Table 15.4

Group (1)	Total number of times a die is thrown in a group (2)	$\frac{\text{Cumulative number of times 1 turned up}}{\text{Total number of times the die is thrown}}$ (3)
1	—	—
2	—	—
3	—	—
4	—	—

The dice used in all the groups should be almost the same in size and appearence. Then all the throws will be treated as throws of the same die.

What do you observe in these tables?

You will find that as the total number of throws gets larger, the fractions in Column (3) move closer and closer to $\frac{1}{6}$.

Activity 4 : (i) Toss two coins simultaneously ten times and record your observations in the form of a table as given below:

Table 15.5

Number of times the two coins are tossed	Number of times no head comes up	Number of times one head comes up	Number of times two heads come up
10	—	—	—

Write down the fractions:

$$A = \frac{\text{Number of times no head comes up}}{\text{Total number of times two coins are tossed}}$$

$$B = \frac{\text{Number of times one head comes up}}{\text{Total number of times two coins are tossed}}$$

$$C = \frac{\text{Number of times two heads come up}}{\text{Total number of times two coins are tossed}}$$

Calculate the values of these fractions.

Now increase the number of tosses (as in Activitiy 2). You will find that the more the number of tosses, the closer are the values of A, B and C to 0.25, 0.5 and 0.25, respectively.

In Activity 1, each toss of a coin is called a *trial*. Similarly in Activity 3, each throw of a die is a *trial*, and each simultaneous toss of two coins in Activity 4 is also a *trial*.

So, *a trial is an action which results in one or several outcomes*. The possible outcomes in Activity 1 were Head and Tail; whereas in Activity 3, the possible outcomes were 1, 2, 3, 4, 5 and 6.

In Activity 1, the getting of a head in a particular throw is an *event with outcome 'head'*. Similarly, *getting a tail is an event with outcome 'tail'*. In Activity 2, the getting of a particular number, say 1, is an *event* with outcome 1.

If our experiment was to throw the die for getting an even number, then the event would consist of three outcomes, namely, 2, 4 and 6.

So, an *event* for an experiment is the collection of some outcomes of the experiment. In Class X, you will study a more formal definition of an event.

So, can you now tell what the events are in Activity 4?

With this background, let us now see what probability is. Based on what we directly observe as the outcomes of our trials, we find the *experimental* or *empirical* probability.

Let n be the total number of trials. The *empirical probability* P(E) of an event E happening, is given by

$$P(E) = \frac{\text{Number of trials in which the event happened}}{\text{The total number of trials}}$$

In this chapter, we shall be finding the empirical probability, though we will write 'probability' for convenience.

Let us consider some examples.

To start with let us go back to Activity 2, and Table 15.2. In Column (4) of this table, what is the fraction that you calculated? Nothing, but it is the empirical probability of getting a head. Note that this probability kept changing depending on the number of trials and the number of heads obtained in these trials. Similarly, the empirical probability of getting a tail is obtained in Column (5) of Table 15.2. This is $\frac{12}{15}$ to start with, then it is $\frac{2}{3}$, then $\frac{28}{45}$, and so on.

So, the empirical probability depends on the number of trials undertaken, and the number of times the outcomes you are looking for coming up in these trials.

Activity 5 : Before going further, look at the tables you drew up while doing Activity 3. Find the probabilities of getting a 3 when throwing a die a certain number of times. Also, show how it changes as the number of trials increases.

Now let us consider some other examples.

Example 1 : A coin is tossed 1000 times with the following frequencies:

Head : 455, Tail : 545

Compute the probability for each event.

Solution : Since the coin is tossed 1000 times, the total number of trials is 1000. Let us call the events of getting a head and of getting a tail as E and F, respectively. Then, the number of times E happens, i.e., the number of times a head come up, is 455.

So, the probability of E = $\frac{\text{Number of heads}}{\text{Total number of trials}}$

i.e., $P(E) = \frac{455}{1000} = 0.455$

Similarly, the probability of the event of getting a tail = $\frac{\text{Number of tails}}{\text{Total number of trials}}$

i.e., $P(F) = \frac{545}{1000} = 0.545$

Note that in the example above, $P(E) + P(F) = 0.455 + 0.545 = 1$, and E and F are the only two possible outcomes of each trial.

Example 2 : Two coins are tossed simultaneously 500 times, and we get

Two heads : 105 times

One head : 275 times

No head : 120 times

Find the probability of occurrence of each of these events.

Solution : Let us denote the events of getting two heads, one head and no head by E_1, E_2 and E_3, respectively. So,

$$P(E_1) = \frac{105}{500} = 0.21$$

$$P(E_2) = \frac{275}{500} = 0.55$$

$$P(E_3) = \frac{120}{500} = 0.24$$

Observe that $P(E_1) + P(E_2) + P(E_3) = 1$. Also E_1, E_2 and E_3 cover all the outcomes of a trial.

Example 3 : A die is thrown 1000 times with the frequencies for the outcomes 1, 2, 3, 4, 5 and 6 as given in the following table :

Table 15.6

Outcome	1	2	3	4	5	6
Frequency	179	150	157	149	175	190

Find the probability of getting each outcome.

Solution : Let E_i denote the event of getting the outcome i, where $i = 1, 2, 3, 4, 5, 6$. Then

Probability of the outcome 1 = $P(E_1) = \dfrac{\text{Frequency of 1}}{\text{Total number of times the die is thrown}}$

$$= \frac{179}{1000} = 0.179$$

Similarly, $P(E_2) = \dfrac{150}{1000} = 0.15$, $P(E_3) = \dfrac{157}{1000} = 0.157$,

$P(E_4) = \dfrac{149}{1000} = 0.149$, $P(E_5) = \dfrac{175}{1000} = 0.175$

and $P(E_6) = \dfrac{190}{1000} = 0.19$.

Note that $P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) + P(E_6) = 1$

Also note that:

(i) The probability of each event lies between 0 and 1.

(ii) The sum of all the probabilities is 1.

(iii) $E_1, E_2, \ldots, E_6$ cover all the possible outcomes of a trial.

Example 4 : On one page of a telephone directory, there were 200 telephone numbers. The frequency distribution of their unit place digit (for example, in the number 25828573, the unit place digit is 3) is given in Table 15.7 :

Table 15.7

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	22	26	22	22	20	10	14	28	16	20

Without looking at the page, the pencil is placed on one of these numbers, i.e., the number is chosen at *random*. What is the probability that the digit in its unit place is 6?

Solution : The probability of digit 6 being in the unit place

$$= \frac{\text{Frequency of 6}}{\text{Total number of selected telephone numbers}}$$

$$= \frac{14}{200} = 0.07$$

You can similarly obtain the empirical probabilities of the occurrence of the numbers having the other digits in the unit place.

Example 5 : The record of a weather station shows that out of the past 250 consecutive days, its weather forecasts were correct 175 times.

(i) What is the probability that on a given day it was correct?

(ii) What is the probability that it was not correct on a given day?

Solution : The total number of days for which the record is available = 250

(i) P(the forecast was correct on a given day)

$$= \frac{\text{Number of days when the forecast was correct}}{\text{Total number of days for which the record is available}}$$

$$= \frac{175}{250} = 0.7$$

(ii) The number of days when the forecast was not correct = 250 – 175 = 75

So, P(the forecast was not correct on a given day) = $\frac{75}{250}$ = 0.3

Notice that:

P(forecast was correct on a given day) + P(forecast was not correct on a given day)

$$= 0.7 + 0.3 = 1$$

Example 6 : A tyre manufacturing company kept a record of the distance covered before a tyre needed to be replaced. The table shows the results of 1000 cases.

Table 15.8

Distance (in km)	less than 4000	4000 to 9000	9001 to 14000	more than 14000
Frequency	20	210	325	445

If you buy a tyre of this company, what is the probability that :

(i) it will need to be replaced before it has covered 4000 km?

(ii) it will last more than 9000 km?

(iii) it will need to be replaced after it has covered somewhere between 4000 km and 14000 km?

Solution : (i) The total number of trials = 1000.

The frequency of a tyre that needs to be replaced before it covers 4000 km is 20.

So, P(tyre to be replaced before it covers 4000 km) = $\frac{20}{1000} = 0.02$

(ii) The frequency of a tyre that will last more than 9000 km is 325 + 445 = 770

So, P(tyre will last more than 9000 km) = $\frac{770}{1000} = 0.77$

(iii) The frequency of a tyre that requires replacement between 4000 km and 14000 km is 210 + 325 = 535.

So, P(tyre requiring replacement between 4000 km and 14000 km) = $\frac{535}{1000} = 0.535$

Example 7 : The percentage of marks obtained by a student in the monthly unit tests are given below:

Table 15.9

Unit test	I	II	III	IV	V
Percentage of marks obtained	69	71	73	68	74

Based on this data, find the probability that the student gets more than 70% marks in a unit test.

Solution : The total number of unit tests held is 5.
The number of unit tests in which the student obtained more than 70% marks is 3.

So, P(scoring more than 70% marks) = $\frac{3}{5}$ = 0.6

Example 8 : An insurance company selected 2000 drivers at random (i.e., without any preference of one driver over another) in a particular city to find a relationship between age and accidents. The data obtained are given in the following table:

Table 15.10

Age of drivers (in years)	Accidents in one year				
	0	**1**	**2**	**3**	**over 3**
18 - 29	440	160	110	61	35
30 - 50	505	125	60	22	18
Above 50	360	45	35	15	9

Find the probabilities of the following events for a driver chosen at random from the city:

(i) being 18-29 years of age *and* having exactly 3 accidents in one year.

