CBSE Test Paper 03

Chapter 10 Circle

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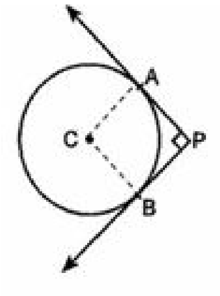
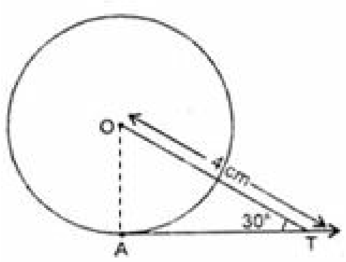
1. In the given figure, the pair of tangents A to a circle with centre O are perpendicular to each other and length of each tangent is 5 cm, then the radius of the circle is : **(1)**



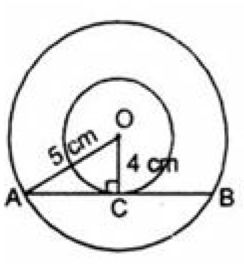
   a. 2.5 cm
   b. 5 cm
   c. 7.5 cm
   d. 10 cm

2. In the given figure, AT is a tangent to the circle with centre O such that OT = 4 cm and $\angle OTA = 30^o$. Then AT is equal to: **(1)**



   a. 2 cm
   b. $2\sqrt{3}$ cm
   c. 4 cm
   d. $4\sqrt{3}$ cm

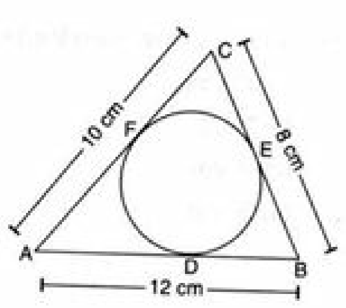
3. If radii of two concentric circles are 4 cm and 5 cm, then the length of the chord of one circle which is tangent to the other circle is: **(1)**



   a. 9 cm

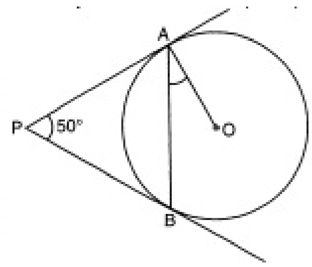
b. 3 cm
c. 1 cm
d. 6 cm

4. A circle is inscribed in $\triangle ABC$ having sides 8 cm, 10 cm and 12 cm as shown in the figure. Then the measure of AD and BE are... **(1)**

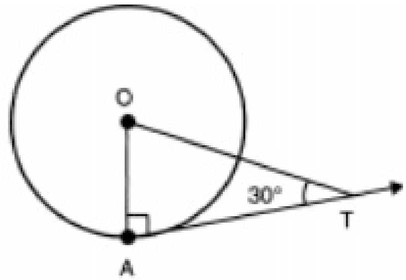


a. AD = 8 cm, BE = 5 cm.
b. AD = 8 cm, BE = 6 cm
c. AD = 5 cm, BE = 7 cm
d. AD = 7 cm, BE = 5 cm

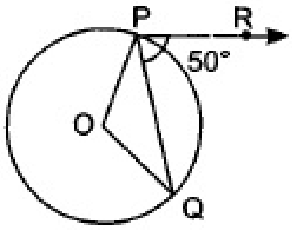
5. PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such $\angle POR = 120^{\circ}$, then $\angle OPQ$ is **(1)**
   a. 60°
   b. 35°
   c. 30°
   d. 45°
6. At which point a tangent is perpendicular to the radius? **(1)**
7. In fig., PA and PB are tangents to the circle with centre O such that ∠APB = 50^{o}. Write the measure of ∠OAB **(1)**



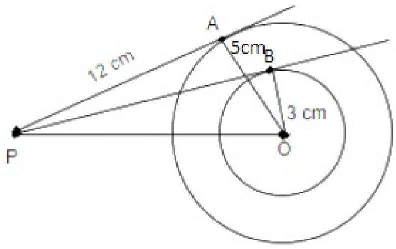
8. In given figure, if AT is a tangent to the circle with centre O, such that OT = 4 cm and ∠OTA = 30^{o}, then find the length of AT (in cm). **(1)**



