

Ex 17.1

Q1(i)

$${}^{14}C_3$$

$$= \frac{14!}{3!(14-3)!}$$

$$\left(\because {}^nC_r = \frac{n!}{r!(n-r)!} \right)$$

$$= \frac{14!}{3!11!}$$

$$= \frac{14 \times 13 \times 12 \times 11!}{3 \times 2 \times 1 \times 11!}$$

$$= \frac{14 \times 13 \times 12}{6}$$

$$= 364$$

Q1(ii)

$${}^{12}C_{10}$$

$$= \frac{12!}{10!(12-10)!}$$

$$\left(\because {}^nC_r = \frac{n!}{r!(n-r)!} \right)$$

$$= \frac{12 \times 11 \times 10!}{10! \times 2 \times 1}$$

$$= 66$$

Q1(iii)

$${}^{35}C_{35}$$

$$= \frac{35!}{35!(35-35)!}$$

$$\left(\because {}^nC_r = \frac{n!}{r!(n-r)!} \right)$$

$$= 1$$

Q1(iv)

$$\begin{aligned} & {}^{n+1}C_r \\ &= \frac{(r+1)!}{(n!) (n+1-r)!} \quad \left(\because {}^nC_r = \frac{n!}{r!(n-r)!} \right) \\ &= \frac{(n+1) \times n!}{n! \times 1!} \\ &= n+1 \end{aligned}$$

Q1(v)

$$\begin{aligned} & \sum_{r=1}^5 {}^5C_r \\ &= {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 \\ &= \frac{5!}{1!4!} + \frac{5!}{2!3!} + \frac{5!}{3!2!} + \frac{5!}{4!1!} + \frac{5!}{5!0!} \quad \left(\because {}^nC_r = \frac{n!}{r!(n-r)!} \right) \\ &= 5 + \frac{5 \times 4}{2} + \frac{5 \times 4}{2} + 5 + 1 \\ &= 5 + 10 + 10 + 5 + 1 \\ &= 31 \end{aligned}$$

Q2

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

$$\text{Hence } n = n$$

$$r = 12 \text{ and } 5$$

Applying formula

$${}^nC_P = {}^nC_Q = n$$

$$\text{Then } P + Q = n$$

$$\rightarrow {}^nC_{12} = {}^nC_5 \\ 12 + 5 = n$$

$$\Rightarrow n = 17$$

Q3

$${}^{12}C_p = {}^nC_q$$

$$\text{Then } p + q = n$$

$$\text{Also } {}^nC_r = \frac{n!}{r!(n-r)!} \dots\dots\dots (i)$$

$$\rightarrow \quad {}^nC_4 = {}^nC_6 \\ 4 + 6 = n$$

$$\Rightarrow \quad n = 10$$

$$\text{then } {}^{12}C_n = {}^{12}C_{10}$$

Applying (i)

$$\begin{aligned} {}^{12}C_{10} &= \frac{12!}{10! 2!} \\ &= \frac{12 \times 11 \times 10!}{10! \times 2 \times 1} \\ &= \frac{12 \times 11}{2 \times 1} = 66 \end{aligned}$$

Q4

$$\text{If } {}^nC_p = {}^nC_q$$

$$\text{Then } p + q = n$$

$$\Rightarrow \quad {}^nC_{10} = {}^nC_{12} \\ 10 + 12 = n$$

$$\Rightarrow \quad n = 22$$

$$\text{Find } {}^{23}C_p$$

$$\begin{aligned} \rightarrow \quad & {}^{23}C_{22} \\ &= \frac{23!}{22! 1!} \\ &= \frac{23 \times 22!}{22!} \\ &= 23 \end{aligned}$$

Q5

If ${}^nC_p = {}^nC_r$ then $p + r = n$

$$\therefore x + 2x + 3 = 24$$

$$3x = 21$$

$$x = 7$$

Q6

If ${}^nC_p = {}^nC_q$

$$\Rightarrow p + q = n$$

also $C_x = {}^{18}C_{x+2}$

$$\Rightarrow x + x + 2 = 18$$

$$2x + 2 = 18$$

$$2x = 18 - 2 = 16$$

$$2x = 16$$

$$x = 8$$

Q7

If ${}^nC_p = {}^nC_q$

Then $p + q = n$

$$\Rightarrow {}^{15}C_{3r} = {}^{15}C_{r+3}$$

$$\Rightarrow 3r + r + 3 = 15$$

$$4r + 3 = 15$$

$$4r = 15 - 3 = 12$$

$$r = 3$$

Q8

$${}^8C_r = {}^7C_2 + {}^7C_3$$

Applying formula ${}^nC_r = \frac{n!}{r!(n-r)!}$

$$\frac{8!}{r!(8-r)!} = \frac{7!}{2!5!} + \frac{7!}{3!4!}$$

$$\frac{8 \times 7!}{r!(8-r)!} = \frac{7!}{2 \times 5 \times 4!} + \frac{7!}{3 \times 2 \times 4!}$$

$$\frac{8 \times 7!}{r!(8-r)!} = \frac{7!}{2 \times 4!} \left(\frac{1}{5} + \frac{1}{3} \right)$$

Cancelling 7! from both sides

$$\frac{8}{r!(8-r)!} = \frac{8}{2 \times 15 \times 4!}$$

Cancelling 8 on both sides

$$2 \times 5 \times 3 \times 4 \times 3 \times 2 \times 1 = r!(8-r)!$$

$$(3 \times 2)(5 \times 4 \times 3 \times 2 \times 1) = r!(8-r)!$$

$$\Rightarrow r! = 3!$$

$$r = 3$$

or $r! = 5!$

$$r = 5$$

Q9

$$\frac{\frac{15!}{(15-r)! r!}}{\frac{15!}{(15-r+1)!(r-1)!}} = \frac{11}{5}$$

$$\frac{\frac{15!}{(15-r)(16-r)! r(r-1)!}}{\frac{15!}{(16-r)!(r-1)!}} = \frac{11}{5}$$

