# CBSE Test Paper 04
## Chapter 5 Continuity and Differentiability

1. $\underset{x \to \frac{\pi}{2}}{Lt}$ [sinx] is equal to
   a. None of these
   b. 1
   c. 0
   d. -1
2. $\frac{d}{dx}(\log|x|)$is equal to $(x \neq 0)$
   a. $\pm\frac{1}{x}$
   b. $\frac{1}{x}$ or $-\frac{1}{x}$
   c. $\frac{1}{|x|}$
   d. $\frac{1}{x}$
3. If f (x) = x $\tan^{-1}$ x then f‘ (1) is equal to
   a. None of these
   b. $\frac{1}{2} - \frac{\pi}{4}$
   c. $\frac{\pi}{4} - \frac{1}{2}$
   d. $\frac{\pi}{4} + \frac{1}{2}$
4. If x = $at^2$ ,y = 2 a t, then $\frac{d^2y}{dx^2}$ is equal to
   a. None of these
   b. 0
   c. $\frac{1}{t^2}$
   d. $-\frac{1}{2a\ t^3}$
5. If a function f is derivable at x = a, then $\underset{h \to 0}{Lt}\ \frac{f(a-h)-f(a)}{h}$ is equal to
   a. f ‘(a)

b. None of these

c. does not exist

d. -f ‘(a)

6. If x = $at^2$ and y = 2at, then $\frac{dy}{dx}$ = ________.
7. If y = A sin x + B cos x, then $\frac{d^2y}{dx^2} + y =$ ________.
8. If $y = \sqrt{\sin\ x + y}$, then $\frac{dy}{dx}$ is equal to _______.
9. Find $\frac{dy}{dx}, y = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$.
10. If $f(x) = |\cos x - \sin x|$. find $f'(\frac{\pi}{6})$.
11. If $f(x) = |\cos x|$ find $f'\left(\frac{3\pi}{4}\right)$.
12. Find $\frac{dy}{dx}$ if $x^3 + x^2y + xy^2 + y^3 = 81$.
13. Show that the function defined by $f(x) = |\cos x|$ is a continuous function.
14. Find $\frac{dy}{dx}$ of each of the following function expressed in parametric form in $x = t + \frac{1}{t}, y = t - \frac{1}{t}$.
15. If $x = a \sin 2t\,(1 + \cos 2t)$ and $y = b \cos 2t\,(1 - \cos 2t)$, show that $\left(\frac{dy}{dx}\right)(at = \frac{\pi}{4}) = \frac{b}{a}$.
16. Find $\frac{dy}{dx}$, If y = $(\cos x)^x + (\sin x)^{1/x}$.
17. Find $\frac{dy}{dx}$, if $(x^2 + y^2)^2 = xy$.
18. Find $\frac{dy}{dx}$ if $x = a(\cos\theta + \theta \sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$.

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## Solution

1. c. 0

   **Explanation:** $\lim_{x\to\frac{\pi}{2}}[\sin x]=0$ [ since $\frac{\pi}{2}-h\leq x\leq\frac{\pi}{2}+h$]

2. d. $\frac{1}{x}$

   **Explanation:** $\frac{d}{dx}(\log|x|)=\frac{1}{|x|}\frac{x}{|x|}=\frac{x}{|x|^2}=\frac{x}{x^2}=\frac{1}{x}$

3. d. $\frac{\pi}{4}+\frac{1}{2}$

   **Explanation:** $f'(x)=\frac{d}{dx}(x\tan^{-1}x)=\frac{x}{1+x^2}+\tan^{-1}x$

   $\Rightarrow f'(1)=\frac{1}{1+1}+\tan^{-1}1=\frac{1}{2}+\frac{\pi}{4}$

4. d. $-\frac{1}{2a\ t^3}$

   **Explanation:** $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{2a}{2at}=\frac{1}{t}\ldots\ldots(1)\Rightarrow\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{1}{t}\right)$

   $=-\frac{1}{t^2}\frac{dt}{dx}=-\frac{1}{t^2}\cdot\frac{1}{2at}=-\frac{1}{2at^3}$

5. d. -f ‘(a)

   **Explanation:** $\lim\limits_{h\to0}\frac{f(a-h)-f(a)}{h}$ =

   $\lim\limits_{t\to0}\frac{f(a+t)-f(a)}{-t}(put..h=-t,as..h\to0,t\to0)$

   $\Rightarrow-\lim\limits_{t\to0}\frac{f(a+t)-f(a)}{t}=-f'(a)$

6. $\frac{1}{t}$
7. 0
8. $\frac{\cos x}{2y-1}$
9. $y=\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$

