Algebra

Polynomial

The word **poly** means 'many' and the word **nomial** means 'terms'. So polynomial is an algebraic expression which consists of many terms involving powers of the variable.

Example: $5x - 5$, $x^2 + 5x + 6$, $x^3 + x^2$.

The general form of a polynomial is

$a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots + a_{n-1}x^{n-1} + a_nx^n$, where $a_0, a_1, a_2 \ldots a_n$ are real numbers and 'n' is a non-negative integer.

Also a polynomial can have more than one variable.

Example: $x^2 + y^2 - 4$, $\frac{2}{3}x^2 - \frac{4}{5}y + 3z$

Types of polynomial

Polynomials can be categorised on the basis of two parameters.

1. By the number of terms

Here, the basis of the classification is the number of terms in the expression.

a. Monomial: Consisting of a single term.
Example: $\sqrt{2}x, -8y^2, 15z$.

b. Binomial: Consisting of two terms.
Example: $2x + 3y$, $y^2 + 3y$.

c. Trinomial: consisting of three terms.
Example: $5x^3 - 3x^2 + 2$, $x + y - 2$.

2. By degree

The highest exponent of any variable of the given polynomial is known as the degree of the polynomial.

Example : $x^6 - 3x^4 + 12$ is a trinomial of degree 6, $x^5 - 3x^4 + 12x^4y^3$ is a trinomial of degree $= 4 + 3 = 7$ and $4xy^3z^5$ is a monomial of degree $1 + 3 + 5 = 9$.

Linear polynomial: A polynomial of degree one is known as a linear polynomial.

Example : $5x - 4$, $5x + 3y$, $\frac{1}{3}x$, $x + \frac{5}{2}$

Quadratic polynomial: A polynomial of degree two is known as a quadratic polynomial.

Example : $x^2 + 3x + 5$, $x^2 - y^2 + xy$

Cubic polynomial: A polynomial of degree three is known as a cubic polynomial.

Example: $x^3 + 3x^2 - 5x + 2$, $4xy^2 + x^2 - 2xy$

Biquadratic polynomial: A polynomial of degree four is known as a biquadratic polynomial.

Example: $x^4 + 6x^3 - x^2 + 1$, $x^3 + 3y^2 - 5xy^3$

Equation

a. Linear equation:

An equation consists of only linear polynomials is known as linear equation.

Example : $3x - 2 = 7x$ is a linear equation in one variable and $3x + 4y = 20$ is a linear equations in two variables.

Solved Examples

1. Is expression $5x^2 - 3\sqrt{x} + 10$ a polynomial?

Solution :

No, because in a polynomial the power of 'x' (or the variable) must be a non-negative integer. Here in term $3\sqrt{x}$, the power of 'x' is not an integer.

2. If $3x + 15 = 60$, then find x.

Solution :

$3x + 15 = 60$

$\therefore 3x = 60 - 15 = 45$

$\Rightarrow x = \frac{45}{3} = 15.$

3. Six years ago, the age of Ram was thrice the age of Shyam and after six years Ram will become twice as old as Shyam. Find the present age of Shyam.

Solution :

Let the present age of Ram is 'x' years, and the present age of Shyam is 'y' years.

Then, $x - 6 = 3 \times (y - 6)$

$\Rightarrow x - 6 = 3y - 18$

$\Rightarrow x - 3y = -12$ … (i)

and $(x + 6) = 2 \times (y + 6)$

$\Rightarrow x + 6 = 2y + 12$

$\Rightarrow x - 2y = 6$ … (ii)

Solving equations (i) and (ii), we get

$$\begin{array}{l} x - 3y = -12 \\ x - 2y = 6 \\ - \quad + \qquad - \\ \hline \quad -y = -18 \end{array}$$

y = 18

Putting y = 18 in either of the two equations we get x = 42

So, Ram's present age is 42 years and Shyam's present age is 18 years.

4. The sum of a two-digit number and the number obtained by interchanging the digits of this number is 33. Find all such possible numbers.

Solution :

Let the two-digit number be xy =10x + y, then the other number will be yx =10y + x.

[Any two digit number can be written as (digit at 10's place) × 10 + (digit at 1's place) × 1]

So their sum = 10x + y + 10y + x

= 11x + 11y = 11(x + y) = 33

$\Rightarrow$ x + y = 3

The value of 'x' can be either 1or 2 or 3. Similarly, the value of 'y' will be either 2 or 1 or 0 respectively.

So, the possible numbers are 12, 21 and 30.

b. Simultaneous equations

When there are two or more than two equations to be solved simultaneously, then they are known as simultaneous equations.

When there are two linear equations in two variables, they are known as simultaneous linear equations in two variables.

Example : 2x + 3y = 12 and 4x + 9y = 15,

x + 2y = 24 and 10x + 13y = 53

Format of simultaneous linear equations:

$a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$

Important rule: Simultaneous equations may be consistent or inconsistent. They may have no solution, or a unique solution, or infinite solutions. The nature of these solutions can be determined by the following rules.

(i) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the system (equations) is consistent and has a unique solution.

(ii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the system (equations) is inconsistent and has no solution.

(iii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the system (equations) is consistent and has infinite solutions.

5. If 2x + y = 35 and 3x + 4y = 65, then find x.

Solution :

2x + y = 35 … (i)

3x + 4y = 65 … (ii)

Multiplying equation (i) by 4, we get

8x + 4y = 140 … (iii)

Subtracting equation (ii) from equation (iii) we get,

5x = 75

$\therefore x = \frac{75}{5} = 15.$

6. Five years ago, A was thrice as old as B and 10 years hence A shall be twice as old as B. Find the present age of A?

Solution :

Let the present ages of A and B be A years and B years respectively, then A – 5 = 3(B – 5)

$\Rightarrow B - 5 = \frac{A-5}{3}$

$\Rightarrow B = \frac{A+10}{3}$

and A + 10 = 2(B + 10)

$\Rightarrow B + 10 = \frac{A+10}{2}$

$\Rightarrow B = \frac{A-10}{2}$

$\therefore \frac{A-10}{2} = \frac{A+10}{3}$

$\Rightarrow$ 3A – 30 = 2A + 20

$\Rightarrow$ A = 30 + 20 = 50

Hence, the present age of A is 50 years.

7. A person has only 25-paisa and 50-paisa coins. In total he has 40 coins amounting Rs.12.50. Find the number of 50-paisa coins available with him.

Solution :

Let the number of 25-paisa coins and 50-paisa coins available with the person be 'x' and 'y' respectively.

Then,

x + y = 40 … (i)

and $\frac{1}{4}x + \frac{1}{2}y = 12.50$

$\Rightarrow$ x + 2y = 50 … (ii)

Subtracting equation (i) from equation (ii), we get

y = 10

Hence, the required number of coins is 10.

8. For what value of 'k', will the following system of equations has no solution?

$2x + 3y = 5$ and $kx - 5y = 7$

Solution :

For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Given, $a_1 = 2$, $b_1 = 3$, $c_1 = 5$, $a_2 = k$, $b_2 = -5$ and $c_2 = 7$ Putting the values we get,

$\frac{2}{k} = \frac{3}{-5} \neq \frac{5}{7} \Rightarrow k = \frac{-10}{3}$.

c. Quadratic equation

An equation of the form $ax^2 + bx + c = 0$, where 'a', 'b' and 'c' are real numbers and $a \neq 0$ is known as a quadratic equation. In the given example above, 'a' is the coefficient of x^2,'b' is the coefficient of 'x' and 'c' is a constant.

Since a quadratic equation is a polynomial of second degree in one variable, it will have two solutions or roots.

For example :

In the equation $x^2 - 4 = 0$, 'x' will have two roots i.e. +2 or –2.

Note:

The roots of a quadratic equation can be real as well as imaginary.

For example:

In the equation $x^2 + 4 = 0$, 'x' will have an imaginary root i.e. $\sqrt{-2}$.

How to find the roots?

There are two ways to find out the roots of a quadratic equation. We hope you know both the ways, so it will be easier for you to understand.

1. Factorization method :

Suppose the equation is:

$x^2 - 5x + 6 = 0$

$x^2 - 3x - 2x + 6 = 0$

$x(x - 3) - 2(x - 3) = 0$

$(x - 3)(x - 2) = 0$

$x - 3 = 0$ or $x - 2 = 0$

$x = 3$ or $x = 2$

Hence, the roots of the equation are 3 and 2.

How to factorize? Split 'b'(coefficient of 'x') in two parts, so that their sum remains 'b' and the product becomes 'ac'.

$x^2 - 5x + 6 = 0$

$x^2 - 3x - 2x + 6 = 0$

Thus, –5 has been broken into two parts i.e. –3 and –2, such that their sum is – 5 and the product is 6.

$x(x - 3) - 2(x - 3) = 0 \Rightarrow x = 3$ and 2.