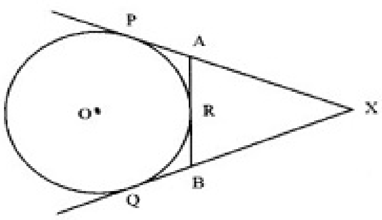
9. In figure, if O is the centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ. Find ∠POQ. **(1)**



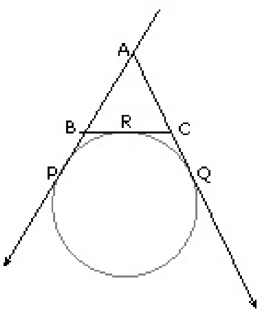
10. Two concentric circles with centre O are of radii 5 cm and 3 cm.From an external point P, two tangents PA and PB are drawn to these circles, respectively. If PA = 12 cm, then find the length of PB **(1)**



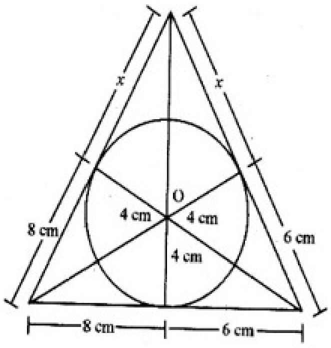
11. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC. **(2)**
12. In given Fig. XP and XQ are tangents from X to the circle with centre O. R is a point on the circle. Prove that XA + AR = XB + BR. **(2)**



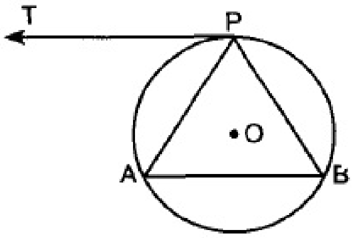
13. In the given figure, find the perimeter of ABC, if AP = 10 cm. **(2)**



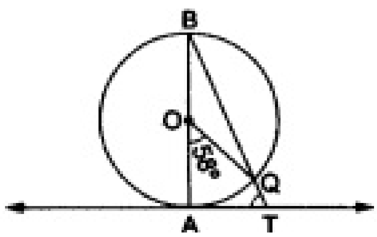
14. Prove that parallelogram circumscribing a circle is a rhombus. **(3)**
15. The radius of the incircle of a triangle is 4 cm and the segment into which one side is divided by the point of contact are 6 cm and 8 cm. Determine the other two sides of the triangle. **(3)**



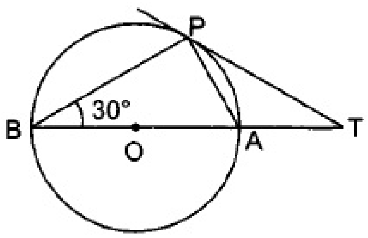
16. A tangent PT is drawn parallel to a chord AB as shown in figure. Prove that APB is an isosceles triangle. **(3)**



17. A chord PQ of a circle is parallel to the tangent drawn at a point R of the circle. Prove that R bisects the arc PRQ. **(3)**
18. In the given figure, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ$ = 58°, find $\angle ATQ$. **(4)**



19. In figure, O is the centre of the circle and TP is the tangent to the circle from an external point T. If ∠PBT = 30°, prove that BA : AT = 2:1. **(4)**



20. If from an external point B of a circle with centre 'O', two tangents BC, BD are drawn such that ∠DBC = 120°, prove that BC + BD = BO, i.e., BO = 2BC. **(4)**

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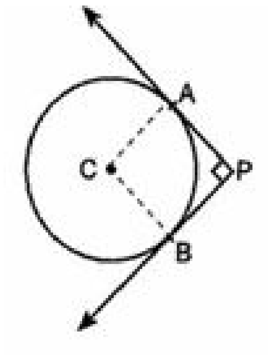
Solution

1. b. 5 cm

   Explanation: 

   Construction: Joined OA and OB.

