3. Polynomials

Exercise 3.1

1 A. Question

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

$4x^2 + 8x$

Answer

To find the zeros of the polynomial let us first solve the polynomial by equating it to zero. Factorizing the given polynomial

$4x^2 + 8x = 0$

$4x(x + 8) = 0$

$4x = 0$ or $x + 8 = 0$

Now solving the first part,

$4x = 0$

$x = 0/4$

$x = 0$

Now solving the second part,

$x + 8 = 0$

$x = -8$

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$\therefore a = 4, b = 8, c = 0$

Sum of zeroes = $-b / a$

$= -8 / 4$

$= -2$

Product of zeroes = c / a

= 0 / 4

= 0

1 B. Question

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

$4x^2 - 4x + 1$

Answer

To find the zeros of the polynomial let us first solve the polynomial by equating it to zero. Factorizing the given polynomial

$4x^2 - 4x + 1 = 0$

To factorize the polynomial we have,

Sum of the value should be equal = -4

Product should be equal to = 4 × 1

= 4

So two numbers are -2, -2

$4x^2 - 2x - 2x + 1 = 0$

$2x(2x - 1) - 1(2x - 1) = 0$

$(2x-1)(2x-1) = 0$

$2x-1 = 0$ or $2x-1 = 0$

Both the parts are same.

Solving them,

$2x-1 = 0$

$2x = 1$

$x = 1/2$

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$\therefore a = 4, b = -4, c = 1$

Sum of zeroes = -b / a

= - (-4) / 4

= 1

Product of zeroes = c / a

= 1 / 4

= 1/4

1 C. Question

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

$6x^2 - x - 2$

Answer

To find the zeros of the polynomial let us first solve the polynomial by equating it to zero. Factorizing the given polynomial

$6x^2 - x - 2 = 0$

To factorize the polynomial we have,

Sum of the value should be equal = -1

Product should be equal to = 6 × (-2)

= -12

So two numbers are -4, 3

$6x^2 - 4x + 3x - 2 = 0$

2x (3x – 2) + 1(3x – 2) = 0

(2x+1)(3x-2) = 0

2x+1 = 0 or 3x-2 = 0

Now Solving first part,

2x+1 = 0

2x = -1

x = -1/2

Now solving the second part,

3x-2 = 0

3x = 2

$x = 2/3$

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

∴ $a = 6, b = -1, c = -2$

Sum of zeroes = $-b / a$

$= - (-1) / 6$

$= 1/6$

Product of zeroes = c / a

$= -2 / 6$

$= -1/3$

1 D. Question

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

$x^2 – 15$

Answer

To find the zeros of the polynomial let us first solve the polynomial by equating it to zero. Factorizing the given polynomial

$x^2 – 15 = 0$

$x^2 = 15$

$x = \sqrt{15}$

When we take square root on both sides, we get two values of a variable

Thus $x = +\sqrt{15}$ and $x = -\sqrt{15}$

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

∴ $a = 1, b = 0, c = -15$

Sum of zeroes = $-b / a$

$= 0 / 4$

$= 0$

Product of zeroes = c / a

= -15 / 1

= -15

1 E. Question

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

$$x^2 - \left(\sqrt{3}+1\right)x + \sqrt{3}$$

Answer

To find the zeros of the polynomial let us first solve the polynomial by equating it to zero. Factorizing the given polynomial

$x^2 – (\sqrt{3} + 1)x + \sqrt{3} = 0$

To factorize the polynomial we have,

Sum of the value should be equal = $-(\sqrt{3} + 1)$

Product should be equal to = $1 \times \sqrt{3}$

= $\sqrt{3}$

$x^2 – [2(\sqrt{3} + 1)x]/2 + \sqrt{3} = 0$

$x^2 - 2(\sqrt{3} + 1)x/2 + (\sqrt{3} + 1)^2/2^2 - (\sqrt{3} + 1)^2/2^2 + \sqrt{3} = 0$

$[x - (\sqrt{3} + 1)/2]^2 - (3 + 1 + 2\sqrt{3})/4 + \sqrt{3} = 0$

$[x - (\sqrt{3} + 1)/2]^2 = (3 + 1 + 2\sqrt{3} - 4\sqrt{3})/4$

$[x - (\sqrt{3} + 1)/2]^2 = (\sqrt{3} - 1)^2/2^2$

$[x - (\sqrt{3} + 1)/2] = \pm (\sqrt{3} - 1)/2$

Solving for positive value,

$x = (\sqrt{3} - 1)/2 + (\sqrt{3} + 1)/2$

$x = (\sqrt{3} - 1 + \sqrt{3} + 1)/2$

$x = 2\sqrt{3}/2 = \sqrt{3}$

Solving for negative value,

$x = -(\sqrt{3} - 1)/2 + (\sqrt{3} + 1)/2$

$x = (-\sqrt{3} + 1 + \sqrt{3} + 1)/2$

$x = 2/2 = 1$

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

∴ $a = 1, b = -(\sqrt{3} + 1), c = \sqrt{3}$

Sum of zeroes = $-b / a$

$= (\sqrt{3} + 1) / 1$

$= \sqrt{3} + 1$

Product of zeroes = c / a

$= \sqrt{3} / 1$

$= \sqrt{3}$

1 F. Question

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

$3x^2 – x – 4$

Answer

To find the zeros of the polynomial let us first solve the polynomial by equating it to zero. Factorizing the given polynomial

$3x^2 – x - 4 = 0$

To factorize the polynomial we have,

Sum of the value should be equal = -1

Product should be equal to = $3 \times (-4)$

= -12

So two numbers are -4, 3

$3x^2 – 4x + 3x - 4 = 0$

$3x (x + 1) -4 (x + 1) = 0$

$(3x - 4)(x + 1) = 0$

$3x - 4 = 0$ or $x + 1 = 0$

Now solving first part,

$3x-4 = 0$

$3x = 4$

$x = 4/3$

Now solving the second part,

$x + 1 = 0$

$x = -1$

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$\therefore a = 3, b = -1, c = -4$

Sum of zeroes = $-b / a$

$= - (-1) / 3$

$= 1/3$

Product of zeroes = c / a

$= -4 / 3$

$= -4/3$

2 A. Question

Find a quadratic polynomial the sum and the product of whose zeroes are respectively the given numbers.

$-3, 2$

Answer

Let the two zeroes be a and b.

The generalized form of the quadratic equation with sum and product of zeroes a and b is as follows:

$ax^2 + bx + c = 0$ ………………… (i)

Now in this case

$\therefore a + b = -b/a = -3$

$\therefore ab = c / a = 2$

If $a = k$ where k is any real number

$b = 3k$ ………………… (ii)

$c = 2k$ ………………… (iii)

Put values from (ii) and (iii) in (i)

$kx^2 + 3kx + 2k = 0$

$k(x^2 + 3x + 2) = 0$

Therefore the quadratic equation is as follows:

$x^2 + 3x + 2 = 0$

2 B. Question

Find a quadratic polynomial the sum and the product of whose zeroes are respectively the given numbers.

$\sqrt{2}, \frac{1}{3}$

Answer

Let the two zeroes be a and b.

The generalized form of the quadratic equation with sum and product of zeroes a and b is as follows:

$ax^2 + bx + c = 0$ ………………… (i)

Now in this case

$\therefore a + b = -b/a = \sqrt{2}$

$\therefore ab = c/a = 1/3$

If $a = k$, where k is any real number

$b = -\sqrt{2}k$ ………………… (ii)

$c = k/3$ ………………… (iii)

Put values from (ii) and (iii) in (i)

$kx^2 - \sqrt{2}kx + k/3 = 0$

$3kx^2 - 3\sqrt{2}kx + k = 0$

$k(x^2 - 3\sqrt{2}x + 1) = 0$

Therefore the quadratic equation is as follows:

$(x^2 - 3\sqrt{2}x + 1) = 0$

2 C. Question

Find a quadratic polynomial the sum and the product of whose zeroes are respectively the given numbers.

$-\frac{1}{4}, \frac{1}{4}$

Answer

Let the two zeroes be a and b.

The generalized form of the quadratic equation with sum and product of zeroes a and b is as follows:

$ax^2 + bx + c = 0$ ………………… (i)

Now in this case

∴ a + b = -b/a = -1/4

∴ ab = 1/4 ………………… (iii)

If a = k, where k is any real number

b = k/4………………… (ii)

c = k/4 ………………… (iii)

Put values from (ii) and (iii) in (i)

$kx^2 + k/4x + k/4 = 0$

$4kx^2 + kx + k = 0$

$k (4x^2 + x + 1) = 0$

Therefore the quadratic equation is as follows:

$(4x^2 + x + 1) = 0$

2 D. Question

Find a quadratic polynomial the sum and the product of whose zeroes are respectively the given numbers.