(ii) being 30-50 years of age *and* having one or more accidents in a year.

(iii) having no accidents in one year.

Solution : Total number of drivers = 2000.

(i) The number of drivers who are 18-29 years old and have exactly 3 accidents in one year is 61.

So, P (driver is 18-29 years old with exactly 3 accidents) = $\frac{61}{2000}$

$= 0.0305 \approx 0.031$

(ii) The number of drivers 30-50 years of age and having one or more accidents in one year = 125 + 60 + 22 + 18 = 225

So, P(driver is 30-50 years of age and having one or more accidents)

$= \frac{225}{2000} = 0.1125 \approx 0.113$

(iii) The number of drivers having no accidents in one year = 440 + 505 + 360

= 1305

Therefore, P(drivers with no accident) = $\frac{1305}{2000}$ = 0.653

Example 9 : Consider the frequency distribution table (Table 14.3, Example 4, Chapter 14), which gives the weights of 38 students of a class.

(i) Find the probability that the weight of a student in the class lies in the interval 46-50 kg.

(ii) Give two events in this context, one having probability 0 and the other having probability 1.

Solution : (i) The total number of students is 38, and the number of students with weight in the interval 46 - 50 kg is 3.

So, P(weight of a student is in the interval 46 - 50 kg) = $\frac{3}{38}$ = 0.079

(ii) For instance, consider the event that a student weighs 30 kg. Since no student has this weight, the probability of occurrence of this event is 0. Similarly, the probability of a student weighing more than 30 kg is $\frac{38}{38}$ = 1.

Example 10 : Fifty seeds were selected at random from each of 5 bags of seeds, and were kept under standardised conditions favourable to germination. After 20 days, the number of seeds which had germinated in each collection were counted and recorded as follows:

Table 15.11

Bag	1	2	3	4	5
Number of seeds germinated	40	48	42	39	41

What is the probability of germination of

(i) more than 40 seeds in a bag?

(ii) 49 seeds in a bag?

(iii) more that 35 seeds in a bag?

Solution : Total number of bags is 5.

(i) Number of bags in which more than 40 seeds germinated out of 50 seeds is 3.

P(germination of more than 40 seeds in a bag) = $\frac{3}{5}$ = 0.6

(ii) Number of bags in which 49 seeds germinated = 0.

P(germination of 49 seeds in a bag) = $\frac{0}{5} = 0$.

(iii) Number of bags in which more than 35 seeds germinated = 5.

So, the required probability = $\frac{5}{5} = 1$.

Remark : In all the examples above, you would have noted that the probability of an event can be any fraction from 0 to 1.

EXERCISE 15.1

1. In a cricket match, a batswoman hits a boundary 6 times out of 30 balls she plays. Find the probability that she did not hit a boundary.

2. 1500 families with 2 children were selected randomly, and the following data were recorded:

Number of girls in a family	2	1	0
Number of families	475	814	211

Compute the probability of a family, chosen at random, having

(i) 2 girls (ii) 1 girl (iii) No girl

Also check whether the sum of these probabilities is 1.

3. Refer to Example 5, Section 14.4, Chapter 14. Find the probability that a student of the class was born in August.

4. Three coins are tossed simultaneously 200 times with the following frequencies of different outcomes:

Outcome	3 heads	2 heads	1 head	No head
Frequency	23	72	77	28

If the three coins are simultaneously tossed again, compute the probability of 2 heads coming up.

5. An organisation selected 2400 families at random and surveyed them to determine a relationship between income level and the number of vehicles in a family. The

information gathered is listed in the table below:

Monthly income (in ₹)	Vehicles per family			
	0	1	2	Above 2
Less than 7000	10	160	25	0
7000 – 10000	0	305	27	2
10000 – 13000	1	535	29	1
13000 – 16000	2	469	59	25
16000 or more	1	579	82	88

Suppose a family is chosen. Find the probability that the family chosen is

(i) earning ₹ 10000 – 13000 per month and owning exactly 2 vehicles.

(ii) earning ₹ 16000 or more per month and owning exactly 1 vehicle.

(iii) earning less than ₹ 7000 per month and does not own any vehicle.

(iv) earning ₹ 13000 – 16000 per month and owning more than 2 vehicles.

(v) owning not more than 1 vehicle.

6. Refer to Table 14.7, Chapter 14.

(i) Find the probability that a student obtained less than 20% in the mathematics test.

(ii) Find the probability that a student obtained marks 60 or above.

7. To know the opinion of the students about the subject *statistics*, a survey of 200 students was conducted. The data is recorded in the following table.

Opinion	Number of students
like	135
dislike	65

Find the probability that a student chosen at random

(i) likes statistics, (ii) does not like it.

8. Refer to Q.2, Exercise 14.2. What is the empirical probability that an engineer lives:

(i) less than 7 km from her place of work?

(ii) more than or equal to 7 km from her place of work?

(iii) within $\frac{1}{2}$ km from her place of work?

9. **Activity :** Note the frequency of two-wheelers, three-wheelers and four-wheelers going past during a time interval, in front of your school gate. Find the probability that any one vehicle out of the total vehicles you have observed is a two-wheeler.

10. **Activity :** Ask all the students in your class to write a 3-digit number. Choose any student from the room at random. What is the probability that the number written by her/him is divisible by 3? Remember that a number is divisible by 3, if the sum of its digits is divisible by 3.

11. Eleven bags of wheat flour, each marked 5 kg, actually contained the following weights of flour (in kg):

    4.97 5.05 5.08 5.03 5.00 5.06 5.08 4.98 5.04 5.07 5.00

    Find the probability that any of these bags chosen at random contains more than 5 kg of flour.

12. In Q.5, Exercise 14.2, you were asked to prepare a frequency distribution table, regarding the concentration of sulphur dioxide in the air in parts per million of a certain city for 30 days. Using this table, find the probability of the concentration of sulphur dioxide in the interval 0.12 - 0.16 on any of these days.

13. In Q.1, Exercise 14.2, you were asked to prepare a frequency distribution table regarding the blood groups of 30 students of a class. Use this table to determine the probability that a student of this class, selected at random, has blood group AB.

15.3 Summary

In this chapter, you have studied the following points:

1. An event for an experiment is the collection of some outcomes of the experiment.
2. The empirical (or experimental) probability P(E) of an event E is given by

$$\text{P(E)} = \frac{\text{Number of trials in which E has happened}}{\text{Total number of trials}}$$

3. The Probability of an event lies between 0 and 1 (0 and 1 inclusive).

APPENDIX 2

INTRODUCTION TO MATHEMATICAL MODELLING

A2.1 Introduction

Right from your earlier classes, you have been solving problems related to the real-world around you. For example, you have solved problems in simple interest using the formula for finding it. The formula (or equation) is a relation between the interest and the other three quantities that are related to it, the principal, the rate of interest and the period. This formula is an example of a **mathematical model**. A **mathematical model** is a mathematical relation that describes some real-life situation.

Mathematical models are used to solve many real-life situations like:

- launching a satellite.
- predicting the arrival of the monsoon.
- controlling pollution due to vehicles.
- reducing traffic jams in big cities.

In this chapter, we will introduce you to the process of constructing mathematical models, which is called **mathematical modelling**. In mathematical modelling, we take a real-world problem and write it as an equivalent mathematical problem. We then solve the mathematical problem, and interpret its solution in terms of the real-world problem. After this we see to what extent the solution is valid in the context of the real-world problem. So, the *stages* involved in mathematical modelling are formulation, solution, interpretation and validation.

We will start by looking at the process you undertake when solving word problems, in Section A2.2. Here, we will discuss some word problems that are similar to the ones you have solved in your earlier classes. We will see later that the steps that are used for solving word problems are some of those used in mathematical modelling also.

In the next section, that is Section A2.3, we will discuss some simple models.

In Section A2.4, we will discuss the overall process of modelling, its advantages and some of its limitations.

A2.2 Review of Word Problems

In this section, we will discuss some word problems that are similar to the ones that you have solved in your earlier classes. Let us start with a problem on direct variation.

Example 1 : I travelled 432 kilometres on 48 litres of petrol in my car. I have to go by my car to a place which is 180 km away. How much petrol do I need?

Solution : We will list the steps involved in solving the problem.

Step 1 : Formulation : You know that farther we travel, the more petrol we require, that is, the amount of petrol we need varies directly with the distance we travel.

Petrol needed for travelling 432 km = 48 litres

Petrol needed for travelling 180 km = ?

Mathematical Description : Let

x = distance I travel

y = petrol I need

y varies directly with x.

So, $y = kx$, where k is a constant.

I can travel 432 kilometres with 48 litres of petrol.

So, $y = 48, x = 432$.

Therefore, $k = \dfrac{y}{x} = \dfrac{48}{432} = \dfrac{1}{9}$.

Since $y = kx$,

therefore, $$y = \frac{1}{9}x \qquad (1)$$

Equation or Formula (1) describes the relationship between the petrol needed and distance travelled.

Step 2 : Solution : We want to find the petrol we need to travel 180 kilometres; so, we have to find the value of y when $x = 180$. Putting $x = 180$ in (1), we have

$$y = \frac{180}{9} = 20.$$

Step 3 : **Interpretation :** Since $y = 20$, we need 20 litres of petrol to travel 180 kilometres.