2. Formula

If the roots of a quadratic equation are α and β, then

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

In the equation $x^2 - 5x + 6 = 0$,

$$\alpha = \frac{-(-5) + \sqrt{(-5)^2 - 4 \times 1 \times 6}}{2 \times 1}$$

$$= \frac{5 + \sqrt{25 - 24}}{2} = \frac{5 + 1}{2} = 3$$

$$\beta = \frac{-(-5) - \sqrt{(-5)^2 - 4 \times 1 \times 6}}{2 \times 1}$$

$$= \frac{5 - \sqrt{25 - 24}}{2} = \frac{5 - 1}{2} = 2$$

$\alpha, \beta = 3, 2$.

Nature of the roots: The nature of the roots of a quadratic equation is determined by the value of the *discriminant*, which is denoted by D.

$D = b^2 - 4ac$

(A) $D \geq 0$, then the roots are real.

a. If D = 0, then both the roots will be equal.

b. If D > 0, then the roots will be distinct and real.

If a, b and c are intergers and:

(i) D is perfect square, then roots are rational.

(ii) D is not a perfect square, then roots are irrational.

(B) D < 0, then both the roots will be non real (imaginary).

Sum of the roots: $\alpha + \beta = -\frac{b}{a}$

Product of the roots: $\alpha \times \beta = \frac{c}{a}$

Formation of a quadratic equation:

If α and β are the roots of a quadratic equation, then it can be written as:

$$(x-\alpha)(x-\beta)=0$$

$$x^2-\alpha x-\beta x+\alpha\beta=0$$

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

x^2 – (sum of roots)x + product of roots = 0

Remainder theoram

Let f(x) be any polynomial of degree greater than or equal to one and let a be any real number. If f(x) is divided by the linear polynomial x – a, then the remainder is f(a).

Example : If $f(x) = x^2 - 5x + 9$ is divided by x – 3, then the remainder will be

$f(3) = 3^2 - 5 \times 3 + 9 = 3.$

Note (i):

If f(a) = 0, then x – a will be the one of the factor of polynomial f(x).

9. Find the nature of the roots of the equation $6x^2 + 7x + 2 = 0$.

Solution :

Given, a = 6, b = 7 and c = 2

Then, $D = b^2 - 4ac = 7^2 - 4 \times 6 \times 2 = 49 - 48 = 1$

As D > 0 and a perfect square

$\therefore$ The roots of the equation will be rational and unequal.

10. Find the roots of the equation $6x^2 + 7x + 2 = 0$.

Solution :

$6x^2 + 7x + 2 = 0$

$\Rightarrow 6x^2 + 3x + 4x + 2 = 0$

$\Rightarrow 3x(2x + 1) + 2(2x + 1) = 0$

$\Rightarrow (2x + 1)(3x + 2) = 0$

$\therefore$ Either 2x + 1 = 0 or 3x + 2 = 0

$\Rightarrow x = -\frac{1}{2}$ or $-\frac{2}{3}$

Note: You can solve the given problem by formula method also.

11. The sum of the roots of a quadratic equation is 5 and their product is 6.Find the quadratic equation.

Solution :

The quadratic equation is

x^2 – (sum of the roots)x + product of the roots= 0

$\therefore$ The required equation will be $x^2 - 5x + 6 = 0$.

12. If one of the roots of a quadratic equation is +4 and the other is –4, then what is the quadratic equation?

Solution :

$\alpha = +4$ and $\beta = -4$

$\Rightarrow \alpha + \beta = 0$ and $\alpha\beta = 4 \times -4 = -16$

$\therefore$ The quadratic equation is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow x^2 - 0 \times x + (-16) = 0$ or $x^2 - 16 = 0$.

13. If one root of a quadratic equation is 6 and the other is 4, then what is the quadratic equation?

Solution :

$\alpha = 4$ and $\beta = 6$

$\Rightarrow \alpha + \beta = 4 + 6 = 10$ and $\alpha\beta = 4 \times 6 = 24$

$\therefore$ The quadratic equation is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow x^2 - 10x + 24 = 0$.

14. Find the factors of the equation $12x^2 - 30x + 18 = 0$.

Solution :

$12x^2 - 30x + 18 = 0$

$2x^2 - 5x + 3 = 0$ [dividing it by 6]

$\Rightarrow 2x^2 - 2x - 3x + 3 = 0$

$\Rightarrow 2x(x - 1) - 3(x - 1) = 0$

$\Rightarrow (x - 1)(2x - 3) = 0$

$\therefore$ The factors of the given equation are (x – 1) and (2x – 3).

15. Find the nature of the roots of the equation $-3x^2 - 4x - 12 = 0$.

Solution :

Given, a = –3, b = –4 and c = –12

Then, $D = b^2 - 4ac = (-4)^2 - 4 \times (-3) \times (-12)$

$= 16 - 144 = -128$

As D < 0,

$\therefore$ The roots of the equation will be imaginary.

16. Find out the remainder when the polynomial $5x^2 - 4x - 12$ is divided by (x – 2).

Solution :

Let $f(x) = 5x^2 - 4x - 12$, then

the remainder $= f(2) = 5 \times 2^2 - 4 \times 2 - 12$

$= 20 - 8 - 12 = 0$

Hence the remainder is 0.

17. Find out the remainder when the polynomial $5x^2 - 4x - 12$ is divided by (5x + 6).

Solution :

Let $f(x) = 5x^2 - 4x - 12$, then

the remainder $= f\left(\frac{-6}{5}\right) = 5\times\left(-\frac{6}{5}\right)^2 - 4\times-\frac{6}{5} - 12$

$$= \frac{36}{5} + \frac{24}{5} - 12 = 0.$$

Hence the remainder is 0.

18. Find out the remainder when the polynomial $5x^2 - 4x - 12$ is divided by $(x + 4)$.

Solution :

Let $f(x) = 5x^2 - 4x - 12$, then

the remainder $= f(-4) = 5(-4)^2 - 4 \times (-4) - 12$

$= 80 + 16 - 12 = 84$

Hence the remainder is 84.

Inequalities

So far, in the cases of linear equations and quadratic equations, we have learnt about equality, which means an expression is equal to some other expression or some numerical value.

Example : $2x + 3y = 0$; or $x^2 + 2x + 4 = 0$.

However, when algebraic expressions are related using any of the signs $<, >, \geq$ or $\leq$, then it results in an inequality.

Example : $2x + 3y \geq 0$, $2x + 3y < 0$; or

$x^2 + 2x + 4 \leq 0$

Notations : For all real numbers x and y

1. $x < y$ means x is less than y.
2. $x > y$ means x is greater than y.
3. $x \leq y$ means x is less than or equal to y.
4. $x \geq y$ means x is greater than or equal to y.
5. $x \not> y$ means x is not greater than y.
6. $x \not< y$ means x is not less than y.

Properties: For real numbers x, y and z.

1. If $x > y$ and $y > z$, then $x > z$

   Example : $6a > 5b$ and $5b > 7c$, then $6a > 7c$.

2. If x and y are both positive or both negative, then

   i. If $x < y$, then $\frac{1}{x} > \frac{1}{y}$

   ii. If $x > y$, then $\frac{1}{x} < \frac{1}{y}$

3. If $x > y$, then $x + z > y + z$ and $x - z > y - z$.

   Example : $5x - 4 > 4x - 1$, then

   $5x - 4 - 4x > 4x - 1 - 4x$

   $\Rightarrow x - 4 > -1$

   $\Rightarrow x - 4 + 4 > -1 + 4$

   $\Rightarrow x > 3$

4. i. If $x > y$ and $z > 0$, then

   $x \times z > y \times z$

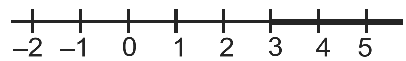
   Example: $3a > 2b$ and $5c > 0$, then $15ac > 10bc$.

   ii. If $x > y$, and $z < 0$, then $x \times z < y \times z$

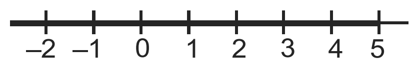
   Example: $3a > 2b$ and $5c < 0$, then $15ac < 10bc$.

Certain other tools that are very handy and would be used very frequently are:

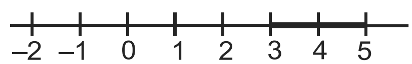
Consider $x > 3$. On a number line the set of all values of x that satisfy this inequality is represented by the bold line in the following figure:



Similarly $x < 5$ is represented on the number line as follows:

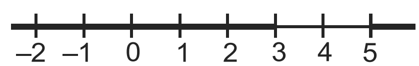


Consider the two inequalities simultaneously $x > 3$ AND $x < 5$. This is basically the intersection of the above two line graphs and the solution set to these two inequalitites simultaneously is:



Similarly the solution for $x > 5$ AND $x < 3$ will be a null set i.e. no value of x will satisfy both the inequalities simultaneously.

However the solution for $x > 5$ OR $x < 3$ is all values of x represented by the bold line in the following line graph.



The difference is because of OR and not AND.

Similarly the solution set of $x > 3$ OR $x > 5$ will be the union i.e. $x > 3$ but the solution set of $x > 3$ AND $x > 5$ will be the intersection $x > 5$.

Practice understanding of inequalities on number lines. This helps you in visualizing the number relationship and would eventually help you do it mentally.

