EXERCISE : 12.2

Q No 1 Find the distance between the following pairs of points :

(i) $(2,3,5)$ and $(4,3,1)$ (ii) $(-3,7,2)$ and $(2,4,-1)$

(iii) $(-1,3,-4)$ and $(1,-3,4)$ (iv) $(2,-1,3)$ and $(-2,1,3)$

Sol. (i) Let d be the distance between given points $(2,3,5)$ and $(4,3,1)$ Using distance formula

$$\therefore d = \sqrt{(4-2)^2+(3-3)^2+(1-5)^2} = \sqrt{4+0+16} = \sqrt{20} = \sqrt{4\times5} = 2\sqrt{5}$$

(ii) Let d be the distance between $(-3,7,2)$ and $(2,4,-1)$

$$\therefore d = \sqrt{(2+3)^2+(4-7)^2+(-1-2)^2} = \sqrt{25+9+9} = \sqrt{43}$$

(iii) Let d be the distance between the given points $(-1,3,-4)$ and $(1,-3,4)$

$$d = \sqrt{(1+1)^2+(-3-3)^2+(4+4)^2} = \sqrt{4+36+64} = \sqrt{104} = \sqrt{4\times26} = 2\sqrt{26}$$

(iv) Let d be the distance between the points $(2,-1,3)$ and $(-2,1,3)$

$$d = \sqrt{(-2-2)^2+(-1+1)^2+(3-3)^2} = \sqrt{16+0+0} = \sqrt{16} = 4.$$

Q No 2 : Show that the points $(-2,3,5)$ $(1,2,3)$ and $(7,0,-1)$ are collinear.

Sol : Given points are $P(-2,3,5)$, $Q(1,2,3)$ and $R(7,0,-1)$

$$|PQ| = \sqrt{(1+2)^2+(2-3)^2+(3-5)^2} = \sqrt{9+1+4} = \sqrt{14}$$

$$|QR| = \sqrt{(7-1)^2+(0-2)^2+(-1-3)^2} = \sqrt{36+4+16} = \sqrt{56} = \sqrt{4\times14} = 2\sqrt{14}$$

$$|RP| = \sqrt{(-2-7)^2+(3-0)^2+(5+1)^2} = \sqrt{81+9+36} = \sqrt{126} = \sqrt{9\times14} = 3\sqrt{14}$$

Now $PQ + QR = \sqrt{14} + 2\sqrt{14} = 3\sqrt{14} = RP$

$\therefore$ P, Q, R are collinear.

Q No 3 : Verify the following :

(i) $(0,7,-10)$ $(1,6,-6)$ and $(4,9,-6)$ are vertices of an isosceles triangle.

(ii) $(0,7,10)$ $(-1,6,6)$ and $(-4,9,6)$ are vertices of right angled triangle.

(iii) $(-1,2,1)$ $(1,-2,5)$ $(4,-7,8)$ and $(2,-3,4)$ are vertices of a parallelogram.

Sol (i) Let $A(0,7,-10)$; $B(1,6,-6)$ and $C(4,9,-6)$ be given points

$$\therefore AB = \sqrt{(1-0)^2+(6-7)^2+(-6+10)^2} = \sqrt{1+1+16} = \sqrt{18} = \sqrt{9\times2} = 3\sqrt{2}$$

$$BC = \sqrt{(4-1)^2+(9-6)^2+(-6+6)^2} = \sqrt{9+9+0} = \sqrt{18} = \sqrt{9\times2} = 3\sqrt{2}$$

$$CA = \sqrt{(0-4)^2+(7-9)^2+(-10+6)^2} = \sqrt{16+4+16} = \sqrt{36} = 6$$

Now $AB = BC$

$\therefore$ ΔABC is isosceles.

(ii) Sol: Let $A(0,7,10)$, $B(-1,6,6)$, $C(-4,9,6)$ be given points.

