3

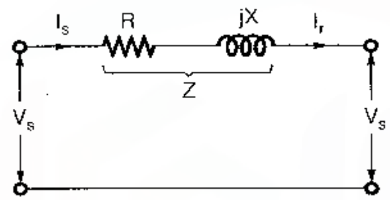
Performance of Transmission Line

Type of Transmission line	Classification based on $f * l$	Classification based on length if line is power line i.e. f = 50 Hz	Based on operating voltage	Effect of capacitance
Short line	$fl < 4000$ Hz km	$l < 80$ km	0-20 kV	Neglected
Medium line	$4000 < fl < 12000$ Hz km	$80 \text{ km} < l < 240$ km	20-100 kV	Capacitor is lumped and constant
Long line	$fl = 12000$ Hz km	$l > 240$ km	> 100 kV	Capacitance is uniformally distributed

where, f = Operating frequency

l = Length of transmission line

Short transmission lines



where, V_s = Sending end voltage

V_r = Receiving end voltage

I_s = Sending end current

I_r = Receiving end current

$$\boxed{I_s = I_r}, \quad \boxed{V_s = V_r + I_r Z}$$

In Matrix form:

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix}$$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix}$$

So, $A = 1, B = Z, C = 0, D = 1$

$$\boxed{V_s = V_r + I_r R \cos\phi_r + I_r X \sin\phi_r}$$

Note:

- The performance of transmission line is determine by efficiency and voltage regulation.
- For rotating machine speed regulation determine and for static machine voltage regulation determine.
- For satisfactory performance lower the regulation and higher the efficiency.

Voltage Regulation

It is the change in receiving end voltage from no load to full load while keeping the sending end voltage constant and made supply frequency constant.

$$\boxed{\text{Voltage regulation} = \frac{V_r' - V_r}{V_r}}$$

where, V_r' = Receiving end voltage under no load condition

V_r = Receiving end voltage under full load condition

Regulation of Short Transmission Line

$$\boxed{\text{Regulation} = \frac{I_r R \cos\phi_r \pm I_r \times \sin\phi_r}{V_R}}$$

where, $+ \rightarrow$ For lagging power factor

$- \rightarrow$ For leading power factor

Note:

- Regulation is always positive for lagging power factor.
- Regulation may be positive, negative or zero for leading power factor.
- In short line sending end power factor always less than receiving end power factor.
- Short line is always symmetrical and reciprocal.
- Regulation maximum when $\boxed{\phi_r = \theta}$.

Maximum voltage regulation occurs when

$$\boxed{\phi_r = \theta}$$

where, ϕ_r = Phase angle of load

θ = Impedance angle of line = $\tan^{-1}\frac{X}{R}$.

Zero regulation occurs when

$$\boxed{\phi_r + \theta = \frac{\pi}{2}}$$

- At leading pf the regulation will generally negative but it also becomes zero provided that

$$\boxed{\phi_r = \tan^{-1}(R/X)}$$ i.e. 0.707 power factor lead.

Medium Length Transmission Line

ABCD parameter in matrix form

(a) For nominal T-circuit

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1+\frac{YZ}{2} & Z\left(1+\frac{YZ}{4}\right) \\ Y & 1+\frac{YZ}{2} \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix}$$

(b) For nominal π-circuit

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1+\frac{YZ}{2} & Z \\ Y\left(1+\frac{YZ}{4}\right) & 1+\frac{YZ}{2} \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix}$$

Note:

For a fix receiving end voltage the sending end voltage which is calculated in nominal-π model will be slightly high when compare to nominal-T so regulation n nominal-π is slightly high when compare to nominal-T.

Farranti Effect

When receiving end of the transmission line is operating under no load condition or lightly load condition, sending end voltage Vs is less than receiving end voltage V_r.

Long Transmission Line

ABCD parameter in matrix form

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \cosh\gamma l & Z_c \sinh\gamma l \\ \frac{1}{Z_c}\sinh\gamma l & \cosh\gamma l \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix}$$

$$\boxed{\gamma = \sqrt{ZY}}$$

also, $$\boxed{\gamma = \alpha + j\beta}$$

and for loss less line $$\boxed{\beta = \omega\sqrt{LC}}$$

where, l = Length of transmission line

γ = Propagation constant

α = Attenuation constant in Neper/sec.