Consider solving the following inequality $(x - 2)(x + 4)(x - 5) > 0$. This is of the type $a \times b \times c > 0$. This is possible if all three of a, b and c is positive OR if one of them is positive and other two are negative. Rather than trying all possibilities, a simpler way also exists. $(x - 2)$ will be positive if $x > 2$ and negative if $x < 2$. Similarly $(x + 4)$ will be positive if $x > -4$ and negative if $x < -4$. Also $(x - 5)$ will be positive if $x > 5$ and negative if $x < 5$.

Combining the above, we will have:

If $x > 5$, all three $(x - 2)$, $(x + 4)$ and $(x - 5)$ will be positive and hence the product will be positive.

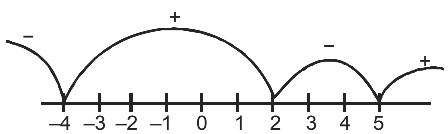
If $2 < x < 5$, $(x - 2)$ and $(x + 4)$ will remain positive but $(x - 5)$ will become negative and hence the product will be negative.

If $-4 < x < 2$, only $(x + 4)$ will be positive and other two $(x - 2)$ and $(x - 5)$ will be negative. Thus the product will be positive.

Finally if $x < -4$, all the terms will be negative and the product will also be negative.

Since we need the product to be positive, the solution will be $x > 5$ or $-4 < x < 2$

Rather than writing all the above, we can simply plot the points where the terms will change the signs, on a number line as follows :



The number line represents all the real numbers from $-\infty$ to $+\infty$. However, it is now broken in various regions, four regions for this inequality. Now we have to identify those values of *x* that satisfy the given inequality,namely$(x - 2)(x + 4)(x - 5) > 0$.

For the right most region i.e. $x > 5$, all terms will be positive and hence the product will be positive. For the second region from the right, one term will turn negative and thus the product will be negative in this range of x. For the third region from right side, two terms will turn negative making the product positive for this range of x. And so on.

If we recollect we could not solve the inequality $x(x - 5) > 3(x - 5)$ by dividing both sides with $(x - 5)$ (unless you took conditions of $x - 5$ being positive or negative).So how do we solve this inequality in a faster way? The inequality can be re-written as $x(x - 5) - 3(x - 5) > 0$ i.e. $(x - 5)(x - 3) > 0$.

And as just seen, the solution set to this inequality will be $x > 5$ or $x < 3$

For a quadratic equation $y = x^2 - 8x + 15$, we see that y is + for all values of $x > 5$ or $x < 3$ and is negative for $3 < x < 5$.

The above method can be used to solve any quadratic or higher degree inequality. In fact the inequality

$x^2 > 9$ is also a quadratic inequality and can be written as $x^2 - 9 > 0$, i.e. $(x + 3)(x - 3) > 0$ and the solution set is $x < -3$ or $x > 3$.

19. If $3x + 5y \le 24$, $x \ge 2$ and $y \ge 3$, then find the values of 'x' and 'y'.

Solution :

Putting the minimum value of 'x' in the equation $3x + 5y \le 24$, we get

$3 \times 2 + 5y \le 24$

$5y \le 18$ or $y \le \frac{18}{5}$

Similarly putting the minimum value of 'y' in the equation, we get $3x + 15 \le 24$ or $x \le 3$.

$\therefore$ 'x' will lie in the range $2 \le x \le 3$ and

'y' will lie in the range $3 \le y \le \frac{18}{5}$.

20. Solve $3x^2 + 13x - 30 \le 0$.

Solution :

$3x^2 + 13x - 30 \le 0$

$\Rightarrow 3x^2 + 18x - 5x - 30 \le 0$

$\Rightarrow 3x(x + 6) - 5(x + 6) \le 0$

$\Rightarrow (x + 6)(3x - 5) \le 0$

$\therefore$ The answer is $-6 \le x \le \frac{5}{3}$.

21. Solve $-6x^2 + 48x - 96 \ge 0$.

Solution :

$-6x^2 + 48x - 96 \ge 0$

Dividing both sides by –6, we get

$x^2 - 8x + 16 \le 0$

$\Rightarrow x^2 - 4x - 4x + 16 \le 0$

$\Rightarrow (x - 4)(x - 4) \le 0$

$\Rightarrow (x - 4)^2 \le 0$

$\Rightarrow x = 4$ is the only possible value as the square can never be less than 0.

Modulas

The absolute value of a real number x is denoted by $|x|$ (modulus). Absolute value of a real number is a always a non negative number.

Example : $|-10| = 10$

Rule:

i. $|x| = x$, if $x \ge 0$

ii. $|x| = -x$, if $x \le 0$

iii. $|x| - |y| \le |x - y|$

Example : $|-2| - |+3| = 2 - 3 = -1$

and $|-2 - 3| = |-5| = 5$

$\therefore |-2| - |+3| < |-2 - 3|$

Similarly, $|4| - |2| = 4 - 2 = 2$

and $|4 - 2| = 2$

$\therefore |4| - |2| = |4 - 2|$

iv. $|x| \times |y| = |xy|$

Example: $|-2| \times |3| = 2 \times 3 = 6$

and $|-2 \times 3| = |-6| = 6$

$\therefore |-2| \times |3| = |-2 \times 3|$

v. $\dfrac{|x|}{|y|} = \left|\dfrac{x}{y}\right|$;

Example : $\dfrac{|-6|}{|2|} = \dfrac{6}{2} = 3$ and $\left|\dfrac{-6}{2}\right| = \dfrac{6}{2} = 3$.

$\therefore \dfrac{|-6|}{|2|} = \left|\dfrac{-6}{2}\right|$

vii. $|x| - |y| \le |x \pm y| \le |x| + |y|$.**(Important)**

22. Solve $|4x| = 20$.

Solution :

Case (i). When $4x < 0$, then $-4x = 20$ or $x = -5$

Case (ii). When $4x \ge 0$, then $4x = 20$ or $x = 5$

$\therefore x = \pm 5$.

23. Solve $|7x - 5| = 40$.

Solution :

Case (i). When $(7x - 5) < 0$ or $x < \dfrac{5}{7}$, then

$7x - 5 = -40$ or $x = -5$

Case (ii). When $(7x - 5) \ge 0$ or $x \ge \dfrac{5}{7}$, then

$7x - 5 = 40$ or $x = \dfrac{45}{7}$

$\therefore x = \dfrac{45}{7}$ or $x = -5$

24. Solve $3x + |x| = -12$.

Solution :

Case (i). When $x < 0$, then

$3x + |x| = 3x + (-x) = 2x = -12$ or $x = -6$

Case (ii). When $x \ge 0$, then

$3x + |x| = 3x + x = 4x = -12$ or $x = -3$

But $x \ge 0$, therefore $x = -3$ is unacceptable

$\therefore x = -6$.

25. Solve $|2x - 1| \ge 9$.

Solution :

Case (i) : When $2x - 1 \ge 0$ or $x \ge \dfrac{1}{2}$, then

$\Rightarrow 2x - 1 \ge 9$ or $2x \ge 10$ or $x \ge 5$.

Case (ii) : When $2x - 1 < 0$ or $x < \dfrac{1}{2}$, then

$-(2x - 1) \ge 9$ or $-2x + 1 \ge 9$ or $-2x \ge 8$ or $x \le -4$

$\therefore x \le -4$ or $x \ge 5$.

Surds and indices

a. Power and Index

If a number 'x' is multiplied by itself 'n' times, the product is called n^{th} power of 'x' and is written as x^n. Here 'x' is called the base and 'n' the index/power/exponent. Example: $2^3 = 2 \times 2 \times 2 = 8$.

Laws of Indices

$x^m \times x^n = x^{m+n}$	Example: $2^5 \times 2^3 = 2^8$
$\dfrac{x^m}{x^n} = x^{m-n}$	Example: $\dfrac{2^3}{2^2} = 2^1 = 2$
$x^{-m} = \dfrac{1}{x^m}$	Example: $2^{-8} = \dfrac{1}{2^8}$
$(x^m)^n = x^{mn}$	Example: $(2^5)^3 = 2^{15}$
$\left(\dfrac{x}{y}\right)^m = \dfrac{x^m}{y^m}$	Example: $\left(\dfrac{2}{3}\right)^2 = \dfrac{2^2}{3^2}$
$x^0 = 1$	

Laws of surds

(i) $\sqrt[n]{a} = a^{\frac{1}{n}}$

(ii) $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b} = (ab)^{\frac{1}{n}}$

(iii) $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \left(\dfrac{a}{b}\right)^{\frac{1}{n}}$

(iv) $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m} = a^{\frac{m}{n}}$

(v) $\sqrt[n]{a^n} = a^{\frac{n}{n}} = a$

(vi) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = a^{\frac{1}{mn}}$

Logarithms

We know that $2^3 = 8$. Suppose we want to express 8 on base 2, then a function is used, called log and it is written $\log_2 8 = 3$ and it is read as log of 8 to base 2 is 3.

If $a^x = y$, where a is a positive number not equal to 1 and x is a rational number; then the log of y on base a is written as $\log_a y = x$. The basic application of logarithm (log) is to make the simplification simpler.