$$\Rightarrow \frac{16-r}{r} = \frac{11}{5}$$

$$80 - 5r = 11r$$

$$80 = 16r$$

$$r = \frac{80}{16}$$

$$= 5$$

$$r = 5$$

Q10

$${}^{n+2}C_8 \cdot {}^{n-2}P_4 = 57 : 16$$

$$\frac{(n+2)!}{8!(n-6)!} = \frac{57}{16}$$

$$\frac{(n-2)!}{(n-6)!}$$

$$\Rightarrow \frac{(n+2)(n+1)(n)(n-1)(n-2)!}{8!(n-2)!} = \frac{57}{16}$$

Cancelling $(n-2)!$ from numerator and denominator

$$\Rightarrow (n+2)(n+1)(n)(n-1) = \frac{57 \times 7 \times 6 \times 5 \times 4 \times 3 \times 1 \times 16}{16}$$

$$\Rightarrow (n+2)(n+1)(n)(n-1) = 21 \times 20 \times 19 \times 10$$

comparing both sides $n = 19$

Q11

$$\begin{aligned}
 & \frac{\frac{20!}{(2r)!(20-2r)!}}{\frac{24}{(2r-4)!(24-(2r-4))!}} = \frac{225}{11} \\
 \Rightarrow & \frac{28 \times 27 \times 26 \times 25 \times 24! (2r-4)!(28-2r)!}{(2r)!(28-2r)! \cdot 24!} = \frac{225}{11} \\
 \Rightarrow & \frac{28 \times 27 \times 26 \times 25}{2r \times (2r-1) \times (2r-2) \times (2r-3)} = \frac{225}{11} \\
 \Rightarrow & \frac{28 \times 27 \times 26 \times 25 \times 11}{15 \times 15} = 2r(2r-1)(2r-2)(2r-3) \\
 \Rightarrow & 11 \times 11 \times 13 \times 14 = 2r(2r-1)(2r-2)(2r-3)
 \end{aligned}$$

Comparing both sides $r = 7$

Q12

$$\begin{aligned}
 & \frac{\frac{4n!}{(2n)!(2n)!}}{\frac{2n!}{n! \cdot n!}} \left(\therefore {}^nC_r = \frac{n!}{r!(n-r)!} \right) \\
 & = \frac{(4n)!}{(2n)!(2n)!} \times \frac{(n!)^2}{(n!)^2} \\
 & = \frac{[1 \cdot 2 \cdot 3 \cdot 4 \dots (4n-1)(4n)](n!)^2}{(2n)! [1 \cdot 2 \cdot 3 \cdot 4 \dots (2n-2)(2n-1)(2n)]^2} \\
 & = \frac{[1 \cdot 3 \cdot 5 \dots (4n-1)] \times [2 \cdot 4 \cdot 6 \dots 4n] \times (n!)^2}{(2n)! [1 \cdot 3 \cdot 5 \dots (2n-1)]^2 \times [2 \cdot 4 \cdot 6 \dots (2n-2)(2n)]^2} \\
 & = \frac{[1 \cdot 3 \cdot 5 \dots (4n-1)] \times 2^{2n} \times [1 \cdot 2 \cdot 3 \dots 2n] \times n!^2}{(2n)! \times [1 \cdot 3 \cdot 5 \dots (2n-1)]^2 \times 2^{2n} \times n!^2} \\
 & = \frac{[1 \cdot 3 \cdot 5 \dots (4n-1)]}{[1 \cdot 3 \cdot 5 \dots (2n-1)]^2}
 \end{aligned}$$

Hence Proved

Q13

$$\frac{\frac{2n!}{3!(2n-3)!}}{\frac{n!}{2!(n-2)!}} = \frac{44}{3}$$

$$\Rightarrow \frac{2n! 2!(n-2)!}{3!(2n-3)!n!} = \frac{44}{3}$$

$$\Rightarrow \frac{2n!}{3n!(n-1)(2n-3)!} = \frac{44}{3}$$

$$\Rightarrow 2n(2n-1)(2n-2) = 44n(n-1)$$

$$\Rightarrow (2n-1)(n-1) = 11(n-1)$$

Q $n = 6$

$$\therefore n = 6$$

Q14

If ${}^nC_r = {}^nC_p$
then $r + p = n$

$$\therefore 16 = r + r + 2$$

$r = 7$

then ${}^rC_4 = {}^7C_4$ ($\because r = 7$)

$$\Rightarrow \frac{7!}{4!(7-4)!} \quad \left(\because {}^nC_r = \frac{n!}{r!(n-r)!} \right)$$

$$\Rightarrow \frac{7 \times 5 \times 6}{3 \times 2}$$

$$= 35$$

Q15

$${}^{20}C_5 + \sum_{r=2}^5 {}^{25-r}C_4$$

$$\Rightarrow ({}^{20}C_5 + {}^{20}C_4) + {}^{21}C_4 + {}^{22}C_4 + {}^{23}C_4$$

$$\Rightarrow ({}^{21}C_5 + {}^{21}C_4) + {}^{22}C_4 + {}^{23}C_4 \quad \left(\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r \right)$$

$$\Rightarrow ({}^{22}C_5 + {}^{22}C_4) + {}^{23}C_4 \quad \left(\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r \right)$$

$$\Rightarrow {}^{23}C_5 + {}^{23}C_4 \quad \left(\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r \right)$$

$$\Rightarrow {}^{24}C_5$$

$$\Rightarrow 42504$$

Q16

$$\text{Product} = [(2n+1)(2n+3)(2n+5)\dots(2n+r)]$$

$$= \frac{(2n)! [(2n+1)(2n+3)\dots(2n+r)]}{(2n)!}$$

$$= \frac{(2n)[(2n-1)(2n-2)\dots 4 \cdot 2(2n+1)(2n+3)]}{(2n)!}$$

$$= \frac{(2n+r)!}{(2n)!}$$

$$\text{Hence } r = 2n$$

$$= \frac{(2n+2n)!}{2n}$$

$$= \frac{(4n)!}{(2n)!}$$

$$= (2n)!$$

Q17

$$\text{L.H.S,} = {}^{2n}C_n + {}^{2n}C_{n-1}$$

$$\frac{2n!}{n!n!} + \frac{2n!}{(n-1)!(n-1)!}$$

$$= (2n)! \left[\frac{1}{n(n-1)!(n-1)!} + \frac{1}{(n-1)!(n-1)!} \right]$$

$$= \frac{(2n)!}{(n-1)!(n-1)!} \left[\frac{1+n^2}{n^2} \right] \dots\dots\dots (i)$$

