Chaper-06
Laplace transform

$$f(t) \rightleftharpoons F(s)$$

where s = complex-variable

$$= \sigma + j\omega$$

σ → damping factor

$j\omega$ → oscillation factor

$$\boxed{F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt}$$

Q → Find LT; of $x(t) = e^{-at}u(t) = X(s)$

Sol^n →

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt = \int_{-\infty}^{\infty} e^{-at} e^{-st} dt$$

$$X(s) = \int_{-\infty}^{\infty} e^{-(a+s)t} dt$$

$$= \frac{\left[e^{-(a+s)t}\right]_0^{\infty}}{-(s+a)} = \frac{e^{-(s+a)\infty} - e^0}{-(s+a)} \quad \text{------ (i)}$$

$$* \; e^{-(s+a)\infty} = e^{-(\sigma+j\omega+a)\infty}$$

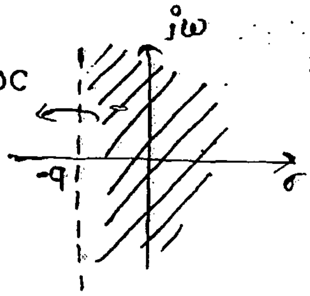
$$= e^{-(\sigma+a)\infty} e^{-j\omega\infty}$$

$$= e^{-(\sigma+a)\infty} \, e^{-j\infty} \text{ undefined}$$

$$* \; e^{-(\sigma+a)\infty} = 0, \quad (\sigma+a) > 0$$

So; from eq^n (i)

$$\boxed{X(s) = \frac{1}{s+a}, \; (\sigma > -a)} \leftarrow \text{ROC}$$



ROC → Range of convergance

Condition for existance of LT →

$$|F(s)| < \infty$$

$$\Rightarrow \left| \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt \right| < \infty$$

$$\Rightarrow \int_{-\infty}^{\infty} \left| f(t)\, e^{-st} \right| dt < \infty$$

$$\Rightarrow \int_{-\infty}^{\infty} \left| f(t) . e^{-(\sigma + j\omega)t} \right| dt < \infty$$

$$\Rightarrow \int_{-\infty}^{\infty} \left| f(t)\, e^{-\sigma t} \right| \left| \cancel{e^{-j\omega t}} \right|^{1} dt < \infty$$

$$\Rightarrow \boxed{\int_{-\infty}^{\infty} \left| f(t)\, e^{-\sigma t} \right| dt < \infty} \longrightarrow \text{LT}$$

$$\sigma = 0 \;,\; s = \sigma + j\omega = j\omega$$

$$\Rightarrow \boxed{\int_{-\infty}^{\infty} |f(t)|\, dt < \infty} \longrightarrow \text{FT}$$

$f(t)$ is absolutely integrable

* The replacement $s = j\omega$ is used for laplace to fourier conversion only for absolutely integrable signal.

Properties of LT →

(1.) Linearity property → $a_1 f_1(t) + a_2 f_2(t) \rightleftharpoons a_1 F_1(s) + a_2 F_2(s)$

(2.) Time-Reversal → $f(-t) \rightleftharpoons F(-s)$

(3.) Conjugation → $f^*(t) \rightleftharpoons F^*(s^*)$

(4.) Time-shifting → $f(t - t_0) \rightleftharpoons F(s) e^{-st_0}$

(5.) Time-scaling → $f(at)_{a \neq 0} \rightleftharpoons \frac{1}{|a|} F\left(\frac{s}{a}\right)$

$$f(t) = \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt \text{; bilateral LT}$$

$$\int_{0}^{\infty} f(t)\, e^{-st}\, dt \text{, Unilateral LT}$$

(6.) Freq. shifting → $e^{-at} f(t) \rightleftharpoons F(s + a)$

(7.) Differentiation in time →

$$\frac{d^n f(t)}{dt^n} \rightleftharpoons \begin{cases} s^n F(s) \text{ ; (Bilateral LT)} \\ s^n F(s) - s^{n-1} f(o^-) - s^{n-2} f'(o^-) - s^{n-3} f''(o^-) \ldots \end{cases}$$

where; $f(0^-) = |f(t)|_{t=0^-}$

$$f'(0^-) = \left.\frac{d f(t)}{dt}\right|_{t=0^-} \qquad f''(0^-) = \left.\frac{d^2 f(t)}{dt^2}\right|_{t=0^-}$$