Important rules

i. $a^x = y$, then $\log_a y = x$

ii. $\log_a(mn) = \log_a m + \log_a n$

iii. $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

iv. $\log_a(m^n) = n\log_a m$

v. $\log_a a^a = a$. [Since $\log_a a = 1$]

vi. $\log_a 1 = 0$

Note: 'm' and 'n' are positive real numbers as 'log' is defined only for positive real numbers.

Logarithms to base 10

$\log_{10} 10 = 1$

$\log_{10} 100 = \log_{10}(10)^2 = 2\log_{10} 10 = 2$

$\log_{10} 0.01 = \log_{10}\frac{1}{10^2} = \log_{10} 10^{-2} = -2$

26. Find the value of $\log_4 256$.

Solution :

$\log_4 256 = \log_4(4)^4 = 4$

27. Find the value of $\log_{10} 0.001$.

Solution :

Let $\log_{10} 0.001 = x$, then

$10^x = 0.001 = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3}$

$\therefore x = -3$.

28. If log 2 = 0.3010 and log 7 = 0.8457, then find (a) log 14 (b) log 28.

Solution :

a. log 14 = log(2 × 7) = log 2 + log 7
= 0.3010 + 0.8457 = 1.1467.

b. log 28 = log (2^2 × 7) = log 2^2 + log 7
= 2 log 2 + log 7 = 2 × 0.3010 + 0.8457
= 0.6020 + 0.8457
= 1.4477.

29. If $\frac{\log 169}{\log 13} = \log x$, find x.

Solution :

$$\frac{\log 169}{\log 13} = \frac{\log 13^2}{\log 13} = \frac{2\times\log 13}{\log 13} = 2$$

$= \log x = \log_{10} x$

$\Rightarrow x = 10^2 = 100$.

Note: If base is not given then it is taken as 10.

Progressions

Progression is a sequence of terms in which the terms follow certain rule or pattern.

a. Arithmetic progression:

Arithmetic progression or arithmetic sequence is a sequence of numbers such that the difference between any two consecutive terms is constant throught the sequence.

The difference between any two consecutive terms is called common difference. Remember that common difference can be positive, negative or zero.

Example: 10, 12, 14, 16, . . .

The common difference = 12 – 10 = 14 – 12 = 2

Example: 16, 15, 14, 13, . . .

The common difference = 15 – 16 = 14 – 15 = –1

If the first term of an AP is **a** and the common difference is **d**, then the n^{th} term = $a_n = a + (n - 1)d$ and (where a_m and a_n are the m^{th} and the n^{th} terms of the AP respectively.

S = Sum of first 'n' terms $= \frac{n}{2}[2a + (n-1)d]$

or S $= \frac{n}{2}[a + a_n]$

Arithmetic mean: Arithmetic mean is the same as the average.

Example : Arithmetic mean of four terms a, b, c, and d is $\frac{(a+b+c+d)}{4}$.

b. Geometric progression:

Geometric progression or geometric sequence is a sequence of numbers where the ratio of any two consecutive terms is constant and the ratio is called as common ratio.

Example : 2, 4, 8, 16, ...

The common ratio $= \frac{4}{2} = \frac{8}{4} = \frac{16}{8} = 2$

Example : 15, 5, $\frac{5}{3}$, $\frac{5}{9}$, ...

The common ratio $= \frac{5}{15} = \frac{\frac{5}{3}}{5} = \frac{\frac{5}{9}}{\frac{5}{3}} = \frac{1}{3}$

If the first term of a G.P. is **a** and the common ratio is **r**, then, $a_2 = ar$, $a_3 = ar^2$, $a_4 = ar^3$ and so on.

By following these pattern we can say that $a_n = ar^{n-1}$

Hence, the G.P. so formed is: a, ar, ar^2, ar^3 ...

$\therefore$ n^{th} term of a G.P. = $a_n = ar^{n-1}$

Sum of *'n'* terms (S_n) = $\dfrac{a\left(1-r^n\right)}{1-r}$ if $r<1$

or $\dfrac{a(r^n-1)}{r-1}$ if $r>1$

If a G.P. have infinite number of terms with common ratio |r| < 1 then the sum of these infinite terms

$$S_\infty = \frac{a}{1-r}$$

Geometric mean: Geometric mean of 'n' numbers is the n^{th} root of the product of n numbers.

Geometric mean of the two numbers a and b is $\sqrt{ab}$.

Similarly, if four terms a, b, c and d are in geometric progression, then their geometric mean is given by $\sqrt[4]{a\times b\times c\times d}$.

Important results

i. Sum of the first 'n' natural numbers
$= \dfrac{n(n+1)}{2}$

ii. Sum of the squares of the first n natural numbers = $\dfrac{n(n+1)(2n+1)}{6}$

iii. Sum of the cubes of the first n natural numbers = $\left\{\dfrac{n(n+1)}{2}\right\}^2$

iv. Sum of the first 'n' even natural numbers
= n(n + 1)

v. Sum of the first 'n' odd natural numbers = n^2

vi. AM $\geq$ GM

30. Find the sum of the sequence:
4 + 6 + 8 + 10 + 12 + . . . + 200.

Solution :

Difference of the two consecutive terms is same, therefore the numbers are in AP.

$a_n = a + (n-1)d$

$\therefore$ 200 = 4 + (n – 1) × 2

$\Rightarrow$ n = 99

Sum of the sequence = $\dfrac{n}{2}[a+a_n]$

$= \dfrac{99}{2}[4+200]$ = 99 × 102 = 10098.

31. Find the sum of all the integers, which lie between 102 and 199 and are divisible by 8.

Solution :

First term is 104 and last term is 192

$\therefore$ 192 = 104 + (n – 1)8

$\Rightarrow n-1 = \dfrac{88}{8} = 11$

$\Rightarrow$ n = 12

Sum = $\dfrac{12}{2}(104+192) = 6\times 296 = 1776.$

32. Find sum $\dfrac{1}{3}+\dfrac{2}{3}+1+\dfrac{4}{3}+\dfrac{5}{3}+2+...$ till 20 terms.

Solution :

The series is $\dfrac{1}{3}+\dfrac{2}{3}+\dfrac{3}{3}+\dfrac{4}{3}+\dfrac{5}{3}+\dfrac{6}{3}+...$ till 20.

a = $\dfrac{1}{3}$ and d = $\dfrac{2}{3}-\dfrac{1}{3}=\dfrac{1}{3}$

$a_n = \dfrac{20}{2}\left[2\times\dfrac{1}{3}+(20-1)\dfrac{1}{3}\right]$

$= 10\left(\dfrac{2}{3}+\dfrac{19}{3}\right) = 70.$

33. If the sum of the n term of the series 25 + 24 + 23 + . . . is 0, then find the value of n.

Solution :

Here, a = 25 and d = –1

$\Rightarrow \dfrac{n}{2}\left[2a+(n-1)d\right]$

$= \dfrac{n}{2}\left[2\times 25+(n-1)(-1)\right] = 0$

Then, either n = 0 or [2 × 25 + (1 – n)]. But, $n \neq 0$

$\therefore$ [2 × 25 + (1 – n)] = 0

$\Rightarrow$ n = 50 + 1 = 51.

34. Find the sum of 4 + 8 + 16 + 32 + . . . upto 10 terms.

Solution :

Here, a = 4, r = $\dfrac{8}{4}$ = 2 and n = 10

Sum = $\dfrac{a\left(r^n-1\right)}{r-1}$

$= \dfrac{4\left(2^{10}-1\right)}{2-1} = 4\times\left(2^{10}-1\right) = 4092.$

35. Find the geometric mean of 3, 9, 27 and 729.

Solution :

Geometric mean of the given numbers

$= \sqrt[4]{3 \times 9 \times 27 \times 729}$

$= \sqrt[4]{3 \times 3 \times 3 \times 3 \times 9 \times 9 \times 9 \times 9}$

$= 3 \times 9 = 27$

36. The arithmetic mean of two numbers is 5 and their geometric mean is 4. Find the numbers.

Solution :

Let the numbers be a and b, then

$\frac{a+b}{2} = 5$ and $\sqrt{a \times b} = 4$

a + b = 10 … (i)

ab = 16 or b = $\frac{16}{a}$ … (ii)

By putting the value of b in equation (i), we get

$a + \frac{16}{a} = 10$

$\Rightarrow a^2 + 16 = 10a$

$\Rightarrow a^2 - 10a + 16 = 0$

$\Rightarrow (a-2)(a-8) = 0$

$\Rightarrow (a-2) = 0$ or $(a-8) = 0$

$\Rightarrow a = 2$, or $a = 8$

If a = 2, then b = 8 and if a = 8, then b = 2.

37. A ball is thrown upwards. It reaches up to a height of 20 m; comes to the ground and bounces back again and again. After each bounce, it reaches up to $\frac{2}{5}$th of its previous height. Find the total distance covered by the ball till it comes to rest.

