CHAPTER 11
Quadratic Functions

11-1. Graphs of Quadratic Equations

The **standard form** of a **quadratic function** $y = f(x)$ can be written in the form $f(x) = ax^2 + bx + c$, in which $a \neq 0$. The graph of a quadratic function is called a **parabola**.

If $a > 0$, the graph opens upward and the vertex is the **minimum** point.
If $a < 0$, the graph opens downward and the vertex is the **maximum** point.
The maximum or minimum point of a parabola is called the **vertex**.

The equation of the **axis of symmetry** is $x = -\frac{b}{2a}$. The coordinates of the vertex are $(-\frac{b}{2a}, f(-\frac{b}{2a}))$.

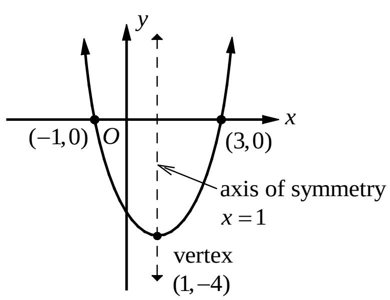
The **vertex form** of a quadratic function can be written in the form $f(x) = a(x-h)^2 + k$, in which (h,k) is the coordinates of vertex of the parabola.

The **solutions** of a quadratic function are the values of x for which $f(x) = 0$. Solutions of functions are also called **roots** or **zeros** of the function. On a graph, the solution of the function is the **x-intercept(s)**.

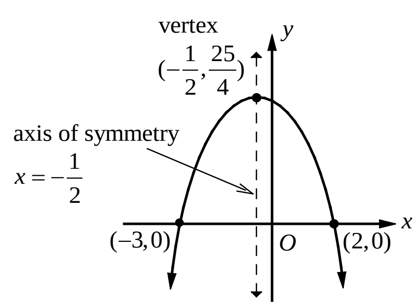
To find the **y-intercept** of a parabola, let x equal to zero in the equation of the parabola and solve for y.

If the parabola has two x-intercepts, then the *x-intercepts are equidistant from the axis of symmetry*.

The **factored form** of a quadratic function can be written in the form $f(x) = a(x-b)(x-c)$.
The x-intercepts or the solutions of the function are $x = b$ and $x = c$.
A parabola may have no x-intercept, one x-intercept, or two x-intercepts.



The graph of the quadratic function $y = x^2 - 2x - 3$ opens upward since $a = 1 > 0$, so the vertex is a minimum point. The equation of the axis of symmetry is $x = -\frac{-2}{2(1)}$ or $x = 1$. The vertex form of the function is $y = (x-1)^2 - 4$, from which the coordinates of the vertex can be identified as $(1, -4)$. The factored form of the function is $y = (x+1)(x-3)$. The x-intercepts are -1 and 3, which are equidistant from the axis of symmetry.



The graph of the quadratic function $y = -x^2 - x + 6$ opens downward since $a = -1 < 0$, so the vertex is a maximum point. The equation of the axis of symmetry is $x = -\frac{-1}{2(-1)}$ or $x = -\frac{1}{2}$. The vertex form of the function is $y = -(x+\frac{1}{2})^2 + \frac{25}{4}$, from which the coordinates of the vertex can be identified as $(-\frac{1}{2}, \frac{25}{4})$. The factored form of the function is $y = -(x+3)(x-2)$. The x-intercepts are -3 and 2, which are equidistant from the axis of symmetry.

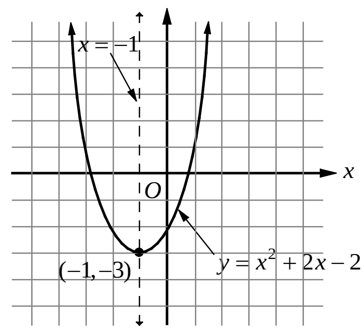
Example 1 □ Given $f(x) = x^2 + 2x - 2$, find the following.

a. The y-intercept.
b. The axis of symmetry.
c. The coordinate of the vertex.
d. Identify the vertex as a maximum or minimum.

Solution □ a. To find the y-intercept, let $x = 0$.

$f(0) = 0^2 + 2(0) - 2 = -2$

The y-intercept is -2.



b. The coefficients are $a = 1$, $b = 2$, and $c = -2$.

The equation of the axis of symmetry is $x = -\dfrac{b}{2a} = -\dfrac{2}{2(1)}$ or $x = -1$.

c. $f(-1) = (-1)^2 + 2(-1) - 2 = -3$.

The coordinates of the vertex are $(-1, -3)$.

d. Since the coefficient of the x^2 term is positive, the parabola opens upward and the vertex is a minimum point.

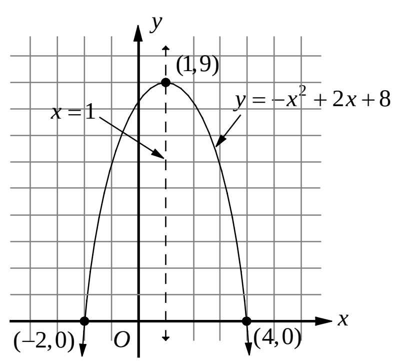
The graph of the parabola $f(x) = x^2 + 2x - 2$ is shown above.

Example 2 □ Given $f(x) = -(x+2)(x-4)$, find the following.

a. The x-intercepts.
b. The axis of symmetry.
c. The coordinate of the vertex.
d. Identify the vertex as a maximum or minimum.

Solution □ a. To find the x-intercept, let $f(x) = 0$.

From $f(x) = -(x+2)(x-4) = 0$, we get x-intercepts, $x = -2$ and $x = 4$.



b. Since the x-intercepts are equidistant from the axis of symmetry, the axis of symmetry is the average of the two x-intercepts.

$x = \dfrac{-2+4}{2}$ or $x = 1$

c. $f(1) = -(1+2)(1-4) = 9$.

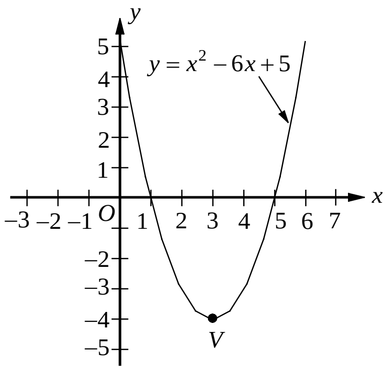
The coordinates of the vertex are $(1, 9)$.

d. Since the coefficient of the x^2 term is negative, the parabola opens downward and the vertex is a maximum point.

The graph of the parabola $f(x) = -(x+2)(x-4)$ is shown above.

Exercises - Graphs of Quadratic Equations

Questions 1 and 2 refer to the following information.



The graph of quadratic function $y = x^2 - 6x + 5$ is shown above.

1

Which of the following is an equivalent form of the equation of the graph shown above, from which the coordinates of vertex V can be identified as constants in the equation?

A) $y = (x-1)(x-5)$

B) $y = (x+1)(x+5)$

C) $y = x(x-6)+5$

D) $y = (x-3)^2 - 4$

2

Which of the following is an equivalent form of the equation of the graph shown above, that displays the x-intercepts of the parabola as constants?

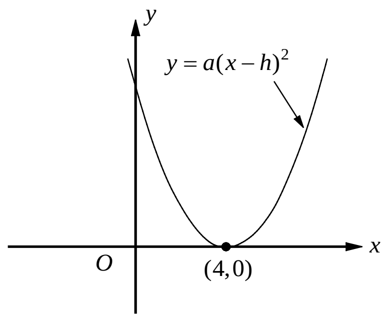
A) $y = (x-1)(x-5)$

B) $y = (x+1)(x+5)$

C) $y = x(x-6)+5$

D) $y = (x-3)^2 - 4$

3



In the xy-plane above, the parabola $y = a(x-h)^2$ has one x-intercept at $(4,0)$. If the y-intercept of the parabola is 9, what is the value of a?

