Exercise 17.1

Chapter 17 Second Order Differential Equations 17.1 1E

Given differential equation is $y'' - y' - 6y = 0$

The auxiliary equation corresponding to the given differential equation is

$r^2 - r - 6 = 0$

$r^2 - 3r + 2r - 6 = 0$

$r(r-3) + 2(r-3) = 0$

$(r-3)(r+2) = 0$

Therefore $r = 3, -2$

Thus the roots of the auxiliary equation are real and distinct.

Hence the general solution to given differential equation is $y = c_1 e^{3x} + c_2 e^{-2x}$

Chapter 17 Second Order Differential Equations 17.1 2E

Given differential equation is $y'' + 4y' + 14y = 0$

The auxiliary equation corresponding to the given differential equation is

$r^2 + 4r + 14 = 0$

Note that the roots of the quadratic equation in x, that is $ax^2 + bx + c = 0$

are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

thus the roots of the auxiliary equation are

$$r = \frac{-4 \pm \sqrt{4^2 - 4(1)(14)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16-56}}{2}$$

$$= \frac{-4 \pm \sqrt{-40}}{2}$$

$$= \frac{-4 \pm \sqrt{40i^2}}{2} \quad (\text{since } i^2 = -1)$$

$$= \frac{-4 \pm 2i\sqrt{10}}{2}$$

$$= -2 \pm i\sqrt{10}$$

Therefore $r = -2 + i\sqrt{10}, -2 - i\sqrt{10}$

Thus the roots of the auxiliary equation are complex conjugates

Hence the general solution to given differential equation is $y = e^{-2x}\left(c_1 \cos\sqrt{10}x + c_2 \sin\sqrt{10}x\right)$

Chapter 17 Second Order Differential Equations 17.1 3E

Given differential equation is $y'' + 16y = 0$

The auxiliary equation corresponding to the given differential equation is

$r^2 + 16 = 0$

Note that the roots of the quadratic equation in x, that is $ax^2 + bx + c = 0$

are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

thus the roots of the auxiliary equation are

$$r = \frac{-0 \pm \sqrt{0^2 - 4(1)(16)}}{2(1)}$$

$$= \frac{0 \pm \sqrt{-64}}{2}$$

$$= \frac{0 \pm \sqrt{(8i)^2}}{2} \quad \left(\text{since } i^2 = -1\right)$$

$$= \frac{0 \pm 8i}{2}$$

$$= 0 \pm 4i$$

Therefore $r = 0 + 4i, 0 - 4i$

Thus the roots of the auxiliary equation are complex conjugates

Hence the general solution to given differential equation is

$y = e^{0x}\left(c_1 \cos 4x + c_2 \sin 4x\right)$ or $y = c_1 \cos 4x + c_2 \sin 4x$ $\left(\text{since } e^0 = 1\right)$

Chapter 17 Second Order Differential Equations 17.1 4E

Given differential equation is $y'' - 8y' + 12y = 0$

The auxiliary equation corresponding to the given differential equation is

$r^2 - 8r + 12 = 0$

$r^2 - 2r - 6r + 12 = 0$

$r(r-2) - 6(r-2) = 0$

$(r-2)(r-6) = 0$

Therefore $r = 2, 6$

Thus the roots of the auxiliary equation are real and distinct.

Hence the general solution to given differential equation is $y = c_1 e^{2x} + c_2 e^{6x}$

Chapter 17 Second Order Differential Equations 17.1 5E

Given differential equation is $9y'' - 12y' + 4y = 0$

The auxiliary equation corresponding to the given differential equation is

$9r^2 - 12r + 4 = 0$

$9r^2 - 6r - 6r + 4 = 0$

$3r(3r-2) - 2(3r-2) = 0$

$(3r-2)(3r-2) = 0$

Therefore $r = \dfrac{2}{3}, \dfrac{2}{3}$

Thus the roots of the auxiliary equation are real and equal.

Hence the general solution to given differential equation is $y = (c_1 + c_2 x)e^{\frac{2}{3}x}$

Chapter 17 Second Order Differential Equations 17.1 6E

Given differential equation is $25y'' + 9y = 0$

The auxiliary equation corresponding to the given differential equation is

$25r^2 + 9 = 0$

Note that the roots of the quadratic equation in x, that is $ax^2 + bx + c = 0$

are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

thus the roots of the auxiliary equation are

$$r = \frac{-0 \pm \sqrt{0^2 - 4(25)(9)}}{2(25)}$$

$$= \frac{0 \pm \sqrt{-2^2 5^2 3^2}}{50}$$

$$= \frac{0 \pm \sqrt{i^2 2^2 5^2 3^2}}{50} \quad \left(\text{since } i^2 = -1\right)$$

$$= \frac{0 \pm i\,2(5)(3)}{50}$$

$$= 0 \pm \frac{3}{5}i$$

Therefore $r = 0 + \frac{3}{5}i, 0 - \frac{3}{5}i$

Thus the roots of the auxiliary equation are complex conjugates

Hence the general solution to given differential equation is

$$y = e^{0x}\left(c_1 \cos\frac{3}{5}x + c_2 \sin\frac{3}{5}x\right) \text{ or } y = c_1 \cos\frac{3}{5}x + c_2 \sin\frac{3}{5}x \quad \left(\text{since } e^0 = 1\right)$$

Chapter 17 Second Order Differential Equations 17.1 7E

Given differential equation is $y' = 2y''$

The auxiliary equation corresponding to the given differential equation is

$r = 2r^2$

implies $2r^2 - r = 0$

$r(2r - 1) = 0$

Therefore $r = 0, \frac{1}{2}$

Thus the roots of the auxiliary equation are real and distinct.

Hence the general solution to given differential equation is

$$y = c_1 e^{0x} + c_2 e^{\frac{1}{2}x} \text{ or } y = c_1 + c_2 e^{\frac{1}{2}x} \quad \left(\text{since } e^0 = 1\right)$$

Chapter 17 Second Order Differential Equations 17.1 8E

Given differential equation is $y'' - 4y' + y = 0$

The auxiliary equation corresponding to the given differential equation is

$r^2 - 4r + 1 = 0$

Note that the roots of the quadratic equation in x, that is $ax^2 + bx + c = 0$

are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

thus the roots of the auxiliary equation are

$$r = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$$
$$= \frac{4 \pm \sqrt{16 - 4}}{2}$$
$$= \frac{4 \pm \sqrt{12}}{2}$$
$$= \frac{4 \pm 2\sqrt{3}}{2}$$
$$= 2 \pm \sqrt{3}$$

Therefore $r = 2 + \sqrt{3}, 2 - \sqrt{3}$

Thus the roots of the auxiliary equation are real and distinct.

Hence the general solution to given differential equation is $y = c_1 e^{(2+\sqrt{3})x} + c_2 e^{(2-\sqrt{3})x}$

Chapter 17 Second Order Differential Equations 17.1 9E

Given differential equation is $y'' - 4y' + 13y = 0$

The auxiliary equation corresponding to the given differential equation is

$r^2 - 4r + 13 = 0$

Note that the roots of the quadratic equation in x, that is $ax^2 + bx + c = 0$

are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

thus the roots of the auxiliary equation are

$$r = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$
$$= \frac{4 \pm \sqrt{16 - 52}}{2}$$
$$= \frac{4 \pm \sqrt{-36}}{2}$$
$$= \frac{4 \pm \sqrt{6^2 i^2}}{2} \quad (\text{since } i^2 = -1)$$
$$= \frac{4 \pm 6i}{2}$$
$$= 2 \pm 3i$$

Therefore $r = 2 + 3i, 2 - 3i$

Thus the roots of the auxiliary equation are complex conjugates

Hence the general solution to given differential equation is $y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x)$

Chapter 17 Second Order Differential Equations 17.1 10E

Given differential equation is $y'' + 3y' = 0$

The auxiliary equation corresponding to the given differential equation is

$r^2 + 3r = 0$

$r(r+3) = 0$

Therefore $r = 0, -3$

Thus the roots of the auxiliary equation are real and distinct.