Did it occur to you that you may not be able to use the formula (1) in all situations? For example, suppose the 432 kilometres route is through mountains and the 180 kilometres route is through flat plains. The car will use up petrol at a faster rate in the first route, so we cannot use the same rate for the 180 kilometres route, where the petrol will be used up at a slower rate. So the formula works if all such conditions that affect the rate at which petrol is used are the same in both the trips. Or, if there is a difference in conditions, the effect of the difference on the amount of petrol needed for the car should be very small. The petrol used will vary directly with the distance travelled only in such a situation. We assumed this while solving the problem.

Example 2 : Suppose Sudhir has invested ₹ 15,000 at 8% simple interest per year. With the return from the investment, he wants to buy a washing machine that costs ₹ 19,000. For what period should he invest ₹ 15,000 so that he has enough money to buy a washing machine?

Solution : Step 1 : **Formulation of the problem :** Here, we know the principal and the rate of interest. The interest is the amount Sudhir needs in addition to 15,000 to buy the washing machine. We have to find the number of years.

Mathematical Description : The formula for simple interest is $I = \frac{Pnr}{100}$,

where P = Principal,

n = Number of years,

r % = Rate of interest

I = Interest earned

Here, the principal = ₹ 15,000

The money required by Sudhir for buying a washing machine = ₹ 19,000

So, the interest to be earned = ₹ (19,000 – 15,000)

= ₹ 4,000

The number of years for which ₹ 15,000 is deposited = n

The interest on ₹ 15,000 for n years at the rate of 8% = I

Then, $$I = \frac{15000 \times n \times 8}{100}$$

So, $$\text{I} = 1200n \qquad (1)$$

gives the relationship between the number of years and interest, if ₹ 15000 is invested at an annual interest rate of 8%.

We have to find the period in which the interest earned is ₹ 4000. Putting I = 4000 in (1), we have

$$4000 = 1200n \qquad (2)$$

Step 2 : Solution of the problem : Solving Equation (2), we get

$$n = \frac{4000}{1200} = 3\frac{1}{3}.$$

Step 3 : Interpretation : Since $n = 3\frac{1}{3}$ and one third of a year is 4 months, Sudhir can buy a washing machine after 3 years and 4 months.

Can you guess the assumptions that you have to make in the example above? We have to assume that the interest rate remains the same for the period for which we calculate the interest. Otherwise, the formula $\text{I} = \frac{\text{P}nr}{100}$ will not be valid. We have also assumed that the price of the washing machine does not increase by the time Sudhir has gathered the money.

Example 3 : A motorboat goes upstream on a river and covers the distance between two towns on the riverbank in six hours. It covers this distance downstream in five hours. If the speed of the stream is 2 km/h, find the speed of the boat in still water.

Solution : Step 1 : Formulation : We know the speed of the river and the time taken to cover the distance between two places. We have to find the speed of the boat in still water.

Mathematical Description : Let us write x for the speed of the boat, t for the time taken and y for the distance travelled. Then

$$y = tx \qquad (1)$$

Let d be the distance between the two places.

While going upstream, the actual speed of the boat

= speed of the boat – speed of the river,

because the boat is travelling against the flow of the river.

So, the speed of the boat upstream = $(x - 2)$ km/h

It takes 6 hours to cover the distance between the towns upstream. So, from (1), we get $$d = 6(x - 2) \qquad (2)$$

When going downstream, the speed of the river has to be *added* to the speed of the boat.

So, the speed of the boat downstream = $(x + 2)$ km/h

The boat takes 5 hours to cover the same distance downstream. So,

$$d = 5(x + 2) \quad (3)$$

From (2) and (3), we have

$$5(x + 2) = 6(x - 2) \quad (4)$$

Step 2 : Finding the Solution

Solving for x in Equation (4), we get $x = 22$.

Step 3 : Interpretation

Since $x = 22$, therefore the speed of the motorboat in still water is 22 km/h.

In the example above, we know that the speed of the river is not the same everywhere. It flows slowly near the shore and faster at the middle. The boat starts at the shore and moves to the middle of the river. When it is close to the destination, it will slow down and move closer to the shore. So, there is a small difference between the speed of the boat at the middle and the speed at the shore. Since it will be close to the shore for a small amount of time, this difference in speed of the river will affect the speed only for a small period. So, we can ignore this difference in the speed of the river. We can also ignore the small variations in speed of the boat. Also, apart from the speed of the river, the friction between the water and surface of the boat will also affect the actual speed of the boat. We also assume that this effect is very small.

So, we have assumed that

1. The speed of the river and the boat remains constant all the time.
2. The effect of friction between the boat and water and the friction due to air is negligible.

We have found the speed of the boat in still water with the *assumptions* (*hypotheses*) above.

As we have seen in the word problems above, there are 3 *steps in solving a word problem. These are*

1. **Formulation :** We analyse the problem and see which factors have a major influence on the solution to the problem. These are the **relevant factors**. In our first example, the relevant factors are the distance travelled and petrol consumed. We ignored the other factors like the nature of the route, driving speed, etc. Otherwise, the problem would have been more difficult to solve. The factors that we ignore are the **irrelevant factors**.

We then describe the problem mathematically, in the form of one or more mathematical equations.

2. **Solution :** We find the solution of the problem by solving the mathematical equations obtained in Step 1 using some suitable method.
3. **Interpretation :** We see what the solution obtained in Step 2 means in the context of the original word problem.

Here are some exercises for you. You may like to check your understanding of the steps involved in solving word problems by carrying out the three steps above for the following problems.

EXERCISE A 2.1

In each of the following problems, clearly state what the relevant and irrelevant factors are while going through Steps 1, 2 and 3 given above.

1. Suppose a company needs a computer for some period of time. The company can either hire a computer for ₹ 2,000 per month or buy one for ₹ 25,000. If the company has to use the computer for a long period, the company will pay such a high rent, that buying a computer will be cheaper. On the other hand, if the company has to use the computer for say, just one month, then hiring a computer will be cheaper. Find the number of months beyond which it will be cheaper to buy a computer.
2. Suppose a car starts from a place A and travels at a speed of 40 km/h towards another place B. At the same instance, another car starts from B and travels towards A at a speed of 30 km/h. If the distance between A and B is 100 km, after how much time will the cars meet?
3. The moon is about 3,84,000 km from the earth, and its path around the earth is nearly circular. Find the speed at which it orbits the earth, assuming that it orbits the earth in 24 hours. (Use $\pi = 3.14$)
4. A family pays ₹ 1000 for electricity on an average in those months in which it does not use a water heater. In the months in which it uses a water heater, the average electricity bill is ₹ 1240. The cost of using the water heater is ₹ 8.00 per hour. Find the average number of hours the water heater is used in a day.

A2.3 Some Mathematical Models

So far, nothing was new in our discussion. In this section, we are going to add another step to the three steps that we have discussed earlier. This step is called *validation.* What does validation mean? Let us see. In a real-life situation, we cannot accept a model that gives us an answer that does not match the reality. This process of checking the answer against reality, and modifying the mathematical description if necessary, is called *validation*. This is a very important step in modelling. We will introduce you to this step in this section.

First, let us look at an example, where we do not have to modify our model after validation.

Example 4 : Suppose you have a room of length 6 m and breadth 5 m. You want to cover the floor of the room with square mosaic tiles of side 30 cm. How many tiles will you need? Solve this by constructing a mathematical model.

Solution : Formulation : We have to consider the area of the room and the area of a tile for solving the problem. The side of the tile is 0.3 m. Since the length is 6 m, we can fit in $\frac{6}{0.3} = 20$ tiles along the length of the room in one row (see Fig. A2.1.).

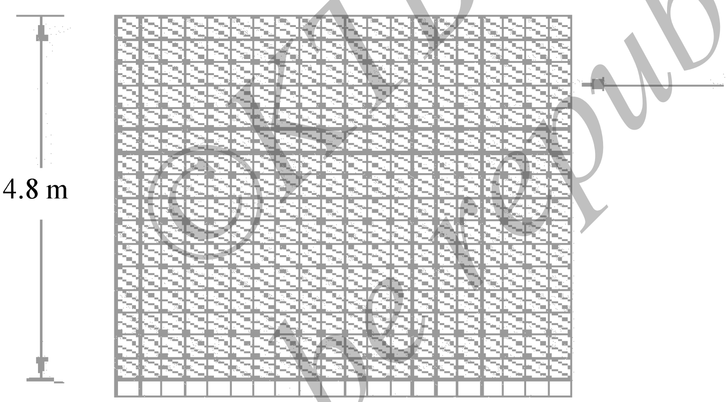


Fig. A2.1

Since the breadth of the room is 5 metres, we have $\frac{5}{0.3} = 16.67$. So, we can fit in 16 tiles in a column. Since $16 \times 0.3 = 4.8$, $5 - 4.8 = 0.2$ metres along the breadth will not be covered by tiles. This part will have to be covered by cutting the other tiles. The breadth of the floor left uncovered, 0.2 metres, is more than half the length of a tile, which is 0.3 m. So we cannot break a tile into two equal halves and use both the halves to cover the remaining portion.

Mathematical Description : We have:

Total number of tiles required = (Number of tiles along the length × Number of tiles along the breadth) + Number of tiles along the uncovered area (1)

Solution : As we said above, the number of tiles along the length is 20 and the number of tiles along the breadth is 16. We need 20 more tiles for the last row. Substituting these values in (1), we get $(20 \times 16) + 20 = 320 + 20 = 340$.