(8.) Integration in time →

$$\int_{-\infty}^{t} f(t)\,dt = \begin{cases} \dfrac{F(s)}{s} & \text{; Bilateral FT.} \\ \dfrac{F(s)}{s} + \dfrac{\int_{-\infty}^{0^-} f(t)\,dt}{s} & \text{, unilateral LT.} \end{cases}$$

(9.) Convolution in time →

$$f_1(t) * f_2(t) = F_1(s) \cdot F_2(s)$$

(10.) multiplication in time →

$$f_1(t) \cdot f_2(t) = \frac{1}{2\pi j}\left[F_1(s) * F_2(s)\right]$$

(11.) Differentiation in freq. →

$$t^n f(t) = (-1)^n \frac{d^n F(s)}{ds^n}$$

(12.) Integration in freq. →

$$\frac{f(t)}{t} = \int_{s}^{\infty} F(s)\,ds$$

(13.) Initial value theorem →

$$x(0) = \lim_{s\to\infty}\left[sX(s)\right]$$

Condition:- It is applicable only for causal type signals.

i.e. $x(t) = 0;\ t<0$

(14.) Final value theorem →

$$x(\infty) = \lim_{s\to 0}\left[sX(s)\right]$$

Condition →

(i.) It is applicable only for causal type signals.

i.e., $x(t) = 0,\ t<0$

** (ii) "$sX(s)$" should have only LHS poles in s-plance

Que.→ $F(s) = \frac{1}{s^2+1} \rightleftharpoons f(t)$. Calculate $f(\infty) = ?$

(a) -1 (b) 0 (c) 1 (d) $-1 \le f(\infty) \le 1$.

Sol^n→ $sF(s) = \frac{s}{s^2+1}$

poles:- $s = \pm j \neq$ LHS poles

* FVT is not applicable because $s = \pm j$ poles

$f(t) = \sin t\, u(t)$

$f(\infty) = \sin\infty\, u(\infty)$

$(-1, 1)$

$f(\infty) = -1 \le f(\infty) \le 1$

Que.→ $F(t) = F(s) = \frac{1}{s(s-1)}$; $f(\infty) = ?$

(a) 0, (b) ∞ ~~(c) -1~~ (d) 1

Sol^n→ (1.) signal is causal because depend on past $(s-1)$

(2.) $sF(s) = \frac{s}{s(s-1)} = \frac{1}{s-1}$

Pole: $s = 1 \neq$ LHS plane

FVT is not applicable.

$F(s) = \frac{1}{s(s-1)} = -\frac{1}{s} + \frac{1}{(s-1)}$

~~$f(t) = e^t u(t) + e$~~ $f(t) = e^t u(t) - u(t)$

$f(\infty) = \infty - 1 + \infty$

$\boxed{F(\infty) \simeq \infty}$

Q→ $f(t) = -e^{-at}u(-t) \rightleftharpoons F(S) = ?,\ ROC = ?$

Sol^n→

$$f(t) = e^{-at}u(-t)$$

$$e^{-at}u(t) \rightleftharpoons \frac{1}{s+a}\ ;\ (\sigma > -a)$$

$$-e^{-at}u(t) \rightleftharpoons \frac{-1}{s+a}\ ;\ (\sigma > -a)$$

$\downarrow(t=-t)$ $\quad\downarrow(s=-s)$ $\quad\downarrow$ (By time Reversal) $(\sigma = -\sigma)$

$\left\{\begin{array}{l} \because s = \sigma + j\omega \\ -s = -\sigma - j\omega \end{array}\right.$