$0, \sqrt{5}$

Answer

Let the two zeroes be a and b.

The generalized form of the quadratic equation with sum and product of zeroes a and b is as follows:

$ax^2 + bx + c = 0$ ………………… (i)

Now in this case

∴ $a + b = -b/a = 0$

∴ $ab = c/a = \sqrt{5}$

If a = k, where k is any real number,

$b = 0$ ………………… (ii)

$c = \sqrt{5}k$ ………………… (iii)

Put values from (ii) and (iii) in (i)

$kx^2 + (0)x + (\sqrt{5}k) = 0$

$k(x^2 + \sqrt{5}) = 0$

Therefore the quadratic equation is as follows:

$x^2 + \sqrt{5} = 0$

2 E. Question

Find a quadratic polynomial the sum and the product of whose zeroes are respectively the given numbers.

4, 1

Answer

Let the two zeroes be a and b.

The generalized form of the quadratic equation with sum and product of zeroes a and b is as follows:

$ax^2 + bx + c = 0$ ………………… (i)

Now in this case

∴ $a + b = -b/a = 4$ ………………… (ii)

∴ $ab = c/a = 1$ ………………… (ii)

If a = k, where k is any real number

$b = -4k$

$c = k$

Put values from (ii) and (iii) in (i)

$kx^2 - 4kx + k = 0$

$k (x^2 - 4x + 1) = 0$

Therefore the quadratic equation is as follows:

$(x^2 - 4x + 1) = 0$

2 F. Question

Find a quadratic polynomial the sum and the product of whose zeroes are respectively the given numbers.

1, 1

Answer

Let the two zeroes be a and b.

The generalized form of the quadratic equation with sum and product of zeroes a and b is as follows:

$ax^2 + bx + c = 0$ ………………… (i)

Now in this case

$\therefore a + b = -b/a = 1$

$\therefore ab = c/a = 1$

If a = k, where k is any real integer,

$b = -k$ ………………… (ii)

$c = k$ ………………… (iii)

Put values from (ii) and (iii) in (i)

$kx^2 - kx + k = 0$

$k (x^2 - x + 1) = 0$

Therefore the quadratic equation is as follows:

$(x^2 - x + 1) = 0$

3. Question

If sum of squares of zeroes of quadratic polynomial $f(x) = x^2 - 8x + k$ is 40, the find the value of k.

Answer

Let the two zeroes be a and b.

Given:

Sum of squares of zeroes is 40

$a^2 + b^2 = 40$

The generalized form of the quadratic equation with sum and product of zeroes a and b is as follows:

$x^2 - (a + b)x + ab = 0$ ………………… (i)

$a + b = 8$

$ab = k$

We also know that,

$(a + b)^2 = a^2 + b^2 + 2ab$

$a^2 + b^2 = (a + b)^2 - 2ab$

$(a + b)^2 - 2ab = 40$

(8)2 – 2k= 40

$2k = 64 - 40$

$2k = 24$

$k = 12$

Therefore the value of k = 12

Exercise 3.2

1 A. Question

Find the quotient and the remainder on dividing f(x) by using division algorithm.

$f(x) = 3x^2 + x^2 + 2x + 5$, $g(x) = 1 + 2x + x^2$

Answer

The solution is as follows:

$$\begin{array}{r|l} & \quad 3x - 5 \\ \hline x^2 + 2x + 1 & 3x^3 + x^2 + 2x + 5 \\ & 3x^3 + 6x^2 + 3x \\ & - \qquad - \qquad - \\ \hline & 0 \quad -5x^2 - x + 5 \\ & \qquad -5x^2 - 10x - 5 \\ & \qquad + \qquad + \qquad + \\ \hline & \qquad 0 \quad 9x + 10 \end{array}$$

Therefore the quotient and remainder are as follows:

Quotient = $3x - 5$

Remainder = $9x + 10$

1 B. Question

Find the quotient and the remainder on dividing f(x) by using division algorithm.

$f(x) = x^3 - 3x^2 + 5x - 3, g(x) = x^2 - 2$

Answer

The solution is as follows:

$$\begin{array}{r|l} & \quad x - 3 \\ \hline x^2 - 2 & x^3 - 3x^2 + 5x - 3 \\ & x^3 \qquad - 2x \\ & - \qquad\quad + \\ \hline & 0 \quad -3x^2 + 7x - 3 \\ & \quad -3x^2 \qquad + 6 \\ & \quad + \qquad\quad - \\ \hline & \qquad 0 \quad 7x - 9 \end{array}$$

Quotient = x – 3

Remainder = 7x – 9

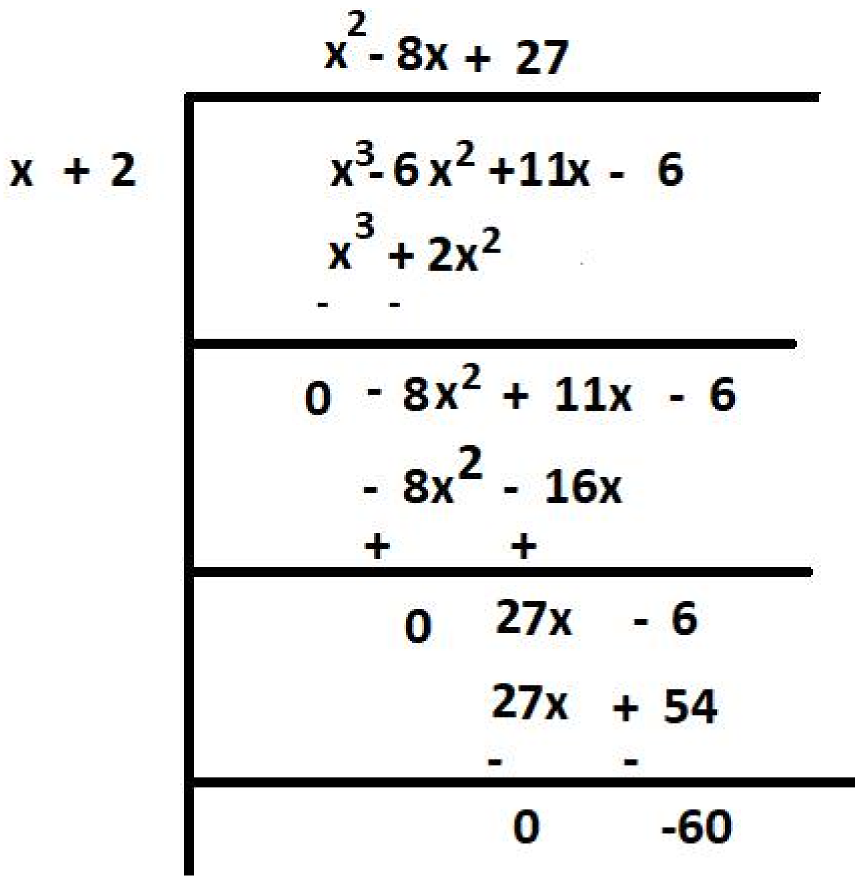
1 C. Question

Find the quotient and the remainder on dividing f(x) by using division algorithm.

$f(x) = x^3 - 6x^2 + 11x - 6, g(x) = x + 2$

Answer

The solution is as follows:



Quotient = $x^2 - 8x + 27$

Remainder = -60

1 D. Question

Find the quotient and the remainder on dividing f(x) by using division algorithm.

$f(x) = 9x^4 - 4x^2 + 4, g(x) = 3x^2 + x - 1$

Answer

The solution is as follows:

$$
\begin{array}{r|l}
 & 3x^2 \;-\; x \\
\hline
3x^2 + x - 1 & 9x^4 - 4x^2 + 4 \\
 & 9x^4 + 3x^3 - 3x^2 \\
 & - \quad\; - \quad\;\; + \\
\hline
 & 0 \quad -3x^3 - x^2 + 4 \\
 & \quad\;\; -3x^3 - x^2 + x \\
 & \quad\quad + \quad\; + \quad\; - \\
\hline
 & \quad\quad 0 \quad 0 \;\; -x + 4
\end{array}
$$