4

In the xy-plane, if the parabola with equation $y = a(x+2)^2 - 15$ passes through $(1,3)$, what is the value of a?

5

The graph of the equation $y = a(x-1)(x+5)$ is a parabola with vertex (h,k). If the minimum value of y is -12, what is the value of a?

11-2. Factoring Trinomials

Factoring Trinomials: $x^2 + bx + c$

To factor trinomials of the form $x^2 + bx + c$, you need to find two integers with sum b and product c.
When two binomials $(x+m)$ and $(x+n)$ are multiplied using FOIL method, $(x+m)(x+n) = x^2 + nx + mx + mn$, or $x^2 + (m+n)x + mn$. So, you can write $x^2 + bx + c$ as $(x+m)(x+n)$, then as $m+n=b$ and $mn=c$.

Example 1 □ Factor each trinomial.

a. $x^2 + 9x + 20$

b. $x^2 - 11x + 18$

c. $x^2 + 5x - 14$

Solution □ a. Make a list of factors of 20, and look for the pair of factors that has a sum of 9.
Both integers must be positive since the sum is positive and the product is positive.

Factors of 20	Sum of Factors
1 and 20	$1 + 20 = 21$
2 and 10	$2 + 10 = 12$
4 and 5	$4 + 5 = 9$

The correct factors are 4 and 5.

$x^2 + 9x + 20 = (x+m)(x+n)$ — Write trinomial as the product of two binomials.

$= (x+4)(x+5)$ — $mn = 20$ and $m+n=9$

b. Make a list of factors of 18, and look for the pair of factors that has a sum of -11.
Both integers must be negative since the sum is negative and the product is positive.

Factors of 18	Sum of Factors
-1 and -18	$-1 + (-18) = -19$
-2 and -9	$-2 + (-9) = -11$
-3 and -6	$-3 + (-6) = -9$

The correct factors are -2 and -9.

$x^2 - 11x + 18 = (x+m)(x+n)$ — Write trinomial as the product of two binomials.

$= (x-2)(x-9)$ — $mn = 18$ and $m+n = -11$

c. Make a list of factors of -14, and look for the pair of factors that has a sum of 5.
One integer is negative and the other is positive since the product is negative.

Factors of -14	Sum of Factors
-1 and 14	$-1 + 14 = 13$
1 and -14	$1 + (-14) = -13$
-2 and 7	$-2 + 7 = 5$
2 and -7	$2 + (-7) = -5$

The correct factors are -2 and 7.

$x^2 + 5x - 14 = (x+m)(x+n)$ — Write trinomial as the product of two binomials.

$= (x-2)(x+7)$ — $mn = -14$ and $m+n=5$

Factoring Trinomials: $ax^2 + bx + c$

To factor trinomials of the form $ax^2 + bx + c$, in which $a \neq \pm 1$, you need to find two integers m and n with sum b and product ac. Then write bx as $mx + nx$ and use the method of factoring by grouping.

Example 2 □ Factor each trinomial.

a. $2x^2 - 7x + 6$

b. $5x^2 - 14x - 3$

c. $6x^2 - 5x - 4$

d. $-16x^2 - 40x + 24$

Solution □ a. Find two numbers with a sum of -7 and a product of $2 \cdot 6$, or 12.
The two numbers are -3 and -4.

$2x^2 - 7x + 6 = 2x^2 - 3x - 4x + 6$	Write $-7x$ as $-3x - 4x$.
$= (2x^2 - 3x) - (4x - 6)$	Group terms with common factors.
$= x(2x - 3) - 2(2x - 3)$	Factor the GCF from each grouping.
$= (2x - 3)(x - 2)$	Distributive Property

b. Find two numbers with a sum of -14 and a product of $5 \cdot -3$, or -15.
The two numbers are 1 and -15.

$5x^2 - 14x - 3 = 5x^2 + x - 15x - 3$	Write $-14x$ as $+x - 15x$.
$= (5x^2 + x) - (15x + 3)$	Group terms with common factors.
$= x(5x + 1) - 3(5x + 1)$	Factor the GCF from each grouping.
$= (5x + 1)(x - 3)$	Distributive Property

c. Find two numbers with a sum of -5 and a product of $6 \cdot -4$, or -24.
The two numbers are 3 and -8.

$6x^2 - 5x - 4 = 6x^2 + 3x - 8x - 4$	Write $-5x$ as $+3x - 8x$.
$= (6x^2 + 3x) - (8x + 4)$	Group terms with common factors.
$= 3x(2x + 1) - 4(2x + 1)$	Factor the GCF from each grouping.
$= (2x + 1)(3x - 4)$	Distributive Property

d. The GCF of the terms $-16x^2$, $-40x$, and 24 is -8. Factor it out first.

$-16x^2 - 40x + 24 = -8(2x^2 + 5x - 3)$	Factor out -8.

Now factor $2x^2 + 5x - 3$. Find two numbers with a sum of 5 and a product of $2 \cdot -3$, or -6. The two numbers are 6 and -1.

$2x^2 + 5x - 3 = 2x^2 + 6x - 1x - 3$	Write $5x$ as $6x - x$.
$= (2x^2 + 6x) - 1 \cdot (x + 3)$	Group terms with common factors.
$= 2x(x + 3) - 1 \cdot (x + 3)$	Factor the GCF from each grouping.
$= (x + 3)(2x - 1)$	Distributive Property

Thus, the complete factorization of $-16x^2 - 40x + 24$ is $-8(x + 3)(2x - 1)$.

Exercises – Factoring Trinomials

1

$$x^2 - 2x - 24$$

Which of the following is equivalent to the expression above?

A) $(x+3)(x-8)$

B) $(x-3)(x+8)$

C) $(x-6)(x+4)$

D) $(x+6)(x-4)$

2

$$x^2 - 17x + 72$$

Which of the following is equivalent to the expression above?

A) $(x+8)(x-9)$

B) $(x-8)(x-9)$

C) $(x-12)(x-6)$

D) $(x-12)(x+6)$

3

$$-x^2 + 5x + 84$$

Which of the following is equivalent to the expression above?

A) $(12-x)(x+7)$

B) $(12+x)(x-7)$

C) $(21+x)(x-4)$

D) $(21-x)(x+4)$

4

$$3x^2 + 7x - 6$$

Which of the following is equivalent to the expression above?

A) $(3x-2)(x+3)$

B) $(3x+2)(x-3)$

C) $(3x-1)(x+6)$

D) $(3x+1)(x-6)$

5

$$2x^2 + x - 15$$

Which of the following is equivalent to the expression above?

A) $(2x+3)(x-5)$

B) $(2x-3)(x+5)$

C) $(2x-5)(x+3)$

D) $(2x+5)(x-3)$

6

$$-6x^2 + x + 2$$

Which of the following is equivalent to the expression above?

A) $-(6x+1)(x-2)$

B) $-(6x-1)(x+2)$

C) $-(3x+2)(2x-1)$

D) $-(3x-2)(2x+1)$

11-3. Factoring Differences of Squares and Perfect Square Trinomials

The polynomial forms $a^2+2ab+b^2$ and $a^2-2ab+b^2$ are called **perfect square trinomials**, and they are the result of squaring $(a+b)$ and $(a-b)$. Also, the polynomial a^2-b^2 is called a **difference of squares**, and is the product of $(a+b)$ and $(a-b)$.