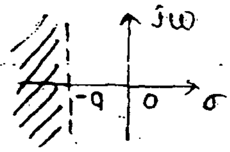
$$-e^{at}u(-t) \rightleftharpoons \frac{-1}{-s+a}\ ;\ (-\sigma > -a)$$

$\downarrow(a=-a)$ $\quad\downarrow(a=-a)$

$$-e^{-at}u(-t) \rightleftharpoons \frac{-1}{-s-a}\ ;\ (-\sigma > a)$$

$$-e^{-at}u(-t) \rightleftharpoons \frac{1}{s+a}\ ;\ (\cancel{-\sigma}\ \sigma < -a)$$

$$-e^{-at}u(-t) \rightleftharpoons \frac{1}{s+a}\ ;\ (\sigma < -a)$$



∴ also;

$$e^{-at}u(t) \rightleftharpoons \frac{1}{s+a}\ ;\ (\sigma > -a)$$

(a=0) $\quad$ (a=0)

$$u(-t) \longrightarrow \frac{1}{s}\ ;\ \sigma < 0$$

$$u(t) \longrightarrow \frac{1}{s}\ ;\ \sigma > 0$$

$$\boxed{\begin{array}{l} e^{-at}u(t) \rightarrow \frac{1}{s+a}\ ;\ (\sigma > -a) \\ -e^{-at}u(-t) \longrightarrow \frac{1}{s+a}\ ;\ (\sigma < -a) \end{array}}$$

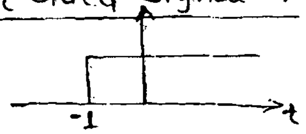
$$\boxed{\begin{array}{l} u(-t) \longrightarrow \frac{1}{s}\ ;\ \sigma < 0 \\ u(t) \longrightarrow \frac{1}{s}\ ;\ \sigma > 0 \end{array}}$$

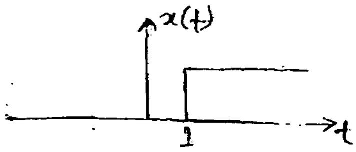
Region of convergance (ROC) → It is defined as the range of complex variable as in s-planes for which LT of signal is convergant (or) finite.

Properties →

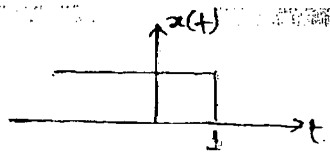
(i) ROC doesn't include any pole.

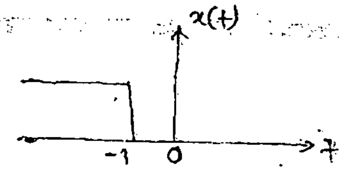
Right sided signal →



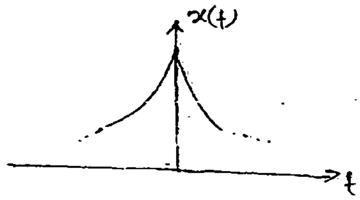


Left sided signal →





Both sided signal →



(ii) For right side signal ROC will be right side to the right most pole. For left sided signal ROC will be left side to the left most pole.

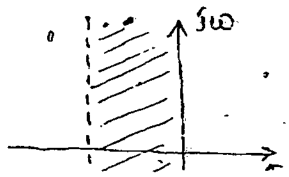
(iii) for stability ROC includes imaginary axis in s-plane.

(iv) For both sided sig. ROC is a strip in s-plane.

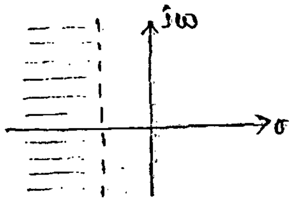
(v.) for ∞ duration signal ROC is entire s-plane excluding possibly $s=0$ or $\pm\infty$.

Q → Check stability of LTI sys of & comment about extention of $h(t)$.