Interpretation : We need 340 tiles to cover the floor.

Validation : In real-life, your mason may ask you to buy some extra tiles to replace those that get damaged while cutting them to size. This number will of course depend upon the skill of your mason! But, we need not modify Equation (1) for this. This gives you a rough idea of the number of tiles required. So, we can stop here.

Let us now look at another situation now.

Example 5 : In the year 2000, 191 member countries of the U.N. signed a declaration. In this declaration, the countries agreed to achieve certain development goals by the year 2015. These are called the *millennium development goals*. One of these goals is to promote gender equality. One indicator for deciding whether this goal has been achieved is the ratio of girls to boys in primary, secondary and tertiary education. India, as a signatory to the declaration, is committed to improve this ratio. The data for the percentage of girls who are enrolled in primary schools is given in Table A2.1.

Table A2.1

Year	Enrolment (in %)
1991-92	41.9
1992-93	42.6
1993-94	42.7
1994-95	42.9
1995-96	43.1
1996-97	43.2
1997-98	43.5
1998-99	43.5
1999-2000	43.6*
2000-01	43.7*
2001-02	44.1*

Source : *Educational statistics, webpage of Department of Education, GOI.*

* *indicates that the data is provisional.*

Using this data, mathematically describe the rate at which the proportion of girls enrolled in primary schools grew. Also, estimate the year by which the enrolment of girls will reach 50%.

Solution : Let us first convert the problem into a mathematical problem.

Step 1 : Formulation : Table A2.1 gives the enrolment for the years 1991-92, 1992-93, etc. Since the students join at the beginning of an academic year, we can take the years as 1991, 1992, etc. Let us assume that the percentage of girls who join primary schools will continue to grow at the same rate as the rate in Table A2.1. So, the number of years is important, not the specific years. (To give a similar situation, when we find the simple interest for, say, ₹ 1500 at the rate of 8% for three years, it does not matter whether the three-year period is from 1999 to 2002 or from 2001 to 2004. What is important is the interest rate in the years being considered). Here also, we will see how the enrolment grows after 1991 by comparing the number of years that has passed after 1991 and the enrolment. Let us take 1991 as the 0th year, and write 1 for 1992 since 1 year has passed in 1992 after 1991. Similarly, we will write 2 for 1993, 3 for 1994, etc. So, Table A2.1 will now look like as Table A2.2.

Table A2.2

Year	Enrolment (in %)
0	41.9
1	42.6
2	42.7
3	42.9
4	43.1
5	43.2
6	43.5
7	43.5
8	43.6
9	43.7
10	44.1

The increase in enrolment is given in the following table :

Table A2.3

Year	Enrolment (in %)	Increase
0	41.9	0
1	42.6	0.7
2	42.7	0.1
3	42.9	0.2
4	43.1	0.2
5	43.2	0.1
6	43.5	0.3
7	43.5	0
8	43.6	0.1
9	43.7	0.1
10	44.1	0.4

At the end of the one-year period from 1991 to 1992, the enrolment has increased by 0.7% from 41.9% to 42.6%. At the end of the second year, this has increased by 0.1%, from 42.6% to 42.7%. From the table above, we cannot find a definite relationship between the number of years and percentage. But the increase is fairly steady. Only in the first year and in the 10th year there is a jump. The mean of the values is

$$\frac{0.7 + 0.1 + 0.2 + 0.2 + 0.1 + 0.3 + 0 + 0.1 + 0.1 + 0.4}{10} = 0.22$$

Let us assume that the enrolment steadily increases at the rate of 0.22 per cent.

Mathematical Description : We have assumed that the enrolment increases steadily at the rate of 0.22% per year.

So, the Enrolment Percentage (EP) in the first year = 41.9 + 0.22

EP in the second year = 41.9 + 0.22 + 0.22 = 41.9 + 2 × 0.22

EP in the third year = 41.9 + 0.22 + 0.22 + 0.22 = 41.9 + 3 × 0.22

So, the enrolment percentage in the nth year = $41.9 + 0.22n$, for $n \geq 1$. (1)

Now, we also have to find the number of years by which the enrolment will reach 50%. So, we have to find the value of n in the equation or formula

$$50 = 41.9 + 0.22n \qquad (2)$$

Step 2 : Solution : Solving (2) for n, we get

$$n = \frac{50 - 41.9}{0.22} = \frac{8.1}{0.22} = 36.8$$

Step 3 : Interpretation : Since the number of years is an integral value, we will take the next higher integer, 37. So, the enrolment percentage will reach 50% in 1991 + 37 = 2028.

In a word problem, we generally stop here. But, since we are dealing with a real-life situation, we have to see to what extent this value matches the real situation.

Step 4 : Validation: Let us check if Formula (2) is in agreement with the reality. Let us find the values for the years we already know, using Formula (2), and compare it with the known values by finding the difference. The values are given in Table A2.4.

Table A2.4

Year	Enrolment (in %)	Values given by (2) (in %)	Difference (in %)
0	41.9	41.90	0
1	42.6	42.12	0.48
2	42.7	42.34	0.36
3	42.9	42.56	0.34
4	43.1	42.78	0.32
5	43.2	43.00	0.20
6	43.5	43.22	0.28
7	43.5	43.44	0.06
8	43.6	43.66	–0.06
9	43.7	43.88	–0.18
10	44.1	44.10	0.00

As you can see, some of the values given by Formula (2) are less than the actual values by about 0.3% or even by 0.5%. This can give rise to a difference of about 3 to 5 years since the increase per year is actually 1% to 2%. We may decide that this much of a difference is acceptable and stop here. In this case, (2) is our mathematical model.

Suppose we decide that this error is quite large, and we have to improve this model. Then we have to go back to Step 1, the formulation, and change Equation (2). Let us do so.

Step 1 : **Reformulation :** We still assume that the values increase steadily by 0.22%, but we will now introduce a correction factor to reduce the error. For this, we find the mean of all the errors. This is

$$\frac{0+0.48+0.36+0.34+0.32+0.2+0.28+0.06-0.06-0.18+0}{10}=0.18$$

We take the mean of the errors, and correct our formula by this value.

Revised Mathematical Description : Let us now add the mean of the errors to our formula for enrolment percentage given in (2). So, our corrected formula is:

Enrolment percentage in the nth year = 41.9 + 0.22n + 0.18 = 42.08 + 0.22n, for $n \geq 1$ (3)

We will also modify Equation (2) appropriately. The new equation for n is:

$$50 = 42.08 + 0.22n \qquad (4)$$

Step 2 : **Altered Solution :** Solving Equation (4) for n, we get

$$n = \frac{50 - 42.08}{0.22} = \frac{7.92}{0.22} = 36$$

Step 3 : **Interpretation:** Since n = 36, the enrolment of girls in primary schools will reach 50% in the year 1991 + 36 = 2027.

Step 4 : **Validation:** Once again, let us compare the values got by using Formula (4) with the actual values. Table A2.5 gives the comparison.

Table A2.5

Year	Enrolment (in %)	Values given by (2)	Difference between values	Values given by (4)	Difference between values
0	41.9	41.90	0	41.9	0
1	42.6	42.12	0.48	42.3	0.3
2	42.7	42.34	0.36	42.52	0.18
3	42.9	42.56	0.34	42.74	0.16
4	43.1	42.78	0.32	42.96	0.14
5	43.2	43.00	0.2	43.18	0.02
6	43.5	43.22	0.28	43.4	0.1
7	43.5	43.44	0.06	43.62	– 0.12
8	43.6	43.66	– 0.06	43.84	– 0.24
9	43.7	43.88	– 0.18	44.06	– 0.36
10	44.1	44.10	0	44.28	– 0.18

As you can see, many of the values that (4) gives are closer to the actual value than the values that (2) gives. The mean of the errors is 0 in this case.

We will stop our process here. So, Equation (4) is our mathematical description that gives a mathematical relationship between years and the percentage of enrolment of girls of the total enrolment. We have constructed a mathematical model that describes the growth.

The process that we have followed in the situation above is called mathematical modelling.

We have tried to construct a mathematical model with the mathematical tools that we already have. There are better mathematical tools for making predictions from the data we have. But, they are beyond the scope of this course. Our aim in constructing this model is to explain the process of modelling to you, not to make accurate predictions at this stage.

You may now like to model some real-life situations to check your understanding of our discussion so far. Here is an Exercise for you to try.

EXERCISE A2.2

1. We have given the timings of the gold medalists in the 400-metre race from the time the event was included in the Olympics, in the table below. Construct a mathematical model relating the years and timings. Use it to estimate the timing in the next Olympics.

Table A2.6

Year	Timing (in seconds)
1964	52.01
1968	52.03
1972	51.08
1976	49.28
1980	48.88
1984	48.83
1988	48.65
1992	48.83
1996	48.25
2000	49.11
2004	49.41

A2.4 The Process of Modelling, its Advantages and Limitations

Let us now conclude our discussion by drawing out aspects of mathematical modelling that show up in the examples we have discussed. With the background of the earlier sections, we are now in a position to give a brief overview of the steps involved in modelling.

Step 1 : Formulation : You would have noticed the difference between the formulation part of Example 1 in Section A2.2 and the formulation part of the model we discussed in A2.3. In Example 1, all the information is in a readily usable form. But, in the model given in A2.3 this is not so. Further, it took us some time to find a mathematical description. We tested our first formula, but found that it was not as good as the second one we got. This is usually true in general, i.e. when trying to model real-life situations; the first model usually needs to be revised. When we are solving a real-life problem, formulation can require a lot of time. For example, Newton's three laws of motion, which are mathematical descriptions of motion, are simple enough to state. But, Newton arrived at these laws after studying a large amount of data and the work the scientists before him had done.