Hence the general solution to given differential equation is

$y = c_1e^{0x} + c_2e^{-3x}$ or $y = c_1 + c_2e^{-3x}$ $\left(\text{since } e^0 = 1\right)$

Chapter 17 Second Order Differential Equations 17.1 11E

Given differential equation is $2\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - y = 0$

The auxiliary equation corresponding to the given differential equation is

$2r^2 + 2r - 1 = 0$

Note that the roots of the quadratic equation in x, that is $ax^2 + bx + c = 0$

are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

thus the roots of the auxiliary equation are

$$r = \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{-2 \pm \sqrt{4+8}}{4}$$

$$= \frac{-2 \pm \sqrt{12}}{4}$$

$$= \frac{-2 \pm 2\sqrt{3}}{4}$$

$$= \frac{-1 \pm \sqrt{3}}{2}$$

Therefore $r = \dfrac{-1+\sqrt{3}}{2}, \dfrac{-1-\sqrt{3}}{2}$

Thus the roots of the auxiliary equation are real and distinct.

Hence the general solution to given differential equation is $y = c_1e^{\left(\frac{-1+\sqrt{3}}{2}\right)t} + c_2e^{\left(\frac{-1-\sqrt{3}}{2}\right)t}$

Chapter 17 Second Order Differential Equations 17.1 12E

Given differential equation is $8\frac{d^2y}{dt^2}+12\frac{dy}{dt}+5y=0$

The auxiliary equation corresponding to the given differential equation is

$8r^2+12r+5=0$

Note that the roots of the quadratic equation in x, that is $ax^2+bx+c=0$

are given by $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

thus the roots of the auxiliary equation are

$$r=\frac{-12\pm\sqrt{12^2-4(8)(5)}}{2(8)}$$
$$=\frac{-12\pm\sqrt{144-160}}{16}$$
$$=\frac{-12\pm\sqrt{-16}}{16}$$
$$=\frac{-12\pm\sqrt{4^2i^2}}{16}\quad\left(\text{since } i^2=-1\right)$$
$$=\frac{-12\pm 4i}{16}$$
$$=\frac{-3}{4}\pm\frac{1}{4}i$$

Therefore $r=\frac{-3}{4}+\frac{1}{4}i,\frac{-3}{4}-\frac{1}{4}i$

Thus the roots of the auxiliary equation are complex conjugates.

Hence the general solution to given differential equation is $y=e^{\frac{-3}{4}t}\left[c_1\cos\left(\frac{1}{4}t\right)+c_2\sin\left(\frac{1}{4}t\right)\right]$

Chapter 17 Second Order Differential Equations 17.1 13E

Given differential equation is $100\frac{d^2P}{dt^2}+200\frac{dP}{dt}+101P=0$

The auxiliary equation corresponding to the given differential equation is

$100r^2+200r+101=0$

Note that the roots of the quadratic equation in x, that is $ax^2+bx+c=0$

are given by $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

thus the roots of the auxiliary equation are

$$r=\frac{-200\pm\sqrt{200^2-4(100)(101)}}{2(100)}$$
$$=\frac{-200\pm\sqrt{40000-40400}}{2(100)}$$
$$=\frac{-200\pm\sqrt{-400}}{2(100)}$$
$$=\frac{-200\pm\sqrt{20^2i^2}}{2(100)}\quad\left(\text{since } i^2=-1\right)$$
$$=\frac{-200\pm 20i}{2(100)}$$
$$=-1\pm\frac{1}{10}i$$

Therefore $r=-1+\frac{1}{10}i,-1-\frac{1}{10}i$

Thus the roots of the auxiliary equation are complex conjugates.

Hence the general solution to given differential equation is $P=e^{-t}\left[c_1\cos\left(\frac{1}{10}t\right)+c_2\sin\left(\frac{1}{10}t\right)\right]$

Chapter 17 Second Order Differential Equations 17.1 14E

Consider the differential equation,

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+20y=0$$

The auxiliary equation is $m^2+4m+20=0$

Solve the auxiliary equation for m .

The quadratic formula is,

The roots are $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ for the Quadratic Equation $ax^2+bx+c=0$

Apply the Quadratic formula to the equation $m^2+4m+20=0$.

The roots are,

$$\begin{aligned} m &= \frac{-4\pm\sqrt{4^2-4(1)(20)}}{2(1)} \\ &= \frac{-4\pm\sqrt{16-80}}{2} \\ &= \frac{-4\pm\sqrt{-64}}{2} \\ &= \frac{-4\pm\sqrt{-1}\cdot\sqrt{64}}{2} \\ &= \frac{-4\pm i\cdot 8}{2} \qquad \sqrt{-1}=i \text{ and } \sqrt{64}=8. \\ &= -2\pm 4i \end{aligned}$$

The general solution is $y=e^{\alpha x}\left(C_1\cos\beta x+C_2\sin\beta x\right)$ of the differential equation $ay''+by'+c=0$, If the roots of the auxiliary equation $am^2+bm+c=0$ are complex numbers $r_1=\alpha+i\beta$ and $r_2=\alpha-i\beta$

So, the general solution of the differential equation $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+20y=0$ is,

$$y=e^{-2x}\left(C_1\cos 4x+C_2\sin 4x\right)$$

Where C_1,C_2 are arbitrary constants.

Let $f(x)=e^{-2x}\cos 4x$ and $g(x)=e^{-2x}\sin 4x$.

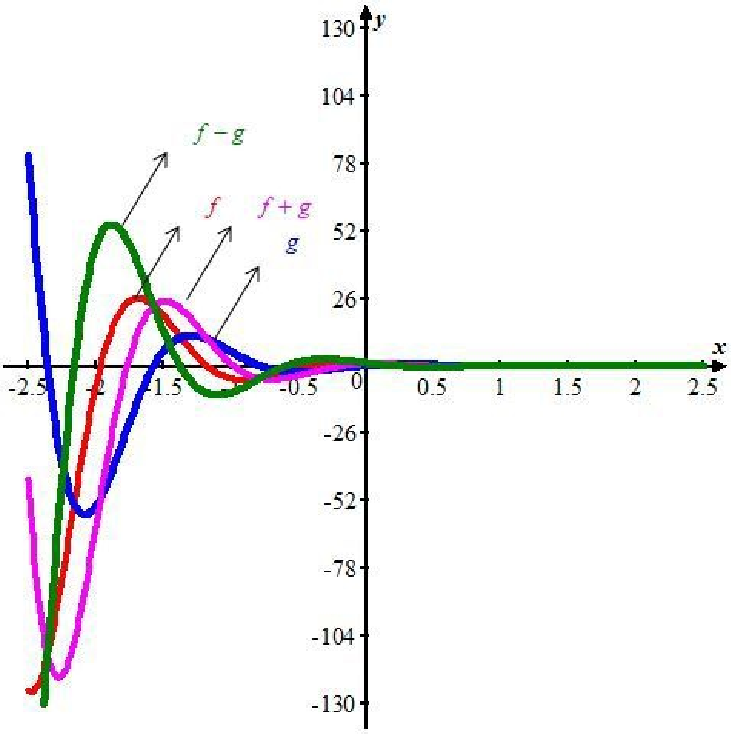
These are the basic solutions.

The other solutions are $f-g$ and $f+g$

$$\begin{aligned} f-g &= e^{-2x}\cos 4x - e^{-2x}\sin 4x \\ &= e^{-2x}\left(\cos 4x - \sin 4x\right) \end{aligned}$$

$$\begin{aligned} f+g &= e^{-2x}\cos 4x + e^{-2x}\sin 4x \\ &= e^{-2x}\left(\cos 4x + \sin 4x\right) \end{aligned}$$

Sketch the graph of solutions.



All solutions are approach to 0 as $x \to \infty$.

Chapter 17 Second Order Differential Equations 17.1 15E

Consider the differential equation,

$$5\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0$$

The auxiliary equation is $5m^2 - 2m - 3 = 0$

Solve the auxiliary equation for m .

The quadratic formula is,

The roots are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for the Quadratic Equation $ax^2 + bx + c = 0$.

Apply the Quadratic formula to the equation $5m^2 - 2m - 3 = 0$.