   $y=\tan^{-1}\left(\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$

   $y=\tan^{-1}\left(\tan\frac{x}{2}\right)$

   $y=\frac{x}{2}$

   $\frac{dy}{dx}=\frac{1}{2}$

10. When $0<x<\frac{\pi}{4}$ ,cos x > sin x, so that cos x – sin x > 0, i.e.,

$f(x)=\cos x-\sin x$
$\Rightarrow f'(x)=-\sin x-\cos x$
Hence, $f'(\frac{\pi}{6})=-\sin\frac{\pi}{6}-cos\frac{\pi}{6}=-\frac{1}{2}-\frac{\sqrt{3}}{2}=-\frac{1}{2}(1+\sqrt{3})$

11. When $\frac{\pi}{2}<x<\pi, \cos x<0$ so that $|\cos x|=-\cos x$, i.e.,
$f(x)=-\cos x\Rightarrow f'(x)=\sin x$
Hence, $f'\left(\frac{3\pi}{4}\right)=\sin\left(\frac{3\pi}{4}\right)=\frac{1}{\sqrt{2}}$

12. we have,
$x^3 + x^2y + xy^2 + y^3 = 81$
Differentiating both sides w.r.t to x,we get,
$3x^2+x^2.\frac{dy}{dx}+y.2x+x.2y\frac{dy}{dx}+y^2.1+3y^2\frac{dy}{dx}=0$
$\left(x^2+2xy+3y^2\right)\frac{dy}{dx}=-3x^2-2xy-y^2$
$\frac{dy}{dx}=\frac{-\left(3x^2+2xy+y^2\right)}{x^2+2xy+3y^2}$

13. Given: $f(x)=|\cos x|$ ....(i)
f(x) has a real and finite value for all $x\in R$.
$\therefore$ Domain of f(x) is R.
Let g(x) = cos x and $h(x)=|x|$
Since g(x) and h(x) being cosine function and modulus function are continuous for all real x
Now, $(goh)x=g\{h(x)\}=g(|x|)=\cos|x|$ being the composite function of two continuous functions is continuous, but not equal to f(x)
Again, $(hog)x=h\{g(x)\}=h(\cos x)=|\cos x|=f(x)$ [Using eq. (i)]
Therefore, $f(x)=|\cos x|=(hog)x$ being the composite function of two continuous functions is continuous.

14. $\because x=t+\frac{1}{t}$ and $y=t-\frac{1}{t}$
$\therefore \frac{dx}{dt}=\frac{d}{dt}\left(t+\frac{1}{t}\right)$ and $\frac{dy}{dt}=\frac{d}{dt}\left(t-\frac{1}{t}\right)$
$\Rightarrow \frac{dx}{dt}=1+(-1)t^{-2}$ and $\frac{dy}{dt}=1-(-1)t^{-2}$
$\Rightarrow \frac{dx}{dt}=1-\frac{1}{t^2}$ and $\Rightarrow \frac{dy}{dt}=1+\frac{1}{t^2}$
$\Rightarrow \frac{dx}{dt}=\frac{t^2-1}{t^2}$ and $\Rightarrow \frac{dy}{dt}=\frac{t^2+1}{t^2}$
$\therefore \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{t^2+1/t^2}{t^2-1/t^2}=\frac{t^2+1}{t^2-1}$

15. $\because x=a\sin 2t(1+\cos 2t)$ and $y=b\cos 2t(1-\cos 2t)$
$\therefore \frac{dx}{dt}=a\left[\sin 2t.\frac{d}{dt}(1+\cos 2t)+(1+\cos 2t).\frac{d}{dt}\sin 2t\right]$

$= a\left[\sin 2t.(-\sin 2t).\frac{d}{dt}2t + (1+\cos 2t).\cos 2t.\frac{d}{dt}2t\right]$

$= -2a\sin^2 2t + 2a\cos 2t\,(1+\cos 2t)$

$\Rightarrow \frac{dx}{dt} = -2a\left[\sin^2 2t - \cos 2t\,(1+\cos 2t)\right]$ ...(i)

and $\frac{dy}{dt} = b\left[\cos 2t.\frac{d}{dt}(1-\cos 2t) + (1-\cos 2t).\frac{d}{dt}\cos 2t\right]$

$= b\left[\cos 2t.(\sin 2t)\frac{d}{dt}2t + (1-\cos 2t)(-\sin 2t).\frac{d}{dt}2t\right]$

$= b\left[2\sin 2t.\cos 2t + 2(1-\cos 2t)(-\sin 2t)\right]$

$= -2b\left[-\sin 2t.\cos 2t + (1-\cos 2t)\sin 2t\right]$..(ii)

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2b[-\sin 2t.\cos 2t+(1-\cos 2t)\sin 2t]}{-2a\left[\sin^2 2t-\cos 2t(1+\cos 2t)\right]}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{t=\pi/4} = \frac{b}{a}\frac{\left[-\sin\frac{\pi}{2}.\cos\frac{\pi}{2}+\left(1-\cos\frac{\pi}{2}\right)\sin\frac{\pi}{2}\right]}{\left[\sin^2\frac{\pi}{2}-\cos\frac{\pi}{2}\left(1+\cos\frac{\pi}{2}\right)\right]}$

$= \frac{b}{a}.\frac{(0+1)}{(1-0)}\left[\because \sin\frac{\pi}{2} = 1 \ and\ \cos\frac{\pi}{2} = 0\right]$

$= \frac{b}{a}$ Hence Proved.