$${}^{2n+2}C_{n+1} = \frac{(2n+2)!}{(n+1)!(n+1)!}$$

$$= \frac{(2n+2)(2n+1)(2n)!}{n(n+1)(n-1)!(n+1)n(n-1)!} \dots\dots\dots (ii)$$

$$\Rightarrow \frac{(2n)!}{(n-1)!(n-1)!} \times \frac{(n+1)^2(n-1)!(n-1)!}{(2n+2)(2n+1)(2n)!} \times \left(\frac{1+n^2}{n^2} \right)$$

$$= \frac{(n+1)^2(1+n^2)}{(2n+2)(2n+1)} = \frac{(n+1)(n+1)(n^2+1)}{2(n+1)(2n+1)}$$

$$= \frac{(n+1)(n^2+1)}{(2n+1)} \times \frac{1}{2}$$

Q18

nC_4 , nC_5 , and nC_6 are in A.P

$$\therefore {}^nC_5 - {}^nC_4 = {}^nC_6 - {}^nC_5$$

$$\frac{n!}{5!(n-5)!} - \frac{n!}{4!(n-4)!} = \frac{n!}{6!(n-6)!} - \frac{n!}{6!(n-5)!}$$

$$\Rightarrow \frac{n!}{4!(n-5)!} \left[\frac{1}{5} - \frac{1}{n-4} \right] = \frac{n!}{5!(n-6)!} \left[\frac{1}{6} - \frac{1}{n-5} \right]$$

$$\Rightarrow \frac{1}{n-5} \left[\frac{n-4-5}{5(n-4)} \right] = \frac{1}{5} \left[\frac{n-5-6}{6(n-5)} \right]$$

$$\Rightarrow \frac{n-9}{n-4} = \frac{n-11}{6}$$

$$\Rightarrow 6n - 54 = n^2 - 15n + 44$$

$$\Rightarrow n^2 - 21n + 98 = 0$$

$$n = 7, 14$$

n is 7 or 14

Q19

We have $\alpha = {}^m C_2 = \frac{m(m-1)}{2} \left(\because {}^n C_r = \frac{n!}{r!(n-r)!} \right)$

$$\begin{aligned} \text{Now } {}^\alpha C_2 &= \frac{\alpha(\alpha-1)}{2} \\ &= \frac{\left(\frac{m(m-1)}{2} \right) \left(\frac{m(m-1)}{2} - 1 \right)}{2} \\ &= \frac{m(m-1)(m^2-m-2)}{2 \times 2 \times 2} = \frac{m(m-1)(m+1)(m-2)}{8} \\ &= \frac{m(m-1)(m+1)(m-2)}{4 \times 2} \end{aligned}$$

multiplying with 3, numerator and denominator to make 4:

$$\begin{aligned} \text{Or } &= \frac{m(m+1)m(m-1)(m-2)}{4 \cdot 3 \cdot 2 \cdot 1} \\ &= \frac{3(m+1)m(m-1)(m-2)}{4!} \\ &= 3 \cdot {}^{m+1}C_4 \quad \left(\because {}^n C_r = \frac{n!}{r!(n-r)!} \right) \end{aligned}$$

Q20(i)

$$\begin{aligned} {}^n C_r &= \frac{n!}{r!(n-r)!} \\ {}^n C_{r+1} &= \frac{n!}{(r+1)!(n-r-1)!} \\ \frac{{}^n C_r}{{}^n C_{r+1}} &= \frac{n!}{r!(n-r)!} \times \frac{(r+1)!(n-r-1)!}{n!} \\ &= \frac{(r+1)!(n-r-1)! \times (n-r)!}{r! \times (r+1)!(n-r)!} \\ &= \frac{n-r+1}{r} \end{aligned}$$

∴ Proved

Q20(ii)

$$\begin{aligned}
& n \times {}^{n-1}C_{r-1} \\
&= n \times \frac{(n-1)!}{(r-1)![(n-1)-(r-1)]!} \\
&= \frac{n! \times (n-r+1)}{(r-1)!(n-r)!(n-r+1)}
\end{aligned}$$

multiplying numerator and denominator by $(n-r+1)$

$$\begin{aligned}
&= \frac{(n-r+1) \times n!}{(r-1)!(n-r+1)!} \\
&= (n-r+1) {}^nC_{r-1}
\end{aligned}$$

Hence Proved

Q20(iii)

$$\begin{aligned}
{}^nC_r &= \frac{n!}{r!(n-r)!} \\
{}^{n-1}C_{r-1} &= \frac{(n-1)!}{(r-1)!(n-1)-(r-1)!} \\
\text{Or } \frac{{}^nC_r}{{}^{n-1}C_{r-1}} &= \frac{n!(r-1)!(n-r)!}{r!(n-r)!(n-1)!} \\
&= \frac{n \times (n-1)!(r-1)! \times (n-r)!}{r \times (n-1)! \times (r-1)! \times (n-r)!} \\
&= \frac{n}{r}
\end{aligned}$$

Hence Proved

Q20(iv)

$$\text{L.H.S} \Rightarrow {}^nC_r + 2 {}^nC_{r-1} + {}^nC_{r-2}$$

$$= ({}^nC_r + {}^nC_{r-1}) + ({}^nC_{r-2} + {}^nC_{r-1})$$

$$= {}^{n+1}C_r + {}^{n+1}C_{r-1} \quad \left[\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r \right]$$

$$= (n+1) + 1 C_r$$

$$= {}^{n+2}C_r$$

Ex 17.2

Q1

No of players = 15

No of players to be selected = 11

Number of combinations

$$= {}^{15}C_{11}$$

$$= \frac{15!}{11! 4!} = \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2}$$

$$= 1365 \text{ ways}$$

Q2

Total boy = 25

Total girls = 10

Party of 8 to be made from 25 boy and 10 girls, selecting 5 boy and 3 girls

$$\Rightarrow {}^{25}C_5 \text{ and } {}^{10}C_3$$

$$= {}^{25}C_5 \times {}^{10}C_3$$

$$\text{Now, } {}^{25}C_5 = \frac{n!}{r!(n-r)!}$$

$$\Rightarrow \frac{25!}{5! 20!} \times \frac{10!}{3! 7!} = \frac{25 \times 24 \times 23 \times 22 \times 21 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 3 \times 2}$$

$$= 6375600$$

Q3

Out of 9 courses 2 are compulsory. So students can choose from 7 courses only. Also out of 5 courses that students need to choose, 2 are compulsory.

So they have to choose 3 courses out of 7 courses. This can be done ${}^7C_3 = 35$ ways.