Formulation involves the following three steps :

(i) Stating the problem : Often, the problem is stated vaguely. For example, the broad goal is to ensure that the enrolment of boys and girls are equal. This may mean that 50% of the total number of boys of the school-going age and 50% of the girls of the school-going age should be enrolled. The other way is to ensure that 50% of the school-going children are girls. In our problem, we have used the second approach.

(ii) Identifying relevant factors : Decide which quantities and relationships are important for our problem and which are unimportant and can be neglected. For example, in our problem regarding primary schools enrolment, the percentage of girls enrolled in the previous year can influence the number of girls enrolled this year. This is because, as more and more girls enrol in schools, many more parents will feel they also have to put their daughters in schools. But, we have ignored this factor because this may become important only after the enrolment crosses a certain percentage. Also, adding this factor may make our model more complicated.

(iii) Mathematical Description : Now suppose we are clear about what the problem is and what aspects of it are more relevant than the others. Then we have to find a relationship between the aspects involved in the form of an equation, a graph or any other suitable mathematical description. If it is an equation, then every important aspect should be represented by a variable in our mathematical equation.

Step 2 : **Finding the solution :** The mathematical formulation does not give the solution. We have to solve this mathematical equivalent of the problem. This is where your mathematical knowledge comes in useful.

Step 3 : **Interpretating the solution :** The mathematical solution is some value or values of the variables in the model. We have to go back to the real-life problem and see what these values mean in the problem.

Step 4 : **Validating the solution :** As we saw in A2.3, after finding the solution we will have to check whether the solution matches the reality. If it matches, then the mathematical model is acceptable. If the mathematical solution does not match, *we go back to the formulation step* again and try to improve our model.

This step in the process is one major difference between solving word problems and mathematical modelling. This is one of the most important step in modelling that is missing in word problems. Of course, it is possible that in some real-life situations, we do not need to validate our answer because the problem is simple and we get the correct solution right away. This was so in the first model we considered in A2.3.

We have given a summary of the order in which the steps in mathematical modelling are carried out in Fig. A2.2 below. Movement from the validation step to the formulation step is shown using a **dotted arrow**. This is because it may not be necessary to carry out this step again.

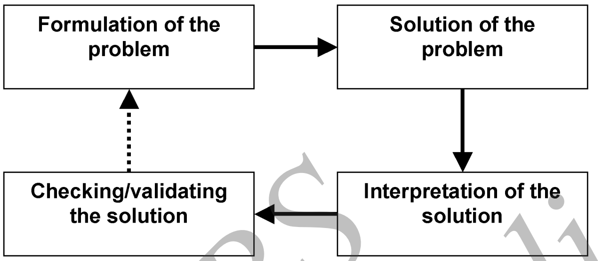


Fig.A2.2

Now that you have studied the stages involved in mathematical modelling, let us discuss some of its aspects.

The *aim* of mathematical modelling is to get some useful information about a real-world problem by converting it into a mathematical problem. This is especially useful when it is not possible or very expensive to get information by other means such as direct observation or by conducting experiments.

You may also wonder why we should undertake mathematical modelling? Let us look at some **advantages of modelling**. Suppose we want to study the corrosive effect of the discharge of the Mathura refinery on the Taj Mahal. We would not like to carry out experiments on the Taj Mahal directly since it may not be safe to do so. Of course, we can use a scaled down physical model, but we may need special facilities for this, which may be expensive. Here is where mathematical modelling can be of great use.

Again, suppose we want to know how many primary schools we will need after 5 years. Then, we can only solve this problem by using a mathematical model. Similarly, it is only through modelling that scientists have been able to explain the existence of so many phenomena.

You saw in Section A2.3, that we could have tried to improve the answer in the second example with better methods. But we stopped because we do not have the mathematical tools. This can happen in real-life also. Often, we have to be satisfied with very approximate answers, because mathematical tools are not available. For example, the model equations used in modelling weather are so complex that mathematical tools to find exact solutions are not available.

You may wonder to what extent we should try to improve our model. Usually, to improve it, we need to take into account more factors. When we do this, we add more variables to our mathematical equations. We may then have a very complicated model that is difficult to use. A model must be simple enough to use. A good model balances two factors:

1. Accuracy, i.e., how close it is to reality.
2. Ease of use.

For example, Newton's laws of motion are very simple, but powerful enough to model many physical situations.

So, is mathematical modelling the answer to all our problems? Not quite! It has its limitations.

Thus, we should keep in mind that a model is *only a simplification* of a real-world problem, and the two are not the same. It is something like the difference between a map that gives the physical features of a country, and the country itself. We can find the height of a place above the sea level from this map, but we cannot find the characteristics of the people from it. So, we should use a model only for the purpose it is supposed to serve, remembering all the factors we have neglected while constructing it. We should apply the model only within the limits where it is applicable. In the later classes, we shall discuss this aspect a little more.

EXERCISE A2.3

1. How are the solving of word problems that you come across in textbooks different from the process of mathematical modelling?

2. Suppose you want to minimise the waiting time of vehicles at a traffic junction of four roads. Which of these factors are important and which are not?
   (i) Price of petrol.
   (ii) The rate at which the vehicles arrive in the four different roads.
   (iii) The proportion of slow-moving vehicles like cycles and rickshaws and fast moving vehicles like cars and motorcycles.

A2.5 Summary

In this Appendix, you have studied the following points :

1. The steps involved in solving word problems.

2. Construction of some mathematical models.

3. The steps involved in mathematical modelling given in the box below.

1. **Formulation :**
   (i) Stating the question
   (ii) Identifying the relevant factors
   (iii) Mathematical description
2. **Finding the solution.**
3. **Interpretation of the solution in the context of the real-world problem.**
4. **Checking/validating to what extent the model is a good representation of the problem being studied.**

4. The aims, advantages and limitations of mathematical modelling.

ANSWERS/HINTS

EXERCISE 8.1

1. $\frac{\sqrt{3}}{4}a^2$, $900\sqrt{3}$ cm^2 **2.** ₹ 1650000 **3.** $20\sqrt{2}$ m^2

4. $21\sqrt{11}$ cm^2 **5.** 9000 cm^2 **6.** $9\sqrt{15}$ cm^2

EXERCISE 8.2

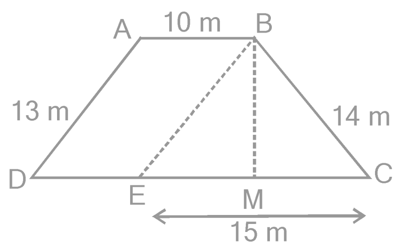
1. 65.5 m^2 (approx.) **2.** 15.2 cm^2 (approx.) **3.** 19.4 cm^2 (approx.)

4. 12 cm **5.** 48 m^2 **6.** $1000\sqrt{6}$ cm^2, $1000\sqrt{6}$ cm^2

7. Area of shade I = Area of shade II = 256 cm^2 and area of shade III = 17.92 cm^2

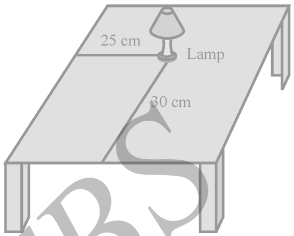
8. ₹ 705.60 **9.** 196 m^2

[See the figure. Find area of Δ BEC = 84 m^2, then find the height BM.]

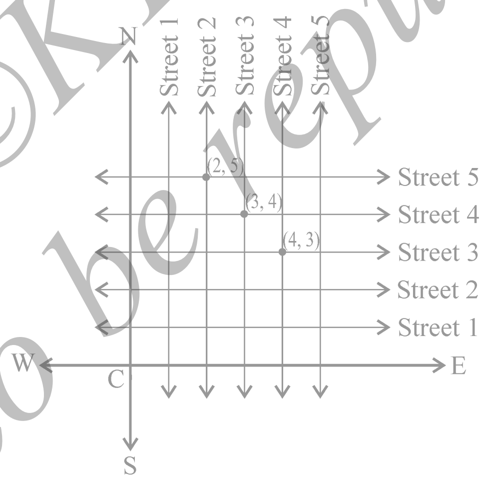


EXERCISE 9.1

1. Consider the lamp as a point and table as a plane. Choose any two perpendicular edges of the table. Measure the distance of the lamp from the longer edge, suppose it is 25 cm. Again, measure the distance of the lamp from the shorter edge, and suppose it is 30 cm. You can write the position of the lamp as (30, 25) or (25, 30), depending on the order you fix.



2. The Street plan is shown in figure given below.



Both the cross-streets are marked in the figure above. They are *uniquely* found because of the two reference lines we have used for locating them.