Quotient = $3x^2 - x$

Remainder = $-x + 4$

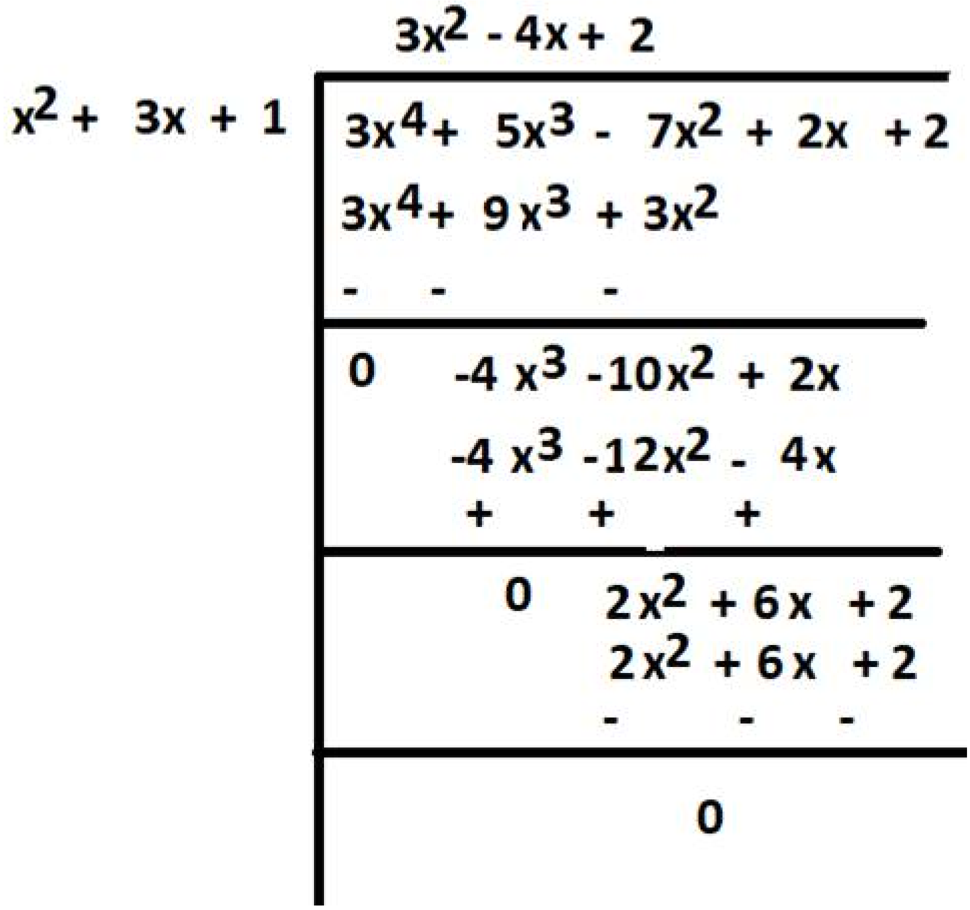
2 A. Question

Dividing the first polynomial, by the second polynomial, check whether the first polynomial is a factor of the second polynomial:

$g(x) = x^2 + 3x + 1$, $f(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 2$

Answer

The solution is as follows:



Since the remainder is zero, it is proved that the first polynomial is the factor of second.

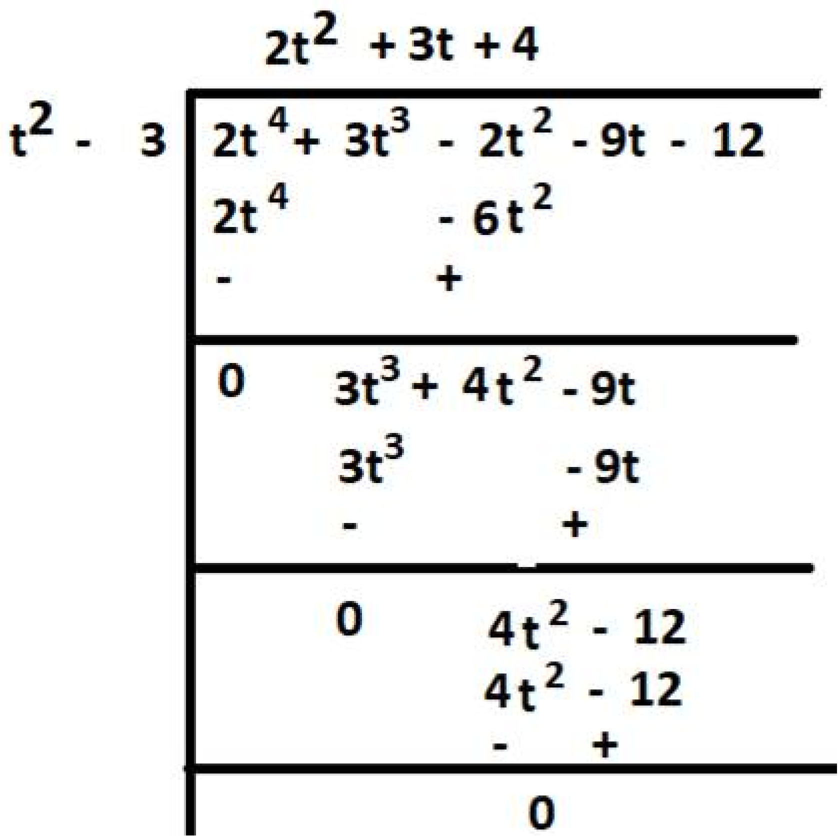
2 B. Question

Dividing the first polynomial, by the second polynomial, check whether the first polynomial is a factor of the second polynomial:

$g(t) = t^2 - 3$, $f(t) = 2t^4 + 3t^3 - 2t^2 - 9t - 12$

Answer

The solution is as follows:



Since the remainder is zero, it is proved that the first polynomial is the factor of second.

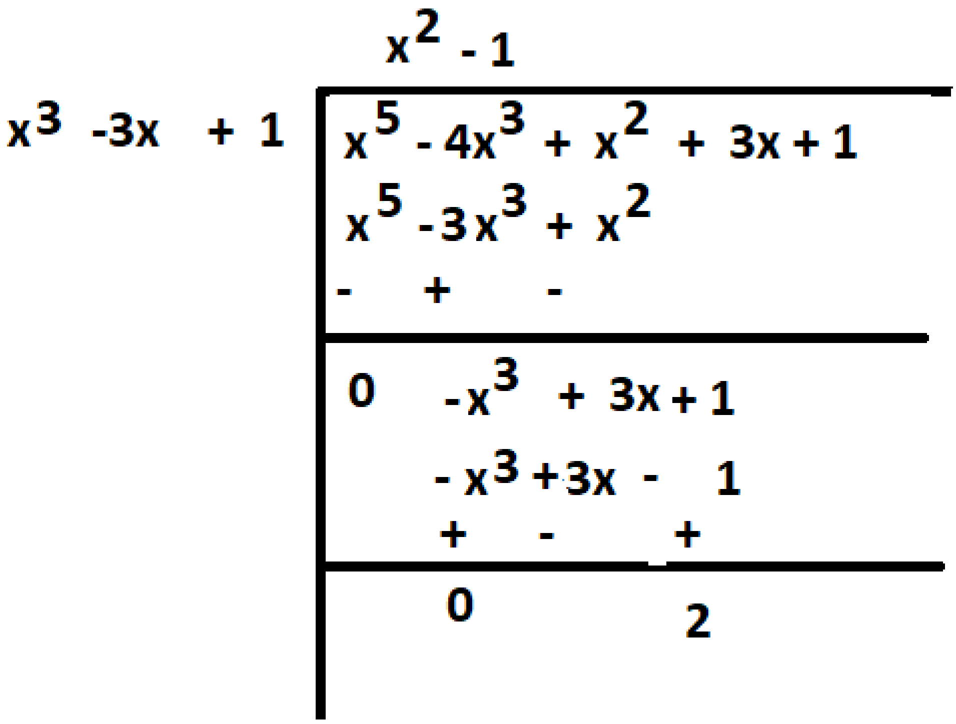
2 C. Question

Dividing the first polynomial, by the second polynomial, check whether the first polynomial is a factor of the second polynomial:

$g(x) = x^3 - 3x + 1$, $f(x) = x^5 - 4x^3 + x^2 + 3x + 1$

Answer

The solution is as follows:



Since the remainder is not zero, it is proved that the first polynomial is not the factor of second.