The roots are,

$$\begin{aligned} m &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)(-3)}}{2(5)} \\ &= \frac{2 \pm \sqrt{4 + 60}}{10} \\ &= \frac{2 \pm \sqrt{64}}{10} \\ &= \frac{2 \pm 8}{10} \\ &= \frac{2+8}{10}, \frac{2-8}{10} \\ &= \frac{10}{10}, \frac{-6}{10} \\ &= 1, \frac{-3}{5} \end{aligned}$$

The general solution is $y = C_1e^{m_1x} + C_2e^{m_2x}$ of the differential equation $ay'' + by' + c = 0$, If the roots of the auxiliary equation $am^2 + bm + c = 0$ are real and distinct.

So, the general solution of the differential equation $5\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0$ is,

$$y = C_1e^x + C_2e^{-\frac{3x}{5}}$$

Where C_1, C_2 are arbitrary constants.

Let $f(x)=e^{x}$ and $g(x)=e^{\frac{-3x}{5}}$.

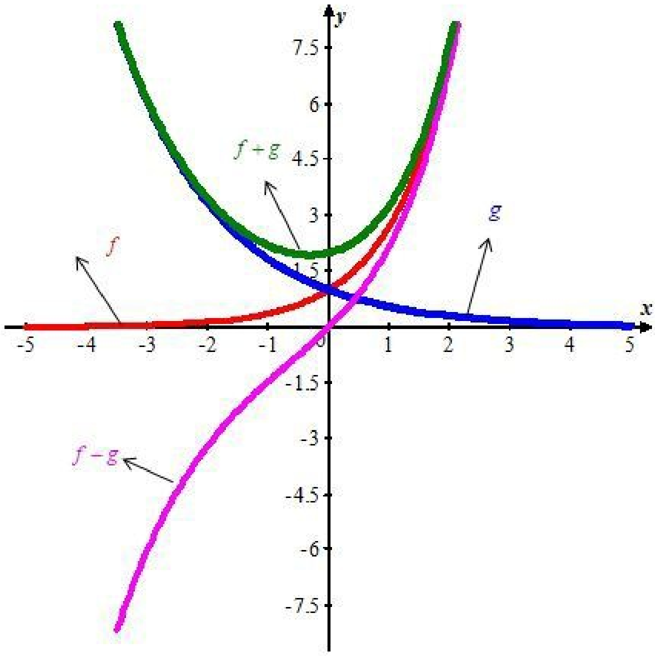
These are the basic solutions.

The other solutions are $f-g$ and $f+g$

$$f-g=e^{x}-e^{\frac{-3x}{5}}$$

$$f+g=e^{x}+e^{\frac{-3x}{5}}$$

Sketch the graph of solutions.



Chapter 17 Second Order Differential Equations 17.1 16E

Consider the following differential equation:

$$9\frac{d^2y}{dx^2}+6\frac{dy}{dx}+y=0$$

The auxiliary equation is $9m^2+6m+1=0$

Solve the auxiliary equation for m.

The quadratic formula is calculated as follows:

The roots are $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ for the Quadratic Equation $ax^2+bx+c=0$.

Apply the Quadratic formula to the equation $9m^2+6m+1=0$.

The roots are calculated as follows:

$$\begin{aligned}
m&=\frac{-(6)\pm\sqrt{(6)^2-4(9)(1)}}{2(9)}\\
&=\frac{-(6)\pm\sqrt{36-36}}{18}\\
&=\frac{-(6)\pm 0}{18}\\
&=\frac{-6}{18},\frac{-6}{18}\\
&=\frac{-1}{3},\frac{-1}{3}
\end{aligned}$$

The general solution is $y = C_1e^{mx} + xC_2e^{mx}$ of the differential equation $ay'' + by' + c = 0$, If the roots of the auxiliary equation $am^2 + bm + c = 0$ are real and equal .

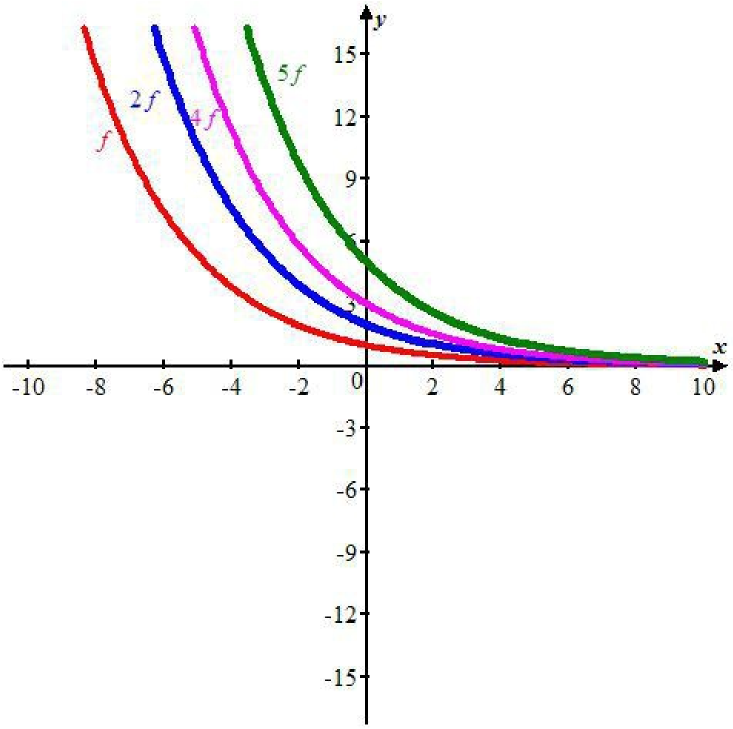
So, the general solution of the differential equation $9\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + y = 0$ is represented as follows:

$$y = C_1e^{\frac{-x}{3}} + xC_2e^{\frac{-x}{3}}$$

Here, C_1, C_2 are the arbitrary constants.

Let $f(x) = e^{\frac{-x}{3}}$. These are the basic solutions. The other solutions are $2f, 4f,$ and $5f$

Sketch the graph of solutions as follows:



The solutions approach to 0 as $x \to \infty$.

Chapter 17 Second Order Differential Equations 17.1 17E

The auxiliary polynomial for the given differential equation is $r^2 - 6r + 8 = 0$ or $(r-2)(r-4) = 0$. The roots of the auxiliary equation are $r = 2$ and $r = 4$ each having multiplicity 1.

If the roots r_1 and r_2 of the auxiliary equation $ar^2 + br + c = 0$ are real and unequal, then the general solution of $ay'' + by' + cy = 0$ is $y = c_1e^{r_1x} + c_2e^{r_2x}$.

On replacing r_1 with 2 and r_2 with 4. we get the general solution of the given differential equation as $y = c_1e^{2x} + c_2e^{4x}$.

For finding the constant c_1, use the initial condition $y(0) = 2$.

$$2 = c_1e^{2(0)} + c_2e^{4(0)}$$
$$2 = c_1(1) + c_2(1)$$
$$c_1 + c_2 = 2$$
$$c_1 = 2 - c_2$$

Differentiate the equation.

$$y' = 2c_1e^{2x} + 4c_2e^{4x}$$

Apply the condition $y'(0) = 2$.

$$2 = 2c_1e^{2(0)} + 4c_2e^{4(0)}$$
$$2 = 2c_1(1) + 4c_2(1)$$
$$c_1 + 2c_2 = 1$$

Replace c_1 with $2 - c_2$ in $c_1 + 2c_2 = 1$.

$$(2 - c_2) + 2c_2 = 1$$
$$2 + c_2 = 1$$
$$c_2 = -1$$

On substituting c_2 with -1 in $c_1 = 2 - c_2$, we get c_1 as 3. Then, $y = 3e^{2x} - e^{4x}$

Thus, the solution to the given differential equation is $\boxed{y = 3e^{2x} - e^{4x}}$.

Chapter 17 Second Order Differential Equations 17.1 18E

The auxiliary polynomial for the given differential equation is $r^2 + 4 = 0$ or $r^2 = -4$. The roots of the auxiliary equation are $r = 2i$ and $r = -2i$ each having multiplicity 1.

If the roots of the auxiliary equation $ar^2 + br + c = 0$ are complex numbers $r_1 = \alpha + i\beta$ and $r_2 = \alpha - i\beta$, then the general solution of $ay'' + by' + cy = 0$ is $y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$.