(i.) ROC : $\sigma > -1$

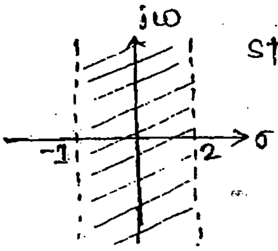


(ii) ROC: $\sigma < -2$



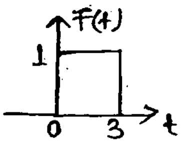
$h(t)$ = Unstable + LS

(3) ROC: $-1 < \sigma < 2$



strip type $h(t)$ = stable + Both side.

Que.→



$\rightleftharpoons F(s) = ?$; ROC = ?

Sol^n→ $F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt$

$$= \int_0^3 e^{-st} dt = \int_0^3 e^{-st} dt = \frac{(e^{-st})_0^3}{-s} = \frac{1 - e^{-3s}}{s}$$

At ($s=0$) $F(0) = \frac{1 - e^{-3s}}{s} = \frac{0}{0}$ then use L-Hospital Rule

diff wrt. s $= 3e^{-3s}\Big|_{s=0} = 3$

at ($s=\infty$) $F(\infty) = 0$

at ($s=-\infty$) $F(-\infty) = 3e^{-3s}\Big|_{(s=-\infty)} = 3e^{\infty} = \infty$

ROC : Entire s-plane excluding $s = -\infty$

$e^{2t} u(-t+3)$ → ROC : $\sigma < 2$ $e^{2t} u(-t-4)$ → ROC : $\sigma < 2$

Que.→ $f(t) = e^{-2t}u(-t) + e^{3t}u(t)$

Sol^n→ $f(t) = \underbrace{e^{-2t}u(-t)}_{\text{ROC}: \sigma < -2} + \underbrace{e^{3t}u(t)}_{\sigma > 3}$

• LT of $f(t)$ will not exist because of no common ROC

Que.→ $f(t) = -e^{-2t}u(-t) + e^{-3t}u(t)$

Sol^n→ $f(t) = -e^{-2t}u(-t) + e^{-3t}u(t)$

common ROC: $\underline{-3 < \sigma < -2}$ ↳ strip

Q → $f(t) = e^{-3|t|}$; ROC = ?

Sol^n → $f(t) = \begin{cases} e^{3t} & ; t<0 \\ e^{-3t} & ; t>0 \end{cases}$

ROC :- $-3 < \sigma < 3$ $\qquad f(t) = \underbrace{e^{3t}u(-t)}_{\sigma<3} + \underbrace{e^{-3t}u(t)}_{\sigma>-3}$

Que → $f(t) = e^{r(t)} = e^{tu(t)}$; ROC = ?

Sol^n → $f(t) = \begin{cases} 1 & ; t<0 \\ e^t & ; t>0 \end{cases}$ (No common ROC)

$= \underbrace{u(-t)}_{\cancel{\sigma<0}} + \underbrace{e^t u(t)}_{\sigma>1}$

Que → $f(t) = e^{-at}u(t)$

Sol^n → where $a = b+jc$

$f(t) = e^{-(b+jc)t}u(t)$

For existance of LT.

$= \int_{-\infty}^{\infty} |f(t)\, e^{-\sigma t}|\, dt < \infty = \int_{-\infty}^{\infty} |e^{-(b+jc)t} e^{-\sigma t}|\, dt < \infty = \int_{-\infty}^{\infty} |e^{-(b+\sigma)t}|\, \underbrace{|e^{-jct}|}_{1}\, dt < \infty$

$= \int_{0}^{\infty} e^{-(\sigma+b)t} dt < \infty = (\sigma+b) > 0 = \sigma > -b = \sigma > -Re(a)$

Que → $f(t) = \cos\omega_0 t\, u(t) \rightleftharpoons F(s) = ?$, ROC = ?