Remainder is 9x – 3

3 A. Question

With the following polynomials their zeroes are given. Find their all other zeroes:

$$f(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2; \sqrt{2} \text{ and} - \sqrt{2}$$

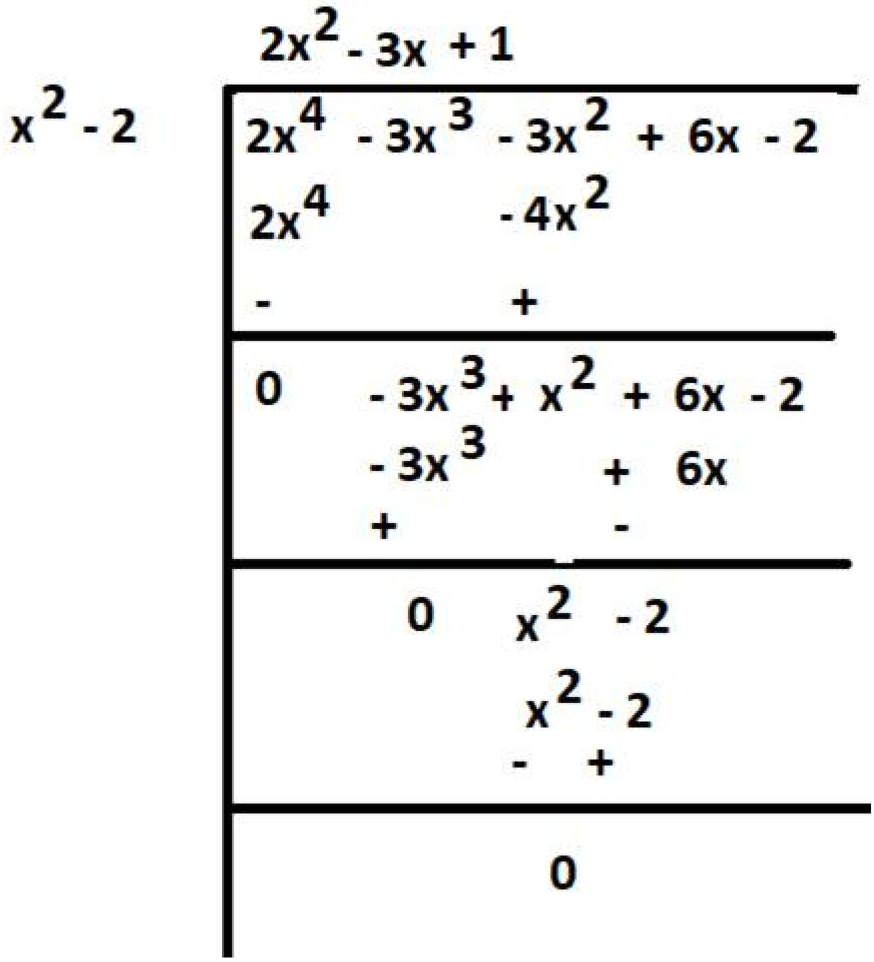
Answer

Given that $\sqrt{2}$ and $-\sqrt{2}$ are the zeroes of the given polynomial.

So $(x - \sqrt{2})(x + \sqrt{2}) = x^2 - 2$

The above relation is same as $(a + b)(a - b) = a^2 - b^2$

Here $a = x$ and $b = \sqrt{2}$



The other zeroes are as follows:

$2x^2 - 3x + 1 = 0$

Solving the above quadratic equation.

Sum = -3

Product = 2

So the numbers which satisfy the above condition are -2 and -1

$2x^2 - 2x - x + 1 = 0$

$2x(x - 1) - 1(x - 1) = 0$

$(2x - 1)(x - 1) = 0$

Solving the first part,

$2x-1 = 0$

$2x = 1$

$x = 1/2$

Solving the second part,

$x - 1 = 0$

x = 1

Therefore the other zeroes of the polynomials are 1/2 and 1.

3 B. Question

With the following polynomials their zeroes are given. Find their all other zeroes:

$$f(x) = x^4 - 6x^3 - 26x^2 + 138x - 35; 2 \pm \sqrt{3}$$

Answer

Given that 2 + √3 and 2 - √3 are the zeroes of the given polynomial.

So (x – (2 + √3)) (x – (2 - √3)) = (x – 2 - √3) (x – 2 + √3)

= x^2 – 2x + √3x – 2x + 4 – 2√3 - √3x + 2√3 – 3

= x^2 – 4x + 1

$$x^2 - 2x - 35$$

$$x^2 - 4x + 1 \;\big)\; x^4 - 6x^3 - 26x^2 + 138x - 35$$

$$x^4 - 4x^3 + x^2$$

$$- \quad + \quad -$$

$$0 \quad -2x^3 - 27x^2 + 138x$$

$$-2x^3 + 8x^2 - 2x$$

$$+ \quad - \quad +$$

$$0 \quad -35x^2 + 140x - 35$$

$$-35x^2 + 140x - 35$$

$$+ \quad - \quad +$$

$$0$$

The other zeroes are as follows:

x^2 – 2x - 35 = 0

Solving the above quadratic equation.

Sum = -2

Product = -35

So the numbers which satisfy the above condition are -7 and 5

$x^2 - 7x + 5x - 35 = 0$

$x(x - 7) + 5(x - 7) = 0$

$(x + 5)(x - 7) = 0$

Solving the first part,

$x + 5 = 0$

$x = -5$

Solving the second part,

$x - 7 = 0$

$x = 7$

Therefore the other zeroes of the polynomials are -5 and 7.

3 C. Question

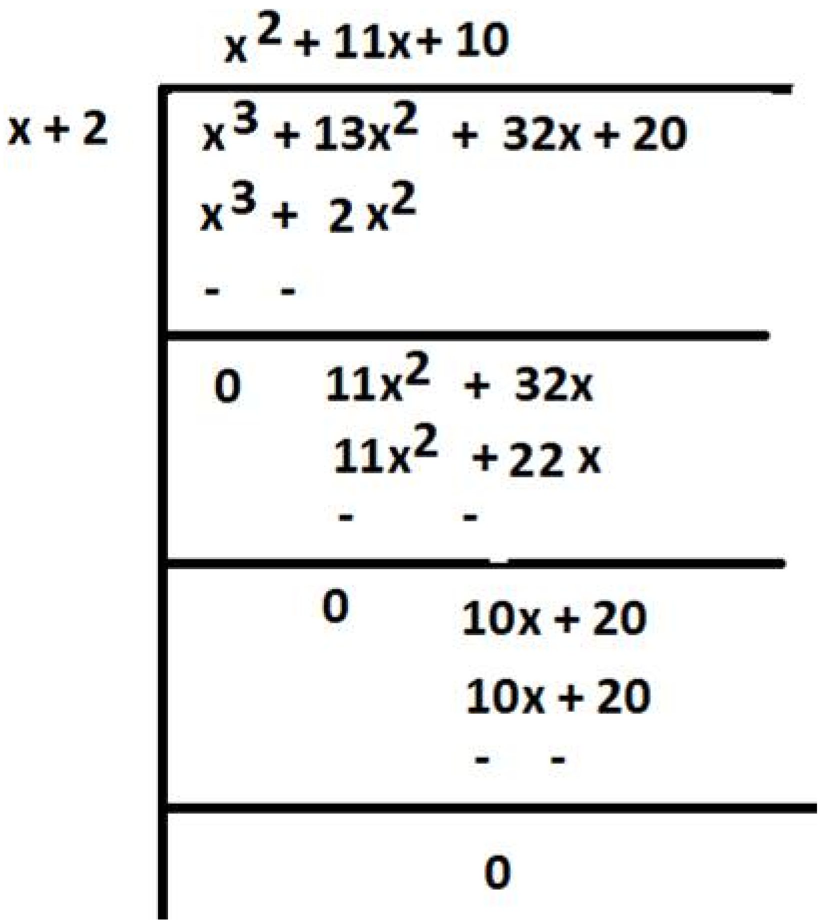
With the following polynomials their zeroes are given. Find their all other zeroes:

$f(x) = x^3 + 13x^2 + 32x + 20; -2$

Answer

Given that -2 is the zero of the polynomial.

So $x = -2$ which is $x + 2 = 0$



The other zeroes are as follows:

$x^2 + 11x + 10 = 0$

Solving the above quadratic equation.

Sum = 11

Product = 10

So the numbers which satisfy the above condition are 10 and 1

$x^2 + 10x + x + 10 = 0$

$x(x + 1) + 10(x + 1) = 0$

$(x + 10)(x + 1) = 0$

Solving the first part,

$x + 10 = 0$

$x = -10$

Solving the second part,

$x + 1 = 0$

$x = -1$

Therefore the other zeroes of the polynomials are -1 and -10.

4. Question

On dividing the polynomial $f(x) = x^3 - 3x^2 + x + 2$ by the polynomial g(x), quotient q(x) and remainder f(x) are respectively obtained as $x - 2$ and $-2x + 4$. Find the polynomial g(x).