Solution :

Here, a = 20, r = $\frac{2}{5}$

So, the total distance covered by the ball will be given by sum of infinite terms = $\frac{a}{1-r}$

$= \frac{20}{\left(1-\frac{2}{5}\right)} = 20 \times \frac{5}{3} = \frac{100}{3}$

Since in one bounce it covers that distance twice, the total distance covered

$= \frac{100}{3} \times 2 = \frac{200}{3} = 66.66$ m.

38. A square is formed by joining the mid-points of the sides of a square. A third square is drawn inside the second square in the same way and this process continues indefinitely. If the side of the first square is 10 cm, find the sum of the areas of all squares.

Solution :

Area of the first square = 10 × 10 = 100 cm^2

Area of the second square = 50 cm^2

Area of the third square = 25 cm^2,

and so on…

Sum of areas of all the squares

= 100 + 50 + 25 + ... till infinity

= Sum of infinite GP with a = 100 and r = $\frac{1}{2}$

$= \frac{100}{1-\frac{1}{2}} = 100 \times \frac{2}{1} = 200$ cm^2.

39. Find the sum of 10 terms of the sequence 5, 5.5, 5.55, 5.555 ...

Solution :

5, 5.5, 5.55, 5.555 ... 10 terms

= 5(1 + 1.1 + 1.11 + ... 10 terms)

$= \frac{5}{9}$ (9 + 9.9 + 9.99 + ... 10 terms)

$= \frac{5}{9}$ (10 – 1 + 10 – 0.1 + 10 – 0.01 + ... 10 terms)

$= \frac{5}{9}$ (10 + 10 + 10 + ... 10 terms) – (1 + 0.1 + 0.01 + ... 10 terms)

$= \frac{5}{9}$ (100 – Sum of GP of 10 terms with a = 1 and r = 0.1)

$= \frac{5}{9}\left[100 - \frac{1 \times \left(1-(0.1)^{10}\right)}{1-0.1}\right]$

$= \frac{5}{9}\left[100 - \frac{10}{9}\left(1 - \frac{1}{10^{10}}\right)\right]$

$= \frac{5 \times 890}{81}\left(1 - 10^{-10}\right)$

40. If $f(x) = \frac{x-2}{x+2}$, then f(3x) is

(a) $\frac{f(x)-2}{f(x)+2}$ (b) $\frac{3f(x)-2}{3f(x)+2}$

(c) $\frac{3f(x)+2}{f(x)+3}$ (d) None of these

Solution :

$f(x) = \frac{x-2}{x+2}$ Then,

$f(3x) = \frac{3x-2}{3x+2} = \frac{3x+2-4}{3x+2} = 1 - \frac{4}{3x+2}$.

Exercise

1. Find the value of x in $\frac{1}{x-1}+\frac{1}{x-2}=\frac{3}{x-3}$.

   (a) $\pm\sqrt{2}$ (b) $\pm\frac{1}{2}$

   (c) $\pm\sqrt{3}$ (d) $\pm\frac{1}{3}$

2. Six years ago, Mohan was 3 times as old as Shyam. At present Mohan is 1.5 times as old as Shyam. Find the present age of Mohan.

   (a) 12 years (b) 11 years

   (c) 60 years (d) 48 years

3. Find the value of 'y' in the folowing system of equations 2x + 4y = 6 and 3x + 15y = 25.

   (a) $\frac{4}{3}$ (b) $\frac{16}{9}$

   (c) $\frac{5}{6}$ (d) $\frac{6}{5}$

4. If $x^2+\frac{1}{x^2}=79$, then find $x+\frac{1}{x}$.

   (a) ± 7 (b) ± 8

   (c) ± 9 (d) ± 10

5. Find $x^4+\frac{1}{x^4}$, if $x^2+\frac{1}{x^2}=10$.

   (a) 100 (b) 50

   (c) 49 (d) 98

6. If (x – 5) is a factor of $2x^2 + 2px - 2p = 0$, then the value of 'p' is:

   (a) –4 (b) $-\frac{25}{4}$

   (c) $\frac{36}{8}$ (d) + 4

7. If $x^4 + 2x^3 - 3x^2 + x - 1$ is divided by x – 2, then the remainder is:

   (a) 12 (b) 14

   (c) 16 (d) 21

8. The sum of the series 101 + 104 + 107 + … + 161 is:

   (a) 2300 (b) 2751

   (c) 2851 (d) 2900

9. 4 log 2 + log 6 = ?

   (a) log 22 (b) log 96

   (c) log 3 (d) log 10

10. log 5 + log 4 + log 30 – log 6 = ?

    (a) 1 (b) 2

    (c) 3 (d) 2.5

11. The range of 'x' in $2|x|+|3x|\geq 5$ is:

    (a) $x\geq 1$ (b) $x\leq -1$

    (c) (a) and (b) both (d) None of these

12. The range of 'x' in $x^2 - 5x - 14 \leq 0$ is:

    (a) x < 5 (b) $x \geq 5$

    (c) $-2 \leq x \leq 7$ (d) 2 < x < 7

13. If (x + p) is the HCF of $(x^2 + bx + a)$ and $(x^2 + cx + d)$, then the value of p is:

    (a) $\frac{d-a}{c-b}$ (b) $\frac{b-c}{c-d}$

    (c) $\frac{b+c}{c+d}$ (d) $\frac{d+a}{b+c}$

14. Find the LCM of $15x^2y^3$ $(x^2 - y^2)$ and $25x^4y$ (x – y).

    (a) $75x^4y^3(x^2 - y^2)$ (b) $60x^4y^3$ (x–y)

    (c) $240x^4y^4$ (d) $375\ x^2y^2\ (x^2 - y^2)$

15. Find the HCF of $2x^3 + x^2 - 3x$ and $x^3 - x$.

    (a) x(x – 1) (b) x(x + 1)

    (c) x^2 (d) $x^2 + 1$

16. A man is engaged in the condition that the day he works he will get Rs. 5 and the day he does not work he will have to pay a penalty of Rs. 7. After 20 days he got Rs. 52. Find out the number of days on which he worked.

    (a) 14 days (b) 16 days

    (c) 12 days (d) 13 days

17. Find the range of 'x' if $x^2 - 4x + 3 \leq 0$ and $x\geq 2$.

    (a) $x\leq 0$ (b) $1\leq x\leq 3$

    (c) $2\leq x\leq 3$ (d) $x\geq 0$

18. If ab + bc + ca = 40 and a + b + c = 10, then find $a^2 + b^2 + c^2$.

    (a) 10 (b) 20

    (c) 30 (d) 40

19. Find the value of ‘x’ that satisfies both the equations: $x^2 = 36$ and $x^2 - x - 30 = 0$.
(a) 5 (b) – 6
(c) 6 (d) – 5

20. Sum of the roots of a quadratic equation is 8 and the product of their roots is also 8, then the equation is
(a) $x^2 + 4 = 0$ (b) $x^2 - 4 = 0$
(c) $x^2 - 4x - 4 = 0$ (d) $x^2 - 8x + 8 = 0$

21. If $2^{x+5} = 2^{x+3} + 6$, then x is
(a) – 2 (b) – 3
(c) 1 (d) 2

22. The sum of the series 11 + 21 + 31 + 41 + ... + 201 is
(a) 1950 (b) 2120
(c) 2420 (d) 2300

23. A ball is thrown in the air up to a height of 40 m and then it falls down. Every time when the ball bounces, it reaches up to 50% of the previous height. Find the total distance covered by the ball before it comes to rest.
(a) 120 m (b) 100 m
(c) 160 m (d) 200 m

24. The sum of three fractions is $2\frac{11}{24}$. When the largest fraction is divided by the smallest, the fraction thus obtained is $\frac{7}{6}$ which is $\frac{1}{3}$ more than the middle one. Find the fractions.
(a) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$ (b) $\frac{3}{4}, \frac{5}{6}, \frac{7}{8}$
(c) $\frac{2}{5}, \frac{2}{3}, \frac{3}{8}$ (d) $\frac{3}{5}, \frac{3}{8}, \frac{3}{11}$

25. The sum of the squares of two positive numbers is 3341 and the difference of their squares is 891. The two numbers are
(a) 46, 35 (b) 45, 36
(c) 11, 40 (d) 36, 15

26. If a, b, c and d are in AP, such that its common difference = 11 and a = 2, then find the arithmetic mean of a, b, c and d.
(a) 15.4 (b) 16.2
(c) 18.5 (d) 13.1

27. For what value of ‘c’, will the equation $9x^2 - 48x + c = 0$ have equal roots?
(a) 24 (b) 44
(c) 54 (d) 64

28. Find the sum of the roots of the expression $\sqrt{5}y^2 - 4\sqrt{5}y - 10 = 0$.
(a) $\frac{\sqrt{5}}{2}$ (b) 4
(c) $\frac{4}{5}$ (d) $\frac{3}{10}$

29. The area of a rectangle is 255 m^2. If its length is decreased by 1 m and its breadth is increased by 1 m, it becomes a square. Find the perimeter of the square.
(a) 45 m (b) 52 m
(c) 64 m (d) 60 m