Q4

No of players = 16

No of players to be selected = 11

∴ No of combination = ${}^{16}C_{11}$

$$= \frac{16!}{11! 5!} = \frac{16 \times 15 \times 14 \times 13 \times 12}{5 \times 4 \times 3 \times 2} = 4368$$

(i) Include 2 particular players

→ Now we have to select 9 more out of remaining 14

∴ Required number of ways

$$= {}^{14}C_9$$

$$= \frac{14!}{9! 5!} = \frac{14 \times 13 \times 12 \times 11 \times 10}{5 \times 4 \times 3 \times 2}$$

$$= 2002$$

(ii) Exclude 2 particular players → now we have to select 11 players out of 14 players

$$= {}^{14}C_{11} = \frac{14!}{11! 3!} = \frac{14 \times 13 \times 12}{3 \times 2}$$

$$= 364$$

Q5

Total number of professor = 10

Total number of student = 20

Committee of 2 professor and 3 student can be selected in $^{10}C_2 \times ^{20}C_3$ ways.

$$= \frac{10!}{2! 8!} \times \frac{20!}{3! 17!}$$

$$= \frac{10 \times 9}{2} \times \frac{20 \times 19 \times 18}{3 \times 2}$$

$$= 51300 \text{ ways}$$

(i) a particular professor is included

$\therefore$ committee is $^9C_1 \times ^{20}C_3$

$$= \frac{9!}{8!} \times \frac{20!}{3! 17!} = \frac{9 \times 20 \times 19 \times 18}{3 \times 2}$$

$$= 10260$$

(ii) a particular student is included

$\therefore$ committee is $^{10}C_2 \times ^{19}C_2$

$$= \frac{10!}{2 \times 8!} \times \frac{19!}{2! 17!} = \frac{10 \times 9 \times 19 \times 18}{2 \times 2 \times 1} = 7695$$

(iii) a particular student is excluded $\rightarrow$ now total student are 19

$\therefore$ committee is $^{10}C_2 \times ^{19}C_3$

$$= \frac{10!}{2 \times 8!} \times \frac{19!}{3! 16!} = \frac{10 \times 9 \times 19 \times 18 \times 17}{2 \times 3 \times 2} = 43605$$

Q6

The we can multiplying 2 or 3 or 4 digits.

Then number of ways of multiplying 4 digits at a time

$$= {}^4C_4 \dots\dots\dots (i)$$

The number of ways of multiplying 3 digits at a time

$$= {}^4C_3 \dots\dots\dots (ii)$$

The number of ways of multiplying 2 digits at a time

$$= {}^4C_2 \dots\dots\dots (iii)$$

$\therefore$ Total number of ways

$$= {}^4C_4 + {}^4C_2 + {}^4C_3$$

$$\Rightarrow = 1 + \frac{4 \times 3}{2} + 4$$

$$\Rightarrow = 11$$

= There are 11 ways

Q7

Total number of boys = 12

Total number of girls = 10

Total number of girls for the competition

$$= 10 + 2 = 12$$

Total students chosen for competition

$$= 10 - 2 \text{ (at least 4 boys and 4 girls)}$$

$\therefore$ Selection can be made in

$${}^{12}C_4 \times {}^8C_4 + {}^{12}C_5 \times {}^8C_3 + {}^{12}C_6 \times {}^8C_2$$

$$= \frac{12!}{4! 8!} \times \frac{8!}{4! 4!} + \frac{12!}{5! 7!} \times \frac{8!}{3! 5!} + \frac{12!}{6! 6!} \times \frac{8!}{2! 6!}$$

$$= \left(\frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 4 \times 3 \times 2} \right) + \left(\frac{12 \times 11 \times 10 \times 9 \times 8 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 3 \times 2} \right) + \left(\frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 8 \times 7}{6 \times 5 \times 4 \times 3 \times 2 \times 2} \right)$$

$$= 55440 + 44352 + 181104$$

$$= 280896$$

$\therefore$ Total number of ways = $385770 - 280896 = 104874$

(385770 = from 10 girls 4 are chosen)

Q8

Total number of books = 10
total books to be selected = 4

(i) there is no restriction

$$= {}^{10}C_4 = \frac{10!}{4! 6!} = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2}$$
$$= 210$$

(ii) two particular books are always selected
these the total books = $10 - 2 = 8$

So out of remaining 8 books selection of 2 books can be done in 8C_2 way

$$= \frac{8!}{2! 6!} = \frac{8 \times 7}{2 \times 1} = 28 \text{ ways}$$

(iii) two particular books are never selected
these the total number of books = $10 - 2 = 8$

so out of remaining 8 books, 4 books can be selected in 8C_4 way

$$= \frac{8!}{4! 4!} = \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2}$$
$$= 14 \times 5$$
$$= 70 \text{ ways}$$

Q9

Total number of officer = 4

Total number of jawans = 8

Total number of selection to be made = 6

(i) to include exactly one officer

This can be done in ${}^4C_1 \times {}^8C_5$ ways

$$= \frac{4!}{1!3!} \times \frac{8!}{5!3!}$$

$$= \frac{4 \times 8 \times 7 \times 6}{5 \times 2} = 224 \text{ ways}$$

(ii) to include at least one officer

This can be done in following ways

$${}^4C_1 \times {}^8C_5 + {}^4C_2 \times {}^8C_4 + {}^4C_3 \times {}^8C_3 + {}^4C_4 \times {}^8C_2$$

$$= \frac{4 \times 8!}{5!3!} + \frac{4!}{2!2!} \times \frac{8!}{4!4!} + \frac{4!}{3!1!} \times \frac{8!}{3!5!} + \frac{1 \times 8!}{2!6!}$$

$$= \left(\frac{4 \times 8 \times 7 \times 6}{3 \times 2} \right) + \left(\frac{4 \times 3 \times 2 \times 7 \times 6 \times 5}{2 \times 4 \times 3 \times 2} \right) + \left(\frac{4 \times 8 \times 7 \times 6}{3 \times 2} \right) + \left(\frac{8 \times 7}{2 \times 1} \right)$$

$$= (4 \times 8 \times 7) + (4 \times 3 \times 7 \times 5) + (4 \times 8 \times 7) + (4 \times 7)$$

$$= 224 + 420 + 224 + 28$$

$$= 896 \text{ ways}$$

Q10

Total number of students is XI = 20

Total number of students is XII = 20

Total number of students to be selected is a team = 11

(at least 5 from XI and 5 from XII)