$AB = \sqrt{(-1-0)^2+(6-7)^2+(6-10)^2} = \sqrt{1+1+16} = \sqrt{18} = \sqrt{2\times 9} = 3\sqrt{2}$

$BC = \sqrt{(-4+1)^2+(9-6)^2+(6-6)^2} = \sqrt{9+9+0} = \sqrt{18} = \sqrt{2\times 9} = 3\sqrt{2}$

$CA = \sqrt{(0+4)^2+(7-9)^2+(10-6)^2} = \sqrt{16+4+16} = \sqrt{36} = 6$

Now $AB = BC$

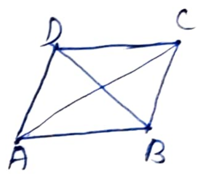
$\therefore$ ΔABC is isosceles.

Also $AB^2 + BC^2 = 18 + 18 = 36$

$\therefore AB^2 + BC^2 = CA^2$

$\therefore$ ΔABC is right-angled at B.

(iii) Let $A(-1,2,1)$; $B(1,-2,5)$, $C(4,-7,8)$, $D(2,-3,4)$ be given points.



$AB = \sqrt{(1+1)^2+(-2-2)^2+(5-1)^2} = \sqrt{4+16+16} = \sqrt{36} = 6$

$BC = \sqrt{(4-1)^2+(-7+2)^2+(8-5)^2} = \sqrt{9+25+9} = \sqrt{43}$

$CD = \sqrt{(2-4)^2+(-3+7)^2+(4-8)^2} = \sqrt{4+16+16} = \sqrt{36} = 6$

$DA = \sqrt{(-1-2)^2+(2+3)^2+(1-4)^2} = \sqrt{9+25+9} = \sqrt{43}$

$AC = \sqrt{(4+1)^2+(-7-2)^2+(8-1)^2} = \sqrt{24+81+49} = \sqrt{155}$

$BD = \sqrt{(2-1)^2+(-3+2)^2+(4-5)^2} = \sqrt{1+1+1} = \sqrt{3}$

$\therefore$ $AB = CD$, $BC = DA$ and $AC \neq BD$

i.e. Opposite sides are equal and diagonal are not equal

$\therefore$ ABCD is a parallelogram.

Q No. 4: Find the equation of the set of points which are equidistance from the points $(1,2,3)$ and $(3,2,-1)$

Sol Let $P(x,y,z)$ be equidistance from the points $A(1,2,3)$ and $B(3,2,-1)$

From given condition.

$$PA = PB \quad \text{or} \quad PA^2 = PB^2$$

$\therefore \quad (x-1)^2+(y-2)^2+(z-3)^2 = (x-3)^2+(y-2)^2+(z+1)^2$

$\Rightarrow \quad x^2-2x+1+y^2-4y+4+z^2-6y+9 = x^2-6x+9+y^2-4y+4+z^2+2z+1$

$\Rightarrow \quad 4x-8z=0 \quad \text{or} \quad x-2z=0$

Which is required equation.

Q No 5 : Find the equation of Set of points P, the sum of whose distances from A(4, 0, 0) and B(−4, 0, 0) is equal to 10.

Sol : Given points are A(4, 0, 0) and B(−4, 0, 0)

Let the coordinates of points P be (x, y, z)

From given condition.

$$PA + PB = 10$$

$$\therefore \sqrt{(x-4)^2+(y-0)^2+(z-0)^2} + \sqrt{(x+4)^2+(y-0)^2+(z-0)^2} = 10.$$

$$\therefore \sqrt{x^2-8x+16+y^2+z^2} + \sqrt{x^2+8x+16+y^2+z^2} = 10$$

or $$\sqrt{x^2-8x+16+y^2+z^2} = 10 - \sqrt{x^2+y^2+z^2+8x+16}$$

Squaring both sides

$$x^2+y^2+z^2-8x+16 = 100 + x^2+y^2+z^2+8x+16 - 20\sqrt{x^2+y^2+z^2+8x+16}$$

$$-16x - 100 = -20\sqrt{x^2+y^2+z^2+8x+16}$$

$$\therefore 4x+25 = 5\sqrt{x^2+y^2+z^2+8x+16}$$

Again Squaring, We get

$$16x^2+625+200x = 25(x^2+y^2+z^2+8x+16)$$

$$\Rightarrow -9x^2-25y^2-25z^2 = -225$$

$$\Rightarrow 9x^2+25y^2+25z^2 = 225$$

Which is required equation.

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