16. Given, $y = (cosx)^x + (sinx)^{1/x}$

Let $u = (cosx)^x \ \ and\ \ v = (sinx)^{1/x}$

Then, given equation becomes

y = u + v

Differentiating both sides w.r.t. x,

$\Rightarrow \quad \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$.......(i)

Taking log both sides,

$\Rightarrow logu = log(cosx)^x$

$\Rightarrow logu = xlog(cosx)$

Differentiating both sides w.r.t x,

$\Rightarrow \frac{1}{u}\frac{du}{dx} = x\cdot\frac{d}{dx}\log(\cos x) + \log(\cos x)\cdot\frac{d}{dx}(x)$[ using product rule of derivative]

$\Rightarrow \frac{1}{u}\frac{du}{dx} = x\cdot\frac{1}{\cos x}(-sinx) + log\ cosx.1$

$\Rightarrow \frac{1}{u}\frac{du}{dx} = -xtanx + log(cosx)$

$\Rightarrow \frac{du}{dx} = u[-x\ tanx + log\ cosx]$

$\Rightarrow \frac{du}{dx} = (cosx)^x[-x\ tanx + log\ cosx]$........(ii)

Now, consider v = $(\sin x)^{1/x}$

log v = $\log(\sin x)^{1/x}$

$\Rightarrow \quad \log v = \frac{1}{x}\log\sin x$

On differentiating both sides w.r.t x, we get

$$\frac{1}{v} \cdot \frac{dv}{dx} = \frac{1}{x} \cdot \frac{d}{dx}(\log \sin x) + \log \sin x \cdot \frac{d}{dx}\left(\frac{1}{x}\right)$$

$$\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = \frac{1}{x} \cdot \frac{1}{\sin x} \cos x + \log \sin x\left(-\frac{1}{x^2}\right)$$

$$\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = \frac{\cot x}{x} - \frac{\log(\sin x)}{x^2}$$

$$\Rightarrow \quad \frac{dv}{dx} = v\left(\frac{\cot x}{x} - \frac{\log(\sin x)}{x^2}\right)$$

$$\Rightarrow \quad \frac{dv}{dx} = (\sin x)^{1/x}\left[\frac{\cot x}{x} - \frac{\log(\sin x)}{x^2}\right] \text{...........(iii)}$$

Now, from Eqs.(i), (ii) and (iii), we get

$$\frac{dy}{dx} = (\cos x)^x[-x \tan x + \log \cos x] + (\sin x)^{1/x}\left[\frac{\cot x}{x} - \frac{\log(\sin x)}{x^2}\right]$$

17. Given, $(x^2 + y^2)^2 = xy$............(i)

Therefore,on differentiating both sides of Eq.(i) w.r.t x, we get,

$$2(x^2 + y^2)\left[2x + 2y\frac{dy}{dx}\right] = x\frac{dy}{dx} + y$$

$$\Rightarrow 4x(x^2 + y^2) + 4y(x^2 + y^2)\frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$\Rightarrow \quad 4y\left(x^2 + y^2\right)\frac{dy}{dx} - x\frac{dy}{dx} = y - 4x(x^2 + y^2)$$

$$\Rightarrow \quad \frac{dy}{dx}[4y(x^2 + y^2) - x] = y - 4x(x^2 + y^2)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y-4x\left(x^2+y^2\right)}{4y(x^2+y^2)-x}$$

$$\therefore \quad \frac{dy}{dx} = \frac{y-4x^3-4xy^2}{4yx^2+4y^3-x}$$

18. $x = a\left(\cos\theta + \theta.\sin\theta\right)$

$$\frac{dx}{d\theta} = a\left[-\sin\theta + \theta.\cos\theta + \sin\theta.1\right]$$

$$\frac{dx}{d\theta} = a\theta.\cos\theta \text{ ...(1)}$$

$$y = a\left(\sin\theta - \theta.\cos\theta\right)$$

$$\frac{dy}{d\theta} = a\left[\cos\theta - \left(-\theta\sin\theta + \cos\theta.1\right)\right]$$

$$= a\left[\cos\theta + \theta.\sin\theta - \cos\theta\right]$$

$$= a\theta.\sin\theta \text{ ...(2)}$$

$(2) \div (1)$

$$\frac{dy}{dx} = \frac{a\,\theta.\sin\theta}{a\,\theta.\cos\theta}$$

$$= \tan\theta$$