this can be done in following ways

$${}^{20}C_5 \times {}^{20}C_6 + {}^{20}C_6 \times {}^{20}C_5$$

$$= 2 \left({}^{20}C_5 \times {}^{20}C_6 \right)$$

$$= 2 \left(\frac{20!}{5!15!} \times \frac{20!}{6!14!} \right)$$

$$\text{or } = \frac{2 \times 20 \times 19 \times 18 \times 17 \times 16 \times 15 \times 20 \times 19 \times 18 \times 17 \times 16}{6 \times 5 \times 4 \times 3 \times 2 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$= 19 \times 17 \times 16 \times 15 \times 2 \times 16 \times 3 \times 17 \times 9$$

$$= 1201870080 \text{ ways}$$

Q11

Total number of questions = 10

Question in part A = 6

Question in part B = 7

Selecting to questions with at least 4 from each part A and part B,
can from done in following way.

$${}^6C_4 \times {}^7C_6 + {}^6C_5 \times {}^7C_5 + {}^6C_6 \times {}^7C_4$$

$$= \left(\frac{6!}{4!2!} \times \frac{7!}{6!1!} \right) + \left(\frac{6!}{5!1!} \times \frac{7!}{5!2!} \right) + \left(\frac{1 \times 7!}{4!3!} \right) \quad \left(\because {}^nC_r = \frac{n!}{r!(n-r)!} \right)$$

$$= \left(\frac{6 \times 5 \times 7}{2} \right) + \left(\frac{6 \times 7 \times 6}{2} \right) + \left(\frac{7 \times 6 \times 5}{3 \times 2} \right)$$

$$= (105) + (126) + (35)$$

$$= 266 \text{ ways}$$

Q12

Total number of question = 5

Total number of question to be answered = 4

Given that 1 and 2 question are compulsory, the number of ways in which a student
can choose the questions will follow the following way.

$$\text{Total question} = 5 - 2 = 3$$

Out of 3 remaining questions a student has to select any 2 for answering

$$\Rightarrow {}^3C_2 = 3 \text{ ways}$$

Q13

Total number of questions = 12

Total number of questions to be answered = 7

Each group has 6 questions (6 + 6) more than 5 question from either group is not permitted, therefore the number of ways a student can choose questions can be done in following ways.

$${}^6C_2 \times {}^6C_5 + {}^6C_4 \times {}^6C_2 + {}^6C_4 \times {}^6C_3 + {}^6C_5 \times {}^6C_2$$

$$= 2 \left({}^6C_2 \times {}^6C_5 + {}^6C_3 \times {}^6C_4 \right)$$

$$= 2 \left(\frac{6!}{2!4!} \times \frac{6!}{5!1!} + \frac{6!}{3!3!} \times \frac{6!}{4!2!} \right)$$

$$= 2 \left(\frac{6 \times 5 \times 6}{2} + \frac{5 \times 5 \times 4 \times 6 \times 5}{3 \times 2 \times 2} \right)$$

$$= \frac{2 \times 6 \times 5 \times 6}{2} \left(1 + \frac{20}{6} \right)$$

$$= 180 \left(\frac{26}{6} \right)$$

$$= 30 \times 26 = 780$$

$$= 780 \text{ ways}$$

Q14

Number of point = 10

Number of collinear points = 4

Since 4 out of 10 points are collinear, so the number of lines will be $\left({}^4C_2 - 1\right)$ lie from ${}^{10}C_2$
(one is subtracted from 4C_2 to count for the line on which 4 collinear points lie)

$$\therefore \text{number of lines} = {}^{10}C_2 - \left({}^4C_2 - 1\right)$$

$$= {}^{10}C_2 - {}^4C_2 + 1$$

$$= \frac{10!}{2! 8!} - \frac{4!}{2! 2!} + 1$$

$$= \frac{10 \times 9}{2} - \frac{4 \times 3}{2} + 1$$

$$= 45 - 6 + 1$$

$$= 40$$

Q15

(i) hexagon $\rightarrow$ A hexagon has 6 angular points. By joining any two angular points we get a line which is either a side or a diagonal.

$$\therefore \text{Number of lines} = {}^6C_2 = \frac{6!}{2! 4!}$$

$$= \frac{6 \times 5}{2} = 15$$

Number of sides = 6

$$\therefore \text{Number of diagonals} = 15 - 6 = 9$$

(ii) Polygon of 16 sides will have 16 angular points. By joining any 2 points we get a line which is either a side or a diagonal.

$$\therefore \text{number of lines} = {}^{16}C_2 = \frac{16!}{2! 14!}$$

$$= \frac{16 \times 15}{2} = 120$$

$$\Rightarrow \text{number of sides} = 16$$

$$\therefore \text{number of diagonals} = 120 - 16 = 104$$

Q16

Since 5 out of 12 points are collinear, so the number of triangles will be 5C_3 less from ${}^{12}C_3$

$$= {}^{12}C_3 - {}^5C_3$$

$$= \frac{12!}{3!9!} - \frac{5!}{3!2!}$$

$$= \frac{12 \times 11 \times 10}{3 \times 2} - \frac{5 \times 4}{2}$$

$$= 220 - 10$$

$$= 210$$

Q17

Total men = 6

Total women = 4

Total persons in committee = 5

(where at least one woman has to be selected)

This can be done in

$${}^4C_1 \times {}^6C_4 + {}^4C_2 \times {}^6C_3 + {}^4C_3 \times {}^6C_2 + {}^4C_4 \times {}^6C_1$$

$$\left({}^nC_r = \frac{n!}{r!(n-r)!} \right) \left({}^nC_r = 1, {}^nC_1 = n \right)$$

$$= \left(\frac{4 \times 6!}{4! \times 2!} \right) + \left(\frac{4!}{2!2!} \times \frac{6!}{3!3!} \right) + \left(\frac{4!}{3!1!} \times \frac{6!}{2!4!} \right) + (1 \times 6)$$

$$= \left(\frac{4 \times 6 \times 5}{2} \right) + \left(\frac{4 \times 3}{2} \times \frac{6 \times 5 \times 4}{3 \times 2} \right) + \left(\frac{4 \times 6 \times 5}{2} \right) + (6)$$

$$= (60) + (120) + 60 + 6$$

$$= 246 \text{ ways}$$

Q18

52 families have at most 2 children, while 35 families have 2 children.