On replacing α with 0, β with 2, we get the general solution of the given differential equation as $y = c_1 \cos 2x + c_2 \sin 2x$.

For finding the constant c_1, use the initial condition $y(\pi) = 5$

$$5 = c_1 \cos 2(\pi) + c_2 \sin 2(\pi)$$
$$5 = c_1(1) + c_2(0)$$
$$c_1 = 5$$

Differentiate the equation.

$$y' = -2c_1 \sin 2x + 2c_2 \cos 2x$$

Apply the condition $y'(\pi) = -4$.

$$-4 = -2c_1 \sin 2(\pi) + 2c_2 \cos 2(\pi)$$
$$= -2c_1(0) + 2c_2(1)$$
$$= 2c_2$$
$$c_2 = -2$$

On substituting c_1 with 5 and c_2 with -2 in $y = c_1 \cos 2x + c_2 \sin 2x$, we get $y = 5\cos 2x - 2\sin 2x$.

Thus, the solution to the given differential equation is $\boxed{y = 5\cos 2x - 2\sin 2x}$.

Chapter 17 Second Order Differential Equations 17.1 19E

The auxiliary polynomial for the given differential equation is $9r^2 + 12r + 4 = 0$ or $(3r + 2)^2 = 0$. The roots of the auxiliary equation are $r = -\frac{2}{3}$ each having multiplicity 1.

If the auxiliary equation $ar^2 + br + c = 0$ has only one real root r, then the general solution of $ay'' + by' + cy = 0$ is $y = c_1 e^{rx} + c_2 x e^{rx}$.

On replacing r with $-\frac{2}{3}$, we get the general solution of the given differential equation as

$$y = c_1 e^{-\frac{2}{3}x} + c_2 x e^{-\frac{2}{3}x}.$$

For finding the constant c_1, use the initial condition $y(0) = 1$.

$$1 = c_1 e^{-\frac{2}{3}(0)} + c_2(0) e^{-\frac{2}{3}(0)}$$
$$1 = c_1 e^{-\frac{2}{3}(0)}$$
$$c_1 = 1$$

Differentiate the equation.

$$y' = -\frac{2}{3}c_1e^{-\frac{2}{3}x} + c_2e^{-\frac{2}{3}x} - \frac{2}{3}c_2xe^{-\frac{2}{3}x}$$

Apply the condition $y'(0) = 0$.

$$0 = -\frac{2}{3}c_1e^{-\frac{2}{3}(0)} + c_2e^{-\frac{2}{3}(0)} - \frac{2}{3}c_2(0)e^{-\frac{2}{3}(0)}$$

$$= -\frac{2}{3}c_1 + c_2$$

$$c_2 = \frac{2}{3}c_1$$

On replacing c_1 with 1 in $c_2 = \frac{2}{3}c_1$, we get $c_2 = \frac{2}{3}$.

We substitute c_1 with 1 and c_2 with $\frac{2}{3}$ in $y = c_1e^{-\frac{2}{3}x} + c_2xe^{-\frac{2}{3}x}$ to get

$$y = e^{-\frac{2}{3}x} + \frac{2}{3}xe^{-\frac{2}{3}x}.$$

Thus, the solution to the given differential equation is $\boxed{y = e^{-\frac{2}{3}x} + \frac{2}{3}xe^{-\frac{2}{3}x}}$.

Chapter 17 Second Order Differential Equations 17.1 20E

The auxiliary polynomial for the given differential equation is $2r^2 + r - 1 = 0$ or $(r+1)(2r-1) = 0$. The roots of the auxiliary equation are $r = -1$ and $r = \frac{1}{2}$ each having multiplicity 1.

If the roots r_1 and r_2 of the auxiliary equation $ar^2 + br + c = 0$ are real and unequal, then the general solution of $ay'' + by' + cy = 0$ is $y = c_1e^{r_1x} + c_2e^{r_2x}$.

On replacing r_1 with -1 and r_2 with $\frac{1}{2}$, we get the general solution of the given differential equation as $y = c_1e^{-x} + c_2e^{\frac{1}{2}x}$.

For finding the constant c_1, use the initial condition $y(0) = 3$.

$$3 = c_1e^{-(0)} + c_2e^{\frac{1}{2}(0)}$$

$$3 = c_1 + c_2$$

$$c_2 = 3 - c_1$$

Differentiate the equation.

$$y' = -c_1e^{-x} + \frac{1}{2}c_2e^{\frac{1}{2}x}$$

Apply the condition $y'(0) = 3$.

$$3 = -c_1e^{-(0)} + \frac{1}{2}c_2e^{\frac{1}{2}(0)}$$

$$3 = -c_1 + \frac{1}{2}c_2$$

Replace c_2 with $3 - c_1$ in $3 = -c_1 + \frac{1}{2}c_2$.

$$3 = -c_1 + \frac{1}{2}(3 - c_1)$$

$$3 = \frac{3}{2} - \frac{3}{2}c_1$$

$$c_1 = -1$$

On substituting c_1 with -1 in $c_2 = 3 - c_1$, we get c_2 as 4. Then, $y = -e^{-x} + 4e^{\frac{1}{2}x}$

Thus, the solution to the given differential equation is $\boxed{y = -e^{-x} + 4e^{\frac{1}{2}x}}$.

Chapter 17 Second Order Differential Equations 17.1 21E

The auxiliary polynomial for the given differential equation is $r^2 - 6r + 10 = 0$. The roots of the auxiliary equation are $r = 3 + i$ and $r = 3 - i$ each having multiplicity 1.

If the roots of the auxiliary equation $ar^2 + br + c = 0$ are complex numbers $r_1 = \alpha + i\beta$ and $r_2 = \alpha - i\beta$, then the general solution of $ay'' + by' + cy = 0$ is $y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$.

On replacing α with 3, β with 1, we get the general solution of the given differential equation as $y = e^{3x}(c_1 \cos x + c_2 \sin x)$.

For finding the constant c_1, use the initial condition $y(0) = 2$.

$$2 = e^{3(0)}(c_1 \cos 0 + c_2 \sin 0)$$
$$2 = [c_1(1) + c_2(0)]$$
$$c_1 = 2$$

Differentiate the equation.

$$y' = 3e^{3x}(c_1 \cos x + c_2 \sin x) + e^{3x}(-c_1 \sin x + c_2 \cos x)$$

Apply the condition $y'(0) = 3$.

$$\begin{aligned} 3 &= 3e^{3(0)}(c_1 \cos 0 + c_2 \sin 0) + e^{3(0)}(-c_1 \sin 0 + c_2 \cos 0) \\ &= 3[c_1(1) + c_2(0)] + [-c_1(0) + c_2(1)] \\ &= 3c_1 + c_2 \end{aligned}$$

Replace c_1 with 2.

$$3(2) + c_2 = 3$$
$$c_2 = 3 - 6$$
$$c_2 = -3$$

On substituting c_1 with 2 and c_2 with -3 in $y = e^{3x}(c_1 \cos x + c_2 \sin x)$, we get $y = e^{3x}(2\cos x - 3\sin x)$.

Thus, the solution to the given differential equation is $\boxed{y = e^{3x}(2\cos x - 3\sin x)}$.

Chapter 17 Second Order Differential Equations 17.1 22E

The auxiliary polynomial for the given differential equation is $4r^2 - 20y + 25 = 0$. On dividing both the sides by 4, we get $r^2 - 5y + \frac{25}{4} = 0$ or $\left(r - \frac{5}{2}\right)^2 = 0$. The root of the auxiliary equation are $r = \frac{5}{2}$.

If the auxiliary equation $ar^2 + br + c = 0$ has only one real root r, then the general solution of $ay'' + by' + cy = 0$ is $y = c_1 e^{rx} + c_2 x e^{rx}$.