Answer

Given:

Dividend = $f(x) = x^3 - 3x^2 + x + 2$

Divisor = g(x)

Quotient = $x - 2$

Remainder = $-2x + 4$

There is an important relation between dividend, divisor, quotient and remainder which is as follows:

Dividend = Divisor × Quotient + Remainder

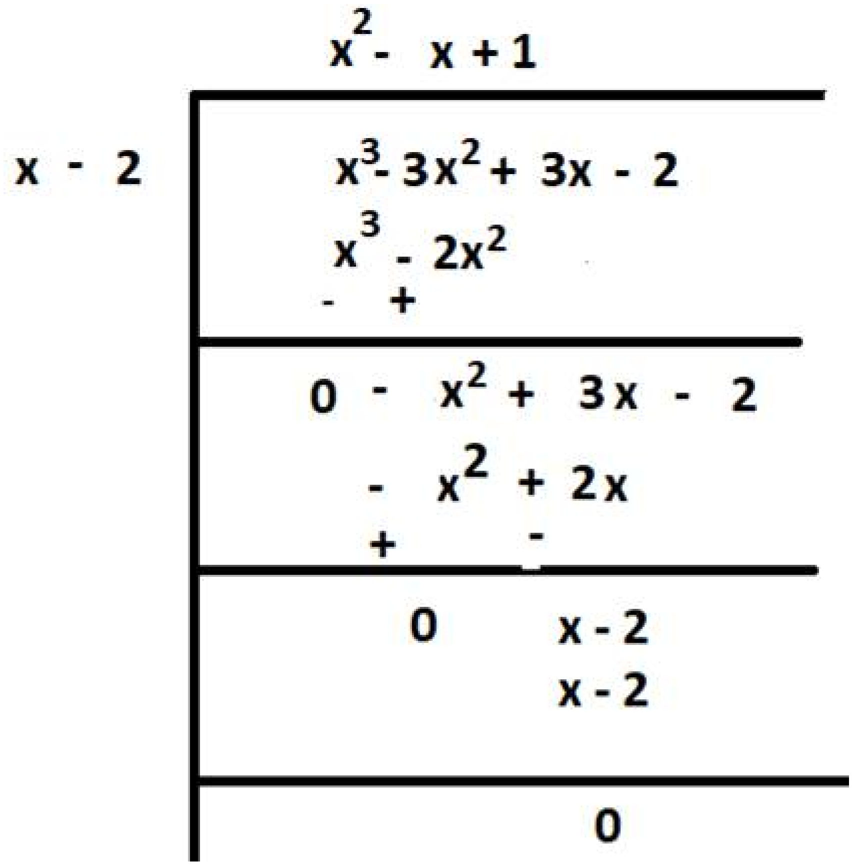
$x^3 - 3x^2 + x + 2 = g(x) \times (x-2) + (4-2x)$

$g(x) \times (x-2) = x^3 - 3x^2 + x + 2 - (4 - 2x)$

$= x^3 - 3x^2 + x + 2 - 4 + 2x$

$g(x) \times (x-2) = x^3 - 3x^2 + 3x - 2$

Therefore $g(x) = (x^3 - 3x^2 + 3x - 2) / (x-2)$



Therefore the $g(x) = x^2 - x - 1$

Exercise 3.3

1 A. Question

Check whether the following are quadratic equations:

$x(x + 1) + 8 = (x + 2)(x - 2)$

Answer

Let us solve the above equation,

$x(x + 1) + 8 = (x + 2)(x - 2)$

$x^2 + x + 8 = (x^2 + 2x - 2x - 4)$

$x^2 + x + 8 = x^2 - 4$

$x^2 + x + 8 - x^2 + 4 = 0$

$x + 12 = 0$

Since on solving the equation we do not get the second power of x which proves that the following set of equation is not quadratic.

1 B. Question

Check whether the following are quadratic equations:

$(x + 2)^3 = x^3 - 4$

Answer

Let us solve the above equation,

$(x + 2)^3 = x^3 - 4$

$x^3 + 3x^2 \times 2 + 3x \times 4 + 8 = x^3 - 4$

$x^3 + 6x^2 + 12x + 8 = x^3 - 4$

$x^3 + 6x^2 + 12x + 8 - x^3 + 4 = 0$

$6x^2 + 12x + 12 = 0$

$6(x^2 + 2x + 1) = 0$

$x^2 + 2x + 1 = 0$

Since on solving the equation we do get the second power of x which proves that the following set of equation is quadratic

1 C. Question

Check whether the following are quadratic equations:

$x^3 + 3x + 1 = (x - 2)^2$

Answer

Let us solve the above equation,

$x^3 + 3x + 1 = (x - 2)^2$

$x^3 + 3x + 1 = x^2 - 2 \times 2 \times x + 4$ …….. $[(a-b)^2 = a^2 - 2ab + b^2]$

$x^3 + 3x + 1 = x^2 - 4x + 4$

$x^3 + 3x + 1 - x^2 + 4x - 4 = 0$

$x^3 - x^2 + 7x - 3 = 0$

Since on solving the equation we do get the third power of x along with second power which proves that the following set of equation is not quadratic. A quadratic equation contain the highest power of x which is 2.

1 D. Question

Check whether the following are quadratic equations:

$$x + \frac{1}{x} + x^2, x \neq 0$$

Answer

Let us solve the above equation by equating it to zero,

$$x + \frac{1}{x} + x^2 = 0$$

$$\frac{x^2 + 1 + x^3}{x} = 0$$

$x^2 + 1 + x^3 = 0 \times x$

$x^2 + 1 + x^3 = 0$

Since on solving the equation we do get the third power of x along with second power which proves that the following set of equation is not quadratic. A quadratic equation contain the highest power of x which is 2.

2 A. Question

Solve the following equations by factorization method:

$2x^2 – 5x + 3 = 0$

Answer

On factorizing the above equation,

$2x^2 – 5x + 3 = 0$

$2x^2 – 2x -3x + 3 = 0$

$2x(x - 1) – 3(x - 1) = 0$

$(2x - 3) (x - 1) = 0$

Solving the first part,

$2x - 3 = 0$

$2x = 3$

$x = 3/2$

Solving the second part,

$x – 1 = 0$

$x = 1$

2 B. Question

Solve the following equations by factorization method:

$9x^2 – 3x – 2 = 0$

Answer

On factorizing the above equation,

$9x^2 – 3x - 2 = 0$

$9x^2 + 3x -6x - 2 = 0$

$3x(3x + 1) – 2(3x + 1) = 0$

$(3x - 2) (3x + 1) = 0$

Solving the first part,

$3x - 2 = 0$

$3x = 2$

$x = 2/3$

Solving the second part,

$3x + 1 = 0$

$3x = -1$

$x = -1/3$

2 C. Question

Solve the following equations by factorization method:

$$\sqrt{3}\, x^2 + 10x + 7\sqrt{3} = 0$$

Answer

On factorizing the above equation,

$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

$\sqrt{3}x^2 +3x + 7x + 7\sqrt{3} = 0$

$\sqrt{3}x(x + \sqrt{3}) + 7(x + \sqrt{3}) = 0$

$(\sqrt{3}x + 7) (x + \sqrt{3}) = 0$

Solving the first part,

$\sqrt{3}x + 7= 0$

$\sqrt{3}x = -7$

$x = -7/\sqrt{3}$

Solving the second part,

$x + \sqrt{3} = 0$

$x = -\sqrt{3}$

2 D. Question

Solve the following equations by factorization method:

$x^2 - 8x + 16 = 0$

Answer

On factorizing the above equation,

$x^2 - 8x + 16 = 0$

$x^2 - 4x - 4x + 16 = 0$

$x(x - 4) - 4(x - 4) = 0$

$(x - 4)(x - 4) = 0$

Here first and second parts are same,

So $x - 4 = 0$

$x = 4$

2 E. Question

Solve the following equations by factorization method:

$$\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x} \text{ where } x \neq 1, 2$$

Answer

On factorizing the above equation,

$$\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$$

$$\frac{(x-1) + 2 \times (x-2)}{(x-2) \times (x-1)} = \frac{6}{x}$$

$$\frac{x - 1 + 2x - 4}{x^2 - x - 2x + 1} = \frac{6}{x}$$

$$\frac{3x-5}{x^2-3x+2}=\frac{6}{x}$$

$(3x - 5) \times (x) = 6 \times (x^2 - 3x + 2)$

$3x^2 - 5x = 6x^2 - 18x + 6$

$6x^2 - 3x^2 - 18x + 5x + 12 = 0$

$3x^2 - 13x + 12 = 0$

$3x^2 - 9x - 4x + 12 = 0$

$3x(x - 3) -4(x - 3) = 0$

$(3x - 4)(x - 3) = 0$

Solving the first part,

$3x - 4 = 0$

$3x = 4$

$x = 4/3$

Solving the second part,

$x - 3 = 0$

$x = 3$

2 F. Question

Solve the following equations by factorization method:

$100x^2 - 20x + 1 = 0$

Answer

On factorizing the above equation,

$100x^2 - 20x + 1 = 0$

$100x^2 - 10x -10x + 1 = 0$

$10x(10x - 1) - 1(10x - 1) = 0$

$(10x - 1) (10x - 1) = 0$

Here both the parts are same,

So $10x - 1 = 0$

$10x = 1$

$x = 1/10$

2 G. Question

Solve the following equations by factorization method:

$$3x^2 - 2\sqrt{6}x + 2 = 0$$

Answer

On factorizing the above equation,

$3x^2 - 2\sqrt{6}x + 2 = 0$

$3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$

$\sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = 0$

$(\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$

Here both the roots are equal.