The selection of 20 families of which at least 18 families must have at most 2 children can be made as under

i) 18 families out of 52 and 2 families out of 35

ii) 19 families out of 52 and 1 family out of 35

iii) 20 families out of 52

Therefore the number of ways are $= {}^{52}C_{18} \times {}^{35}C_2 + {}^{52}C_{19} \times {}^{35}C_1 + {}^{52}C_{20} \times {}^{35}C_0$

Q19

i) Since, the team does not include any girl therefore, only boys are to be selected.

5 boys out of 7 boys can be selected in 7C_5 ways.

$$= {}^7C_5 = \frac{7!}{5!2!} = \frac{6 \times 7}{2} = 21$$

ii) Since, at least one boy and one girl are to be there in every team. The team consist of

a) 1 boy and 4 girls i.e. ${}^7C_1 \times {}^4C_4$

b) 2 boys and 3 girls i.e. ${}^7C_2 \times {}^4C_3$

c) 3 boys and 2 girls i.e. ${}^7C_3 \times {}^4C_2$

d) 4 boys and 1 girls i.e. ${}^7C_4 \times {}^4C_1$

$\therefore$ The required number of ways

$$= {}^7C_1 \times {}^4C_4 + {}^7C_2 \times {}^4C_3 + {}^7C_3 \times {}^4C_2 + {}^7C_4 \times {}^4C_1$$

$$= 7 + 84 + 210 + 140$$

$$= 441$$

iii) Since, the team has to consist of at least 3 girls, the team can consist of

a) 3 girls and 2 boys $= {}^7C_2 \times {}^4C_3$ ways

b) 4 girls and 1 boy $= {}^4C_4 \times {}^7C_1$, ways

$\therefore$ The required number of ways

$$= {}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1$$

$$= 84 + 7$$

$$= 91$$

Q20

The number of ways selecting of 3 people out of 5

$$= {}^5C_3 = \frac{5!}{3!2!} = \frac{5 \times 4}{2} = 10.$$

1 man can be selected from 2 men in 2C_1 ways and 2 women can be selected from 3 women in 3C_2 ways.

∴ The required number of committees

$$= {}^2C_1 \times {}^3C_2$$

$$= \frac{2!}{1!1!} \times \frac{3!}{2!1!}$$

$$= 6$$

Q21

A decagon has 10 sides

By joining any two angular points

we get a line which is either a side or a diagonal

$$\therefore \text{number of lines} = {}^{10}C_2 = \frac{10!}{2!8!} = \frac{10 \times 9}{2} = 45$$

$$\therefore \text{number of sides} = 10$$

$$\therefore \text{number of diagonals} = 45 - 10 = 35$$

Also, by joining 3 angular points a triangle is formed

$$= {}^{10}C_3$$

$$= \frac{10!}{3!7!} = \frac{10 \times 9 \times 8}{3 \times 2} = \frac{720}{6} = 120$$

$$= 120$$

Q22

Out of the 52 cards 4 are kings and 48 are Non-kings.

Five cards with at least one king

= {one king and 4 non-kings} or {two kings and 3 non kings} or
{3 kings and 2 non kings} or {4 kings and 1 non kings}

$$= {}^4C_1 \times {}^{48}C_4 + {}^4C_2 \times {}^{48}C_3 + {}^4C_3 \times {}^{48}C_2 + {}^4C_4 \times {}^{48}C_1$$

$$= 4 \times \frac{48 \times 47 \times 46 \times 45}{4 \times 3 \times 2} + \frac{4 \times 3}{2} \times \frac{48 \times 47 \times 46}{3 \times 2} + 4 \times \frac{48 \times 47}{2} + 1 \times 48$$

$$= 778320 + 103776 + 4512 + 48$$

$$= 886656$$

Required Number of ways = 886656

Q23

Total persons = 8 Selection to be made = 6 person.

If A is chosen then B must be chosen.

$\Rightarrow A$ and B are chosen together

$\therefore$ Selection can be made in

$${}^6C_4 = \frac{6!}{4!2!} = \frac{6 \times 5}{2} = 15 \text{ ways}$$

Also the number of selections in which A and B are not chosen are

$${}^7C_6 = \frac{7!}{6!1!} = 7 \text{ ways}$$

Total number of ways in which selection is made = $15 + 7$

$$= 22 \text{ ways}$$

Q24

There are 5 boys and 4 girls.

The team consists of 3 boys and 3 girls.

Number of ways to form the team

$$= {}^5C_3 \times {}^4C_3$$

$$= \frac{5!}{3!2!} \times \frac{4!}{3!1!}$$

$$= \frac{5 \times 4}{2} \times 4$$

$$= 40$$

Number of ways = 40

Q25

There are 6 red balls, 5 white balls and 5 blue balls.

Number of ways to select 9 balls consisting of 3 balls of each colour.

= (3 red out of 6 red) and
(3 white out of 5 white) and
(3 blue out of 5 blue balls)

$$= {}^6C_3 \times {}^5C_3 \times {}^5C_3$$

$$= \frac{6 \times 5 \times 4}{3 \times 2 \times 1} \times \frac{5 \times 4}{2} \times \frac{5 \times 4}{2}$$

$$= 2000$$

Required Number of ways = 2000

Q26

Out of 52 cards 4 are ace and
and 48 are Non ace.

Number of ways to select 5 cards with exactly one ace.

$$= \{ \text{one ace out of 4 ace} \} \text{ and } \\ \{ \text{4 non-ace out of 48 Non-ace} \} \\ = {}^4C_1 \times {}^{48}C_4$$

$$= 4 \times \frac{48 \times 47 \times 46 \times 45}{4 \times 3 \times 2 \times 1}$$

$$= 778320$$

Required Number of ways = 778320

Q27

There are total 5 bowlers and 12 batsman are available to select from.

Number of ways to select a team of 11 that includes exactly 4 bowlers.

$$= \{ 7 \text{ batsman out of 12 batsman} \} \text{ and } \\ \{ 4 \text{ bowlers out of 5 bowlers} \} \\ = {}^{12}C_7 \times {}^5C_4$$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} \times 5$$

$$= 1960$$

Required number of ways = 1960

Q28

Bag contains 5 black and 6 red balls.