30. Find the mean proportional of 4, 16 and 64.
(a) 42 (b) 24
(c) 20 (d) 16

31. If (x – 6) (4x + 3) = 0, then the possible values of 2x are
(a) 12 or $-\frac{3}{2}$ (b) 12 or 0
(c) 6 or $-\frac{3}{4}$ (d) 6 only

32. If one root of the equation $ax^2 + bx + c = 0$ is ‘c’, then the other root is:
(a) $\frac{1}{b}$ (b) $\frac{1}{a}$
(c) $\frac{1}{c}$ (d) $\frac{1}{x}$

33. Find the equation whose roots are $2+\sqrt{2}$ and $2-\sqrt{2}$.
(a) $x^2 + 4x + 2 = 0$ (b) $x^2 - 4x + 2 = 0$
(c) $x^2 + 6x - 3 = 0$ (d) $x^2 - 6x + 3 = 0$

34. Find the remainder when the polynomial $2x^2 + 14x - 15 = 0$ is divided by (x – 4).
(a) 65 (b) 0
(c) 73 (d) 45

35. If (x – 2) and (x + 1) are the factors of the expression $2x^4 - ax^3 + x + b$, then the value of b is:
(a) $-\frac{14}{3}$ (b) –11
(c) $-\frac{22}{3}$ (d) +11

36. What must be subtracted from the numerator as well as from the denominator of $\frac{2}{x}$ so that it becomes equivalent to 40?

(a) $\frac{40x-2}{39}$ (b) $\frac{16-3x}{5}$

(c) $\frac{12x-5}{3}$ (d) $\frac{18-2x}{7}$

37. 5 + 2 |x| = 12. Find the value of x.

(a) $\pm\frac{7}{2}$ (b) –2.5

(c) ± 3 (d) 3.5

38. How many pair of $\{(x, y) \in W\}$ are possible for the equation 2x + 3y = 10?

(a) 1 (b) 2

(c) 3 (d) 4

39. The sum of the series 1 + 4 + 9 + 25 +... + 100 is:

(a) 295 (b) 385

(c) 425 (d) 625

40. The sum of the series 4 + 8 + 16 + 32 + ... till 10 terms is:

(a) $6 \times 2^{10} - 1$ (b) $4 \times 2^{10} + 1$

(c) 6×2^{10} (d) $4(2^{10} - 1)$

41. Find the range of 'x' if |2x| + 5|x| $\leq$ 30.

(a) $-5 < x < 5$ (b) $\frac{-30}{7} \leq x \leq \frac{30}{7}$

(c) $x < \frac{30}{7}$ (d) $\frac{-30}{7} < x$

42. The HCF of $x^2 - 6x + 9$ and $x^2 - 5x + 6$ is:

(a) (x – 3) (b) (x – 2)

(c) $(x-3)^2$ (d) (x – 3) (x – 2)

43. Find the value of $\left(\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}\right)$.

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$

(c) $\frac{(n-1)}{(n+1)}$ (d) None

44. For what value of 'm' will 2m – 1, m + 3 and 3 m be three consecutive terms of an AP?

(a) $\frac{5}{3}$ (b) $\frac{7}{3}$

(c) 3 (d) 2

45. The sum of three consecutive terms of an AP is 6. If the square of the second term is equal to the sum of the other two terms, then the common difference of the AP is:

(a) 12 (b) 4

(c) 6 (d) Cannot be determined

46. Which term will be the first positive term of the AP: –96, –91, –86 . . .. ?

(a) 20^{th} (b) 22^{nd}

(c) 21^{st} (d) None of these

47. If the sum of 'n' terms of the AP: –40, –38, –36 … is positive, then the least possible value of 'n' is.

(a) 41 (b) 42

(c) 43 (d) 44

48. The 15^{th} term of the GP $\frac{7}{2}, \frac{7}{6}, \frac{7}{18} \ldots$ is:

(a) $\frac{7}{2 \times 3^{13}}$ (b) $\frac{7}{2 \times 3^{14}}$

(c) $\frac{7}{2 \times 3^{12}}$ (d) $\frac{7}{2 \times 3^{15}}$

49. The sum of three consecutive term of a GP is 9 and their product is – 216. The largest term among them is.

(a) 12 (b) –6

(c) 6 (d) 9

50. If p^{th}, q^{th} and r^{th} term of a GP are x, y and z respectively, such that p, q and r are in AP, and p < q < r, then which of the following must be true.

(a) $y^3 = xyz$ (b) $y^2 = xz$

(c) z = xy (d) None of these

51. Find the value of the series: $10^3 + 11^3 + 12^3 + 13^3 + \ldots + 20^3$.

(a) 39525 (b) 41075

(c) 45025 (d) 42075

52. The integral value of 'k' for which the pair of linear equations kx – y + 2 = 0 and 3x – (2k + 1)y + 5 = 0 have no solutions is:

(a) 1 (b) 2

(c) –1 (d) $\frac{7}{3}$

53. The cost of two kg apples and five kg oranges is Rs.510 and that of 5 kg apples and two kg oranges is Rs.540. Find the cost of two kg apples.
    (a) Rs.80 (b) Rs.160
    (c) Rs.70 (d) Rs.140
54. Taxi charges in Delhi consist of a fixed charge for the first two kilometres and additional charges for the each subsequent kilometre. For a distance of 12 km the charge paid is Rs 80, and for a journey of 20 km the charge paid is Rs 128. What is the fixed charge for the the first two kilometres?
    (a) Rs. 20 (b) Rs. 12
    (c) Rs. 22 (d) Rs. 6
55. If $\left(\frac{3}{5}\right)^{-2} \times \left(\frac{25}{9}\right)^{2x-5} \times \left(\frac{27}{125}\right)^{2} = \frac{25}{9}$, then the value of 'x' is:
    (a) 3
    (b) 2
    (c) $\frac{7}{2}$
    (d) 4

Answer Key

1. (c)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (b)	**7.** (d)	**8.** (b)	**9.** (b)	**10.** (b)
11. (c)	**12.** (c)	**13.** (a)	**14.** (a)	**15.** (a)	**16.** (b)	**17.** (c)	**18.** (b)	**19.** (c)	**20.** (d)
21. (a)	**22.** (b)	**23.** (c)	**24.** (b)	**25.** (a)	**26.** (c)	**27.** (d)	**28.** (b)	**29.** (c)	**30.** (d)
31. (a)	**32.** (b)	**33.** (b)	**34.** (c)	**35.** (a)	**36.** (a)	**37.** (a)	**38.** (b)	**39.** (b)	**40.** (d)
41. (b)	**42.** (a)	**43.** (b)	**44.** (b)	**45.** (d)	**46.** (c)	**47.** (b)	**48.** (b)	**49.** (a)	**50.** (b)
51. (d)	**52.** (a)	**53.** (b)	**54.** (a)	**55.** (d)					

Explanations

1. c $\frac{1}{x-1}+\frac{1}{x-2}=\frac{3}{x-3}$

$\frac{x-2+x-1}{x^2-2x-x+2}=\frac{3}{x-3}=\frac{2x-3}{x^2-3x+2}$

$\Rightarrow 3x^2 - 9x + 6 = 2x^2 - 6x - 3x + 9$

$\Rightarrow x^2 + 6 - 9 = 0$

$\Rightarrow x^2 - 3 = 0$

$\Rightarrow x^2 = 3$

$\therefore\ x=\pm\sqrt{3}$

2. a Let the present age of Mohan be 'x' years and the present age of Shyam be 'y' years.

So $(x - 6) = 3 \times (y - 6)$

$\Rightarrow x - 6 = 3y - 18$

or $x = 3y - 12$ ... eq (i)

And $x = y \times 1.5 \Rightarrow y = \frac{x\times 2}{3}$

Putting $y = \frac{x\times 2}{3}$ in $x = 3y - 12$, we get

$x = 3\times\frac{2x}{3}-12$ or $x = 2x - 12$

Hence, x = 12 years

3. b $2x + 4y = 6$ ... (i)

$3x + 15y = 25$ ... (ii)

Multiplying equation (i) by 1.5, we get

$3x + 6y = 9$ ... (iii)

Subtracting (iii) from (ii)

$$\begin{array}{r} 3x+15y=25 \\ \underline{-3x\pm\ 6y=-9} \\ 9y=16 \end{array}$$

$y=\frac{16}{9}$

4. c $\left(x+\frac{1}{x}\right)^2=x^2+x\cdot 2\cdot\frac{1}{x}+\frac{1}{x^2}$

$\Rightarrow\left(x+\frac{1}{x}\right)^2=x^2+2+\frac{1}{x^2}$

$\Rightarrow\left(x+\frac{1}{x}\right)^2=79+2=81$

$\Rightarrow x+\frac{1}{x}=\sqrt{81}=\pm 9$

5. d $\left(x^2+\frac{1}{x^2}\right)^2=x^4+\frac{1}{x^4}+2$

$(10)^2=x^4+\frac{1}{x^4}+2$

$100-2=x^4+\frac{1}{x^4}$

$98=x^4+\frac{1}{x^4}$

6. b If $x - 5$ is a factor of the given equation then 5 is the root of the equation (Remainder theorem).