So $\sqrt{3}x - \sqrt{2} = 0$

$\sqrt{3}x = \sqrt{2}$

$x = \sqrt{2} / \sqrt{3}$

2 H. Question

Solve the following equations by factorization method:

$x^2 + 8x + 7$

Answer

On factorizing the above equation,

$x^2 + 8x + 7 = 0$

$x^2 + 7x + x + 7 = 0$

$x(x + 7) + 1(x + 7) = 0$

$(x + 7)(x + 1) = 0$

Solving the first part,

$x + 7 = 0$

$x = -7$

Solving the second part,

$x + 1 = 0$

$x = -1$

2 I. Question

Solve the following equations by factorization method:

$$\frac{x+3}{x+2} = \frac{3x-7}{2x-3}$$

Answer

Cross Multiplying in the above equation,

$(x + 3) \times (2x - 3) = (3x - 7) \times (x + 2)$

$2x^2 - 3x + 6x - 9 = 3x^2 + 6x - 7x - 14$

$2x^2 + 3x - 9 = 3x^2 - x - 14$

$3x^2 - 2x^2 - x - 3x - 14 + 9 = 0$

$x^2 - 4x - 5 = 0$

On factorizing the above equation,

$x^2 + x - 5x - 5 = 0$

$x(x + 1) - 5(x + 1) = 0$

$(x - 5)(x + 1) = 0$

Solving the first part,

$x - 5 = 0$

$x = 5$

Solving the second part,

$x + 1 = 0$

$x = -1$

2 J. Question

Solve the following equations by factorization method:

$4x^2 - 4a^2x + (a^4 - b^4) = 0$

Answer

On factorizing the above equation,

$(a^4 - b^4) = [(a^2 + b^2)(a^2 - b^2)]$

The above relation is same as $(a + b)(a - b) = a^2 - b^2$

Here $a = (a^2 + b^2)$ and $b = (a^2 - b^2)$

$4x^2 - 4a^2x + (a^4 - b^4) = 0$

$4x^2 - [2(a^2 + b^2) + 2(a^2 - b^2)]x + [(a^2 + b^2)(a^2 - b^2)] = 0$

$4x^2 - 2x(a^2 + b^2) - 2x(a^2 - b^2) + [(a^2 + b^2)(a^2 - b^2)] = 0$

$2x[2x - (a^2 + b^2)] - (a^2 - b^2)[2x - (a^2 + b^2)] = 0$

$[2x - (a^2 - b^2)][2x - (a^2 + b^2)] = 0$

Solving the first part,

$[2x - (a^2 - b^2)] = 0$

$2x = (a^2 - b^2)$

$x = (a^2 - b^2) / 2$

Solving the second part,

$[2x - (a^2 + b^2)] = 0$

$2x = (a^2 + b^2)$

$x = (a^2 + b^2) / 2$

2 K. Question

Solve the following equations by factorization method:

$abx^2 + (b^2 - ac)x - bc = 0$

Answer

On factorizing the above equation,

$abx^2 + (b^2 - ac)x - bc = 0$

$abx^2 + b^2x - acx - bc = 0$

$bx(ax + b) - c(ax + b) = 0$

$(bx - c)(ax + b) = 0$

Solving the first part,

$bx - c = 0$

$bx = c$

$x = c/b$

Solving the second part,

$ax + b = 0$

$ax = -b$

$x = -b / a$

Exercise 3.4

1 A. Question

Solve the following quadratic equations by the method of completing the square:

$3x^2 – 5x + 2 = 0$

Answer

Now in the above quadratic equation the coefficient of x^2 is 3. Let us make it unity by dividing the entire quadratic equation by 3.

$x^2 – 5/3x + 2/3 = 0$

$x^2 – 5/3x = -2/3$

Now by taking half of the coefficient of x and then squaring it and adding on both LHS and RHS sides.

Coefficient of x = 5/3

Half of 5/3 = 5/6

Squaring the half of 5/3 = 25/36

$$x^2 - \frac{5}{3}x + \frac{25}{36} = -\frac{2}{3} + \frac{25}{36}$$

Now the LHS term is a perfect square and can be expressed in the form of $(a-b)^2 = a^2 – 2ab + b^2$ where a = x and b = 5/6

$$\left[x - \frac{5}{6}\right]^2 = \frac{-24 + 25}{36}$$

On simplifying both RHS and LHS we get an equation of following form,

$(x \pm A)^2 = k^2$

$$\left[x - \frac{5}{6}\right]^2 = \frac{1}{36}$$

Taking Square root of both sides.

$$\left[x - \frac{5}{6}\right] = \pm\frac{1}{6}$$

Now taking the positive part,

$$x - \frac{5}{6} = \frac{1}{6}$$

$$x = \frac{1}{6} + \frac{5}{6}$$

x = 6/6

x = 1

Now taking the negative part,

$$x - \frac{5}{6} = -\frac{1}{6}$$

$$x = -\frac{1}{6} + \frac{5}{6}$$

x = 4/6

x = 2/3

1 B. Question

Solve the following quadratic equations by the method of completing the square:

$5x^2 - 6x - 2 = 0$

Answer

Now in the above quadratic equation the coefficient of x^2 is 5. Let us make it unity by dividing the entire quadratic equation by 5.

$x^2 - 6/5x - 2/5 = 0$

$x^2 - 6/5x = 2/5$

Now by taking half of the coefficient of x and then squaring it and adding on both LHS and RHS sides.

Coefficient of x = 6/5

Half of 6/5 = 6/10

Squaring the half of 6/10 = 36/100

$$x^2 - \frac{6}{5}x + \frac{36}{100} = \frac{2}{5} + \frac{36}{100}$$

Now the LHS term is a perfect square and can be expressed in the form of $(a-b)^2 = a^2 - 2ab + b^2$ where a = x and b = 6/10

$$\left[x - \frac{6}{10}\right]^2 = \frac{40 + 36}{100}$$

On simplifying both RHS and LHS we get an equation of following form,

$(x \pm A)^2 = k^2$

$$\left[x - \frac{6}{10}\right]^2 = \frac{76}{100}$$

Taking Square root of both sides.

$$\left[x - \frac{6}{10}\right] = \pm \frac{2\sqrt{19}}{10}$$

Now taking the positive part,

$$x - \frac{6}{10} = \frac{2\sqrt{19}}{10}$$

$$x = \frac{2\sqrt{19}}{10} + \frac{6}{10}$$

$$x = \frac{6 + 2\sqrt{19}}{10}$$

$$x = \frac{2(3 + \sqrt{19})}{10}$$

$$x = \frac{3 + \sqrt{19}}{5}$$

Now taking the negative part,

Now taking the positive part,

$$x - \frac{6}{10} = -\frac{2\sqrt{19}}{10}$$

$$x = -\frac{2\sqrt{19}}{10} + \frac{6}{10}$$

$$x = \frac{6 - 2\sqrt{19}}{10}$$

$$x = \frac{2(3 - \sqrt{19})}{10}$$

$$x = \frac{3 - \sqrt{19}}{5}$$

1 C. Question

Solve the following quadratic equations by the method of completing the square:

$4x^2 + 3x + 5 = 0$

Answer

Now in the above quadratic equation the coefficient of x^2 is 4. Let us make it unity by dividing the entire quadratic equation by 4.

$x^2 - 3/4x + 5/4 = 0$

$x^2 - 3/4x = -5/4$

Now by taking half of the coefficient of x and then squaring it and adding on both LHS and RHS sides.

Coefficient of x = 3/4

Half of 3/4 = 3/8

Squaring the half of 3/4 = 9/64

$$x^2 - \frac{3}{4}x + \frac{9}{64} = -\frac{5}{4} + \frac{9}{64}$$

Now the LHS term is a perfect square and can be expressed in the form of $(a-b)^2 = a^2 - 2ab + b^2$ where a = x and b = 3/8

$$\left[x - \frac{3}{8}\right]^2 = \frac{-80 + 9}{64}$$

On simplifying both RHS and LHS we get an equation of following form,

$(x \pm A)^2 = k^2$

$$\left[x-\frac{3}{8}\right]^2=\frac{-71}{64}$$

It is observed that the term obtained on RHS is a negative term and taking square root of a negative term will give imaginary roots for the given quadratic equation.

Therefore the given quadratic equation does not has real roots.

1 D. Question

Solve the following quadratic equations by the method of completing the square:

$$4x^2+4\sqrt{3}x+3=0$$

Answer

Now in the above quadratic equation the coefficient of x^2 is 4. Let us make it unity by dividing the entire quadratic equation by 4.

$x^2 + \sqrt{3}x + 3/4 = 0$

$x^2 + \sqrt{3}x = -3/4$

Now by taking half of the coefficient of x and then squaring it and adding on both LHS and RHS sides.