Perfect Square Trinomials

$a^2+2ab+b^2=(a+b)^2$

$a^2-2ab+b^2=(a-b)^2$

Difference of Squares

$a^2-b^2=(a+b)(a-b)$

Example 1 □ Factor each polynomial.

a. $27x^2-3$ b. $4x^2+12x+9$ c. $x^2-\frac{1}{2}x+\frac{1}{16}$

Solution □ a. $27x^2-3=3(9x^2-1)$ Factor out the GCF.

$=3((3x)^2-(1)^2)$ Write in the form a^2-b^2.

$=3(3x+1)(3x-1)$ Factor the difference of squares.

b. Since $4x^2=(2x)^2$, $9=3^2$, and $12x=2(2x)(3)$, $4x^2+12x+9$ is a perfect square trinomial.

$4x^2+12x+9=(2x)^2+2(2x)(3)+3^2$ Write as $a^2+2ab+b^2$.

$=(2x+3)^2$ Factored as $(a+b)^2$.

c. Since $x^2=(x)^2$, $\frac{1}{16}=(\frac{1}{4})^2$, and $\frac{1}{2}x=2(x)(\frac{1}{4})$, $x^2-\frac{1}{2}x+\frac{1}{16}$ is a perfect square trinomial.

$x^2-\frac{1}{2}x+\frac{1}{16}=(x)^2-2(x)(\frac{1}{4})+(\frac{1}{4})^2$ Write as $a^2-2ab+b^2$.

$=(x-\frac{1}{4})^2$ Factored as $(a-b)^2$.

Zero Product Property

If the product $a\cdot b=0$, then either $a=0$, $b=0$, or both a and b equal zero. Factoring and zero product property allow you to solve a quadratic equation by converting it into two linear equations.

Example 2 □ Solve each equation.

a. $(2x-5)(3x-10)=0$ b. $x^2-3x+28=0$

Solution □ a. $(2x-5)(3x-10)=0$

$2x-5=0$ or $3x-10=0$ Zero Product Property

$x=\frac{5}{2}$ or $x=\frac{10}{3}$ Solve each equation.

b. $x^2-3x+28=0$

$(x+4)(x-7)=0$ Factor.

$x+4=0$ or $x-7=0$ Zero Product Property

$x=-4$ or $x=7$ Solve each equation.

Exercises – Factoring Differences of Squares and Perfect Square Trinomials

1

$$3x^2 - 48$$

Which of the following is equivalent to the expression above?

A) $3(x-4)(x+4)$

B) $3(x-4)^2$

C) $(3x-4)(x+4)$

D) $(3x+4)(x-4)$

2

$$x - 6\sqrt{x} - 16$$

Which of the following is equivalent to the expression above?

A) $(\sqrt{x}-4)^2$

B) $(\sqrt{x}-4)(\sqrt{x}+4)$

C) $(\sqrt{x}+8)(\sqrt{x}-2)$

D) $(\sqrt{x}-8)(\sqrt{x}+2)$

3

If $x^2 + y^2 = 10$ and $xy = -3$, what is the value of $(x-y)^2$?

A) 12

B) 16

C) 20

D) 25

4

If $x + y = 10$ and $x - y = 4$, what is the value of $x^2 - y^2$?

A) 20

B) 24

C) 36

D) 40

5

$$6x^2 + 7x - 24 = 0$$

If r and s are two solutions of the equation above and $r > s$, which of the following is the value of $r - s$?

A) $\frac{7}{6}$

B) $\frac{16}{3}$

C) $\frac{25}{6}$

D) $\frac{20}{3}$

6

$$x^2 - 3x = 28$$

If r and s are two solutions of the equation above, which of the following is the value of $r + s$?

A) -3

B) 3

C) 6

D) 9

11-4. Solving Quadratic Equations by Completing the Square

Definition of Square Root

For any number $a > 0$, if $x^2 = a$, then x is a square root of a and $x = \pm\sqrt{a}$.

A method called **completing the square** can be used to solve a quadratic equation. To complete the square for a quadratic equation of the form $ax^2 + bx + c = 0$, you can follow the steps below.

1. Subtract c from each side: $ax^2 + bx = -c$
2. If $a \neq 1$, divide both sides by a: $x^2 + \frac{b}{a}x = -\frac{c}{a}$
3.Add $\frac{1}{2}$ of $\frac{b}{a}$ square which is $(\frac{b}{2a})^2$, to both sides: $x^2 + \frac{b}{a}x + (\frac{b}{2a})^2 = -\frac{c}{a} + (\frac{b}{2a})^2$
4. You have completed the square: $(x + \frac{b}{2a})^2 = -\frac{c}{a} + (\frac{b}{2a})^2$
5. Square root each side: $x + \frac{b}{2a} = \pm\sqrt{-\frac{c}{a} + (\frac{b}{2a})^2}$ or $x = -\frac{b}{2a} \pm \sqrt{-\frac{c}{a} + (\frac{b}{2a})^2}$

Definition of Equal Polynomials

If $ax^2 + bx + c = px^2 + qx + r$ for all values of x, then $a = p$, $b = q$, and $c = r$.

Example 1 □ Solve $2x^2 - 6x - 7 = 0$ by completing the square.

Solution □

$2x^2 - 6x = 7$	Add 7 to each side.
$x^2 - 3x = \frac{7}{2}$	Divide each side by 2.
$x^2 - 3x + \frac{9}{4} = \frac{7}{2} + \frac{9}{4}$	$\frac{1}{2}$ of -3 is $-\frac{3}{2}$. Add $(-\frac{3}{2})^2 = \frac{9}{4}$, to each side.
$(x - \frac{3}{2})^2 = \frac{23}{4}$	Complete the square and simplify.
$x - \frac{3}{2} = \pm\frac{\sqrt{23}}{\sqrt{4}}$	Take the square root of each side and simplify.
$x = \frac{3}{2} \pm \frac{\sqrt{23}}{2}$	Add $\frac{3}{2}$ to each side.

Example 2 □ If $s > 0$ and $4x^2 - rx + 9 = (2x - s)^2$ for all values of x, what is the value of $r - s$?

Solution □

$4x^2 - rx + 9 = 4x^2 - 4sx + s^2$	FOIL right side of the equation.
$4x^2 - rx + 9 = 4x^2 - 4sx + s^2$	By the Definition of Equal Polynomials $4 = 4$, $r = 4s$, and $9 = s^2$.
$s = \pm\sqrt{9} = \pm 3$	Definition of square root

Since $s > 0$ is given, $s = 3$.

$r = 4s = 4(3) = 12$

$r - s = 12 - 3 = 9$

Exercises - Solving Quadratic Equations by Completing the Square

1

If $x^2 - 10x = 75$ and $x < 0$, what is the value of $x + 5$?

A) -15

B) -10

C) -5

D) 0

2

If $x^2 - kx = 20$ and $x - \frac{k}{2} = 6$, which of the following is a possible value of x?

A) 2

B) 4

C) 6

D) 8

3

$$x^2 - \frac{k}{3}x = 5$$

Which of the following is an equivalent form of the equation shown above, from which the equation could be solved by completing the square?

A) $x^2 - \frac{k}{3}x + \frac{k}{6} = \frac{k}{6} + 5$

B) $x^2 - \frac{k}{3}x + \frac{k^2}{9} = \frac{k^2}{9} + 5$

C) $x^2 - \frac{k}{3}x + \frac{k^2}{36} = \frac{k^2}{36} + 5$

D) $x^2 - \frac{k}{3}x + \frac{k^2}{6} = \frac{k^2}{6} + 5$

4

$$x^2 - rx = \frac{k^2}{4}$$

In the quadratic equation above, k and r are are constants. What are the solutions for x?