On replacing r with $\frac{5}{2}$, we get the general solution of the given differential equation as

$$y = c_1 e^{\left(\frac{5}{2}\right)x} + c_2 x e^{\left(\frac{5}{2}\right)x}.$$

For finding the constant c_1, use the initial condition $y(0) = 2$.

$$2 = c_1 e^{\left(\frac{5}{2}\right)(0)} + c_2(0)e^{\left(\frac{5}{2}\right)(0)}$$
$$2 = c_1(1) + 0$$
$$c_1 = 2$$

Differentiate the equation.

$$y' = \frac{5}{2}c_1e^{\frac{5}{2}x} + c_2e^{\frac{5}{2}x} + \frac{5}{2}c_2xe^{\frac{5}{2}x}$$

Apply the condition $y'(0) = -3$.

$$-3 = \frac{5}{2}c_1e^{\frac{5}{2}(0)} + c_2e^{\frac{5}{2}(0)} + \frac{5}{2}c_2(0)e^{\frac{5}{2}(0)}$$

$$-3 = \frac{5}{2}c_1(1) + c_2(1) + \frac{5}{2}(0)$$

$$-3 = \frac{5}{2}c_1 + c_2$$

$$-6 = 5c_1 + 2c_2$$

Replace c_1 with 2 in $5c_1 + 2c_2 = -6$.

$$5(2) + 2c_2 = -6$$

$$2c_2 = -16$$

$$c_2 = -8$$

On substituting c_1 with 2 and c_2 with −8 in $y = c_1e^{\left(\frac{5}{2}\right)x} + c_2xe^{\left(\frac{5}{2}\right)x}$, we get

$$y = 2e^{\frac{5}{2}x} - 8xe^{\frac{5}{2}x}.$$

Thus, the solution to the given differential equation is $\boxed{y = 2e^{\frac{5}{2}x} - 8xe^{\frac{5}{2}x}}$.

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Chapter 17 Second Order Differential Equations 17.1 23E

The auxiliary polynomial for the given differential equation is $r^2 - r - 12 = 0$ or $(r+3)(r-4) = 0$. The roots of the auxiliary equation are $r = -3$ and $r = 4$ each having multiplicity 1.

If the roots r_1 and r_2 of the auxiliary equation $ar^2 + br + c = 0$ are real and unequal, then the general solution of $ay'' + by' + cy = 0$ is $y = c_1e^{r_1x} + c_2e^{r_2x}$.

On replacing r_1 with −3 and r_2 with 4, we get the general solution of the given differential equation as $y = c_1e^{-3x} + c_2e^{4x}$.

For finding the constant c_1, use the initial condition $y(1) = 0$.

$$0 = c_1e^{-3(1)} + c_2e^{4(1)}$$

$$0 = c_1e^{-3} + c_2e^{4}$$

$$c_1e^{-3} = -c_2e^{4}$$

$$c_1 = -c_2e^{7}$$

Differentiate the equation.

$$y' = -3c_1e^{-3x} + 4c_2e^{4x}$$

Apply the condition $y'(1) = 1$.

$$1 = -3c_1e^{-3(1)} + 4c_2e^{4(1)}$$

$$1 = -3c_1e^{-3} + 4c_2e^{4}$$

On substituting c_2 with $\frac{1}{7e^4}$ in $c_1 = -c_2e^7$, we get c_2 as $-\frac{1}{7}e^3$. Then,

$$y = -\frac{1}{7}e^3e^{-3x} + \frac{1}{7e^4}e^{4x}.$$

Thus, the solution to the given differential equation is $\boxed{y = \frac{1}{7}e^{-4+4x} - \frac{1}{7}e^{3-3x}}$.

Chapter 17 Second Order Differential Equations 17.1 24E

The auxiliary polynomial for the given differential equation is $4r^2+4r+3=0$. The roots of the auxiliary equation are $r = -\frac{1}{2}+\frac{\sqrt{2}}{2}\mathbf{i}$ and $r = -\frac{1}{2}-\frac{\sqrt{2}}{2}\mathbf{i}$ each having multiplicity 1.

If the roots of the auxiliary equation $ar^2+br+c=0$ are complex numbers $r_1 = \alpha+i\beta$ and $r_2 = \alpha-i\beta$, then the general solution of $ay''+by'+cy = 0$ is $y = e^{\alpha x}(c_1\cos\beta x+c_2\sin\beta x)$.

On replacing α with $-\frac{1}{2}$ and β with $\frac{\sqrt{2}}{2}$, we get the general solution of the given differential equation as $y = e^{-\frac{1}{2}x}\left(c_1\cos\frac{\sqrt{2}}{2}x+c_2\sin\frac{\sqrt{2}}{2}x\right)$.

For finding the constant c_1, use the initial condition $y(0)=0$.

$$0 = e^{-\frac{1}{2}(0)}\left(c_1\cos\frac{\sqrt{2}}{2}(0)+c_2\cos\frac{\sqrt{2}}{2}(0)\right)$$
$$0 = c_1+0$$
$$c_1 = 0$$

Differentiate the equation.

$$y' = -\frac{1}{2}e^{-\frac{1}{2}x}\left(c_1\cos\frac{\sqrt{2}}{2}x+c_2\sin\frac{\sqrt{2}}{2}x\right)+e^{-\frac{1}{2}x}\left(-\frac{c_1\sqrt{2}}{2}\sin\frac{\sqrt{2}}{2}x+\frac{c_2\sqrt{2}}{2}\cos\frac{\sqrt{2}}{2}x\right)$$
$$= \frac{e^{-\frac{1}{2}x}}{2}\left[-\left(c_1\cos\frac{\sqrt{2}}{2}x+c_2\sin\frac{\sqrt{2}}{2}x\right)+\left(-c_1\sqrt{2}\sin\frac{\sqrt{2}}{2}x+c_2\sqrt{2}\cos\frac{\sqrt{2}}{2}x\right)\right]$$
$$= \frac{e^{-\frac{1}{2}x}}{2}\left[-c_1\cos\frac{\sqrt{2}}{2}x-c_2\sin\frac{\sqrt{2}}{2}x-\sqrt{2}c_1\sin\frac{\sqrt{2}}{2}x+\sqrt{2}c_2\cos\frac{\sqrt{2}}{2}x\right]$$

Apply the condition $y'(0)=1$.

$$1 = \frac{e^{-\frac{1}{2}(0)}}{2}\left[-c_1\cos\frac{\sqrt{2}}{2}(0)-c_2\sin\frac{\sqrt{2}}{2}(0)-\sqrt{2}c_1\sin\frac{\sqrt{2}}{2}(0)+\sqrt{2}c_2\cos\frac{\sqrt{2}}{2}(0)\right]$$
$$1 = \frac{1}{2}\left[-c_1(1)-c_2(0)-\sqrt{2}c_1(0)+\sqrt{2}c_2(1)\right]$$
$$-c_1+\sqrt{2}c_2 = 2$$

Replace c_1 with 0 in $-c_1+\sqrt{2}c_2 = 2$.

$$-0+\sqrt{2}c_2 = 2$$
$$c_2 = \frac{2}{\sqrt{2}}$$
$$c_2 = \sqrt{2}$$

On substituting c_1 with 1 and c_2 with $\sqrt{2}$ in $y = e^{-\frac{1}{2}x}\left(c_1\cos\frac{\sqrt{2}}{2}x+c_2\sin\frac{\sqrt{2}}{2}x\right)$, we get the general solution as $y = e^{-\frac{1}{2}x}\left(\cos\frac{\sqrt{2}}{2}x+\sqrt{2}\sin\frac{\sqrt{2}}{2}x\right)$.

Thus, the solution to the given differential equation is

$$y = e^{-\frac{1}{2}x}\left(\cos\frac{\sqrt{2}}{2}x+\sqrt{2}\sin\frac{\sqrt{2}}{2}x\right).$$

Chapter 17 Second Order Differential Equations 17.1 25E

The auxiliary polynomial for the given differential equation is $r^2+4=0$ or $r^2=-4$. The roots of the auxiliary equation are $r=2i$ and $r=-2i$ each having multiplicity 1.

If the roots of the auxiliary equation $ar^2+br+c=0$ are complex numbers $r_1 = \alpha + i\beta$ and $r_2 = \alpha - i\beta$, then the general solution of $ay''+by'+cy = 0$ is $y = e^{\alpha x}(c_1\cos\beta x + c_2\sin\beta x)$.

On replacing α with 0, β with 2, we get the general solution of the given differential equation as $y = c_1\cos 2x + c_2\sin 2x$.