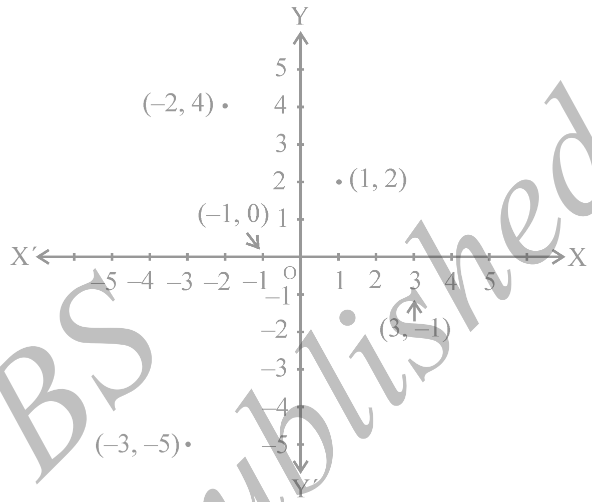
EXERCISE 9.2

1. (i) The x - axis and the y - axis (ii) Quadrants (iii) The origin

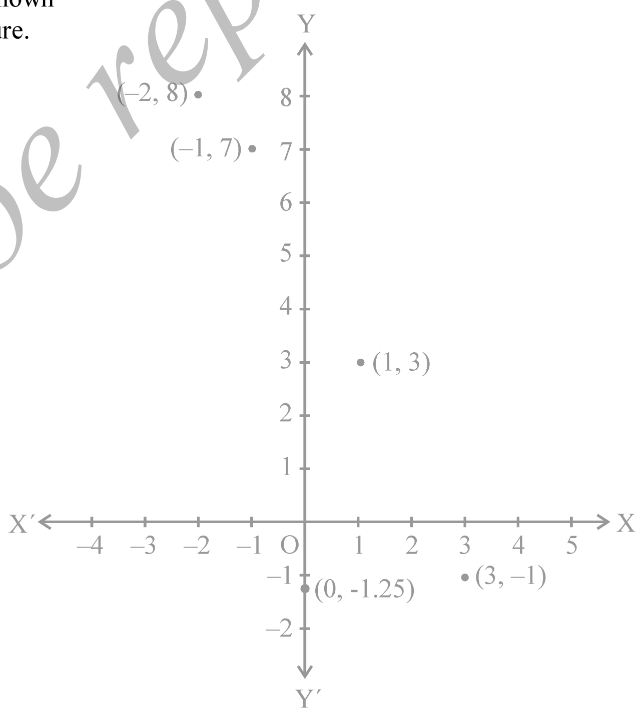
2. (i) (– 5, 2) (ii) (5, –5) (iii) E (iv) G (v) 6 (vi) – 3 (vii) (0, 5) (viii) (–3, 0)

EXERCISE 9.3

1. The point (–2, 4) lies in quadrant II, the point (3, –1) lies in the quadrant IV, the point (– 1, 0) lies on the negative *x* - axis, the point (1, 2) lies in the quadrant I and the point (–3, –5) lies in the quadrant III. Locations of the points are shown in the adjoining figure.



2. Positions of the points are shown by dots in the adjoining figure.



EXERCISE 10.1

1. $x - 2y = 0$

2. (i) $2x + 3y - 9.3\overline{5} = 0$; $a = 2, b = 3, c = -9.3\overline{5}$

   (ii) $x - \frac{y}{5} - 10 = 0$; $a = 1, b = \frac{-1}{5}, c = -10$

   (iii) $-2x + 3y - 6 = 0$; $a = -2, b = 3, c = -6$

   (iv) $1.x - 3y + 0 = 0$; $a = 1, b = -3, c = 0$

   (v) $2x + 5y + 0 = 0$; $a = 2, b = 5, c = 0$

   (vi) $3x + 0.y + 2 = 0$; $a = 3, b = 0, c = 2$

   (vii) $0.x + 1.y - 2 = 0$; $a = 0, b = 1, c = -2$

   (viii) $-2x + 0.y + 5 = 0$; $a = -2, b = 0, c = 5$

EXERCISE 10.2

1. (iii), because for every value of x, there is a corresponding value of y and vice-versa.

2. (i) (0, 7), (1, 5), (2, 3), (4, –1)

   (ii) $(1, 9 - \pi), (0, 9), (-1, 9 + \pi), \left(\frac{9}{\pi}, 0\right)$

   (iii) $(0, 0), (4, 1), (-4, 1), \left(2, \frac{1}{2}\right)$

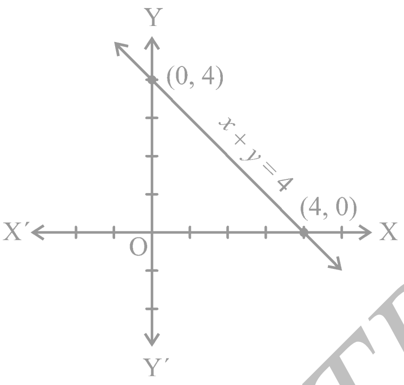
3. (i) No (ii) No

   (iii) Yes (iv) No (v) No

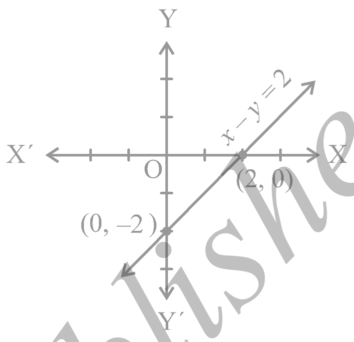
4. 7

EXERCISE 10.3

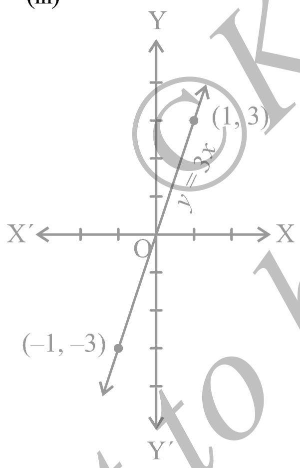
1. (i)



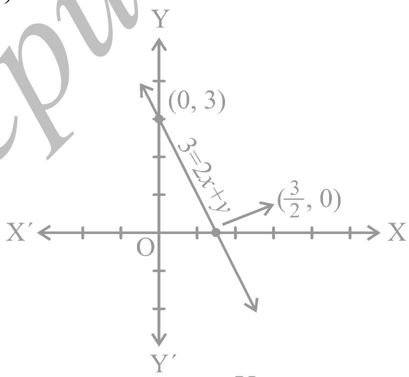
(ii)



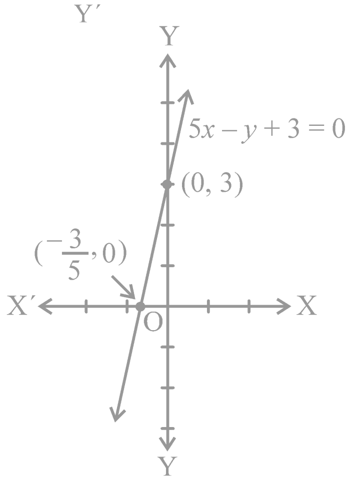
(iii)



(iv)



2. $7x - y = 0$ and $x + y = 16$; infintely many [Through a point infinitely many lines can be drawn]



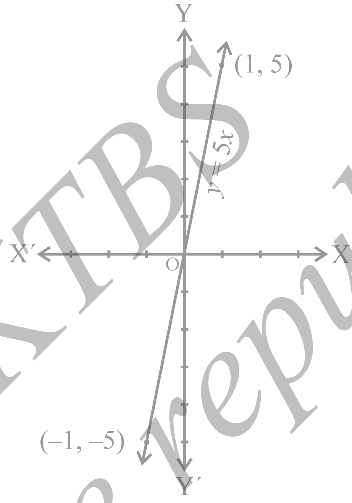
3. $\frac{5}{3}$

4. $5x - y + 3 = 0$

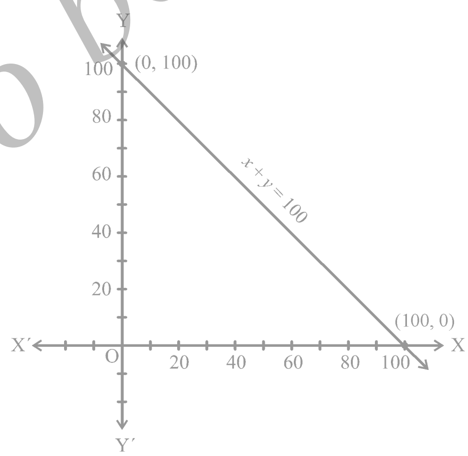
5. For Fig. 4.6, $x + y = 0$ and for Fig. 4.7, $y = -x + 2$.