Sol^n → $\cos\omega_0 t = \dfrac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$

$f(t) = \frac{1}{2}\left(e^{j\omega_0 t}u(t) + e^{-j\omega_0 t}u(t)\right)$

$e^{-at}u(t) \rightleftharpoons \dfrac{1}{s+a}$; $\sigma > -Re(a)$

$a = 0 + j\omega_0$

$e^{-j\omega_0 t}u(t) \rightleftharpoons \dfrac{1}{s+j\omega_0}$; $\sigma > 0$

$a = 0 - j\omega_0$

$e^{j\omega_0 t}u(t) \rightleftharpoons \dfrac{1}{s-j\omega_0}$; $\sigma > 0$

Important signals →

$F(t)$	$F(s)$	ROC
$\delta(t)$	1	entire s-plane
$u(t)$	$1/s$	$\sigma>0$
$e^{-at}u(t)$	$\frac{1}{s+a}$	$\sigma>-a$
$e^{-at}u(-t)$	$\frac{1}{s+a}$	$\sigma<-a$
$tu(t)$	$\frac{1}{s^2}$	
$e^{-at}t\cdot u(t)$	$\frac{1}{(s+a)^2}$	$\sigma>-a$
$\cos\omega_0 t\, u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\sigma>0$
$-te^{-at}u(-t)$	$\frac{1}{(s+a)^2}$	$\sigma<-a$
$\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\sigma>0$
$t^n u(t), n\geq 0$	$\frac{\lfloor n}{s^{n+1}}$	$\sigma>0$

Que.→ $r(t)=tu(t) \rightleftharpoons R(s) = 1/s^2$

Que.→ $f(t)=tu(t-1) \rightleftharpoons [(t-1)+1]\cdot u(t-1) = (t-1)u(t-1)+u(t-1)$

$= \frac{1}{s^2}e^{-s} + \frac{e^{-s}}{s}$

$y(t)=r(t-1) \rightleftharpoons Y(s)=R(s)e^{-s} = \frac{1}{s^2}e^{-s}$

$=(t-1)u(t-1)$

Que.→ $f(t)=(t^2+5t-2)u(t-1)$

Sol^n→ $[(t-1)^2+7t-3]\,u(t-1)$

$=[(t-1)^2+7(t-1)+4]\,u(t-1)$

$=(t-1)^2u(t-1)+7(t-1)u(t-1)+4u(t-1)$

$F(s) = \frac{2}{s^3}e^{-s}+\frac{7}{s^2}e^{-s}+\frac{4}{s}e^{-s}$

Que.→ $f(t)=(t^3+5t^2+3t+1)u(t-1)$

Sol^n→ $t=t+1$

$f(t+1)=[(t+1)^3+5(t+1)^2+3(t+1)+1]u(t)$ ...

$f(t+1)=[t^3+8t^2+16t+10]u(t)$

$\downarrow$ LT

$F(s)=\left(\frac{6}{} + \frac{16}{} + \frac{16}{} + \frac{10}{}\right)e^{-s}$

Q.→ $f(t) = F(s) = \log\left(\frac{s+5}{s+6}\right)$

find $f(t)$:- (a) $\frac{1}{t}(e^{-6t} - e^{-5t})u(t)$ (c.) $t(e^{-6t} - e^{-5t})u(t)$

(b.) $\frac{1}{t}(e^{-5t} - e^{-6t})u(t)$ (d.) $t(e^{-5t} - e^{-6t})u(t)$

Soln→ Diff. in Freq. domain;

$$f(t) \rightleftharpoons F(s)$$

$$t f(t) \rightleftharpoons \frac{-dF(s)}{ds} = -\left[\frac{1}{s+5} - \frac{1}{s+6}\right]$$