For finding the constant c_1, use the initial condition $y(0)=5$.

$$5 = c_1\cos 2(0) + c_2\sin 2(0)$$
$$5 = c_1(1) + c_2(0)$$
$$c_1 = 5$$

Now, apply the condition $y\left(\frac{\pi}{4}\right) = 3$.

$$3 = c_1\cos 2\left(\frac{\pi}{4}\right) + c_2\sin 2\left(\frac{\pi}{4}\right)$$
$$3 = c_1(0) + c_2(1)$$
$$c_2 = 3$$

On substituting c_1 with 5 and c_2 with 3 in $y = c_1\cos 2x + c_2\sin 2x$, we get $y = 5\cos 2x + 3\sin 2x$.

Thus, the solution to the given differential equation is $\boxed{y = 5\cos 2x + 3\sin 2x}$.

Chapter 17 Second Order Differential Equations 17.1 26E

The auxiliary polynomial for the given differential equation is $r^2=4$. The roots of the auxiliary equation are $r=2$ and $r=-2$.

If the roots r_1 and r_2 of the auxiliary equation $ar^2+br+c=0$ are real and unequal, then the general solution of $ay''+by'+cy = 0$ is $y = c_1e^{r_1x} + c_2e^{r_2x}$.

On replacing r_1 with 2 and r_2 with -2, we get the general solution of the given differential equation as $y = c_1e^{2x} + c_2e^{-2x}$.

For finding the constant c_1, use the initial condition $y(0)=1$.

$$1 = c_1e^{2(0)} + c_2e^{-2(0)}$$
$$1 = c_1(1) + c_2(1)$$
$$c_1 + c_2 = 1$$
$$c_2 = 1 - c_1$$

Now, apply the condition $y(1)=0$.

$$0 = c_1e^{2(1)} + c_2e^{-2(1)}$$
$$0 = c_1e^2 + c_2e^{-2}$$
$$c_1e^2 = -c_2e^{-2}$$

Replace c_2 with $1-c_1$ in $c_1e^2 = -c_2e^{-2}$.

$$c_1e^2 = -(1-c_1)e^{-2}$$
$$c_1e^2 = -e^{-2} + c_1e^{-2}$$
$$c_1\left(e^2 - e^{-2}\right) = -e^{-2}$$
$$c_1 = \frac{-e^{-2}}{e^2 - e^{-2}}$$

On substituting c_1 with $c_1 = \dfrac{-e^{-2}}{e^2 - e^{-2}}$ in $c_2 = 1 - c_1$, we get c_2 as $\dfrac{e^2}{e^2 - e^{-2}}$. Then,

$$y = \frac{-e^{-2}e^{2x}}{e^2 - e^{-2}} + \frac{e^2 e^{-2x}}{e^2 - e^{-2}}.$$

Thus, the solution to the given differential equation is $\boxed{y = \dfrac{-e^{-2}e^{2x}}{e^2 - e^{-2}} + \dfrac{e^2 e^{-2x}}{e^2 - e^{-2}}}$.

Chapter 17 Second Order Differential Equations 17.1 27E

The auxiliary polynomial for the given differential equation is $r^2 + 4r + 4 = 0$. The roots of the auxiliary equation are $r = -2$ each having multiplicity 1.

If the auxiliary equation $ar^2 + br + c = 0$ has only one real root r, then the general solution of $ay'' + by' + cy = 0$ is $y = c_1 e^{rx} + c_2 x e^{rx}$.

On replacing r with -2, we get the general solution of the given differential equation as $y = c_1 e^{-2x} + c_2 x e^{-2x}$.

For finding the constant c_1, use the initial condition $y(0) = 2$.

$$\begin{aligned} 2 &= c_1 e^{-2(0)} + c_2 (0) e^{-2(0)} \\ &= c_1 + 0 \\ c_1 &= 2 \end{aligned}$$

Apply the condition $y(1) = 0$.

$$\begin{aligned} 0 &= c_1 e^{-2(1)} + c_2 (1) e^{-2(1)} \\ c_1 e^{-2} &= -c_2 e^{-2} \end{aligned}$$

Replace c_1 with 2 in $c_1 e^{-2} = -c_2 e^{-2}$.

$$\begin{aligned} 2e^{-2} &= -c_2 e^{-2} \\ c_2 &= -2 \end{aligned}$$

On substituting c_1 with 2 in $c_2 = -2$ in $y = c_1 e^{-2x} + c_2 x e^{-2x}$, we get $y = 2e^{-2x} - 2xe^{-2x}$

Thus, the solution to the given differential equation is $\boxed{y = 2e^{-2x} - 2xe^{-2x}}$.

Chapter 17 Second Order Differential Equations 17.1 28E

The auxiliary polynomial for the given differential equation is $r^2 - 8r + 17 = 0$. The roots of the auxiliary equation are $r = 4 + i$ and $r = 4 - i$ each having multiplicity 1.

If the roots of the auxiliary equation $ar^2 + br + c = 0$ are complex numbers $r_1 = \alpha + i\beta$ and $r_2 = \alpha - i\beta$, then the general solution of $ay'' + by' + cy = 0$ is $y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$.

On replacing α with 4, β with 1, we get the general solution of the given differential equation as $y = e^{4x}(c_1 \cos x + c_2 \sin x)$.

For finding the constant c_1, use the initial condition $y(0) = 3$.

$$\begin{aligned} 3 &= e^{4(0)}\left[c_1 \cos(0) + c_2 \sin(0)\right] \\ 3 &= (1)\left[c_1 (1) + c_2 (0)\right] \\ c_1 &= 3 \end{aligned}$$

Now, apply the condition $y(\pi) = 2$.

$$2 = e^{4(\pi)}\left[c_1 \cos(\pi) + c_2 \sin(\pi)\right]$$
$$2 = e^{4(\pi)}\left[c_1(-1) + (0)\right]$$
$$c_1 = -\frac{2}{e^{4\pi}}$$

Since we cannot determine the value of c_2, we can say that the given problem has no solution.

Chapter 17 Second Order Differential Equations 17.1 29E

The auxiliary polynomial for the given differential equation is $r^2 = r$ or $r(r-1) = 0$. The roots of the auxiliary equation are $r = 0$ and $r = 1$.

If the roots r_1 and r_2 of the auxiliary equation $ar^2 + br + c = 0$ are real and unequal, then the general solution of $ay'' + by' + cy = 0$ is $y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$.

On replacing r_1 with 0 and r_2 with 1. we get the general solution of the given differential equation as $y = c_1 + c_2 e^x$.

For finding the constant c_1, use the initial condition $y(0) = 1$.

$$1 = c_1 + c_2 e^{(0)}$$
$$c_1 = 1 - c_2$$

Now, apply the condition $y(1) = 2$.

$$2 = c_1 + c_2 e^{(1)}$$
$$2 = c_1 + c_2 e$$

Replace c_1 with $1 - c_2$ in $2 = c_1 + c_2 e$.

$$2 = (1 - c_2) + c_2 e$$
$$2 = 1 - c_2 + c_2 e$$
$$1 = c_2(e - 1)$$
$$c_2 = \frac{1}{e-1}$$

On substituting c_2 with $\frac{1}{e-1}$ in $c_1 = 1 - c_2$, we get c_1 as $\frac{e-2}{e-1}$. Then,

$$y = \frac{e-2}{e-1} + \frac{e^x}{e-1}.$$

Thus, the solution to the given differential equation is $\boxed{y = \frac{e-2}{e-1} + \frac{e^x}{e-1}}$.

Chapter 17 Second Order Differential Equations 17.1 30E

The auxiliary polynomial for the given differential equation is $4r^2 - 4r + 1 = 0$. On dividing both the sides by 4, we get $r^2 - r + \frac{1}{4} = 0$ or $\left(r - \frac{1}{2}\right)^2 = 0$. The root of the auxiliary equation is $r = \frac{1}{2}$.

If the auxiliary equation $ar^2 + br + c = 0$ has only one real root r, then the general solution of $ay'' + by' + cy = 0$ is $y = c_1 e^{rx} + c_2 x e^{rx}$.