So $2x^2 + 2px - 2p = 0$

$\Rightarrow 2\cdot(5)^2+2p\cdot 5-2p=0$

$50 + 10p - 2p = 0$

$8p = -50$ pr $p = \frac{-50}{8}=\frac{-25}{4}$

7. d Putting x = 2 in the expression, we get

$x^4 + 2x^3 - 3x^2 + x - 1$

$2^4 + 2\cdot 2^3 - 3\cdot 2^2 + 2 - 1$

$= 16 + 16 - 12 + 2 - 1$

$= 34 - 13 = 21$

8. b In the series 101 + 104 + … + 161

a = 101 and d = 3

$a_n = a + (n - 1) d$

$161 = 101 + (n - 1)3$

$161 = 101 + 3n - 3$

$161 - 98 = 3n$

$3n = 63$

$n = 21$

So the sum is $=\frac{n}{2}\times\{a+a_n\}$

$=\frac{21}{2}\{101+161\}$

$=\frac{21}{2}(262)=21\times 131 = 2751$

9. b $4 \log 2 + \log 6 = \log 2^4 + \log 6$ [$m \log n = \log n^m$]

$= \log(16 \times 6) = \log 96$ [$\log m + \log n = \log(m \times n)$]

10. b $\log 5 + \log 4 + \log 30 - \log 6$

$\log(5 \times 4 \times 30) - \log 6 = \log 600 - \log 6$

$\log\frac{600}{6} = \log 100 = 2$

11. c Since 'x' is in modulus so 'x' can either be positive or negative.

Case (i):

If $x\ge 0$

Then $2x+3x\ge 5$

$\Rightarrow 5x\ge 5$

or $x \ge 1$

Case (ii):

If $x < 0$

Then $2(-x) + -(3x) \geq 5$

$\Rightarrow -2x - 3x \geq 5$

$-5x \geq 5$

$5x \leq -5$ or $x \leq -1$

$\therefore x \geq 1$ or $x \leq -1$

12. c $x^2 - 5x - 14 \leq 0$

$x^2 - 7x + 2x - 14 \leq 0$

$x(x - 7) + 2(x - 7) \leq 0$

$(x + 2)(x - 7) \leq 0$

So $-2 \leq x \leq 7$

[Because if $(x - a)(x - b) \leq 0$ then the value of x lies from a to b]

13. a x + p is the HCF of both expressions, means x + p is the factor and –p is root. So substituting –p in place of x.

$(-p)^2 + b(-p) + a = p^2 - bp + a = 0$

Similarly

$(-p)^2 + c(-p) + d = 0$

$p^2 - cp + d = 0$

and $p^2 - bp + a = p^2 - cp + d = 0$

So $cp - bp = d - a$

and $p(c - b) = d - a$

and $p = \frac{d-a}{c-b}$

14. a LCM of 15 and 25 is 75.

LCM of x^2y^3 and x^4y is x^4y^3.

LCM of $x^2 - y^2$ and $x - y$ is $x^2 - y^2$.

So, the required LCM is $75(x^4y^3)(x^2 - y^2)$

15. a $2x^3 + x^2 - 3x$ and $x^3 - x$

$2x^3 + x^2 - 3x = 2x^3 + 3x^2 - 2x^2 - 3x$

$= x^2(2x + 3) - x(2x + 3)$

$= (x^2 - x)(2x + 3)$

$= x(x - 1)(2x + 3)$

$x^3 - x = x(x^2 - 1) = x(x + 1)(x - 1)$

$\therefore$ the highest common factor is $x(x - 1)$.

16. b Suppose that he worked for x days then

$5x - (20 - x)7 = 52$

$\Rightarrow 5x - 140 + 7x = 52$

$\Rightarrow$ So $12x = 140 + 52$

$\Rightarrow x = \frac{192}{12} = 16$.

17. c $x^2 - 4x + 3 \leq 0$

$x^2 - 3x - x + 3 \leq 0$

$x(x - 3) - 1(x - 3) \leq 0$

$(x - 1)(x - 3) \leq 0$

$\therefore 1 \leq x \leq 3$ but it is given that $x \geq 2$

Hence $2 \leq x \leq 3$

18. b $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$(10)^2 = a^2 + b^2 + c^2 + 2(40)$

$100 - 80 = a^2 + b^2 + c^2$

Hence $a^2 + b^2 + c^2 = 20$

19. c $x^2 = 36$

$\Rightarrow x^2 - 36 = 0$

$\Rightarrow (x - 6)(x + 6) = 0$

$\Rightarrow x = \pm 6$

Similarly, $x^2 - x - 30 = 0$

$\Rightarrow x^2 + 5x - 6x - 30 = 0$

$\Rightarrow x(x + 5) - 6(x + 5) = 0$

$\Rightarrow (x - 6)(x + 5) = 0$

$\Rightarrow x = +6, -5$

Since x = +6 satisfies both the equations, that is the correct value of x.

20. d Quadratic equation = $x^2 - (\alpha + \beta)x + \alpha \times \beta$

$= x^2 - 8x + 8 = 0$

21. a $2^{x+5} = 2^{x+3} + 6$

$2^x \times 2^5 = 2^x \times 2^3 + 6$

$2^x \times 2^5 - 2^x \times 2^3 = 6$

$2^x \times 2^3 (2^2 - 1) = 6$

$2^x \times 8 \times 3 = 6$

$2^x = \frac{6}{24} = \frac{1}{4}$

$\Rightarrow 2^x = \frac{1}{4} = 4^{-1} = (2^2)^{-1} = 2^{-2}$ or $2^x = 2^{-2}$

Hence $x = -2$

Shortcut: Pick up the value of 'x' from the option and check the conditions.

22. b In the given series a = 11, l = 201 and d = 10.

Number of terms is not known, so the nth term

$= a + (n - 1) \times d$

$201 = 11 + (n - 1)10 = 11 + 10n - 10$

$= 10n + 1$

$200 = 10n$

$n = 20$

So sum = $\frac{a+a_n}{2} \times n = \frac{11+201}{2} \times 20$

= 212 × 10 = 2120.

23. c It is a case of infinite series

(40 + 20 + 10 + 5 + 2.5 + 1.25 + ...) × 2

[as the ball travels up and down]

So total distance covered

$= \left(\frac{a}{1-r}\right) \times 2 = \left(\frac{40}{1-\frac{1}{2}}\right) \times 2 = (40 \times 2) \times 2$ = 160 metres.

24. b Let the numbers in the ascending order be x, y and z

So x + y + z = $\frac{59}{24}$, $\frac{z}{x} = \frac{7}{6}$ and $y = \frac{7}{6} - \frac{1}{3} = \frac{5}{6}$

So $x = \frac{6z}{7}$

Putting the values of 'x' and 'y' in x + y + z

= $\frac{59}{24}$, we get

$\frac{6z}{7} + \frac{5}{6} + z = \frac{59}{24}$ or $\frac{13z}{7} = \frac{59}{24} - \frac{5}{6} = \frac{39}{24} = \frac{13}{8}$

or $\frac{13z}{7} = \frac{13}{8} \Rightarrow z = \frac{7}{8}$

Also $\frac{z}{x} = \frac{7}{6}$ So $\frac{\frac{7}{8}}{x} = \frac{7}{6}$ or $\frac{7}{8} \times \frac{1}{x} = \frac{7}{6}$

$\Rightarrow x = \frac{6}{8} = \frac{3}{4}$

Hence, $x = \frac{3}{4}$, $y = \frac{5}{6}$ and $z = \frac{7}{8}$

25. a Let the number be x and y

$x^2 + y^2 = 3341$ and $x^2 - y^2 = 891$

So $2x^2 = 4232$

$x^2 = 2116$

x = 46

$2116 - 891 = y^2$

So $y^2 = 1225$ and y = 35

26. c a = 2, b = 13, c = 24 and d = 35

$\Rightarrow AM = \frac{a+b+c+d}{4} = \frac{74}{4} = 18.5$

27. d For equal roots D = 0

So $D = b^2 - 4ac = (-48)^2 - 4 \times 9 \times c = 0$

= 2304 – 36c = 0

$\Rightarrow$ 36c = 2304 and c = $\frac{2304}{36} = 64$

28. b Sum of the roots = $-\frac{b}{a}$ = $\frac{-(-4\sqrt{5})}{\sqrt{5}} = \frac{4\sqrt{5}}{\sqrt{5}} = 4$

29. c The length and the breadth of the rectangle should be of the form (x + 2) and (x) respectively.

$\Rightarrow$ (x + 2) × x = 255 or $x^2 + 2x - 255 = 0$

$x^2 + 17x - 15x - 255 = 0$

x(x + 17) – 15 (x + 17) = 0

(x – 15)(x + 17) = 0

x – 15 = 0

x = 15

$\Rightarrow$ Breadth = 15 m and length = 17 m.

So the side of the square = 15 + 1 = 16 m (or 17 – 1 = 16)

Hence its perimeter = 16 × 4 = 64 metres.