$$t f(t) = \frac{1}{s+6} - \frac{1}{s+5}$$

$$t f(t) = e^{-6t}u(t) - e^{-5t}u(t)$$

$$f(t) = \frac{1}{t}(e^{-6t} - e^{-5t})u(t)$$

Q.→ $f(t) = \frac{(1 - e^{t})u(t)}{t}$; $F(s) = ?$

(a.) $\log\left(\frac{s}{s-1}\right)$ (c) $\log\left(\frac{s-1}{s+1}\right)$

(b.) $\log\left(\frac{s-1}{s}\right)$ (d) $\log\left(\frac{s+1}{s-1}\right)$

Soln→ Integration in freq. domain :-

$$y(t) = (1 - e^{t})u(t) \rightleftharpoons Y(s)$$

$$f(t) = \frac{y(t)}{t} \rightleftharpoons F(s) = \int_s^\infty Y(s)\,ds$$

$$F(s) = \int_s^\infty Y(s)\,ds = \int_s^\infty \left(\frac{1}{s} - \frac{1}{s-1}\right) ds$$

$$F(s) = \left[\log(s) - \log(s-1)\right]_s^\infty = \left[\log\left(\frac{s}{s-1}\right)\right]_s^\infty$$

$$= \log\left[\lim_{s\to\infty}\left(\frac{s}{s-1}\right)\right] - \log\left(\frac{s}{s-1}\right)$$

$$= \log(1) - \log\left(\frac{s}{s-1}\right)$$

$$= 0 - \log\left(\frac{s}{s-1}\right)$$

Que→ Find inverse LT of $F(S) = \frac{S^2+2S+5}{(S+3)(S+5)^2}$

For (i) $\sigma > -3$ (ii) $\sigma < -5$ (iii) $-5 < \sigma < -3$

Soln→

$$F(S) = \frac{A}{(S+3)} + \frac{B}{(S+2)} + \frac{C}{(S+2)^2}$$

$$A = 2,\ B = -1,\ C = -10$$

$$F(S) = \frac{2}{S+3} - \frac{1}{S+5} - \frac{10}{(S+5)^2}$$

Poles:- $S = -3, -5$

(i.) $\sigma > -3$; $f(t)$ will be Right sided.

$$f(t) = 2e^{-3t}u(t) - e^{-5t}u(t) - 10te^{-5t}u(t)$$

(ii) $\sigma < -5$; $f(t)$ will be left sided.

$$f(t) = -2e^{-3t}u(-t) - [-e^{-5t}u(-t)] - 10[-te^{-5t}u(-t)]$$

(iii) $\underline{-5 < \sigma < -3}$; $f(t)$ will be both sided.
↳ strip

ROC; $\sigma > -5$ ↓ Right sided

ROC; $\sigma < -3$ ↓ left sided

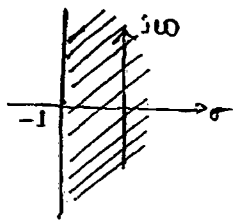
$$f(t) = -2e^{-3t}u(-t) - e^{-5t}u(t) - 10te^{-5t}u(t).$$

*Causal system→

(1.) $h(t) = 0;\ t < 0$

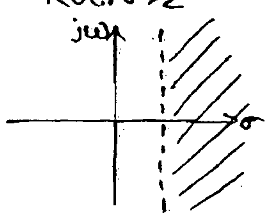
For causal sys. ROC will be right side to the right most pole.

Eg:- ROC: $\sigma > -1$



(Stable)

ROC: $\sigma > 2$



(Unstable)

$H(S) = \frac{1}{S+1},\ \sigma > -1$

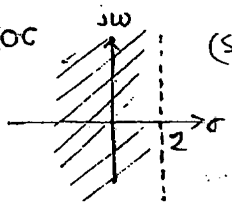
$h(t) = e^{-t}u(t)$ (energy)

$H(S) = \frac{1}{S-2};\ \sigma > 2$

$h(t) = e^{2t}u(t)$ (NENP)

Note → For stability of causal sys., poles of TF should lie in the LHS of s-plane.

Eg:- ROC

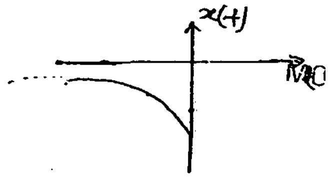


(stable)

$$H(s) = \frac{1}{(s-2)} \quad \sigma < 2$$

$$h(t) = -e^{2t}u(-t)$$

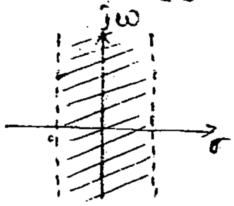
(Energy)
↓
stable.