On replacing r with $\frac{1}{2}$, we get the general solution of the given differential equation as

$$y = c_1 e^{\left(\frac{1}{2}\right)x} + c_2 x e^{\left(\frac{1}{2}\right)x}.$$

For finding the constant c_1, use the initial condition $y(0) = 4$.

$$4 = c_1 e^{\left(\frac{1}{2}\right)(0)} + c_2(0)e^{\left(\frac{1}{2}\right)(0)}$$
$$4 = c_1(1) + 0$$
$$c_1 = 4$$

Now, apply the condition $y(2) = 0$.

$$0 = c_1 e^{\left(\frac{1}{2}\right)(2)} + c_2(2)e^{\left(\frac{1}{2}\right)(2)}$$
$$0 = c_1 e + 2c_2 e$$
$$c_1 e = -2c_2 e$$
$$c_1 = -2c_2$$

Replace c_1 with 4 in $c_1 = -2c_2$.

$$(4)e = -2c_2 e$$
$$c_2 = -2$$

On substituting c_1 with 4 and c_2 with -2 in $y = c_1 e^{\left(\frac{1}{2}\right)x} + c_2 x e^{\left(\frac{1}{2}\right)x}$, we get

$$y = 4e^{\frac{x}{2}} - 2c_2 x e^{\frac{x}{2}}.$$

Thus, the solution to the given differential equation is $\boxed{y = 2e^{\frac{5}{2}x} - 8xe^{\frac{5}{2}x}}$.

Chapter 17 Second Order Differential Equations 17.1 31E

The auxiliary polynomial for the given differential equation is $r^2 + 4r + 20 = 0$.
The roots of the auxiliary equation are $r = -2 + 4i$ and $r = -2 - 4i$ each having multiplicity 1.

If the roots of the auxiliary equation $ar^2 + br + c = 0$ are complex numbers $r_1 = \alpha + i\beta$ and $r_2 = \alpha - i\beta$, then the general solution of $ay'' + by' + cy = 0$ is
$y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$.
On replacing α with -2, β with 4, we get the general solution of the given differential equation as $y = e^{-2x}(c_1 \cos 4x + c_2 \sin 4x)$.

For finding the constant c_1, use the initial condition $y(0) = 1$.

$$1 = e^{-2(0)}[c_1 \cos 4(0) + c_2 \sin 4(0)]$$
$$1 = c_1(1) + c_2(0)$$
$$c_1 = 1$$

Now, apply the condition $y(\pi) = 2$.

$$2 = e^{-2(\pi)}[c_1 \cos 4(\pi) + c_2 \sin 4(\pi)]$$
$$2 = e^{-2\pi}[c_1(1) + c_2(0)]$$
$$c_1 = \frac{2}{e^{-2\pi}}$$

Since we cannot determine the value of c_2, we can say that the given problem has no solution.

Chapter 17 Second Order Differential Equations 17.1 32E

The auxiliary polynomial for the given differential equation is $r^2+4r+20=0$. The roots of the auxiliary equation are $r=-2+4i$ and $r=-2-4i$ each having multiplicity 1.

If the roots of the auxiliary equation $ar^2+br+c=0$ are complex numbers $r_1 = \alpha + i\beta$ and $r_2 = \alpha - i\beta$, then the general solution of $ay''+by'+cy = 0$ is $y = e^{\alpha x}\left(c_1\cos\beta x + c_2\sin\beta x\right)$.

On replacing α with -2, β with 4, we get the general solution of the given differential equation as $y = e^{-2x}\left(c_1\cos 4x + c_2\sin 4x\right)$.

For finding the constant c_1, use the initial condition $y(0)=1$.

$$1 = e^{-2(0)}\left[c_1\cos 4(0) + c_2\sin 4(0)\right]$$
$$1 = c_1(1) + c_2(0)$$
$$c_1 = 1$$

Now, apply the condition $y(\pi) = e^{-2\pi}$.

$$e^{-2\pi} = e^{-2(\pi)}\left[c_1\cos 4(\pi) + c_2\sin 4(\pi)\right]$$
$$e^{-2\pi} = e^{-2\pi}\left[c_1(1) + c_2(0)\right]$$
$$c_1 = 1$$

Since we cannot determine the value of c_2, we can say that the given problem has no solution.

Chapter 17 Second Order Differential Equations 17.1 33E

(A) Given equation is $y''+\lambda y=0$

If $\lambda=0$ then the given equation reduces to

$y''=0$

Integrating both sides, we get, $y'=c_1$ where c_1 is a constant.

Again integrating both sides, we get, $y=c_1x+c_2$ where c_2 is another constant.

Now applying boundary conditions when $x=0, y=0$

Therefore $0=c_1\times 0+c_2 \quad \Rightarrow c_2=0$

when x =L , y=0

therefore $0=c_1 L+c_2$

$\Rightarrow \quad c_1L=0$

$\Rightarrow \quad c_1=0 \quad$ since $L\neq 0$.

Thus the solution of given equation is

$$\begin{aligned} y &= c_1x+c_2 \\ &= 0x+0 \\ &= 0 \end{aligned}$$

When λ is negative, then the given equation can be written as

$$y''-(-\lambda)y=0$$

Its auxiliary equation is

$$r^2-(-\lambda)=0$$

$\Rightarrow \quad r^2-\left(\sqrt{-\lambda}\right)^2=0$

$\Rightarrow \quad \left(r+\sqrt{-\lambda}\right)\left(r-\sqrt{-\lambda}\right)=0$

$\Rightarrow \quad r=-\sqrt{-\lambda}, \sqrt{-\lambda}=\pm\sqrt{\lambda}i$

Therefore, the general solution of given equation is,

$$y=c_1e^{\sqrt{-\lambda}x}+c_2e^{-\sqrt{-\lambda}x}$$

Applying boundary conditions
When $x = 0, \quad y = 0$

Therefore $0 = c_1 e^o + c_2 e^o$

$\Rightarrow \quad 0 = c_1 + c_2$

$\Rightarrow \quad c_2 = -c_1$

Also when $x = L, y = 0$

Therefore

$$0 = c_1 e^{\sqrt{-\lambda}L} + c_2 e^{-\sqrt{-\lambda}L}$$

$$\Rightarrow \quad c_1 e^{\sqrt{-\lambda}L} - c_1 e^{-\sqrt{\lambda}L} = 0$$

$$\Rightarrow \quad c_1\left(e^{\sqrt{-\lambda}L} - e^{-\sqrt{\lambda}L}\right) = 0$$

$$\Rightarrow \quad c_1 = \frac{0}{\left(e^{\sqrt{-\lambda}L} - e^{-\sqrt{-\lambda}L}\right)} = 0$$

And $c_2 = -c_1 = 0$

Thus the solution of given equation is

$$y = c_1 e^{\sqrt{-\lambda}x} + c_2 e^{-\sqrt{-\lambda}x}$$

$$= 0 \times e^{\sqrt{-\lambda}x} + 0 \times e^{-\sqrt{-\lambda}x}$$

Hence

For the cases $\lambda = 0$ and $\lambda < 0$ solution of given equation is $y = 0$

(B) The given differential equation is

$$y'' + \lambda y = 0, \quad y(0) = 0 = y(L)$$

The auxiliary equation of given differential equation is

$$r^2 + \lambda = 0$$

$$\Rightarrow \quad r^2 + \left(\sqrt{\lambda}\right)^2 = 0$$

$$\Rightarrow \quad \left(r + i\sqrt{\lambda}\right)\left(r - i\sqrt{\lambda}\right) = 0$$

$$\Rightarrow \quad r = i\sqrt{\lambda}, -i\sqrt{\lambda}$$

Therefore, the general solution of given equation is

$$y = e^{ox}\left[c_1 \cos\sqrt{\lambda}x + c_2 \sin\sqrt{\lambda}x\right]$$

$$= c_1 \cos\sqrt{\lambda}x + c_2 \sin\sqrt{\lambda}x$$

Given, when $x = 0, y = 0$

Therefore $0 = c_1 \cos\sqrt{\lambda} \times 0 + c_2 \sin\sqrt{\lambda} \times 0$

$$0 = c_1$$

Thus $\quad y = c_2 \sin\sqrt{\lambda}x$

Also, when $x = L, y = 0$

Therefore $0 = c_2 \sin\sqrt{\lambda}L$

Since the given equation has non trivial solution and $c_2 = 0$ makes $y = 0$ i. e. Trivial solution so $c_2 \neq 0$