30. d Mean proportional of a, b and c is $\sqrt[3]{abc}$

So, mean proportional of 4, 16 and 64 is $\sqrt[3]{4 \times 16 \times 64}$

= $\sqrt[3]{4 \times 4 \times 4 \times 4 \times 4 \times 4}$ = 4 × 4 = 16

Note: Mean proportional is also known as Geometric Mean.

31. a (x – 6)(4x + 3) = 0

$\Rightarrow$ either x – 6 = 0 or 4x + 3 = 0

$\Rightarrow$ either x = 6 or x = $\frac{-3}{4}$

so 2x = 12 or $2x = \frac{-3}{4} \times 2 = \frac{-3}{2}$

32. b $ax^2 + bx + c = 0$

Product of the roots = $\frac{c}{a}$

$\alpha \times \beta = \frac{c}{a}$

$c \times \beta = \frac{c}{a}$

So $\beta = \frac{1}{a}$

33. b Product of the roots:

$\alpha \times \beta = (2+\sqrt{2})(2-\sqrt{2}) = 4 - 2\sqrt{2} + 2\sqrt{2} - (\sqrt{2})^2$

= 4 – 2 = 2 and sum of the roots = 4

So the equation is $x^2 - 4x + 2 = 0$

34. c In $2x^2 + 14x - 15$ take the value of x = 4

$2(4)^2 + 14 \times 4 - 15$

32 + 56 – 15 = 73,

Note : Here, you can say that $2x^2+14x-15-73$ is exactly divisible by x – 4 or 4 is the root of this equation.

35. a As x – 2 and x + 1 are the factors of the given expression, then 2 and –1 will be its roots.

$2(2)^4-a(2)^3+2+b=0$

$\Rightarrow 32 - 8a + 2 + b = 0$

$\Rightarrow 34 + b = 8a$ … (i)

$2 + a - 1 + b = 0$ … (ii)

$34 + b = -8b - 8$

$9b = -42$

$b = \frac{-42}{9} = \frac{-14}{3}$

36. a Let 'y' be the number to be subtracted,

then $\frac{2-y}{x-y} = \frac{40}{1} \Rightarrow 2 - y = 40x - 40y$

So $39y = 40x - 2$ and $y = \frac{40x-2}{39}$

37. a $5 + 2|x| = 12$

First case $x \ge 0$: $5 + 2x = 12$

So $2x = 12 - 5 = 7$ or $x = \frac{7}{2}$

Second case $x < 0$: $5 + 2(-x) = 12$

So $5 - 2x = 12$ or $x = \frac{-7}{2}$

Hence $x = \pm\frac{7}{2}$.

38. b $2x + 3y = 10$

$y = \frac{10-2x}{3}$

x	5	2
y	0	2

only two values satisfy.

39. b The series is the sum of the squares of first 10 natural numbers $= \frac{n(n+1)(2n+1)}{6} = \frac{10\times11\times21}{6} = 385$

40. d The given series is in geometric progression.

Here, a = 4 and common ratio r = 2 and n = 10

So sum $= \frac{a(r^n-1)}{r-1} = \frac{4(2^{10}-1)}{1}$

41. b Case(i):

If $x \ge 0$

Then, $2x + 5x \le 30$ or $7x \le 30$

$x \le \frac{30}{7}$

If x < 0

Then, $-2x + 5(-x) \le 30$ or $-7x \le 30$

$x \ge \frac{-30}{7}$

Hence the range of x = $\frac{-30}{7} \le x \le \frac{30}{7}$

42. a x^2-6x+9 can be factorised as (x – 3)(x – 3)

and x^2-5x+6 can be factorised as (x – 3) (x – 2)

So their HCF is x – 3.

43. b Taking 2^{n-1} common from numerator and 2^n from the denominator of the expression, we get $\frac{2^{n-1}(2+1)}{2^n(2-1)} = \frac{3}{2}$

44. b 2m – 1, m + 3 and 3 m are three consecutive terms of an AP

$\therefore m + 3 - (2m - 1) = 3m - (m + 3)$

$\Rightarrow -m + 4 = 2m - 3$

$\Rightarrow 3m - 7 = 0$

Hence $m = \frac{7}{3}$

45. d Let the three consecutive terms of the AP are a – d, a and a + d

According to the question

$a-d+a+a+d=6$

$\Rightarrow 3a = 6 \quad \therefore a = 2$ … (i)

It is given that

$a^2 = a-d+a+d$

$\Rightarrow a^2 = 2a$

$\Rightarrow a = 2,\ 0$, but $a \ne 0$ as a = 2 [from (i)]

Using the given information, we cannot determine the value of 'd'.

46. c Let the n^{th} term of the given AP the first positive term

$\therefore t_n > 0$

$\Rightarrow a + (n - 1)d > 0$

$\Rightarrow -96 + (n - 1)5 > 0$

$\Rightarrow 5n > 101$

$\therefore n > 20$

Hence, 21^{st} term of the given AP will be the first positive term.

47. b It is given that sum of first 'n' terms is positive

$\therefore \frac{n}{2}(2a+(n-1)d) > 0$

$\Rightarrow \frac{n}{2}(-80+(n-1)2) > 0$

$\Rightarrow \frac{n}{2}(2n-82)>0$

$\Rightarrow n(n-41)>0$

$\Rightarrow n>41$ or n < 0, but n < 0 is not possible.

Hence, the least value of 'n' is 42.

48. b n^{th} term of a GP = ar^{n-1}

Here, $a = \frac{7}{2}$, and $r = \frac{1}{3}$

$\therefore$ 15^{th} term of given GP = $ar^{14} = \frac{7}{2}\left(\frac{1}{3}\right)^{14} = \frac{7}{2\times 3^{14}}$

49. a Let three consecutive term of GP are $\frac{a}{r}$, a and ar

According to question

$\frac{a}{r}\times a\times ar=-216$

$\Rightarrow a^3=-216$

$\therefore a=-6$

It is given

$\frac{a}{r}+a+ar=9$

$\Rightarrow \frac{-6}{r}+(-6)+(-6)r=9$

$\Rightarrow 6r^2+15r+6=0$

$\Rightarrow 2r^2+5r+2=0$

$\Rightarrow (r+2)(2r+1)=0$

$\therefore r=-2,-\frac{1}{2}$

$\therefore$ The terms are 3, –6, 12 or 12, –6, 3

Hence, the largest term is 12.

50. b Let first term of GP is 'a' and common ratio is 'm'

$\therefore$ $am^{p-1} = x$, $am^{q-1} = y$ and $am^{r-1} = z$

p, q, r in AP and p < q < r

$\therefore$ 2q = p + r

Now $y^2 = (am^{q-1})^2 = a^2 m^{2q-2}$ and xz

$= (am^{p-1})(am^{r-1}) = a^2m^{p+r-2} = a^2m^{2q-2}$

Hence, $y^2 = xz$

51. d $10^3+11^3+12^3+13^3+...+20^3$

$=\left(1^3+2^3+3^3+...+20^3\right)-\left(1^3+2^3+3^3+...+9^3\right)$

$=\left(\frac{20\times 21}{2}\right)^2-\left(\frac{9\times 10}{2}\right)^2$

$=(210)^2-(45)^2$

$=(210+45)(210-45)$

$=255\times 165=42075$

52. a For no solution

$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

$\therefore \frac{k}{3}=\frac{-1}{-(2k+1)}\neq\frac{2}{5}$

Taking $\frac{k}{3}=\frac{-1}{-(2k+1)}$, we get

$2k^2+k-3=0$

$\Rightarrow 2k^2+3k-2k-3=0$

$\Rightarrow (2k+3)(k-1)=0$

$\Rightarrow k=1,\frac{-3}{2}$

Hence, the integral value of k is 1.

53. b Let the cost of one kg apple be Rs. x and that of the orange be Rs. y.

$\therefore$ 2x + 5y = 510 and 5x + 2y = 540

Solving these equations we get, x = 80, y = 70

$\therefore$ Cost of two kg apples = 2 × 80 = Rs. 160.

54. a Let fixed charge for two km be Rs. x and additional charges for the each subsequent km be Rs. y

$\therefore$ x + (12 – 2)y = 80

$\Rightarrow$ x + 10y = 80 … (i)

and x + (20 – 2)y = 128

$\Rightarrow$ x + 18y = 128 … (ii)

solving (i) and (ii) we get, x = 20 and y = 6

Hence, the fix charge for first two kms is Rs. 20.

55. d $\left(\frac{3}{5}\right)^{-2}\times\left(\frac{25}{9}\right)^{2x-5}\times\left(\frac{27}{125}\right)^{2}=\frac{25}{9}$

$\Rightarrow \left(\frac{5}{3}\right)^{2}\times\left(\frac{5}{3}\right)^{4x-10}\times\left(\frac{5}{3}\right)^{-6}=\left(\frac{5}{3}\right)^{2}$

$\Rightarrow \left(\frac{5}{3}\right)^{2+4x-10-6}=\left(\frac{5}{3}\right)^{2}$

$\Rightarrow 2+4x-10-6=2$

$\therefore x=4$