A) $x = \frac{r}{4} \pm \frac{\sqrt{k^2 + 2r^2}}{4}$

B) $x = \frac{r}{2} \pm \frac{\sqrt{k^2 + 8r^2}}{4}$

C) $x = \frac{r}{4} \pm \frac{\sqrt{k^2 + r^2}}{2}$

D) $x = \frac{r}{2} \pm \frac{\sqrt{k^2 + r^2}}{2}$

5

If $(x - 7)(x - s) = x^2 - rx + 14$ for all values of x, what is the value of $r + s$?

6

If $x^2 - \frac{3}{2}x + c = (x - k)^2$, what is the value of c?

11-5. Quadratic Formula and the Discriminant

Solving Quadratic Equations by Using the Quadratic Formula

The solutions of the quadratic equation $ax^2 + bx + c$, in which $a \neq 0$, are given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Example 1 □ Use the quadratic formula to solve $2x^2 - 4x - 3 = 0$.

Solution □ For this equation $a = 2$, $b = -4$, and $c = -3$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

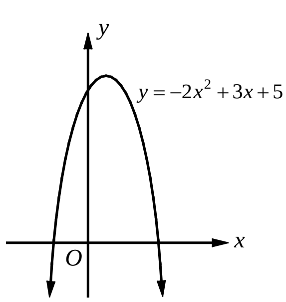
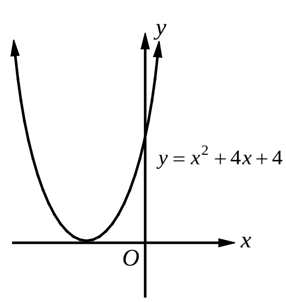
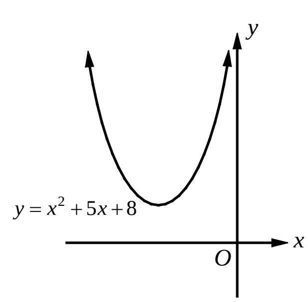
$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-3)}}{2(2)}$$ Substitute $a = 2$, $b = -4$, and $c = -3$.

$$= \frac{4 \pm \sqrt{16 + 24}}{4} = \frac{4 \pm \sqrt{40}}{4} = \frac{2 \pm \sqrt{10}}{2}$$ Simplify.

$$x = \frac{2 + \sqrt{10}}{2} \text{ or } x = \frac{2 - \sqrt{10}}{2}$$ Separate the solutions.

The Discriminant

In a Quadratic Formula, the expression under the radical sign $b^2 - 4ac$ is called the **discriminant**.
The value of the discriminant can be used to determine the number of real roots for the quadratic equation.

Example	$-2x^2 + 3x + 5 = 0$	$x^2 + 4x + 4 = 0$	$x^2 + 5x + 8 = 0$
Discriminant	$b^2 - 4ac = 3^2 - 4(-2)(5) = 49$ Positive	$b^2 - 4ac = 4^2 - 4(1)(4) = 0$ Zero	$b^2 - 4ac = 5^2 - 4(1)(8) = -7$ Negative
Number of Real Roots	2	1	0
x-intercepts or Roots of the function	$x = \frac{-3 \pm \sqrt{49}}{2(-2)} = \frac{-3 \pm 7}{-4}$ $x = 2.5$ or $x = -1$ two real roots	$x = \frac{-4 \pm \sqrt{0}}{2(1)} = -2$ one root	$x = \frac{-5 \pm \sqrt{-7}}{2(1)}$ no real roots
Graph of Related Function	 The graph crosses the x-axis twice.	 The graph touches the x-axis once.	 The graph does not cross the x-axis.

Example 2 □ Find the number of solutions for each system of equations.

a. $\begin{cases} y = 5x - 7 \\ y = x^2 + 6x + 1 \end{cases}$ b. $\begin{cases} y = -x + 2 \\ y = x^2 - 3x - 8 \end{cases}$

Solution □ a. Substitute $5x - 7$ for y in the quadratic equation.

$5x - 7 = x^2 + 6x + 1$ Substitution

$x^2 + x + 8 = 0$ Standard form of a quadratic equation.

For the quadratic equation above $a = 1$, $b = 1$, and $c = 8$.

Discriminant $= b^2 - 4ac = (1)^2 - 4(1)(8) = -31 < 0$

Since the discriminant is negative, the system of equations has no solution.

b. Substitute $-x + 2$ for y in the quadratic equation.

$-x + 2 = x^2 - 3x - 8$ Substitution

$x^2 - 2x - 10 = 0$ Standard form of a quadratic equation.

For the quadratic equation above $a = 1$, $b = -2$, and $c = -10$

Discriminant $= b^2 - 4ac = (-2)^2 - 4(1)(-10) = 44 > 0$

Since the discriminant is positive, the system of equations has two solutions.

Sum of Roots and **Product of Roots**

If r_1 and r_2 are roots of the quadratic equation $ax^2 + bx + c = 0$, then

$r_1 + r_2 = \text{sum of roots} = -\dfrac{b}{a}$ and $r_1 \cdot r_2 = \text{product of roots} = \dfrac{c}{a}$.

Example 3 □ Find the sum and product of all values x that satisfy $2x^2 - 5x - 1 = 0$.

Solution □ Method 1

Use the quadratic formula to find the roots of the given equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-1)}}{2(2)} = \frac{5 \pm \sqrt{25 + 8}}{4} = \frac{5 \pm \sqrt{33}}{4}$$

The two roots are $x = \dfrac{5 + \sqrt{33}}{4}$ and $x = \dfrac{5 - \sqrt{33}}{4}$

Sum of the roots $= \dfrac{5 + \sqrt{33}}{4} + \dfrac{5 - \sqrt{33}}{4} = \dfrac{10}{4} = \dfrac{5}{2}$

Product of the roots $= (\dfrac{5 + \sqrt{33}}{4})(\dfrac{5 - \sqrt{33}}{4}) = \dfrac{(5 + \sqrt{33})(5 - \sqrt{33})}{16}$

$$= \frac{25 - \cancel{5\sqrt{33}} + \cancel{5\sqrt{33}} - 33}{16} = -\frac{8}{16} = -\frac{1}{2}$$

Method 2

Use the sum and products formula.

$r_1 + r_2 = \text{sum of roots} = -\dfrac{b}{a} = -\dfrac{-5}{2} = \dfrac{5}{2}$

$r_1 \cdot r_2 = \text{product of roots} = \dfrac{c}{a} = \dfrac{-1}{2}$

Exercises - Quadratic Formula and the Discriminant

1

$$(p-1)x^2-2x-(p+1)=0$$

In the quadratic equation above, p is a constant. What are the solutions for x?

A) $\frac{1+\sqrt{2-p^2}}{p-1}$ and $\frac{1-\sqrt{2-p^2}}{p-1}$

B) $\frac{1+2p}{p-1}$ and -1

C) $\frac{p+1}{p-1}$ and -1

D) $\frac{p+1}{p-1}$ and $\frac{2p+1}{p-1}$

2

What is the sum of all values of x that satisfy $3x^2+12x-29=0$?

A) -4

B) -2

C) 2

D) 4

3

If the quadratic equation $kx^2+6x+4=0$ has exactly one solution, what is the value of k?

A) $\frac{3}{2}$

B) $\frac{5}{2}$

C) $\frac{7}{4}$

D) $\frac{9}{4}$

4

$$\begin{cases} y=bx-3 \\ y=ax^2-7x \end{cases}$$

In the system of equations above, a and b are constants. For which of the following values of a and b does the system of equations have exactly two real solutions?

A) $a=3,\ b=-2$

B) $a=5,\ b=0$

C) $a=7,\ b=2$

D) $a=9,\ b=4$

5

What are the solutions to $x^2+4=-6x$?