Therefore $\sin\sqrt{\lambda}L = 0$

$\Rightarrow \quad \sqrt{\lambda}L = n\pi \quad$ Where n is an integer.

$$\Rightarrow \quad \sqrt{\lambda} = \frac{n\pi}{L}$$

$$\Rightarrow \quad \lambda = \frac{n^2\pi^2}{L^2}$$

The corresponding solution for $\lambda = \frac{n^2\pi^2}{L^2}$ is

$$y = c_2 \sin\sqrt{\lambda}x$$

$$= c_2 \sin\frac{n\pi}{L}x.$$

Hence

value of $\lambda = \frac{n^2\pi^2}{L^2}$
corresponding solution $y = c_2 \sin\frac{n\pi}{L}x$

Chapter 17 Second Order Differential Equations 17.1 34E

The given differential equation is,

$$ay''+by'+cy=0$$

The corresponding auxiliary equation is,

$$ar^2+br+c=0$$

$$\Rightarrow \quad r=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Let $r_1=\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $r_2=\frac{-b-\sqrt{b^2-4ac}}{2a}$

The solution of given differential equation will depend upon the solution of auxiliary equation and solution of auxiliary equation will depend upon the discriminant of it.

Let $b^2-4ac>0$

Then the roots (r_1,r_2) of auxiliary equation will be real and distinct. And solution of given differential equation will be

$$y=c_1e^{\frac{\left(-b+\sqrt{b^2-4ac}\right)x}{2a}}+c_2e^{\frac{\left(-b-\sqrt{b^2-4ac}\right)x}{2a}}$$

$$=e^{-\frac{b}{2a}x}\left[c_1e^{\frac{\sqrt{b^2-4ac}}{2a}x}+c_2e^{\frac{-\sqrt{b^2-4ac}}{2a}x}\right]$$

Since a,b,c are positive. So $\frac{b}{2a}$ will be positive. And $\lim_{x\to\infty}e^{-\frac{b}{2x}x}=\lim_{x\to\infty}\frac{1}{e^{\frac{b}{2a}x}}=0$

Now, $\lim_{x\to\infty}y(x)=\lim_{x\to\infty}e^{-\frac{b}{2a}x}\left[c_1e^{\frac{\sqrt{b^2-4ac}}{2a}x}+c_2e^{\frac{-\sqrt{b^2-4ac}}{2a}x}\right]$

$$=\lim_{x\to\infty}e^{-\frac{b}{2a}x}\times\lim_{x\to\infty}\left[c_1e^{\frac{\sqrt{b^2-4ac}}{2a}x}+c_2e^{\frac{-\sqrt{b^2-4ac}}{2a}x}\right]$$

$$=0\times\left[\lim_{x\to\infty}c_1e^{\frac{\sqrt{b^2-4ac}}{2a}x}+c_2e^{\frac{-\sqrt{b^2-4ac}}{2a}x}\right]=0$$

Let $b^2-4ac=0$ then the roots of auxiliary equation will be real and equal. And we have

$$r_1=\frac{-b}{2a},\qquad r_2=\frac{-b}{2a}$$

The solution of given differential equation will be

$$y=e^{-\frac{b}{2a}x}\left[c_1+c_2x\right]$$

Since $a,b,$ are positive. So $\frac{b}{2a}$ will be positive and $\lim_{x\to\infty}e^{-\frac{b}{2a}x}=\lim_{x\to\infty}e^{\frac{1}{\frac{b}{2a}x}}=0$

Therefore

$$\lim_{x\to\infty}y(x)=\lim_{x\to\infty}e^{-\frac{b}{2a}x}\left[c_1+c_2x\right]$$

$$=\lim_{x\to\infty}e^{-\frac{b}{2a}x}\times\lim_{x\to\infty}(c_1+c_2x)$$

$$=0\times\lim_{x\to\infty}(c_1+c_2x)$$

$$=0$$

Let $b^2-4ac<0$, then the roots of auxiliary equation will be imaginary and distinct. And we have

$$r_1=\frac{-b+i\sqrt{b^2-4ac}}{2a},\quad r_2=\frac{-b-i\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow r_1=-\frac{b}{2a}+i\frac{\sqrt{b^2-4ac}}{2a},\quad r_2=-\frac{b}{2a}-i\frac{\sqrt{b^2-4ac}}{2a}$$

Therefore, the solution of given differential equation will be,

$$y=e^{-\frac{b}{2a}x}\left[c_1\cos\frac{\sqrt{b^2-4ac}}{2a}x+c_2\sin\frac{\sqrt{b^2-4ac}}{2a}x\right]$$

Now

$$\begin{aligned}\lim_{x\to\infty} y(x)&=\lim_{x\to\infty} e^{-\frac{b}{2a}x}\left[c_1\cos\frac{\sqrt{b^2-4ac}}{2a}x+c_2\sin\frac{\sqrt{b^2-4ac}}{2a}x\right]\\&=\lim_{x\to\infty} e^{-\frac{b}{2a}x}\times\lim_{x\to\infty}\left[c_1\cos\frac{\sqrt{b^2-4ac}}{2a}x+c_2\sin\frac{\sqrt{b^2-4ac}}{2a}x\right]\\&=0\times\lim_{x\to\infty}\left[c_1\cos\frac{\sqrt{b^2-4ac}}{2a}x+c_2\sin\frac{\sqrt{b^2-4ac}}{2a}x\right]\\&=0\end{aligned}$$

Hence

In all the three cases we found that
$\lim_{x\to\infty} y(x)0$

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The auxiliary polynomial for the given differential equation is $r^2-2r+2=0$. The roots of the auxiliary equation are $r=1+i$ and $r=1-i$ each having multiplicity 1.

If the roots of the auxiliary equation $ar^2+br+c=0$ are complex numbers $r_1=\alpha+i\beta$ and $r_2=\alpha-i\beta$, then the general solution of $ay''+by'+cy=0$ is $y=e^{\alpha x}(c_1\cos\beta x+c_2\sin\beta x)$.

On replacing α with 1, β with 1, we get the general solution of the given differential equation as $y=e^x(c_1\cos x+c_2\sin x)$.

Since $y(a)=c$, we get $c=e^a(c_1\cos a+c_2\sin a)$ or $c=c_1e^a\cos a+c_2e^a\sin a$. Also, we have $y(b)=d$. Then, $d=e^b(c_1\cos b+c_2\sin b)$ or $d=c_1e^b\cos b+c_2e^b\sin b$.

(a)If the given problem has a unique solution, then $\begin{vmatrix}e^a\cos a & e^a\sin a\\ e^b\cos b & e^b\sin b\end{vmatrix}\neq 0$.

$$(e^a\cos a)(e^b\sin b)-(e^a\sin a)(e^b\cos b)\neq 0$$

$$e^{a+b}\cos a\sin b-e^{a+b}\sin a\cos b\neq 0$$

Since exponential function is always positive, we can say that $\cos a\sin b-\sin a\cos b\neq 0$ or $\cos(a-b)\neq 0$.

This means that $a-b\neq(2n+1)\frac{\pi}{2}$ or $a\neq b\pm(2n+1)\frac{\pi}{2}$.

(b)If the given problem has no solution, then rank of $\begin{vmatrix}e^a\cos a & e^a\sin a\\ e^b\cos b & e^b\sin b\end{vmatrix}$ and rank of $\begin{vmatrix}e^a\cos a & e^a\sin a & c\\ e^b\cos b & e^b\sin b & d\end{vmatrix}$ are not equal. This means that when $b=a$ and $c\neq d$, the given problem has no solution.

(c)Now, if the problem has infinitely many solutions, then rank of $\begin{vmatrix}e^a\cos a & e^a\sin a\\ e^b\cos b & e^b\sin b\end{vmatrix}$ and $\begin{vmatrix}e^a\cos a & e^a\sin a & c\\ e^b\cos b & e^b\sin b & d\end{vmatrix}$ should be 1. This means that when $b=a$ and $c=d$, the given problem has infinitely many solutions.