β = Phase constant in rad/km

$$\boxed{\sin\gamma l = \sqrt{YZ}\left[1+\frac{YZ}{3!}+\frac{(YZ)^2}{5!}+\cdots\right]}$$

$$\boxed{\cos\gamma l = 1+\frac{YZ}{2!}+\frac{(YZ)^2}{4!}+\cdots}$$

Wavelength (λ)

The distance corresponding to which there is a phase changes of 2π or 360°.

$$\boxed{\lambda = \frac{2\pi}{\beta}}$$

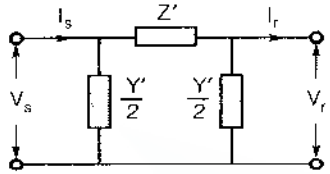
Velocity of wave Propagation

$$\boxed{V_C = f\lambda}$$

where, f = Frequency

λ = Wavelength

Equivalent π Circuit and ABCD Parameters



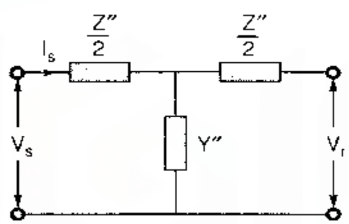
$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1+\frac{Y'Z'}{2} & Z \\ Y'\left(1+\frac{Y'Z'}{4}\right) & 1+\frac{Y'Z'}{2} \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix}$$

where,

$$Z' = Z_c(\sinh \gamma l) = Z \cdot \frac{(\sinh \gamma l)}{\gamma l}$$

$$Y' = \frac{2}{Z_c}\tanh\left(\frac{\gamma l}{2}\right) = Y\frac{\tanh(\gamma l/2)}{(\gamma l/2)}$$

Equivalent T Circuit and ABCD Parameter



$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1+\frac{Y''Z''}{2} & Z''\left(1+\frac{Z''Y''}{4}\right) \\ Y'' & 1+\frac{Y''Z''}{2} \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix}$$

where,

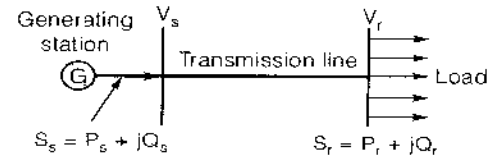
$$\frac{Z''}{2} = Z_c\left(\frac{\cosh \gamma l - 1}{\sinh \gamma l}\right) = \frac{Z}{2}\frac{\tanh(\gamma l/2)}{(\gamma l/2)}$$

$$Y'' = \frac{1}{Z_c}\sinh \gamma l = Y\frac{\sin \gamma l}{\gamma l}$$

Note: ....

The equivalence is valid for only one frequency and only for the terminal conditions.

Power for Transmission Lines



(A two bus power system)

Let $\quad V_s = |V_s|\angle\delta, \quad V_r = |V_r|\angle 0$

$D = A = |A|\angle\alpha,\ B = |B|\angle\beta$

- Complex power per phase at the receiving end

$$S_r = \frac{|V_s||V_r|}{|B|}\angle(\beta-\delta) - \frac{|A||V_r|^2}{|B|}\angle(\beta-\alpha)$$

Surge Impedance

The impedance of transmission line with losses, is known as characteristic impedance (Z_c) and the impedance of transmission line without losses is known as surge impedance (Z_s) or natural impedance (Z_n)

$$Z_c = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$Z_c = \sqrt{\frac{B}{C}} = \sqrt{Z_{O.C.}\,Z_{S.C.}}$$

where, Y, Z = Shunt admittance and series impedance per unit length respectively

Z_{OC} = Open circuit impedance

Z_{SC} = Short circuit impedance

For lossless transmission line $R = 0$, $G = 0$

$$Z_s = \sqrt{\frac{L}{C}}$$

Note: ....

Surge impedance does not depend upon length of the line.

Remember:

- $Z_c = 400\ \Omega$ for transmission line
- $Z_c = 40\ \Omega$ for cable
- Z

Flat Line (or) Infinite Line

A lossless transmission line terminated with surge impedance at the receiving end is known as infinite or flat line.

Surge Impedance Loading (SIL)

The power delivered by the line to a purely resistive load equal to its surge impedance (or) the load at which the inductive and capacitive reactive voltamperes are equal and opposite, is called surge impedance loading of the line.

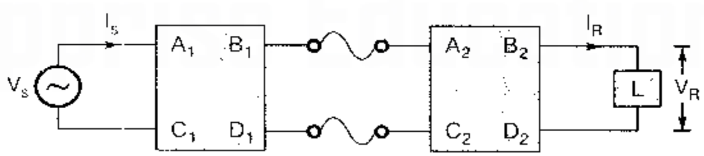
$$\text{SIL} = \frac{V_s V_R}{Z_s} = \frac{(kV)^2}{Z_o}\ \text{MW}$$

Note:

- SIL is independent of the distance and depends on the voltage.
- SIL is always less than the rated capacity of a line.
- If load on line is < SIL then power factor will be leading.
- If load on line is > SIL then power factor will be lagging.
- If load on line is = SIL then power factor will be unity.