   Here OA$\perp$ AP and OB$\perp$ BP and PA $\perp$ PB

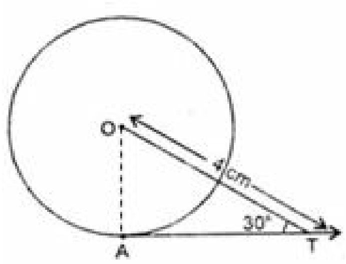
   Also AP = PB

   Therefore, APBO is a square.

   $\Rightarrow$ AP = OA = OB = 5 cm

2. b. $2\sqrt{3}$ cm

   Explanation: Construction: Joined OA.

   Since OA is perpendicular to AT, then

   $\angle$OAT = 90° In right angled triangle OAT

   $\cos 30^{\circ} = \frac{\mathrm{AT}}{\mathrm{OT}} \Rightarrow$

   $\frac{\sqrt{3}}{2} = \frac{\mathrm{AT}}{4} \Rightarrow \mathrm{AT} = 2\sqrt{3}$ cm

3. d. 6 cm

   Explanation: Here OC is perpendicular to AB.

   Then OC bisects AB i.e., AC = BC

   Now, in triangle OAC, $\mathrm{OA}^2 = \mathrm{AC}^2 + \mathrm{OC}^2$

$\Rightarrow (5)^2 = AC^2 + (4)^2 \Rightarrow AC^2 = 25 - 16$

$\Rightarrow AC = 3$ Therefore, length of tangent AB = AC + BC = 3 + 3 = 6 cm

4. d. AD = 7 cm, BE = 5 cm

Explanation: Let AD = x and BE = y

$\therefore$ BD = 12 - x $\Rightarrow$ BE = y

But BD = BE (Tangents to a circle from an external point B)

$\Rightarrow$ y = 12 - x $\Rightarrow$ x + y = 12 ....(i)

Also, AF = x

and CF = 10 - x

and CE = 8 - y

$\therefore$ 10 - x = 8 - y

x - y = 2........(ii)

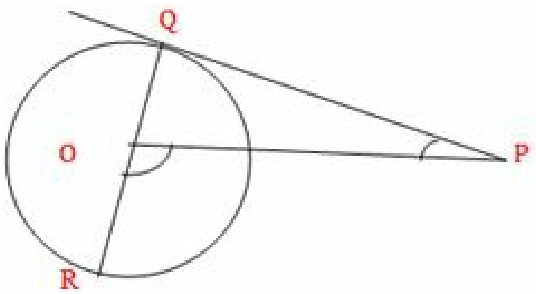
On solving eq. (i) and (ii), we get

x = 7 and y = 5

Therefore AD = 7 cm and BE = 5 cm

5. c. 30°

Explanation:



Here $\angle PQO = 90^\circ$ Since, $\angle QOR = 180^\circ$

$\therefore \angle POQ + \angle POR = 180^\circ$

$\Rightarrow \angle POQ + 120^\circ = 180^\circ$

$\Rightarrow \angle POQ = 60^\circ$

Now, in triangle OPQ,

$\angle OPQ + \angle PQO + \angle QOP = 180^\circ$

$\Rightarrow \angle OPQ + 90^\circ + 60^\circ = 180^\circ$

$\Rightarrow \angle OPQ = 30^\circ$

6. A line which intersects a circle at any one point is called the tangent. The tangent at any point of a circle is perpendicular to the radius through the all point of contact.

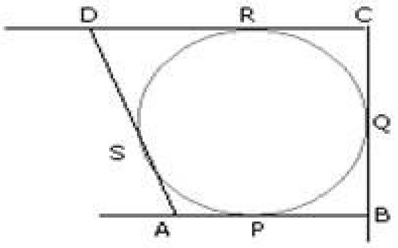
7. Here, $\angle APB = 50^\circ$
$\angle PAB = \angle PBA = \frac{180^\circ - 50^\circ}{2} = 65^\circ$
$\angle OAB = 90^\circ - \angle PAB$
$= 90^\circ - 65^\circ = 25^\circ$

8. In given figure, AT is a tangent to the circle with centre O, such that OT = 4 cm and $\angle$OTA = 30^0, then we have to find the length of AT (in cm).
$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$
$\frac{AT}{OT} = \cos 30^\circ$
$\therefore \text{AT} = \text{ OT } \cos 30^\circ$
or, $AT = 4 \times \frac{\sqrt{3}}{2}$
$= 2\sqrt{3}\text{cm}$