A) $-3\pm\sqrt{13}$

B) $-3\pm\sqrt{5}$

C) $-6\pm\sqrt{5}$

D) $-6\pm\sqrt{13}$

6

Which of the following equations has no real solution?

A) $5x^2-10x=6$

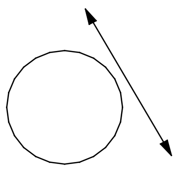
B) $4x^2+8x+4=0$

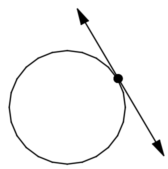
C) $3x^2-5x=-3$

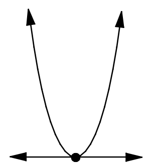
D) $-\frac{1}{3}x^2+2x-2=0$

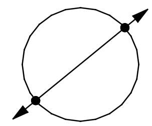
11-6. Solving Systems Consisting Linear and Quadratic Equations

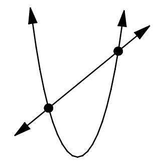
A system containing only quadratic equations or a combination of linear and quadratic equations in the same two variables is called a **quadratic system**. The substitution and elimination methods used to solve linear systems can also be used to solve quadratic systems algebraically. You can use graphs to find the number of real solutions of a quadratic system. If the graphs of a system of equations are a quadratic and a linear, the system will have 0, 1, or 2 solutions. If the graphs of a system of equations are two quadratic equations, the system will have 0, 1, 2, 3, or 4 solutions.

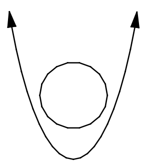

no solution


one solution


one solution


two solutions


two solutions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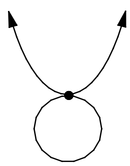
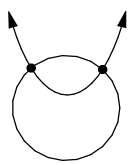
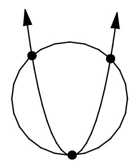
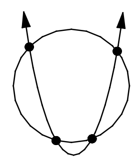

one solution


two solutions


three solutions


four solutions

Example 1 □ Solve the system of equations.

$y = x^2 - 5$

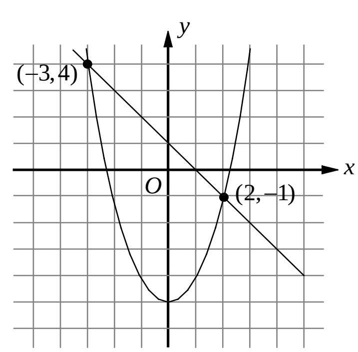
$x + y = 1$

Solution □ Rewrite $x + y = 1$ as $y = 1 - x$.

$y = x^2 - 5$	
$1 - x = x^2 - 5$	Substitute $1 - x$ for y.
$x^2 + x - 6 = 0$	Simplify.
$(x+3)(x-2) = 0$	Factor.
$x + 3 = 0$ or $x - 2 = 0$	Zero Product Property
$x = -3$ or $x = 2$	Solve for x.
$-3 + y = 1$ or $2 + y = 1$	Substitute -3 and 2 for x in $x + y = 1$.
$y = 4$ or $y = -1$	Solve for y.

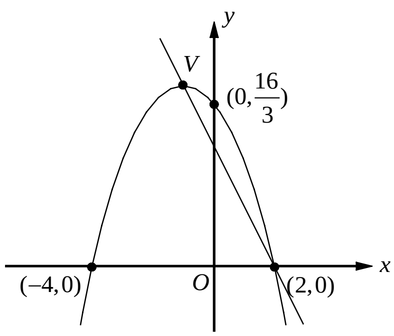
The solutions of the system of equations are $(-3, 4)$ and $(2, -1)$.

The equations are graphed in the diagram at the right. As you can see, the graphs have two points of intersection at $(-3, 4)$ and $(2, -1)$.



Exercises - Solving Systems Consisting Linear and Quadratic Equations

1



The xy-plane above shows two x-intercepts, a y-intercept and vertex V of a parabola. If the line passes through the points $(2,0)$ and V, which of the following must be the y-intercept of the line?

A) 3

B) $\frac{7}{2}$

C) 4

D) $\frac{9}{2}$

2

$$\begin{cases} y = x^2 + x \\ y = ax - 1 \end{cases}$$

In the system of equations above, $a > 0$. If the system of equations has exactly one real solution, what is the value of a?

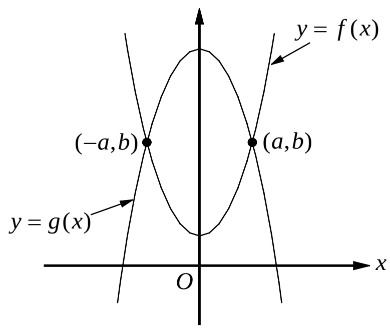
A) $\frac{5}{2}$

B) 3

C) $\frac{7}{2}$

D) 4

3



The function f and g, defined by $f(x) = 2x^2 + 2$ and $g(x) = -2x^2 + 18$, are graphed in the xy-plane above. The two graphs intersect at the points (a,b) and $(-a,b)$. What is the value of b?

A) 6

B) 8

C) 10

D) 12

4

$$\begin{cases} x^2 + y^2 = 14 \\ x^2 - y = 2 \end{cases}$$

If (x, y) is a solution to the system of equations above, what is the value of x^2?

A) 2

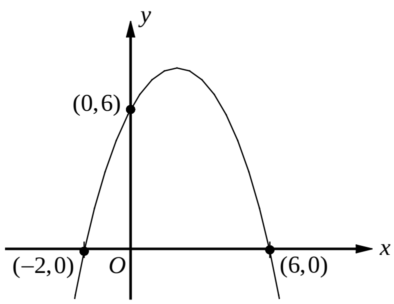
B) 3

C) 4

D) 5

Chapter 11 Practice Test

1



The graph of the quadratic function above shows two x-intercepts and a y-intercept. Which of the following equations represents the graph of the quadratic function above?

A) $y=-\frac{1}{2}(x-1)^2+9$

B) $y=-\frac{1}{2}(x-2)^2+8$

C) $y=-\frac{1}{2}(x-2)^2+9$

D) $y=-\frac{1}{2}(x-3)^2+8$

2

If $(x+y)^2=324$ and $(x-y)^2=16$, what is the value of xy?

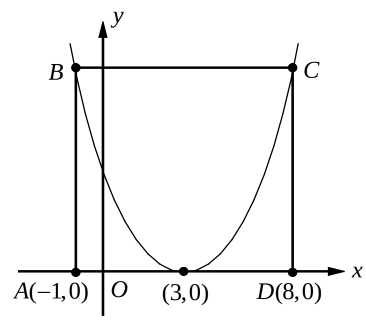
A) 33

B) 55

C) 77

D) 99

3



In the figure above, the vertex of the graph of the quadratic function is at $(3,0)$. The points B and C lie on the parabola. If $ABCD$ is a rectangle with perimeter 38, which of the following represents the equation of the parabola?

A) $y=\frac{2}{5}(x-3)^2$

B) $y=\frac{5}{8}(x-3)^2$

C) $y=\frac{3}{4}(x-3)^2$

D) $y=\frac{7}{8}(x-3)^2$

4

If $(ax+b)(2x-5)=12x^2+kx-10$ for all values of x, what is the value of k?

A) -26

B) -10

C) 24

D) 32

Questions 5-8 refer to the following information.

$$h = -\frac{1}{2}gt^2 + v_0 t + h_0$$

The equation above describes the motion of an object thrown upward into the air. In the equation, g is the acceleration due to gravity (9.8m/s^2), t is the time elapsed since the object was thrown upward, v_0 is the initial speed of the object, h_0 is the initial height from which the object was thrown, and h is the height of the object above the ground t seconds after the object was thrown.

