ ${}^8_5\text{B} \rightarrow {}^8_4\text{Be} + e^+ + \nu_e$ ${}^8_4\text{Be} \rightarrow 2{}^4_2\text{He}$
CNO cycle	triple- α process	
${}^{12}_6\text{C} + p^+ \rightarrow {}^{13}_7\text{N} + \gamma$ ${}^{13}_7\text{N} \rightarrow {}^{13}_6\text{C} + e^+ + \nu_e$ ${}^{13}_6\text{C} + p^+ \rightarrow {}^{14}_7\text{N} + \gamma$ ${}^{14}_7\text{N} + p^+ \rightarrow {}^{15}_8\text{O} + \gamma$ ${}^{15}_8\text{O} \rightarrow {}^{15}_7\text{N} + e^+ + \nu_e$ ${}^{15}_7\text{N} + p^+ \rightarrow {}^{12}_6\text{C} + {}^4_2\text{He}$	${}^4_2\text{He} + {}^4_2\text{He} \rightleftharpoons {}^8_4\text{Be} + \gamma$ ${}^8_4\text{Be} + {}^4_2\text{He} \rightleftharpoons {}^{12}_6\text{C}^*$ ${}^{12}_6\text{C}^* \rightarrow {}^{12}_6\text{C} + \gamma$	γ photon p^+ proton e^+ positron e^- electron ν_e electron neutrino

^aAll species are taken as fully ionised.

Pulsars

Braking index	$\dot{\omega} \propto -\omega^n$ (9.65) $n = 2 - \frac{P\ddot{P}}{\dot{P}^2}$ (9.66)	ω rotational angular velocity P rotational period ($=2\pi/\omega$) n braking index
Characteristic age ^a	$T = \frac{1}{n-1} \frac{P}{\dot{P}}$ (9.67)	T characteristic age L luminosity μ_0 permeability of free space c speed of light
Magnetic dipole radiation	$L = \frac{\mu_0 \ddot{m} ^2 \sin^2 \theta}{6\pi c^3}$ (9.68) $= \frac{2\pi R^6 B_p^2 \omega^4 \sin^2 \theta}{3c^3 \mu_0}$ (9.69)	m pulsar magnetic dipole moment R pulsar radius B_p magnetic flux density at magnetic pole θ angle between magnetic and rotational axes
Dispersion measure	$\text{DM} = \int_0^D n_e dl$ (9.70)	DM dispersion measure D path length to pulsar dl path element n_e electron number density
Dispersion ^b	$\frac{d\tau}{dv} = \frac{-e^2}{4\pi^2 \epsilon_0 m_e c v^3} \text{DM}$ (9.71) $\Delta\tau = \frac{e^2}{8\pi^2 \epsilon_0 m_e c} \left(\frac{1}{v_1^2} - \frac{1}{v_2^2} \right) \text{DM}$ (9.72)	τ pulse arrival time $\Delta\tau$ difference in pulse arrival time v_i observing frequencies m_e electron mass

^aAssuming $n \neq 1$ and that the pulsar has already slowed significantly. Usually n is assumed to be 3 (magnetic dipole radiation), giving $T = P/(2\dot{P})$.

^bThe pulse arrives first at the higher observing frequency.

Compact objects and black holes

Schwarzschild radius	$r_s = \frac{2GM}{c^2} \simeq 3 \frac{M}{M_\odot} \text{ km} \quad (9.73)$	r_s Schwarzschild radius G constant of gravitation M mass of body c speed of light $M_\odot$ solar mass
Gravitational redshift	$\frac{v_\infty}{v_r} = \left(1 - \frac{2GM}{rc^2}\right)^{1/2} \quad (9.74)$	r distance from mass centre v_∞ frequency at infinity v_r frequency at r
Gravitational wave radiation ^a	$L_g = \frac{32}{5} \frac{G^4}{c^5} \frac{m_1^2 m_2^2 (m_1 + m_2)}{a^5} \quad (9.75)$	m_i orbiting masses a mass separation L_g gravitational luminosity
Rate of change of orbital period	$\dot{P} = -\frac{96}{5} (4\pi^2)^{4/3} \frac{G^{5/3}}{c^5} \frac{m_1 m_2 P^{-5/3}}{(m_1 + m_2)^{1/3}} \quad (9.76)$	P orbital period
Neutron star degeneracy pressure (nonrelativistic)	$p = \frac{(3\pi^2)^{2/3}}{5} \frac{\hbar^2}{m_n} \left(\frac{\rho}{m_n}\right)^{5/3} = \frac{2}{3} u \quad (9.77)$	p pressure $\hbar$ (Planck constant)/(2 π) m_n neutron mass ρ density
Relativistic ^b	$p = \frac{\hbar c (3\pi^2)^{1/3}}{4} \left(\frac{\rho}{m_n}\right)^{4/3} = \frac{1}{3} u \quad (9.78)$	u energy density
Chandrasekhar mass ^c	$M_{\text{Ch}} \simeq 1.46 M_\odot \quad (9.79)$	M_{Ch} Chandrasekhar mass
Maximum black hole angular momentum	$J_m = \frac{GM^2}{c} \quad (9.80)$	J_m maximum angular momentum
Black hole evaporation time	$\tau_e \sim \frac{M^3}{M_\odot^3} \times 10^{66} \text{ yr} \quad (9.81)$	τ_e evaporation time
Black hole temperature	$T = \frac{\hbar c^3}{8\pi G M k} \simeq 10^{-7} \frac{M_\odot}{M} \text{ K} \quad (9.82)$	T temperature k Boltzmann constant

^aFrom two bodies, m_1 and m_2 , in circular orbits about their centre of mass. Note that the frequency of the radiation is twice the orbital frequency.

^bParticle velocities $\sim c$.

^cUpper limit to mass of a white dwarf.