9. OP $\perp$ PR [$\because$ Tangent and radius are $\perp$ to each other at the point of contact]
$\angle OPQ = 90 - 50 = 40$
$OP = OQ$ [By isosceles triangle's property]
$\angle$OPQ = $\angle OQP = 40$
In $\triangle$OPQ,
$\Rightarrow \angle$O + $\angle$P + $\angle$Q = 180°
$\Rightarrow \angle O + 40 + 40 = 180$
$\angle O = 180 - 80 = 100$

10. Two concentric circles with centre O are of radii 5 cm and 3 cm. From an external point P, two tangents PA and PB are drawn to these circles, respectively. If PA = 12 cm, then we have to find the length of PB.
We know that radius is perpendicular to the tangent at the point of contact therefore, $AO \perp AP$.
Now, in right-angled triangle PAO, $\angle PAO = 90^\circ$
$OP^2 = (PA)^2 + (AO)^2$ [by Pythagoras theorem]
$\Rightarrow OP = \sqrt{(PA)^2 + (AO)^2}$
$\Rightarrow OP = \sqrt{169} = 13\ cm$
Similarly, in right-angled triangle PBO, $\angle PBO = 90^\circ$
$PO^2 = (PB)^2 + (OB)^2$ [by Pythagoras theorem]
$\Rightarrow PB = \sqrt{(OP)^2 - (OB)^2}$

$\Rightarrow PB = \sqrt{(13)^2 - (3)^2} = \sqrt{160} = 4\sqrt{10}\ cm$ [∵ radius, OB = 3 cm, given]

11. 

∵ AP, AS are tangents from a point A (Outside the circle) to the circle.

∴ AP = AS

Similarly BP = BQ

CQ = CR

DR = DS

Now AB + CD = AP + PB + CR + RD

= AS + BQ + CQ +DS

= (AS +DS) + (BQ + CQ)

= AD + BC

AB + CD = AD + BC

12. Since the lengths of tangents from an exterior point to a circle are equal.

∴ XP = XQ ....(i)

AP = AR .... (ii)

BQ = BR .... (iii)

Now Xp = XQ i.e. XA + AP = XB + BQ

$\Rightarrow$ XA + AR = XB + bR

Hence proved.

13. ∵ BC touches the circle at R

∵ Tangents drawn from external point to the circle are equal.

∴ AP = AQ, BR = BP

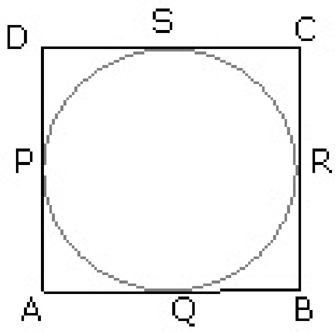
And CR = CQ

∴ Perimeter of $\triangle$ABC = AB + BC + AC

= AB + (BR + RC) + AC

= AB + BP + CQ + AC

= AP + AQ = 2AP = 2 $\times$ 10 = 20 cm

14. 

Given ABCD is a parallelogram in which all the sides touch a given circle

To prove:- ABCD is a rhombus

Proof:-

∵ ABCD is a parallelogram

∴ AB = DC and AD = BC

Again AP, AQ are tangents to the circle from the point A

∴ AP = AQ

Similarly, BR = BQ

CR = CS

DP = DS

∴(AP + DP) + (BR + CR) = AQ + DS + BQ + CS = (AQ + BQ) + (CS + DS)

⇒ AD + BC = AB + DC

⇒ BC + BC = AB + AB [∵ AB = DC, AD = BC]

⇒ 2BC = 2AB

⇒ BC = AB

Hence, parallelogram ABCD is a rhombus

15. Equate the areas of the triangle formed by using the formula $\sqrt{s(s-a)(s-b)(s-c)}$ and also found by dividing it into three triangles.

Let the length of tangent from third vertex = x.