Transmission Line Connection

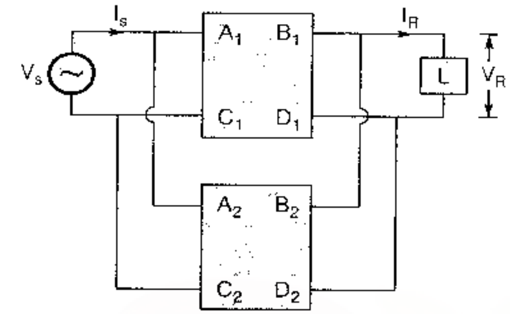
1. Transmission Line Connected Either in Series or Cascaded



$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A_1A_2 + B_1C_2 & A_1B_2 + B_1D_2 \\ C_1A_2 + D_1C_2 & C_1B_2 + D_1D_2 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

2. Transmission Line Connected in Parallel



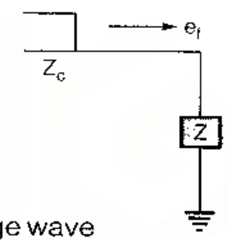
$$A = \frac{A_1B_2 + A_2B_1}{B_1 + B_2} \qquad B = \frac{B_1B_2}{B_1 + B_2}$$

$$C = (C_1 + C_2) + \frac{(D_1 - D_2)}{B_1 + B_2}, \qquad D = \frac{D_1B_2 + D_2B_1}{B_1 + B_2}$$

Reflection and Refraction of Waves

1. Line Terminated by an Impedance (Z)

$$e_t = \frac{2Z}{Z + Z_c} \cdot e_f$$



$$e_r = \frac{Z - Z_c}{Z + Z_c} \cdot e_f$$

where, e_f = Incident or forward voltage wave

e_t = Refracted or transmitted voltage wave

e_r = Reflected voltage

$\frac{2Z}{Z + Z_c}$ = Refraction coefficient of a line with surge impedance Z_c terminated by an impedance Z.

$\frac{Z - Z_c}{Z + Z_c}$ = Reflection coefficient.

- Transmitted current

$$I_t = \frac{2}{Z + Z_c} \cdot e_f$$

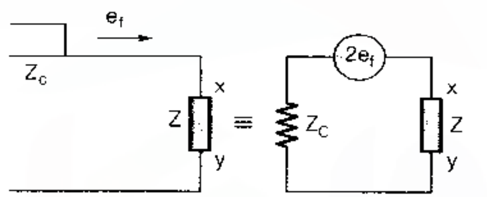
- Reflected current

$$i_r = \frac{Z_c - Z}{Z_c + Z} \cdot i_f$$

Conditions

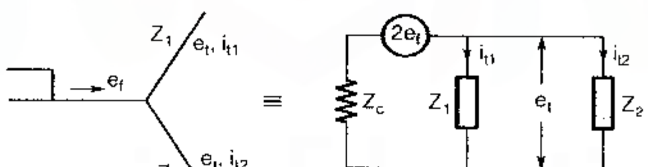
1. If line is terminated by a resistance, equal, to the surge impedance.
   then, $e_t = e_f$, $i_t = i_f$, $e_r = 0$ and $i_r = 0$
   The incident wave continues as it is and there is no reflection.
2. If line is open circuited line
   then, $e_t = 2e_f$, $i_t = 0$, $e_r = e_f$, $i_r = -i_f$
   An open circuit at the end of a line demands, that the current at that point be zero at all times.
3. If line is short circuited line
   then, $e_t = 0$, $i_t = 2i_f$, $e_r = -e_f$, $i_r = i_f$

Equivalent Circuit of Travelling Waves



Actual circuit   Equivalent circuit

2. Forked line



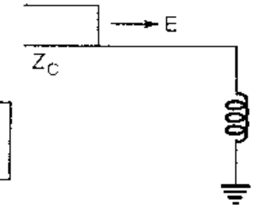
- Refracted or Transmitted voltage

$$e_t = \frac{2e_f}{Z_c + \frac{Z_1Z_2}{Z_1+Z_2}}\left(\frac{Z_1Z_2}{Z_1+Z_2}\right)$$

- Refracted or Transmitted currents

$$i_{t_1} = \frac{e_t}{Z_1} \text{ and } i_{t_2} = \frac{e_t}{Z_2}$$

3. Line terminated by inductance



- Transmitted current

$$i_t = \frac{2E}{Z_c}\left[1 - \exp\left(-\frac{Z_c t}{L}\right)\right]$$

where, E = A step wave travelling on a line of surge impedance Z_c

- Transmitted voltage

$$e_t = 2E \cdot \exp\left(\frac{-Z_c t}{L}\right)$$

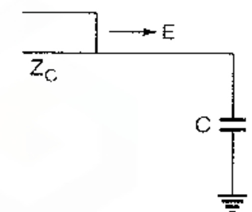
- Reflected voltage

$$e_r = E\left[2\exp\left(-\frac{Z_c t}{L}\right) - 1\right]$$

- Reflected current

$$i_r = \frac{E}{Z_c}\left[1 - 2\exp\left(-\frac{Z_c t}{L}\right)\right]$$

4. Line Terminated by Capacitance



- Transmitted current

$$i_t = \frac{2E}{Z_c}\exp\left(-\frac{t}{CZ_c}\right)$$

- Transmitted voltage

$$e_t = 2E\left[1 - \exp\left(-\frac{t}{CZ_c}\right)\right]$$

- Reflected voltage

$$e_r = E\left[1 - 2 \cdot \exp\left(-\frac{t}{CZ_c}\right)\right]$$

- Reflected current

$$i_r = \frac{E}{Z_c}\left[2\exp\left(-\frac{t}{CZ_c}\right) - 1\right]$$

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