Coefficient of x = √3

Half of √3= √3/2

Squaring the half of √3= 3/4

$$x^2+\sqrt{3}x+\frac{3}{4}=-\frac{3}{4}+\frac{3}{4}$$

Now the LHS term is a perfect square and can be expressed in the form of $(a-b)^2 = a^2 - 2ab + b^2$ where a = x and b = √3/2

$$\left[x+\frac{\sqrt{3}}{2}\right]^2=0$$

On simplifying both RHS and LHS we get an equation of following form,

$(x \pm A)^2 = k^2$

Here RHS term is zero which implies that the roots of the quadratic equation are real and equal.

Taking square root of both sides,

$$\left[x + \frac{\sqrt{3}}{2}\right] = 0$$

$$x = -\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$$

1 E. Question

Solve the following quadratic equations by the method of completing the square:

$2x^2 + x - 4 = 0$

Answer

Now in the above quadratic equation the coefficient of x^2 is 2. Let us make it unity by dividing the entire quadratic equation by 2.

$x^2 + 1/2x - 2 = 0$

$x^2 + 1/2\ x = 2$

Now by taking half of the coefficient of x and then squaring it and adding on both LHS and RHS sides.

Coefficient of x = 1/2

Half of 1/2 = 1/4

Squaring the half of 1/2 = 1/16

$$x^2 + \frac{1}{2}x + \frac{1}{16} = 2 + \frac{1}{16}$$

Now the LHS term is a perfect square and can be expressed in the form of $(a-b)^2 = a^2 - 2ab + b^2$ where a = x and b = 1/4

$$\left[x + \frac{1}{4}\right]^2 = \frac{32 + 1}{16}$$

On simplifying both RHS and LHS we get an equation of following form,

$(x \pm A)^2 = k^2$

$$\left[x + \frac{1}{4}\right]^2 = \frac{33}{16}$$

Taking Square root of both sides.

$$\left[x + \frac{1}{4}\right] = \pm\frac{\sqrt{33}}{4}$$

Now taking the positive part,

$$x + \frac{1}{4} = \frac{\sqrt{33}}{4}$$

$$x = \frac{\sqrt{33}}{4} + \frac{1}{4}$$

$$x = \frac{1 + \sqrt{33}}{4}$$

Now taking the negative part,

$$x + \frac{1}{4} = -\frac{\sqrt{33}}{4}$$

$$x = -\frac{\sqrt{33}}{4} + \frac{1}{4}$$

$$x = \frac{1 - \sqrt{33}}{4}$$

1 F. Question

Solve the following quadratic equations by the method of completing the square:

$2x^2 + x + 4 = 0$

Answer

Now in the above quadratic equation the coefficient of x^2 is 2. Let us make it unity by dividing the entire quadratic equation by 2.

$x^2 + 1/2\ x + 2 = 0$

$x^2 + 1/2\ x = -\ 2$

Now by taking half of the coefficient of x and then squaring it and adding on both LHS and RHS sides.

Coefficient of x = 1/2

Half of 1/2 = 1/4

Squaring the half of 1/2 = 1/16

$$x^2 + \frac{1}{2}x + \frac{1}{16} = -2 + \frac{1}{16}$$

Now the LHS term is a perfect square and can be expressed in the form of (a-b) ^{2} = a^2 – 2ab + b^2 where a = x and b = 1/4

$$\left[x + \frac{1}{4}\right]^2 = \frac{-32 + 1}{16}$$

On simplifying both RHS and LHS we get an equation of following form,

$(x \pm A)^2 = k^2$

$$\left[x + \frac{1}{4}\right]^2 = -\frac{31}{16}$$

It is observed that the term obtained on RHS is a negative term and taking square root of a negative term will give imaginary roots for the given quadratic equation.

Therefore the given quadratic equation does not has real roots.

1 G. Question

Solve the following quadratic equations by the method of completing the square:

$4x^2 + 4bx - (a^2 - b^2) = 0$

Answer

Now in the above quadratic equation the coefficient of x^2 is 4. Let us make it unity by dividing the entire quadratic equation by 4.

$4x^2 + 4bx - (a^2 - b^2) = 0$

$x^2 + bx = (a^2 - b^2)/4$

Now by taking half of the coefficient of x and then squaring it and adding on both LHS and RHS sides.

Coefficient of x = b

Half of b = b/2

Squaring the half of b = b/4

$$x^2 + bx + \frac{b^2}{4} = \frac{(a^2 - b^2)}{4} + \frac{b^2}{4}$$

Now the LHS term is a perfect square and can be expressed in the form of (a-b) ^{2} = a^2 – 2ab + b^2 where a = x and b = b/2

$$\left[x + \frac{b}{2}\right]^2 = \frac{a^2 - b^2 + b^2}{4}$$

$$\left[x + \frac{b}{2}\right]^2 = \frac{a^2}{4}$$

On simplifying both RHS and LHS we get an equation of following form,

$(x \pm A)^2 = k^2$

$$\left[x + \frac{b}{2}\right]^2 = \frac{a^2}{4}$$

Taking Square root of both sides.

$$\left[x + \frac{b}{2}\right] = \pm \frac{a}{2}$$

Now taking the positive part,

$$x + \frac{b}{2} = \frac{a}{2}$$

$$x = \frac{a}{2} - \frac{b}{2}$$

$x = (a - b) / 2$

Now taking the negative part,

$$x + \frac{b}{2} = -\frac{a}{2}$$

$$x = -\frac{a}{2} - \frac{b}{2}$$

$x = - (a + b) / 2$

2 A. Question

Find the root of the following quadratic equations, if they exist, by using the quadratic formula by Shridharacharya Method:

$$2x^2 - 2\sqrt{2} + 1 = 0$$

Answer

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$a = 2$

$b = -2\sqrt{2}$

c = 1

There is one formula developed by Shridharacharya to determine the roots of a quadratic equation which is as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Before putting the values in the formula let us check the nature of roots by $b^2 - 4ac > 0$

$\Rightarrow (-2\sqrt{2})^2 - (4 \times 2 \times 1)$

$\Rightarrow (4 \times 2) - 8$

$\Rightarrow 8 - 8$

$\Rightarrow 0$

Since $b^2 - 4ac = 0$ the roots are real and equal.

Now let us put the values in the above formula

$$x = \frac{-(-2\sqrt{2}) \pm \sqrt{(-2\sqrt{2})^2 - (4 \times 2 \times 1)}}{2 \times 2}$$

$$x = \frac{2\sqrt{2} \pm \sqrt{8 - 8}}{4}$$

$$x = \frac{2\sqrt{2}}{4}$$

$$x = \frac{\sqrt{2}}{2}$$

$x = 1/\sqrt{2}\ ,\ 1/\sqrt{2}$

2 B. Question

Find the root of the following quadratic equations, if they exist, by using the quadratic formula by Shridharacharya Method:

$9x^2 + 7x - 2 = 0$

Answer

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

a = 9

b = 7

c = -2

There is one formula developed by Shridharacharya to determine the roots of a quadratic equation which is as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Before putting the values in the formula let us check the nature of roots by $b^2 - 4ac > 0$

$\Rightarrow (7)^2 - (4 \times 9 \times -2)$

$\Rightarrow 49 - (-72)$

$\Rightarrow 49 + 72$

$\Rightarrow 121$

Since $b^2 - 4ac = 121$ the roots are real and distinct.

Now let us put the values in the above formula

$$x = \frac{-(7) \pm \sqrt{(7)^2 - (4 \times 9 \times -2)}}{2 \times 9}$$

$$x = \frac{-7 \pm \sqrt{49 + 72}}{18}$$

$$x = \frac{-7 \pm \sqrt{121}}{18}$$

$$x = \frac{-7 \pm 11}{18}$$

Solving with positive value first,

$$x = \frac{-7 + 11}{18}$$

x = 4 / 18

x = 2/9

Solving with negative value second,

$$x = \frac{-7 - 11}{18}$$

x = -18 / 18

x = -1

2 C. Question

Find the root of the following quadratic equations, if they exist, by using the quadratic formula by Shridharacharya Method:

$$x + \frac{1}{x} = 3, x \neq 0$$

Answer

Let us solve the above equation by equating it to zero,

$$x + \frac{1}{x} = 3$$

$$\frac{x^2 + 1}{x} = 3$$

$x^2 + 1 = 3x$

$x^2 - 3x + 1 = 0$

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

a = 1

b = -3

c = 1

There is one formula developed by Shridharacharya to determine the roots of a quadratic equation which is as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Before putting the values in the formula let us check the nature of roots by $b^2 - 4ac > 0$

$\Rightarrow (-3)^2 - (4 \times 1 \times 1)$

$\Rightarrow 9 - 4$

$\Rightarrow 5$

Since $b^2 - 4ac = 5$ the roots are real and distinct.