Number of ways to select 2 black balls out of 5 black and 3 red balls out of 6 red balls.

$$= {}^5C_2 \times {}^6C_3$$

$$= \frac{5 \times 4}{2} \times \frac{6 \times 5 \times 4}{3 \times 2}$$

$$= 200$$

Required number of ways = 200

Q29

There are total 9 courses are available and out of these 2 subjects are compulsory. So,

Number of ways to select 2 compulsory and 3 option out of 9 - 2 = 7 subjects

$$= {}^7C_2 \times {}^7C_3$$

$$= 1 \times \frac{7 \times 6 \times 5}{3 \times 2}$$

$$= 35$$

Required number of ways = 35

Q30

- i) The committee consists of exactly 3 girls.
∴ We have to select 4 boys from 9 boys.

This can be done in 9C_4 ways and 3 girls out of 4 girls can be selected in 4C_3 ways.

∴ The required number ways = ${}^9C_4 \times {}^4C_3$

$$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \times 4$$

$$= 504$$

ii) At least 3 girls are there.

∴ There are 3 or more than i.e. 3 or 4 girls

a) 3 girls and 4 boys i.e. ${}^4C_3 \times {}^9C_4$ ways

b) 4 girls and 3 boys i.e. ${}^4C_4 \times {}^9C_3$ ways

∴ The required number of ways

$$= {}^4C_3 \times {}^9C_4 + {}^4C_4 \times {}^9C_3$$

$$= 504 + 84$$

$$= 588$$

iii) For at most 3 girls there are 3, 2, 1 or 0 girls

i.e. a) 0 girls and 7 boys = ${}^4C_0 \times {}^9C_7$

b) 1 girls and 6 boys = ${}^4C_1 \times {}^9C_6$

c) 2 girls and 5 boys = ${}^4C_2 \times {}^9C_5$

d) 3 girls and 4 boys = ${}^4C_3 \times {}^9C_4$

∴ Total number of required ways

$$= {}^4C_0 \times {}^9C_7 + {}^4C_1 \times {}^9C_6 + {}^4C_2 \times {}^9C_5 + {}^4C_3 \times {}^9C_4$$

$$= 0 \times \frac{9 \times 8}{2} + 4 \times \frac{9 \times 8 \times 7}{3 \times 2} + \frac{4 \times 3}{2} \times \frac{9 \times 8 \times 7 \times 5}{4 \times 3 \times 2} + 504$$

$$= 36 + 48 \times 7 + 18 \times 42 + 504$$

$$= 1632$$

Q31

Here, part I has 5 questions and part II has 7 questions.

Student has to attempt 8 questions selecting at least 3 from each section.

So,

Number of ways to select at least 3 from each section and a total of 8 questions.

= {3 from part I and 5 from part II} or

{4 from part I and 4 from part II} or

{5 from part I and 3 from part II}

$$= \left({}^5C_3 \times {}^7C_5 \right) + \left({}^5C_4 \times {}^7C_4 \right) + \left({}^5C_3 \times {}^7C_3 \right)$$

$$= \left(\frac{5 \times 4}{2 \times 1} \times \frac{7 \times 6}{2 \times 1} \right) + \left(5 \times \frac{7 \times 6 \times 5}{3 \times 2} \right) + \left(1 \times 7 \times \frac{5 \times 6 \times 5}{3 \times 2} \right)$$

$$= 210 + 175 + 35$$

$$= 420$$

Required number of ways = 420

Q32

In a parallelogram, there are 2 sets of parallel lines. Each set of parallel lines consists of $(m+2)$ lines and, each parallelogram is formed by choosing two lines from the first set and two straight lines from the second set.

Hence, the total number of parallelogram = ${}^{m+2}C_2 \times {}^{m+2}C_2$

$$= \left({}^{m+2}C_2 \right)^2$$

Q33

There are 18 points in a plane out of which 5 points are collinear.

Then number of straight lines joining these points are

$$\Rightarrow {}^nC_2 - ({}^pC_2 - 1)$$

$$\Rightarrow {}^nC_2 - {}^pC_2 + 1 \left(\begin{array}{l} \text{where } n = 18 \\ p = 5 \end{array} \right)$$

$$\Rightarrow {}^{18}C_2 - {}^5C_2 + 1$$

$$\Rightarrow \frac{18 \times 17}{2} - \frac{5 \times 4}{2} + 1$$

$$\Rightarrow 144$$

number of triangle = ${}^{13}C_3$

$$= \frac{13!}{3! 10!} = \frac{13 \times 12 \times 11}{3 \times 2}$$

$$= 13 \times 2 \times 11$$

$$= 13 \times 22$$

$$= 806$$

Ex 17.3

Q1

Total vowels are 5

Total consonants are 17

Vowels formed from 5 vowels and 17 consonants by selecting 2 vowels and 3 consonants are,

$$\begin{aligned} &= {}^5C_2 \times {}^{17}C_3 \times 5! \\ &= \frac{5!}{2! 3!} \times \frac{17!}{3! 4!} \times 120 \\ &= \frac{5 \times 4}{2} \times \frac{17 \times 16 \times 15}{3 \times 2} \times 120 \\ &= 10 \times 17 \times 8 \times 5 \times 120 \\ &= 400 \times 17 \times 120 \\ &= 6800 \times 120 \\ &= 816000 \end{aligned}$$

Q2

Total persons=10

Number of persons to be selected=5

Condition = p_1 must and p_4, p_5 must not be there

Remaining number of persons required is 4 out of $10-3=7$

$${}^7C_4 \times 5!$$

Q3

(i) Total number of 4 letter words formed from the letters of the word 'MONDAY' is = ${}^6C_4 \times 4! = 360$

(ii) Total number of words formed by using all letters of the word 'MONDAY' is = $6! = 720$

(iii)

There are two vowels A and O . So, first place can be filled in 2 ways and the remaining 5 places can be filled in $5!$ ways.

So, total number of words beginning with a vowel = $2 \times 5! = 240$

Q4

First separate the 3 and then arrange the remaining things

$${}^{n-3}C_{r-3} (r-2)! \times 3!$$

Q5

I N V O L U T E

Number of letters = 8

Vowels = I, O, U, E

Consonants = N, V, L, T,

Number of ways to select 3 vowels = 4C_3

Number of ways to select 2 consonants = 4C_2

Number of ways to arrange these five letters

$$= {}^4C_3 \times {}^4C_2 \times 5!$$

$$= 4 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 2880$$

Required number of ways = 2880

Q6

There are x things

Two specific things are to occur together, so remaining things are $(r-2)$.