5

Which of the following equations represents the motion of the object, if the object was thrown upward from 40 meters above the ground with an initial speed of 35 meters per second (m/s)?

A) $h = -9.8t^2 + 40t + 35$

B) $h = -9.8t^2 + 35t + 40$

C) $h = -4.9t^2 + 40t + 35$

D) $h = -4.9t^2 + 35t + 40$

6

How many seconds will it take the object to reach its maximum height? (hint: The function has a maximum point at $t = -\frac{b}{2a}$.)

A) $\frac{15}{7}$

B) $\frac{20}{7}$

C) $\frac{25}{7}$

D) $\frac{30}{7}$

7

What is the maximum height from the ground the object will reach, to the nearest meter?

A) 103

B) 112

C) 125

D) 133

8

How long will it take the object to hit the ground, to the nearest second? (hint: Height of the object is zero when the object hits the ground.)

A) 7

B) 8

C) 9

D) 10

9

$$h = -16t^2 + h_0$$

The equation above describes the height of an object t seconds after it dropped from a height of h_0 feet above the ground. If a hiker dropped a water bottle from a cliff 150 feet above the ground, how many seconds will it take to hit the ground? (Round your answer to the nearest second.)

A) 2

B) 3

C) 4

D) 5

Answer Key

Section 11-1

1. D	2. A	3. $\frac{9}{16}$	4. 2	5. $\frac{4}{3}$

Section 11-2

1. C	2. B	3. A	4. A	5. C
6. D				

Section 11-3

1. A	2. D	3. B	4. D	5. C
6. B				

Section 11-4

1. D	2. A	3. C	4. D	5. 11
6. $\frac{9}{16}$				

Section 11-5

1. C	2. A	3. D	4. D	5. B
6. C				

Section 11-6

1. C	2. B	3. C	4. D

Chapter 11 Practice Test

1. B	2. C	3. A	4. A	5. D
6. C	7. A	8. B	9. B	

Answers and Explanations

Section 11-1

1. D

Change the given equation into the vertex form $y = a(x-h)^2 + k$, in which (h,k) is the vertex of the parabola, by completing the square.

$y = x^2 - 6x + 5$

$= x^2 - 6x + (\frac{-6}{2})^2 - (\frac{-6}{2})^2 + 5$

$= (x^2 - 6x + 9) - 9 + 5$

$= (x-3)^2 - 4$

The coordinate of the vertex can be read as $(3,-4)$.

2. A

Change the given equation into the factored form $y = (x-a)(x-b)$, in which $x = a$ and $x = b$ are the x-intercepts of the parabola. Find two numbers with a sum of -6 and a product of 5. The two numbers are -1 and -5.

$y = x^2 - 6x + 5$ can be written in the factored form $y = (x-1)(x-5)$. The x-intercepts are 1 and 5.

3. $\frac{9}{16}$

$y = a(x-h)^2$

$0 = a(4-h)^2$ $\quad$ x-intercept at $(4,0)$

Since $a \neq 0$, $4 - h = 0$, or $h = 4$.

The graph of the parabola passes through $(0,9)$, since the y-intercept of the parabola is 9.

$9 = a(0-h)^2$ $\quad$ y-intercept at $(0,9)$

$9 = ah^2$ $\quad$ Simplify.

$9 = a(4)^2$ $\quad$ Substitute 4 for h.

$\frac{9}{16} = a$

4. 2

$y = a(x+2)^2 - 15$

$3 = a(1+2)^2 - 15$ $\quad$ $x = 1$ and $y = 3$

$3 = 9a - 15$

$18 = 9a$

$2 = a$

5. $\frac{4}{3}$

The x-intercepts of the graph of the equation $y = a(x-1)(x+5)$ are -5 and 1. The x-coordinate of the vertex is the average of the two x-intercepts.

Therefore, $h = \frac{-5+1}{2} = -2$. The value of k is -12 because the minimum value of y is -12. So the coordinate of the vertex is $(-2,-12)$. Substitute $x = -2$ and $y = -12$ in the given equation.

$-12 = a(-2-1)(-2+5)$

$-12 = -9a$

$\frac{12}{9} = a$ or $a = \frac{4}{3}$

Section 11-2

1. C

$x^2 - 2x - 24$

Find two numbers with a sum of -2 and a product of -24. The two numbers are -6 and 4.
Therefore, $x^2 - 2x - 24 = (x-6)(x+4)$.

2. B

$x^2 - 17x + 72$

Find two numbers with a sum of -17 and a product of 72. The two numbers are -8 and -9.
Therefore, $x^2 - 17x + 72 = (x-8)(x-9)$.

3. A

$-x^2 + 5x + 84 = -(x^2 - 5x - 84)$

Find two numbers with a sum of -5 and a product of -84. The two numbers are -12 and 7.

$-x^2 + 5x + 84 = -(x^2 - 5x - 84)$
$= -(x-12)(x+7) = (12-x)(x+7)$

4. A

$3x^2 + 7x - 6$

Find two numbers with a sum of 7 and a product of $3 \cdot -6$ or -18. The two numbers are -2 and 9.

$3x^2 + 7x - 6$	
$= 3x^2 - 2x + 9x - 6$	Write $7x$ as $-2x + 9x$.
$= (3x^2 - 2x) + (9x - 6)$	Group terms.
$= x(3x-2) + 3(3x-2)$	Factor out the GCF.
$= (3x-2)(x+3)$	Distributive Property

5. C

$2x^2 + x - 15$

Find two numbers with a sum of 1 and a product of $2 \cdot -15$ or -30. The two numbers are -5 and 6.

$2x^2 + x - 15$	
$= 2x^2 - 5x + 6x - 15$	Write x as $-5x + 6x$.
$= (2x^2 - 5x) + (6x - 15)$	Group terms.
$= x(2x-5) + 3(2x-5)$	Factor out the GCF.
$= (2x-5)(x+3)$	Distributive Property

6. D

$-6x^2 + x + 2 = -(6x^2 - x - 2)$

Find two numbers with a sum of -1 and a product of $6 \cdot -2$ or -12. The two numbers are -4 and 3.

$-6x^2 + x + 2$	
$= -(6x^2 - x - 2)$	
$= -(6x^2 - 4x + 3x - 2)$	Write $-x$ as $-4x + 3x$.
$= -[(6x^2 - 4x) + (3x - 2)]$	Group terms.
$= -[2x(3x-2) + (3x-2)]$	Factor out the GCF.
$= -(3x-2)(2x+1)$	Distributive Property

Section 11-3

1. A

$3x^2 - 48$	
$= 3(x^2 - 16)$	Factor out the GCF.
$= 3((x)^2 - (4)^2)$	Write in the form $a^2 - b^2$.
$= 3(x-4)(x+4)$	Difference of Squares

2. D

$x - 6\sqrt{x} - 16$
Let $y = \sqrt{x}$, then $y^2 = x$.

$x - 6\sqrt{x} - 16$	
$= y^2 - 6y - 16$	$y = \sqrt{x}$ and $y^2 = x$
$= (y-8)(y+2)$	
$= (\sqrt{x} - 8)(\sqrt{x} + 2)$	$y = \sqrt{x}$ and $y^2 = x$

3. B

$(x-y)^2$	
$= (x-y)(x-y)$	
$= x^2 - 2xy + y^2$	
$= (x^2 + y^2) - 2xy$	
$= 10 - 2(-3) = 16$	$x^2 + y^2 = 10$ and $xy = -3$

4. D

$x^2 - y^2$	
$= (x+y)(x-y)$	
$= (10)(4)$	
$= 40$	$x + y = 10$ and $x - y = 4$

5. C

$6x^2 + 7x - 24 = 0$

$(3x+8)(2x-3) = 0$	Factor.
$3x+8=0$ or $2x-3=0$	Zero Product Property
$x = -\frac{8}{3}$ or $x = \frac{3}{2}$	Solve each equation.