6. Supposing x is the distance and y is the work done. Therefore according to the problem the equation will be $y = 5x$.

(i) 10 units (ii) 0 unit



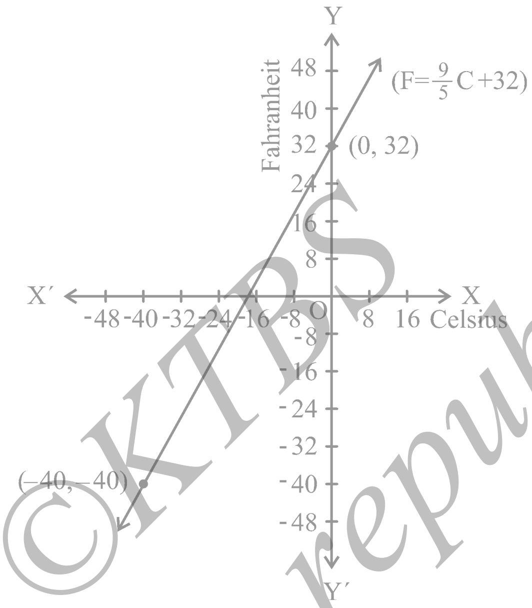
7. $x + y = 100$



8. (i) See adjacent figure. (ii) 86° F

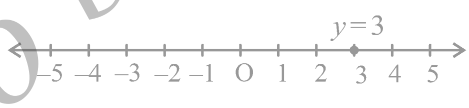
(iii) 35° C (iv) 32° F, –17.8° C (approximately)

(v) Yes, – 40° (both in F and C)

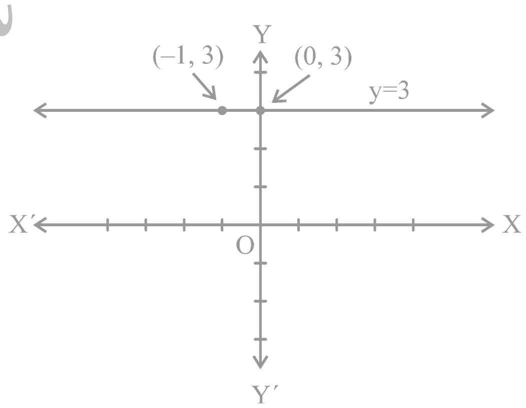


EXERCISE 10.4

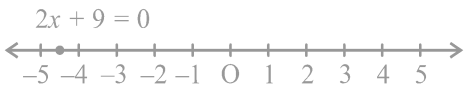
1. (i)



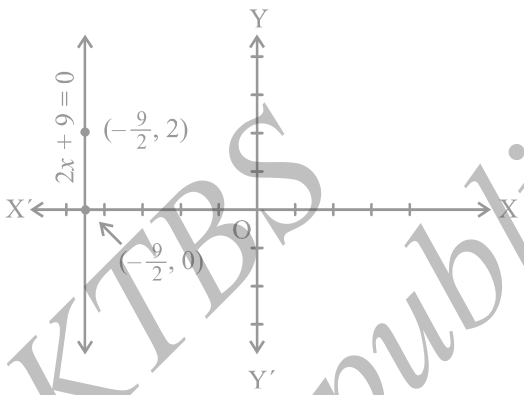
(ii)



2. (i)



(ii)



EXERCISE 11.1

1. (i) Base DC, parallels DC and AB; (iii) Base QR, parallels QR and PS;

(v) Base AD, parallels AD and BQ

EXERCISE 11.2

1. 12.8 cm. **2.** Join EG; Use result of Example 2.

6. Wheat in Δ APQ and pulses in other two triangles or pulses in Δ APQ and wheat in other two triangles.

EXERCISE 11.3

4. Draw CM $\perp$ AB and DN $\perp$ AB. Show CM = DN. **12.** See Example 4.

EXERCISE 11.4 (Optional)

7. Use result of Example 3 repeatedly.

EXERCISE 12.1

1. (i) Interior (ii) Exterior (iii) Diameter

(iv) Semicircle (v) The chord (vi) Three

2. (i) True (ii) False (iii) False

(iv) True (v) False (vi) True

EXERCISE 12.2

1. Prove exactly as Theorem 10.1 by considering chords of congruent circles.
2. Use SAS axiom of congruence to show the congruence of the two triangles.

EXERCISE 12.3

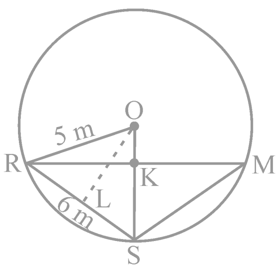
1. 0, 1, 2. Two 2. Proceed as in Example 1.
3. Join the centres O, O′ of the circles to the mid-point M of the common chord AB. Then, show $\angle$ OMA = 90° and $\angle$ O′MA = 90°.

EXERCISE 12.4

1. 6 cm. First show that the line joining centres is perpendicular to the radius of the smaller circle and then that common chord is the diameter of the smaller circle.
2. If AB, CD are equal chords of a circle with centre O intersecting at E, draw perpendiculars OM on AB and ON on CD and join OE. Show that right triangles OME and ONE are congruent.
3. Proceed as in Example 2.
4. Draw perpendicular OM on AD.
5. Represent Reshma, Salma and Mandip by R, S and M respectively. Let KR = x m (see figure).

   Area of Δ ORS = $\frac{1}{2}x \times 5$. Also, area of Δ ORS = $\frac{1}{2}$ RS × OL = $\frac{1}{2} \times 6 \times 4$.

   Find x and hence RM.



6. Use the properties of an equilateral triangle and also Pythagoras Theorem.

EXERCISE 12.5

1. 45° 2. 150°, 30° 3. 10°
4. 80° 5. 110° 6. $\angle$BCD = 80° and $\angle$ECD = 50°
8. Draw perpendiculars AM and BN on CD (AB || CD and AB < CD). Show ΔAMD ≅ ΔBNC. This gives $\angle$ C = $\angle$ D and, therefore, $\angle$ A + $\angle$ C = 180°.

EXERCISE 12.6 (Optional)

2. Let O be the centre of the circle. Then perpendicular bisector of both the chords will be same and passes through O. Let r be the radius, then $r^2 = \left(\frac{11}{2}\right)^2 + x^2 = \left(\frac{5}{2}\right)^2 + (6-x)^2$, where x is length of the perpendicular from O on the chord of length 11 cm. This gives $x = 1$. So, $r = \frac{5\sqrt{5}}{2}$ cm. **3.** 3 cm.

4. Let $\angle$ AOC = x and $\angle$ DOE = y. Let $\angle$ AOD = z. Then $\angle$ EOC = z and $x + y + 2z = 360°$.

$\angle ODB = \angle OAD + \angle DOA = 90° - \frac{1}{2}z + z = 90° + \frac{1}{2}z$. Also $\angle OEB = 90° + \frac{1}{2}z$

8. $\angle ABE = \angle ADE$, $\angle ADF = \angle ACF = \frac{1}{2}\angle C$.

Therefore, $\angle EDF = \angle ABE + \angle ADF = \frac{1}{2}(\angle B + \angle C) = \frac{1}{2}(180° - \angle A) = 90° - \frac{1}{2}\angle A$.

9. Use Q. 1, Ex. 10.2 and Theorem 10.8.

10. Let angle-bisector of $\angle$ A intersect circumcircle of Δ ABC at D. Join DC and DB. Then $\angle BCD = \angle BAD = \frac{1}{2}\angle A$ and $\angle DBC = \angle DAC = \frac{1}{2}\angle A$.
Therefore, $\angle BCD = \angle DBC$ or, DB = DC. So, D lies on the perpendicular bisector of BC.

EXERCISE 13.1

1. (i) 5.45 m^2 (ii) ₹ 109 **2.** ₹ 555 **3.** 6 m **4.** 100 bricks.

5. (i) Lateral surface area of cubical box is greater by 40 cm^2.

(ii) Total surface area of cuboidal box is greater by 10 cm^2.

6. (i) 4250 cm^2 of glass

(ii) 320 cm of tape. [Calculate the sum of all the edges (The 12 edges consist of 4 lengths, 4 breadths and 4 heights)].

7. ₹ 2184 **8.** 47 m^2

EXERCISE 13.2

1. 2 cm **2.** 7.48 m^2 **3.** (i) 968 cm^2 (ii) 1064.8 cm^2 (iii) 2038.08 cm^2

[Total surface area of a pipe is (inner curved surface area + outer curved surface area + areas of the two bases). Each base is a ring of area given by $\pi (R^2 - r^2)$, where R = outer radius and r = inner radius].

4. 1584 m^2 **5.** ₹ 68.75 **6.** 1 m

7. (i) 110 m^2 (ii) ₹ 4400 **8.** 4.4 m^2

9. (i) 59.4 m^2 (ii) 95.04 m^2

[Let the actual area of steel used be x m^2. Since $\frac{1}{12}$ of the actual steel used was wasted, the area of steel which has gone into the tank = $\frac{11}{12}$ of x. This means that the actual area of steel used = $\frac{12}{11} \times 87.12\,m^2$]

10. 2200 cm^2; Height of the cylinder should be treated as (30 + 2.5 + 2.5) cm

11. 7920 cm^2

EXERCISE 13.3

1. 165 cm^2 **2.** 1244.57 m^2 **3.** (i) 7 cm (ii) 462 cm^2

4. (i) 26 m (ii) ₹ 137280 **5.** 63 m **6.** ₹ 1155

7. 5500 cm^2 **8.** ₹ 384.34 (approx.)

EXERCISE 13.4

1. (i) 1386 cm^2 (ii) 394.24 cm^2 (iii) 2464 cm^2

2. (i) 616 cm^2 (ii) 1386 cm^2 (iii) 38.5 m^2

3. 942 cm^2 **4.** 1 : 4 **5.** ₹ 27.72

6. 3.5 cm **7.** 1 : 16 **8.** 173.25 cm^2

9. (i) $4\pi r^2$ (ii) $4\pi r^2$ (iii) 1 : 1

EXERCISE 13.5

1. 180 cm^3 **2.** 135000 litres **3.** 4.75 m **4.** ₹ 4320 **5.** 2 m

6. 3 days **7.** 16000 **8.** 6 cm, 4 : 1 **9.** 4000 m^3

EXERCISE 13.6

1. 34.65 litres

2. 3.432 kg [Volume of a pipe = $\pi h \times (R^2 - r^2)$, where R is the outer radius and r is the inner radius].