Now, number of ways to arrange $(r-2)$ things out of $(n-2) = {}^{(n-2)}P_{(r-2)}$

Two things can be arranged in $(r-1)$ ways.

and these two can be placed in 2 ways.

Therefore,

$$\text{Required number of ways} = 2(r-1) {}^{(n-2)}P_{(r-2)}$$

Q7

The given word is P R O P O R T I O N.

Total letters = 10

Number of P = 2, Number of R = 2

Number of O = 3, Number of T = 1

Number of I = 1, Number of N = 1

(i) Case I: There are 6 different letters in which all the four are distinct to selected.

$$\begin{aligned}\text{Number of ways to select therefour} &= {}^6C_4 \\ &= 15\end{aligned}$$

Case II: Two same and two distinct letters are selected there are three pairs which more than, letters.

$$\begin{aligned}\text{Number of ways to select therefour} \\ &= {}^3C_1 \times {}^5C_2 \\ &= 3 \times 10 \\ &= 30\end{aligned}$$

Case III: Two alike of one kind and two alike of other kind.

There are 3 pairs of letters in the more than one letters. Any 2 of these 3 letters.

$$\begin{aligned}\text{Number of ways to select these letters} \\ &= {}^3C_2 \\ &= 3\end{aligned}$$

Case IV: Three alike and one different.

$$\begin{aligned}\text{Number of ways to select these letters} \\ &= 1 \times {}^5C_1 \\ &= 5\end{aligned}$$

Therefore,

$$\begin{aligned}\text{Number of ways to select four letters} \\ &= 15 + 30 + 3 + 5 \\ &= 53\end{aligned}$$

Required number of ways to select = 53

(ii) For case I:

$$\begin{aligned}\text{Number of arrangements of four letters all distinct} &= {}^6C_4 \times 4! \\ &= 15 \times 24 \\ &= 360\end{aligned}$$

For case II:

$$\begin{aligned}\text{Number of arrangements of four letters two same kind and two of different kind} &= {}^3C_1 \times {}^5C_2 \times \frac{4!}{2!1!1!} \\ &= 3 \times 10 \times 12 \\ &= 360\end{aligned}$$

For case III:

$$\begin{aligned}\text{Number of arrangements of four letters two alike of one kind and two of other kind} &= {}^3C_2 \times \frac{4!}{2!2!} \\ &= 3 \times 6 \\ &= 18\end{aligned}$$

Case IV:

Number of arrangements of four letters 3 alike and 1 other kind

$$\begin{aligned}&= 1 \times {}^5C_1 \times \frac{4!}{3!1!} \\ &= 20\end{aligned}$$

Therefore,

Total number of arrangements of four letters selected = $360 + 360 + 18 + 20$

Required number of arrangement = 758

Q8

M O R A D A B A D

Number of M = 1, Number of O = 1

Number of R = 1, Number of A = 3

Number of D = 2, Number of B = 1

(i) $\frac{\text{Four distinct letters}}{\text{There are 6 letters}}$

Number of arrangement of 4 letters

selected from these 6 = ${}^6C_4 \times 4!$

$$= 15 \times 24$$

$$= 360$$

(ii) Two alike and two different letters

There are 2 pairs with more than one

So, one pair from these and 2 from letters from rest 5 letters.

Number of ways to arrange therefour

$$= {}^2C_1 \times {}^5C_2 \times \frac{4!}{2!}$$

$$= 2 \times 10 \times 12$$

$$= 240$$

(iii) Two alike and two alike of other kinds.

Number of ways to arrange therefour

$$= {}^2C_2 \times {}^5C_2 \times \frac{4!}{2!2!}$$

$$= 6$$

(iv) Three alike and one different number of ways to arrange therefour

$$= 1 \times {}^5C_1$$

$$= 5 \times \frac{4!}{3!1!}$$

$$= 20$$

Therefore,

$$\text{Required number of ways} = 240 + 360 + 6 + 20$$

$$\text{Required number ways} = 626$$

Q9

In one round table the business man can accommodate the guests in ${}^{21}C_{15}$ ways. In the second round table he can accommodate the guests in 6C_6 ways. Keeping one guest as fixed in the first round table, the other 14 guests can be arrange in $14!$ ways. Keeping one guest as fixed in the second round table, the other 5 guests can be arrange in $5!$ ways.
Therefore the total number of ways in which the guests can be arrange is
 $= {}^{21}C_{15} \times {}^6C_6 \times 14! \times 5!$ ways

Q10

The word EXAMINATION has letters E, X, A, M, I, N, T, O where A, I, N repeat twice.

∴ The total number of letter = 11

The number of ways of selecting 4 letters.

$$= {}^{11}C_4 = \frac{11!}{4!7!} = \frac{11 \times 10 \times 9 \times 8}{4 \times 3 \times 2}$$

$$= 330.$$

The number of arranging 4 letters

$$\begin{aligned} \text{a) All different } {}^8C_4 \times 4! &= {}^8P_4 = \frac{8!}{4!} \\ &= 8 \times 7 \times 6 \times 5 \\ &= 56 \times 30 \\ &= 1680 \end{aligned}$$

b) 2 distinct and 2 alike

$$\begin{aligned} &= {}^3C_1 \times {}^7C_2 = \frac{3 \times 7 \times 6}{2} = 63 \times \frac{4!}{2!} \\ &= 378 \end{aligned}$$

c) 2 alike of one kind and 2 alike of other kind

$${}^3C_2 \times \frac{4!}{2 \times 2} = 3 \times 6 = 18$$

d) 3 alike and 1 distinct letter

$${}^3C_1 \times {}^7C_2 = \frac{3 \times 7 \times 6}{2} = 378$$

$$\begin{aligned} \therefore \text{Total number of ways in which 4 letter words are formed} &= 1680 + 378 + 18 + 378 \\ &= 2454 \text{ ways} \end{aligned}$$

Q11

No of persons = 16

Condition on specific persons = 4 and 2 = 6

Remaining people = $16 - 6 = 10$

So let's fill 8 people on both sides first from these 10.

First side, we can select 4 out of 10.

$${}^{10}C_4 \times {}^6C_6$$

Now we can arrange these 8 people on both sides in $8! \times 8!$ ways

$$\text{Answer} = {}^{10}C_4 \times {}^6C_6 \times 8! \times 8!$$