* For anticausal sys. ROC will be left side to the left most pole.

* For stablity of anticausal sys., poles of TF should lie in the RHS of s-plane.

Eg:-(4) ROC $-1 < \sigma < 3$



(stable) + NC

Que → Consider a contineous time LTI sys. whose i/p $x(t)$ & o/p $y(t)$ are related by the differential eqn.-

$$\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t)$$

determine $h(t)$ of sys.

(a) when the sys. is causal.

(b) when the sys is ~~NC~~ stable.

(c) Neither stable nor causal.

Soln →

$$s^2Y(s) - sY(s) - 2Y(s) = X(s)$$

$$Y(s)[s^2 - s - 2] = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 - s - 2}$$

$$= \frac{1}{}$$

$$H(S) = \frac{\left(\frac{1}{3}\right)}{(S-2)} + \frac{\left(\frac{1}{-3}\right)}{(S+1)}$$

Poles:- $-1, 2$

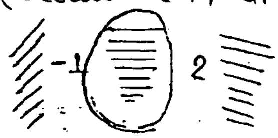
(i) when the sys is causal $h(t)$ will be Right sided.

$$h(t) = -\frac{1}{3}e^{-t}u(t) + \frac{1}{3}e^{2t}u(t)$$

(ii) When sys is stable. $h(t)$ will be both sided.

ROC:- $-1<\sigma<2$ (because img axis is included)

Poles:-



$S+1 \longrightarrow$ Right sided.

$S-2 \longrightarrow$ Left sided.

$$h(t) = -\frac{1}{3}e^{-t}u(t) - \frac{1}{3}e^{2t}u(-t)$$

(iii) ROC: $\sigma<-1$; $h(t)$ will be left sided.

$$h(t) = \left(\frac{1}{3}\right)e^{-t}u(-t) - \frac{1}{3}e^{2t}u(-t)$$

<u>Que</u> → For diff eqn

$$\frac{d^2y(t)}{dt^2} + \frac{6dy(t)}{dt} + 8y(t) = 0$$

with initial condn $y(0^-)=1$, $y'(0^-)=0$; the soln of $y(t)$ is:

(a) $2e^{-2t}-e^{-4t}$ (b) $2e^{-6t}-e^{-2t}$ (c) $-e^{-6t}+2e^{-4t}$ (d) $e^{-2t}+2e^{-4t}$

