MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 48

Total Marks : 32

Max. Time : 40 min.

Topics : Vector, Application of Derivatives

Type of Questions	**M.M., Min.**
Subjective Questions (no negative marking) Q.1 to Q.8 (4 marks, 5 min.)	**[32, 40]**

1. A segment of a line PQ with its extremities on AB and AC bisects a triangle ABC with sides a, b, c into two equal areas. Find the shortest length of the segment PQ.

2. Let f, g : [a, b] $\to$ R be two continuous function, differentiable on (a, b). Assume in addition that g and g' are no where zero on (a, b) and $\frac{f(a)}{g(a)} = \frac{f(b)}{g(b)}$. Prove that there exists c $\in$ (a, b) such that $\frac{f(c)}{g(c)} = \frac{f'(c)}{g'(c)}$

3. Let a, b, c, d, e, f $\in$ R such that ad + be + cf = $\sqrt{(a^2+b^2+c^2)\,(d^2+e^2+f^2)}$

Use vector to prove that $\frac{a+b+c}{\sqrt{a^2+b^2+c^2}} = \frac{d+e+f}{\sqrt{d^2+e^2+f^2}}$

4. Show that f(x) = $\left(1+\frac{1}{x}\right)^x$ is always an increasing function for all x in its domain.

5. With usual notation in ΔABC if 2b = 3a and $\tan^2 A = \frac{3}{5}$, prove that there are two values of third side, one of which is double the other.

6. Prove that the locus of the centre of a circle , which intercepts a chord of given length '2 a' on the axis of x and passes through a given point on the axis of y, distance b from the origin , is curve, $x^2 \pm 2yb + b^2 = a^2$.

7. Find the sum $\tan\theta + \frac{1}{2}\tan\frac{\theta}{2} + \frac{1}{2^2}\tan\frac{\theta}{2^2} + \frac{1}{2^3}\tan\frac{\theta}{2^3} + ...\infty$ and hence the sum of the series

$$\frac{1}{2^2}\tan\frac{\pi}{2^2} + \frac{1}{2^3}\tan\frac{\pi}{2^3} + \frac{1}{2^4}\tan\frac{\pi}{2^4} + ...\infty$$

8. The two adjacent sides of a paralelogram are represented by the lines x – y + 1 = 0 and 4x–3y–2 = 0. If one of the diagonals of the parallelogram is along the line y = $\frac{3x}{2}$, then find the equation of the other diagonal without finding the vertices of the parallelogram.

Answers Key

1. $\sqrt{\dfrac{(c+a-b)(a+b-c)}{2}}$

7. $\dfrac{1}{\theta}-2\cot 2\theta$, $\dfrac{1}{\pi}$

8. $5x - 4y - 1 = 0$