Since $\frac{3}{2} > -\frac{8}{3}$, $r = \frac{3}{2}$ and $s = -\frac{8}{3}$.

$r - s = \frac{3}{2} - (-\frac{8}{3}) = \frac{9}{6} + \frac{16}{6} = \frac{25}{6}$

6. B

$x^2 - 3x = 28$

$x^2 - 3x - 28 = 0$	Make one side 0.
$(x-7)(x+4) = 0$	Factor.
$x-7=0$ or $x+4=0$	Zero Product Property
$x=7$ or $x=-4$	Solve each equation.

Therefore, $r + s = 7 + (-4) = 3$.

Section 11-4

1. D

$x^2 - 10x = 75$

Add $(\frac{-10}{2})^2$ to each side.

$x^2 - 10x + (-\frac{10}{2})^2 = 75 + (-\frac{10}{2})^2$

$x^2 - 10x + 25 = 75 + 25$	Simplify.
$(x-5)^2 = 100$	Factor $x^2 - 10x + 25$.
$x - 5 = \pm 10$	Take the square root.
$x = 5 \pm 10$	Add 5 to each side.
$x = 5+10$ or $x = 5-10$	Separate the solutions.
$x = 15$ or $x = -5$	Simplify.

If $x < 0$, $x = -5$. Therefore, $x + 5 = -5 + 5 = 0$.

2. A

$x^2 - kx = 20$

Add $(\frac{-k}{2})^2$ to each side.

$x^2 - kx + (\frac{-k}{2})^2 = 20 + (\frac{-k}{2})^2$

$x^2 - kx + \frac{k^2}{4} = 20 + \frac{k^2}{4}$	Simplify.
$(x - \frac{k}{2})^2 = 20 + \frac{k^2}{4}$	Factor $x^2 - kx + \frac{k^2}{4}$.
$(6)^2 = 20 + \frac{k^2}{4}$	Substitute 6 for $x - \frac{k}{2}$.

$16 = \frac{k^2}{4}$

Solving for k gives $k = \pm 8$.

Solving the given equation $x - \frac{k}{2} = 6$ for x

gives $x = 6 + \frac{k}{2}$.

If $k = 8$, $x = 6 + \frac{k}{2} = 6 + \frac{8}{2} = 10$.

If $k = -8$, $x = 6 + \frac{k}{2} = 6 + \frac{-8}{2} = 2$.

Of the answer choices, 2 is a possible value of x. Therefore, Choice A is correct.

3. C

$x^2 - \frac{k}{3}x = 5$

The equation could be solved by completing the square by adding $(\frac{1}{2} \cdot \frac{k}{3})^2$, or $\frac{k}{36}^2$, to each side.

Choice C is correct.

4. D

$x^2 - rx = \frac{k^2}{4}$

Add $(\frac{-r}{2})^2$, or $\frac{r^2}{4}$, to each side.

$x^2 - rx + \frac{r^2}{4} = \frac{k^2}{4} + \frac{r^2}{4}$

$(x - \frac{r}{2})^2 = \frac{k^2 + r^2}{4}$	Factor $x^2 - rx + \frac{r^2}{4}$.
$x - \frac{r}{2} = \pm\sqrt{\frac{k^2 + r^2}{4}}$	Take the square root.
$x - \frac{r}{2} = \pm\frac{\sqrt{k^2 + r^2}}{2}$	Simplify.
$x = \frac{r}{2} \pm \frac{\sqrt{k^2 + r^2}}{2}$	Add $\frac{r}{2}$ to each side.

Choice D is correct.

5. 11

$(x-7)(x-s) = x^2 - rx + 14$

$x^2 - (s+7)x + 7s = x^2 - rx + 14$

Since the x-terms and constant terms have to be equal on both sides of the equation,

$r = s + 7$ and $7s = 14$.
Solving for s gives $s = 2$.
$r = s + 7 = 2 + 7 = 9$
Therefore, $r + s = 9 + 2 = 11$.

6. $\dfrac{9}{16}$

$x^2 - \dfrac{3}{2}x + c = (x - k)^2 \;\Rightarrow$

$x^2 - \dfrac{3}{2}x + c = x^2 - 2kx + k^2$

Since the x-terms and constant terms have to be equal on both sides of the equation,

$2k = \dfrac{3}{2}$ and $c = k^2$.

Solving for k gives $k = \dfrac{3}{4}$.

Therefore, $c = k^2 = (\dfrac{3}{4})^2 = \dfrac{9}{16}$.

Section 11-5

1. C

$(p-1)x^2 - 2x - (p+1) = 0$

Use the quadratic formula to find the solutions for x.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(p-1)(-(p+1))}}{2(p-1)}$$
$$= \frac{2 \pm \sqrt{4 + 4(p-1)(p+1)}}{2(p-1)}$$
$$= \frac{2 \pm \sqrt{4 + 4p^2 - 4}}{2(p-1)}$$
$$= \frac{2 \pm \sqrt{4p^2}}{2(p-1)} = \frac{2 \pm 2p}{2(p-1)}$$
$$= \frac{2(1 \pm p)}{2(p-1)} = \frac{1 \pm p}{p-1}$$

The solutions are $\dfrac{1+p}{p-1}$ and $\dfrac{1-p}{p-1}$, or -1.

Choice C is correct.

2. A

Let r_1 and r_2 be the solutions of the quadratic equation $3x^2 + 12x - 29 = 0$.
Use the sum of roots formula.

$r_1 + r_2 = -\dfrac{b}{a} = -\dfrac{12}{3} = -4$.

3. D

$kx^2 + 6x + 4 = 0$

If the quadratic equation has exactly one solution, then $b^2 - 4ac = 0$.

$b^2 - 4ac = 6^2 - 4(k)(4) = 0 \;\Rightarrow\; 36 - 16k = 0$

$\Rightarrow\; k = \dfrac{36}{16} = \dfrac{9}{4}$

4. D

$y = bx - 3$ and $y = ax^2 - 7x$

Substitute $bx - 3$ for y in the quadratic equation.

$bx - 3 = ax^2 - 7x$

$ax^2 + (-7 - b)x + 3 = 0$ Make one side 0.

The system of equations will have exactly two real solutions if the discriminant of the quadratic equation is positive.

$(-7 - b)^2 - 4a(3) > 0$, or $(7 + b)^2 - 12a > 0$.

We need to check each answer choice to find out for which values of a and b the system of equations has exactly two real solutions.

A) If $a = 3$ and $b = -2$, $(7 - 2)^2 - 12(3) < 0$.

B) If $a = 5$ and $b = 0$, $(7 + 0)^2 - 12(5) < 0$.

C) If $a = 7$ and $b = 2$, $(7 + 2)^2 - 12(7) < 0$.

D) If $a = 9$ and $b = 4$, $(7 + 4)^2 - 12(9) > 0$.

Choice Dis correct.

5. B

$x^2 + 4 = -6x$

$x^2 + 6x - 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$$
$$= \frac{-6 \pm \sqrt{20}}{2} = \frac{-6 \pm 2\sqrt{5}}{2}$$
$$= -3 \pm \sqrt{5}$$

6. C

If the quadratic equation has no real solution, the discriminant, $b^2 - 4ac$, must be negative. Check each answer choice.