Then the sides of triangle are 14, x + 6, x + 8

$\therefore s=\frac{14+x+6+x+8}{2}=\frac{28+2x}{2}=14+x$

$\therefore \sqrt{(14+x)(14+x-x-6)(14+x-x-8)(14+x-14)}$

$=\frac{1}{2}\times 4\times 14+\frac{1}{2}\times 4\times (x+6)+\frac{1}{2}\times 4(x+8)$

$\Rightarrow \sqrt{(14+x)\times 8\times 6\times x}=2(14+x+6+8)$

$\Rightarrow \sqrt{42x+3x^2}=14+x$

⇒ $42x + 3x^2 = (14 + x)^2$

$\Rightarrow 42x + 3x^2 = 196 + x^2 + 28x$

$\Rightarrow 2x^2 + 14x - 196 = 0$

$\Rightarrow x^2 + 7x - 98 = 0$

$\Rightarrow x^2 + 14x - 7x - 98 = 0$

$\Rightarrow x(x + 14) -7(x + 14) = 0$

$\Rightarrow (x + 14)(x - 7) = 0$

$\Rightarrow x - 7 = 0$

Or $x + 14 = 0$

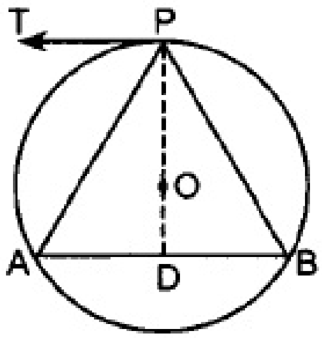
$\Rightarrow x = 7$

Or $x = -14$ extraneous

$\therefore$ The remaining two sides of the triangle are (x + 6) cm and (x - 8)cm,

i.e., (7 + 6)cm or 13 cm and 15 cm.

16. Given,



Construction: Join PO and produce it to D.

Proof: Here, $OP \perp TP$

$\angle OPT = 90^\circ$

Also, $TP \parallel AB$

$\therefore \angle \text{TPD} + \angle \text{ADP} = 180^\circ$

$\Rightarrow \angle \text{ADP} = 90^\circ$

OD bisects AB [Perpendicular from the centre bisects the chord]

In $\triangle$ADP and $\triangle$BDP

AD = BD

$\angle ADP = \angle BDP$ [Each 90°]

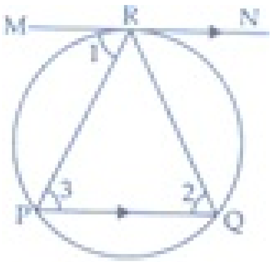
$PD = PD$

$\therefore \triangle ADP \cong \triangle BDP$ [SAS]

$\angle PAB = \angle PBA$ [C.P.C.T.]

$\therefore$ $\triangle$PAB is isosceles triangle.

17. Given: In a circle a chord PQ and a tangent MRN at R such that QP || MRN



To prove: R bisects the arc PRQ.

Construction: Join RP and RQ.

Proof: Chord RP subtends $\angle$1 with tangent MN and $\angle$2 in alternates segment of circle so $\angle$1 = $\angle$2.

MRN || PQ

$\therefore$ $\angle$1 = $\angle$3 [Alternate interior angles]

$\Rightarrow$ $\angle$2 = $\angle$3

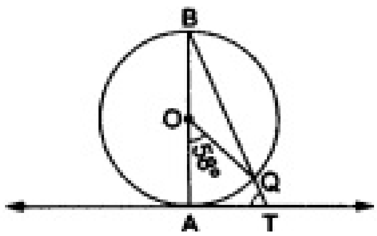
$\Rightarrow$ PR = RQ [Sides opp. to equal $\angle$s in Δ RPQ]

$\because$ Equal chords subtend equal arcs in a circle so

arc PR = arc RQ

or R bisect the arc PRQ. Hence, proved.

18.



$\angle$AOQ = 58°

$\Rightarrow \quad \angle ABQ = \frac{1}{2}\angle AOQ = 29^\circ$ [$\therefore$ the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle]

$\Rightarrow \quad \angle ABT = 29^\circ$

Now, AT is a tangent at A and OA is the radius through the point of contact A.