<u>Soln.</u>

$$\cancel{s^2Y(s)+6Y(s)s+8Y(s)=0}$$

$$\cancel{Y(s)(s^2+6s+8)=0}$$

By applying LT on given diff eqn

$$[s^2Y(s) - sy(0^-) - y'(0^-)] + 6[sY(s) - y(0^-)] + Y(s) = 0$$

$$s^2Y(s) - s + 6[sY(s) \underset{-1}{}] + Y(s) = 0$$

$$Y(s)[s^2+6s+1] = s+6$$

$$Y(s) = \frac{s+6}{s^2+6s+1} = \frac{s+6}{(s+2)(s+4)}$$

$$Y(s) = \frac{2}{(s+2)} - \frac{1}{(s+4)}$$

$$\boxed{y(t) = 2e^{-2t} - e^{-4t}}$$

Que.→ For DE $\frac{d^2y(t)}{dt^2} + \frac{2dy(t)}{dt} + y(t) = \delta(t)$

with initial condn; $y(0^-) = -2$, $y'(0^-) = 0$

the value of $\left.\frac{dy(t)}{dt}\right|_{t=0^+}$ is

(a) -1 (b) 1 (c) 0 (d) 2

Soln→

$$s^2Y(s) - sy(0^-) - y'(0^-) + 2[sY(s) - y(0^-)] + Y(s) = 1$$

$$Y(s)[s^2+2s+1] - (-2s + 2\times(2)) = 1$$

$$Y(s) = \frac{-2s-3}{s^2+2s+1}$$

$$Y(s) = \frac{-2s-3}{(s+1)^2} = \frac{-2(s+1)-1}{(s+1)^2}$$

$$Y(s) = \frac{-2}{(s+1)} - \frac{1}{(s+1)^2}$$

$$y(t) = -2e^{-t} - te^{-t}$$

$$\frac{dy(t)}{dt} = 2e^{-t} - (e^{-t} - te^{-t})$$

$$\left.\frac{dy(t)}{dt}\right|_{t=0^+} = 2(2) - (e1 - 0)$$

$$\boxed{\text{Ans.} = 1}$$

Que.→ $y(t) = \sum_{n=0}^{\infty} f(t-nT_0)$ Find $Y(s)$ in terms of $F(s)$

Soln→ $y(t) = \sum_{n=0}^{\infty} f(t-nT_0)$

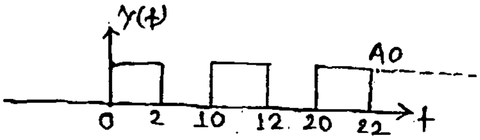
$$y(t) = f(t) + f(t-T_0) + f(t-2T_0) + \cdots$$

$\big\downarrow$ LT

$$Y(s) = F(s)\left[1 + e^{-sT_0} + (e^{-sT_0})^2 + \cdots\right]$$

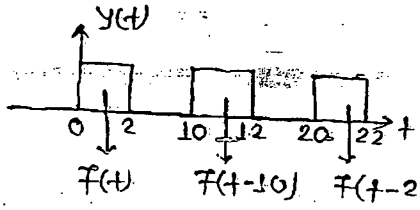
$$\boxed{Y(s) = \frac{F(s)}{1 - e^{-sT_0}}}$$

Que.→



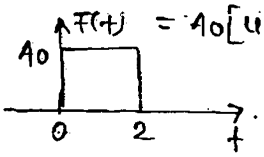
Find $Y(s)$

Soln→



$$y(t) = f(t) + f(t-10) + f(t-2\times10) + \cdots\cdots$$

$$= \sum_{n=0}^{\infty} f(t - nT_0) \; ; \quad T_0 = 10$$



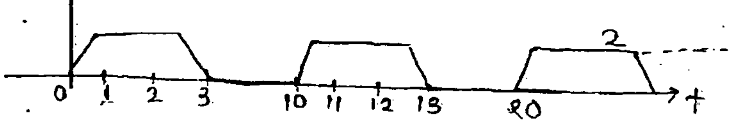
$$f(t) = A_0[u(t) - u(t-2)]$$

$$F(s) = \frac{A_0}{s}(1 - e^{-2s})$$

$$Y(s) = \frac{F(s)}{(1 - e^{-sT_0})}$$

$$\boxed{Y(s) = \frac{A_0(1 - e^{-2s})}{s(1 - e^{-10s})}}$$

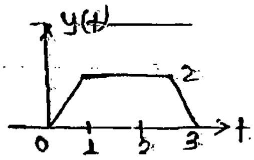
Que→



Find LT of the signal.

Soln→

$$y(t) = \sum_{n=0}^{\infty} f(t - nT_0)$$



$$f(t) = 2r(t) - 2r(t-1) - 2r(t-2) + 2r(t-3)$$

$$F(s) = \frac{2}{s^2} - \frac{2e^{-s}}{s^2} - \frac{2e^{-2s}}{s^2} + \frac{2e^{-3s}}{s^2}$$

$$F(s) = \frac{2}{s^2}\left[1 - e^{-s} + e^{-2s} + e^{-3s}\right]$$

$$Y(s) = \frac{F(s)}{1 - e^{-sT_0}}$$

$$Y(s) = \frac{\frac{2}{s^2}\left(1 - e^{-s} + e^{-2s} + e^{-3s}\right)}{1 - e^{-10s}}$$