A) $5x^2 - 10x = 6 \Rightarrow 5x^2 - 10x - 6 = 0$
$b^2 - 4ac = (-10)^2 - 4(5)(-6) > 0$

B) $4x^2 + 8x + 4 = 0$
$b^2 - 4ac = (8)^2 - 4(4)(4) = 0$

C) $3x^2 - 5x = -3 \Rightarrow 3x^2 - 5x + 3 = 0$
$b^2 - 4ac = (-5)^2 - 4(3)(3) < 0$

Choice C is correct.

Section 11-6

1. C

Since the two x-intercepts are -4 and 2, the equation of the parabola can be written as $y = a(x+4)(x-2)$. Substitute $x = 0$ and $y = \frac{16}{3}$ in the equation, since the graph of the parabola passes through $(0, \frac{16}{3})$.

$\frac{16}{3} = a(0+4)(0-2)$

Solving the equation for a gives $a = -\frac{2}{3}$.

Thus the equation of the parabola is $y = -\frac{2}{3}(x+4)(x-2)$.

The x-coordinate of the vertex is the average of the two x-intercepts: $\frac{-4+2}{2}$, or -1.

The y-coordinate of the vertex can be found by substituting -1 for x in the equation of the parabola: $y = -\frac{2}{3}(-1+4)(-1-2) = 6$.

The line passes through $(2,0)$ and $(-1,6)$.

The slope of the line is $\frac{6-0}{-1-2} = -2$. The equation of the line in point-slope form is $y - 0 = -2(x-2)$. To find the y-intercept of the line, substitute 0 for x. $y = -2(0-2) = 4$

Choice C is correct.

2. B

$y = x^2 + x$ and $y = ax - 1$
Substitute $ax - 1$ for y in the quadratic equation.

$ax - 1 = x^2 + x$
$x^2 + (-a+1)x + 1 = 0$ Make one side 0.

If the system of equations has exactly one real solution, the discriminant $b^2 - 4ac$ must be equal to 0.

$(-a+1)^2 - 4(1)(1) = 0$	$b^2 - 4ac = 0$
$a^2 - 2a + 1 - 4 = 0$	Simplify.
$a^2 - 2a - 3 = 0$	Simplify.
$(a-3)(a+1) = 0$	Factor.
$a = 3$ or $a = -1$	Solutions

Since $a > 0$, $a = 3$.

3. C

One can find the intersection points of the two graphs by setting the two functions $f(x)$ and $g(x)$ equal to one another and then solving for x. This yields $2x^2 + 2 = -2x^2 + 18$. Adding $2x^2 - 2$ to each side of the equation gives $4x^2 = 16$. Solving for x gives $x = \pm 2$.
$f(2) = 2(2)^2 + 2 = 10$ and also $f(-2) = 10$.
The two point of intersections are $(2,10)$ and $(-2,10)$. Therefore, the value of b is 10.

4. D

$x^2 + y^2 = 14$	First equation
$x^2 - y = 2$	Second equation
$x^2 = y + 2$	Second equation solved for x^2.
$y + 2 + y^2 = 14$	Substitute $y + 2$ for x^2 in first equation.
$y^2 + y - 12 = 0$	Make one side 0.
$(y+4)(y-3) = 0$	Factor.
$y = -4$ or $y = 3$	Solve for y.

Substitute -4 and 3 for y and solve for x^2.

$x^2 = y + 2 = -4 + 2 = -2$.

Since x^2 cannot be negative, $y = -4$ is not a solution.

$x^2 = y + 2 = 3 + 2 = 5$

The value of x^2 is 5.

Chapter 11 Practice Test

1. B

The x-coordinate of the vertex is the average of the x-intercepts. Thus the x-coordinate of the vertex is $x = \frac{-2+6}{2} = 2$. The vertex form of the parabola can be written as $y = a(x-2)^2 + k$.
Choices A and D are incorrect because the x-coordinate of the vertex is not 2.
Also, the parabola passes through $(0,6)$.
Check choices B and C.

B) $y = -\frac{1}{2}(x-2)^2 + 8$

$6 = -\frac{1}{2}(0-2)^2 + 8$ Correct.

C) $y = -\frac{1}{2}(x-2)^2 + 9$

$6 = -\frac{1}{2}(0-2)^2 + 9$ Not correct.

Choice B is correct.

2. C

$(x+y)^2 = 324 \Rightarrow x^2 + 2xy + y^2 = 324$
$x^2 + y^2 = 324 - 2xy$
$(x-y)^2 = 16 \Rightarrow x^2 - 2xy + y^2 = 16$
$\Rightarrow x^2 + y^2 = 16 + 2xy$
Substituting $16 + 2xy$ for $x^2 + y^2$ in the equation $x^2 + y^2 = 324 - 2xy$ yields
$16 + 2xy = 324 - 2xy$.
Solving this equation for xy yields $xy = 77$.

3. A

From the graph we read the length of AD, which is 9. Let the length of $CD = w$.
Perimeter of rectangle $ABCD$ is 38.
$2 \cdot 9 + 2w = 38 \Rightarrow 2w = 20 \Rightarrow w = 10$
Therefore, the coordinates of B are $(-1,10)$ and the coordinates of C are $(8,10)$.
The equation of the parabola can be written in vertex form as $y = a(x-3)^2$.
Now substitute 8 for x and 10 for y in the equation. $10 = a(8-3)^2$. Solving for a gives $a = \frac{10}{25} = \frac{2}{5}$. Choice A is correct.

4. A

$(ax+b)(2x-5) = 12x^2 + kx - 10$

FOIL the left side of the equation.
$2ax^2 + (-5a+2b)x - 5b = 12x^2 + kx - 10$

By the definition of equal polynomials, $2a = 12$, $-5a + 2b = k$, and $5b = 10$. Thus, $a = 6$ and $b = 2$, and $k = -5a + 2b = -5(6) + 2(2) = -26$.

5. D

$h = -\frac{1}{2}gt^2 + v_0 t + h_0$
In the equation, $g = 9.8$, initial height $h_0 = 40$, and initial speed $v_0 = 35$. Therefore, the equation of the motion is $h = -\frac{1}{2}(9.8)t^2 + 35t + 40$.
Choice D is correct.

6. C

In the quadratic equation, $y = ax^2 + bx + c$, the x-coordinate of the maximum or minimum point is at $x = -\frac{b}{2a}$.
Therefore, the object reaches its maximum height when $t = -\frac{35}{2(-4.9)} = \frac{25}{7}$.

7. A

The object reaches to its maximum height when $t = \frac{25}{7}$. So substitute $t = \frac{25}{7}$ in the equation.

$h = -4.9(\frac{25}{7})^2 + 35(\frac{25}{7}) + 40 = 102.5$

To the nearest meter, the object reaches a maximum height of 103 meters.

8. B

Height of the object is zero when the object hits the ground.

$0 = -4.9t^2 + 35t + 40$

Use quadratic formula to solve for t.

$$t = \frac{-35 \pm \sqrt{35^2 - 4(-4.9)(40)}}{2(-4.9)}$$

$$= \frac{-35 \pm \sqrt{2009}}{-9.8} \approx \frac{-35 \pm 44.82}{-9.8}$$

Solving for t gives $t \approx -1$ or $t \approx 8.1$.
Since time cannot be negative, the object hits the ground about 8 seconds after it was thrown.

9. B

When an object hits the ground, $h = 0$.
$h_0 = 150$ is given.

$0 = -16t^2 + 150$	Substitution
$16t^2 = 150$	Add $16t^2$ to each side.
$t^2 = \frac{150}{16}$	Divide each side by 16.
$t = \sqrt{\frac{150}{16}} \approx 3.06$	