3. The cylinder has the greater capacity by 85 cm^3.

4. (i) 3 cm (ii) 141.3 cm^3

5. (i) 110 m^2 (ii) 1.75 m (iii) 96.25 *kl* **6.** 0.4708 m^2

7. Volume of wood = 5.28 cm^3, Volume of graphite = 0.11 cm^3.

8. 38500 cm^3 or 38.5*l* of soup

EXERCISE 13.7

1. (i) 264 cm^3 (ii) 154 cm^3 **2.** (i) 1.232 *l* (ii) $\frac{11}{35}$ *l*

3. 10 cm **4.** 8 cm **5.** 38.5 *kl*

6. (i) 48 cm (ii) 50 cm (iii) 2200 cm^2

7. 100π cm^3 **8.** 240π cm^3; 5 : 12

9. 86.625x m^3, 99.825 m^2

EXERCISE 13.8

1. (i) $1437\frac{1}{3}$ cm^3 (ii) 1.05 m^3 (approx.)

2. (i) $11498\frac{2}{3}$ cm^3 (ii) 0.004851 m^3 **3.** 345.39 g (approx.) **4.** $\frac{1}{64}$

5. 0.303*l* (approx.) **6.** 0.06348 m^3 (approx.)

7. $179\frac{2}{3}$ cm^3

8. (i) 249.48 m^2 (ii) 523.9 m^3 (approx.)

9. (i) $3r$ (ii) 1 : 9

10. 22.46 mm^3 (approx.)

EXERCISE 13.9 (Optional)

1. ₹ 6275

2. ₹ 2784.32 (approx.) [Rememeber to subtract the part of the sphere that is resting on the support while calculating the cost of silver paint].

3. 43.75%

EXERCISE 14.1

1. Five examples of data that we can gather from our day-to-day life are :

(i) Number of students in our class.

(ii) Number of fans in our school.

(iii) Electricity bills of our house for last two years.

(iv) Election results obtained from television or newspapers.

(v) Literacy rate figures obtained from Educational Survey.

Of course, remember that there can be many more different answers.

2. Primary data; (i), (ii) and (iii)

Secondary data; (iv) and (v)

EXERCISE 14.2

1.

Blood group	Number of students
A	9
B	6
O	12
AB	3
Total	30

Most common – O , Rarest – AB

2.

Distances (in km)	Tally Marks	Frequency
0 - 5	𝍸	5
5 - 10	𝍸 𝍸 𝍷	11
10 - 15	𝍸 𝍸 𝍷	11
15 - 20	𝍸 𝍷𝍷𝍷𝍷	9
20 - 25	𝍷	1
25 - 30	𝍷	1
30 - 35	𝍷𝍷	2
Total		40

3. (i)

Relative humidity (in %)	Frequency
84 - 86	1
86 - 88	1
88 - 90	2
90 - 92	2
92 - 94	7
94 - 96	6
96 - 98	7
98 - 100	4
Total	30

(ii) The data appears to be taken in the rainy season as the relative humidity is high.

(iii) Range = 99.2 – 84.9 = 14.3

4. (i)

Heights (in cm)	Frequency
150 - 155	12
155 - 160	9
160 - 165	14
165 - 170	10
170 - 175	5
Total	50

(ii) One conclusion that we can draw from the above table is that more than 50% of students are shorter than 165 cm.

5. (i)

Concentration of Sulphur dioxide (in ppm)	Frequency
0.00 - 0.04	4
0.04 - 0.08	9
0.08 - 0.12	9
0.12 - 0.16	2
0.16 - 0.20	4
0.20 - 0.24	2
Total	30

(ii) The concentration of sulphur dioxide was more than 0.11 ppm for 8 days.

6.

Number of heads	Frequency
0	6
1	10
2	9
3	5
Total	30

7. (i)

Digits	Frequency
0	2
1	5
2	5
3	8
4	4
5	5
6	4
7	4
8	5
9	8
Total	50

(ii) The most frequently occurring digits are 3 and 9. The least occurring is 0.

8. (i)

Number of hours	Frequency
0 - 5	10
5 - 10	13
10 - 15	5
15 - 20	2
Total	30

(ii) 2 children.

9.

Life of batteries (in years)	Frequency
2.0 - 2.5	2
2.5 - 3.0	6
3.0 - 3.5	14
3.5 - 4.0	11
4.0 - 4.5	4
4.5 - 5.0	3
Total	40

EXERCISE 14.3

1. (ii) Reproductive health conditions.

3. (ii) Party A **4.** (ii) Frequency polygon (iii) No **5.** (ii) 184

8.

Age (in years)	Frequency	Width	Length of the rectangle
1 - 2	5	1	$\frac{5}{1} \times 1 = 5$
2 - 3	3	1	$\frac{3}{1} \times 1 = 3$
3 - 5	6	2	$\frac{6}{2} \times 1 = 3$
5 - 7	12	2	$\frac{12}{2} \times 1 = 6$
7 - 10	9	3	$\frac{9}{3} \times 1 = 3$
10 - 15	10	5	$\frac{10}{5} \times 1 = 2$
15 - 17	4	2	$\frac{4}{2} \times 1 = 2$

Now, you can draw the histogram, using these lengths.

9. (i)

Number of letters	Frequency	Width of interval	Length of rectangle
1 - 4	6	3	$\frac{6}{3} \times 2 = 4$
4 - 6	30	2	$\frac{30}{2} \times 2 = 30$
6 - 8	44	2	$\frac{44}{2} \times 2 = 44$
8 - 12	16	4	$\frac{16}{4} \times 2 = 8$
12 - 20	4	8	$\frac{4}{8} \times 2 = 1$

Now, draw the histogram.

(ii) 6 - 8

EXERCISE 14.4

1. Mean = 2.8; Median = 3; Mode = 3
2. Mean = 54.8; Median = 52; Mode = 52
3. $x = 62$ **4.** 14
5. Mean salary of 60 workers is Rs 5083.33.

EXERCISE 15.1

1. $\frac{24}{30}$, i.e., $\frac{4}{5}$ **2.** (i) $\frac{19}{60}$ (ii) $\frac{407}{750}$ (iii) $\frac{211}{1500}$ **3.** $\frac{3}{20}$ **4.** $\frac{9}{25}$

5. (i) $\frac{29}{2400}$ (ii) $\frac{579}{2400}$ (iii) $\frac{1}{240}$ (iv) $\frac{1}{96}$ (v) $\frac{1031}{1200}$ **6.** (i) $\frac{7}{90}$ (ii) $\frac{23}{90}$

7. (i) $\frac{27}{40}$ (ii) $\frac{13}{40}$ **8.** (i) $\frac{9}{40}$ (ii) $\frac{31}{40}$ (iii) 0 **11.** $\frac{7}{11}$ **12.** $\frac{1}{15}$ **13.** $\frac{1}{10}$

EXERCISE A2.1

1. Step 1: **Formulation :**

The relevant factors are the time period for hiring a computer, and the two costs given to us. We assume that there is no significant change in the cost of purchasing or hiring the computer. So, we treat any such change as irrelevant. We also treat all brands and generations of computers as the same, i.e. these differences are also irrelevant.

The expense of hiring the computer for x months is ₹ $2000x$. If this becomes more than the cost of purchasing a computer, we will be better off buying a computer. So, the equation is

$$2000x = 25000 \qquad (1)$$

Step 2 : **Solution :** Solving (1), $x = \frac{25000}{2000} = 12.5$

Step 3 : **Interpretation :** Since the cost of hiring a computer becomes more **after** 12.5 months, it is cheaper to buy a computer, if you have to use it for more than 12 months.

2. Step1 : **Formulation :** We will assume that cars travel at a constant speed. So, any change of speed will be treated as irrelevant. If the cars meet after x hours, the first car would have travelled a distance of $40x$ km from A and the second car would have travelled $30x$ km, so that it will be at a distance of $(100 - 30x)$ km from A. So the equation will be $40x = 100 - 30x$, i.e., $70x = 100$.

Step 2 : **Solution :** Solving the equation, we get $x = \frac{100}{70}$.

Step 3 : **Interpretation :** $\frac{100}{70}$ is approximately 1.4 hours. So, the cars will meet after 1.4 hours.

3. Step1 : **Formulation :** The speed at which the moon orbits the earth is

$$\frac{\text{Length of the orbit}}{\text{Time taken}}.$$

Step 2 : **Solution :** Since the orbit is nearly circular, the length is $2 \times \pi \times 384000$ km $= 2411520$ km

The moon takes 24 hours to complete one orbit.

So, speed $= \frac{2411520}{24} = 100480$ km/hour.

Step 3 : **Interpretation :** The speed is 100480 km/h.

4. **Formulation :** An assumption is that the difference in the bill is only because of using the water heater.

Let the average number of hours for which the water heater is used $= x$

Difference per month due to using water heater = ₹ 1240 – ₹ 1000 = ₹ 240

Cost of using water heater for one hour = ₹ 8

So, the cost of using the water heater for 30 days $= 8 \times 30 \times x$

Also, the cost of using the water heater for 30 days = Difference in bill due to using water heater

So, $240x = 240$

Solution : From this equation, we get $x = 1$.

Interpretation : Since $x = 1$, the water heater is used for an average of 1 hour in a day.

EXERCISE A2.2

1. We will not discuss any particular solution here. You can use the same method as we used in last example, or any other method you think is suitable.

EXERCISE A2.3

1. We have already mentioned that the formulation part could be very detailed in real-life situations. Also, we do not validate the answer in word problems. Apart from this word problem have a 'correct answer'. This need not be the case in real-life situations.

2. The important factors are (ii) and (iii). Here (i) is not an important factor although it can have an effect on the number of vehicles sold.