$\therefore \quad OA \perp AT$, i.e,

$\angle OAT = 90^\circ$

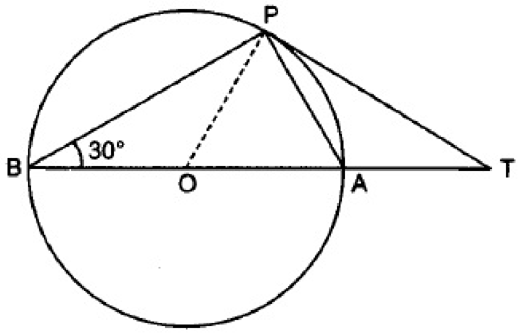
$\Rightarrow \angle BAT = 90^\circ$.

in $\triangle$BAT,

we have

$\angle BAT + \angle ABT + \angle ATB = 180^\circ$
$\Rightarrow 90^\circ + 29^\circ + \angle ATB = 180^\circ$
$\angle ATB = 61^\circ.$
$\therefore \quad \angle ATQ = \angle ATB = 61^\circ.$

19. According to the question,



AB is the chord passing through the centre
So, AB is the diameter
Since, angle in a semi circle is a right angle
∠APB= 90°
By using alternate segment theorem
We have ∠APB = ∠PAT = 30°
Now, in APB
∠BAP + ∠APB + ∠BAP = 180° (Angle sum property of triangle)
∠BAP = 180° – 90° – 30° = 60°
Now, ∠BAP = ∠APT + ∠PTA (Exterior angle property)
60° = 30° + ∠PTA
∠PTA = 60° – 30° = 30°
We know that sides opposite to equal angles are equal
AP = AT
In right triangle ABP:
sin ∠ABP$= \frac{AP}{BA}$
sin30° $= \frac{AP}{BA}$
$\frac{1}{2} = \frac{AP}{BA}$
BA : AT = 2 : 1

20. According to the question, we are a given that from an external point B of a circle with centre ‘O’, two tangents BC, BD are drawn such that ∠DBC = 120°, we have to

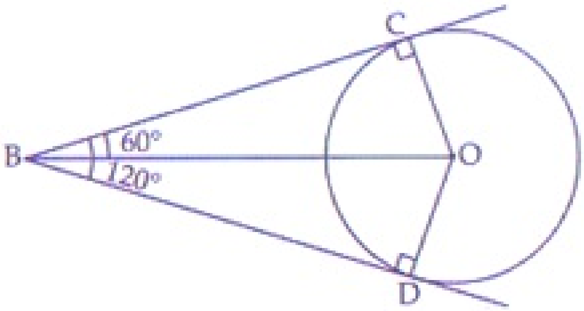
prove that BC + BD = BO, i.e., BO = 2BC.

Given: A circle with centre O.

Tangents BC and BD are drawn from an external point B such that $\angle$DBC = 120°

To prove: BC + BD = BO, i.e., BO = 2BC

Construction: Join OB, OC and OD.



Proof: In $\triangle$OBC and $\triangle$OBD, we have

OB = OB [Common]

OC = OD [Radi of same circle]

BC = BD [Tangents from an external point are equal in length] ...(i)

$\therefore$ $\triangle$OBC $\cong$ $\triangle$OBD [By SSS criterion of congruence]

$\Rightarrow$ $\angle$OBC = $\angle$OBD (CPCT)

$\therefore$ $\angle$OBC = $\frac{1}{2}$ $\angle$DBC = $\frac{1}{2} \times$ 120° [$\because$ $\angle$CBD = 120° given]

$\Rightarrow$ $\angle$OBC = 60°

OC and BC are radius and tangent respectively at contact point C.

Hence, $\angle$OCB = 90°

Now, in right angle $\triangle$OCB, $\angle$OBC = 60°

$\therefore$ cos 60° = $\frac{BC}{BO}$

$\Rightarrow \frac{1}{2} = \frac{BC}{BO}$

$\Rightarrow$ OB = 2BC

$\Rightarrow$ OB = BC + BC

$\Rightarrow$ OB = BC + BD [$\because$ BC = BD from (i)]

Hence, proved.