Now let us put the values in the above formula

$$x = \frac{-(-3)\ \pm\sqrt{(-3)^2-(4\times 1\times 1)}}{2\times 1}$$

$$x = \frac{3\ \pm\sqrt{9-4}}{2}$$

$$x = \frac{3\ \pm\sqrt{5}}{2}$$

2 D. Question

Find the root of the following quadratic equations, if they exist, by using the quadratic formula by Shridharacharya Method:

$$\sqrt{2}\,x^2+7x+5\sqrt{2}=0$$

Answer

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$a = \sqrt{2}$

$b = 7$

$c = 5\sqrt{2}$

There is one formula developed by Shridharacharya to determine the roots of a quadratic equation which is as follows:

$$x = \frac{-b\ \pm\sqrt{b^2-4ac}}{2a}$$

Before putting the values in the formula let us check the nature of roots by $b^2 - 4ac > 0$

$\Longrightarrow (7)^2 - (4 \times \sqrt{2} \times 5\sqrt{2})$

$\Longrightarrow 49 - (20 \times 2)$

$\Longrightarrow 49 - 40$

$\Longrightarrow 9$

Since $b^2 - 4ac = 121$ the roots are real and distinct.

Now let us put the values in the above formula

$$x = \frac{-(7) \pm \sqrt{(7)^2 - (4 \times \sqrt{2} \times 5\sqrt{2})}}{2 \times \sqrt{2}}$$

$$x = \frac{-7 \pm \sqrt{49 - 40}}{2\sqrt{2}}$$

$$x = \frac{-7 \pm \sqrt{9}}{2\sqrt{2}}$$

$$x = \frac{-7 \pm 3}{2\sqrt{2}}$$

Solving with positive value first,

$$x = \frac{-7 + 3}{2\sqrt{2}}$$

$$x = \frac{-4}{2\sqrt{2}}$$

$$x = \frac{-2}{\sqrt{2}}$$

$$x = \frac{-2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$x = \frac{-2\sqrt{2}}{2}$$

$x = -\sqrt{2}$

Solving with negative value second,

$$x = \frac{-7 - 3}{2\sqrt{2}}$$

$$x = \frac{-10}{2\sqrt{2}}$$

$$x = \frac{-5}{\sqrt{2}}$$

2 E. Question

Find the root of the following quadratic equations, if they exist, by using the quadratic formula by Shridharacharya Method:

$x^2 + 4x + 5 = 0$

Answer

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

a = 1

b = 4

c = 5

There is one formula developed by Shridharacharya to determine the roots of a quadratic equation which is as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Before putting the values in the formula let us check the nature of roots by $b^2 - 4ac > 0$

$\Rightarrow (4)^2 - (4 \times 1 \times 5)$

$\Rightarrow 16 - 20$

$\Rightarrow -4$

Since $b^2 - 4ac = -4$ the roots are imaginary.

Therefore the given quadratic equation does not has real roots.

2 F. Question

Find the root of the following quadratic equations, if they exist, by using the quadratic formula by Shridharacharya Method:

$$\frac{1}{x} - \frac{1}{x-2} = 3, x \neq 0, 2$$

Answer

Let us solve the above equation by equating it to zero,

$$\frac{1}{x} - \frac{1}{x-2} = 3$$

$$\frac{x - 2 - x}{x(x-2)} = 3$$

$$\frac{-2}{x^2 - 2x} = 3$$

$-2 = 3(x^2 - 2x)$

$3x^2 - 6x + 2 = 0$

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$a = 3$

$b = -6$

$c = 2$

There is one formula developed by Shridharacharya to determine the roots of a quadratic equation which is as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Before putting the values in the formula let us check the nature of roots by $b^2 - 4ac > 0$

$\Longrightarrow (-6)^2 - (4 \times 3 \times 2)$

$\Longrightarrow 36 - 24$

$\Longrightarrow 12$

Since $b^2 - 4ac = 5$ the roots are real and distinct.

Now let us put the values in the above formula

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - (4 \times 3 \times 2)}}{2 \times 3}$$

$$x = \frac{6 \pm \sqrt{36 - 24}}{6}$$

$$x = \frac{6 \pm \sqrt{12}}{6}$$

$$x = \frac{6 \pm 2\sqrt{3}}{6}$$

$$x = \frac{2\,(3 \pm \sqrt{3})}{6}$$

$$x = \frac{3 \pm \sqrt{3}}{3}$$

3. Question

Find two consecutive odd positive integers, sum of whose squares is 290.

Answer

Let the two consecutive odd integers be a and a + 2.

Given:

Sum of squares of numbers = 290

$(a)^2 + (a + 2)^2 = 290$

$a^2 + 4a + 4 + a^2 = 290$

$2a^2 + 4a + 4 = 290$

$2a^2 + 4a - 290 + 4 = 0$

$2a^2 + 4a - 286 = 0$

$2(a^2 + 2a - 143) = 0$

$a^2 + 2a - 143 = 0$

On factorizing the above equation,

$a^2 + 2a - 143 = 0$

Sum = 2

Product = 143

Therefore the two numbers satisfying the above conditions are -11 and 13.

$a^2 + 13a - 11a - 143 = 0$

$a(a + 13) - 11(a + 13) = 0$

$(a - 11)(a + 13) = 0$

Solving first part,

$a - 11 = 0$

$a = 11$

Solving second part,

$a + 13 = 0$

$a = -13$

Given that the numbers are positive. So a = -13 is not possible.

So First number is 11 and consecutive positive odd number is 13.

4. Question

The difference of squares of two numbers is 45 and the square of the smaller number is four times the larger number. Find the two numbers.

Answer

Let larger number be x and smaller number be y.

Given:

(Large Number)^2 – (Smaller Number)2 = 45

$x^2 - y^2 = 45$

$y^2 = x^2 - 45$

Also Smaller Number^2, $y^2 = 4 \times$ Large Number

$x^2 - 45 = 4x$

$x^2 - 4x - 45 = 0$

On factorizing the above equation,

Sum = -4

Product = -45

Therefore the two numbers satisfying the above conditions are -9 and 5.

$x^2 - 9x + 5x - 45 = 0$

$x(x - 9) + 5(x - 9) = 0$

$(x - 9)\ (x + 5) = 0$

Solving first part,

$x - 9 = 0$

$x = 9$

Solving second part,

$x + 5 = 0$

$x = -5$

If x = 9, then

$y^2 = 4 \times 9$

$y^2 = 36$

$y = \pm 6$

If x = -5, then

$y^2 = 4 \times -5$

$y^2 = -20$

Value of y becomes imaginary which is not possible.

So possible values of x and y are 9, 6 or 9, -6.

5. Question

Divide 16 into two parts such that two times the square of the larger part is 164 more than the square of the smaller part.

Answer

Let the smaller part be x and the larger part be 16 – x.

Given:

$2 \times (\text{Larger Part})^2 = (\text{Smaller Part})^2 + 164$

$2 \times (16 - x)^2 = (x)^2 + 164$

$2 \times (256 - 32x + x^2) = x^2 + 164$

$512 - 64x + 2x^2 = x^2 + 164$

$x^2 - 64x + 512 - 164 = 0$

$x^2 - 64x + 348 = 0$

On factorizing the above equation,

Sum = -64

Product = 348

Therefore the two numbers satisfying the above conditions are -58 and -6.

$x^2 - 6x - 58x + 348 = 0$

$x(x - 6) - 58(x - 6) = 0$

$(x - 6)(x - 58) = 0$

Solving first part,

$x - 6 = 0$

a = 6

Solving second part,

x - 58 = 0

x = 58

Since x is the smaller it cannot be greater than 16. Hence x cannot be 58

So the smaller part is x = 6

So the larger Part = 16 – 6

= 10

Exercise 3.5

1 A. Question

Find the nature of the roots of the following quadratic equations:

$2x^2 - 3x + 5 = 0$

Answer

Nature of the quadratic equation can be found out using the following formula

$b^2 - 4ac > 0$ for real roots

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

a = 2

b = -3

c = 5

$\Rightarrow (-3)^2 - (4 \times 2 \times 5)$

$\Rightarrow 9 - 40$

$\Rightarrow -31$

The answer is negative which implies that the quadratic equation does not have real roots that is they have imaginary roots.

1 B. Question

Find the nature of the roots of the following quadratic equations:

$2x^2 - 4x + 3 = 0$

Answer

Nature of the quadratic equation can be found out using the following formula

$b^2 - 4ac > 0$ for real roots

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

a = 2

b = -4

c = 3

$\Longrightarrow (-4)^2 - (4 \times 2 \times 3)$

$\Longrightarrow 16 - 24$

$\Longrightarrow -8$

The answer is negative which implies that the quadratic equation does not have real roots that is they have imaginary roots.

1 C. Question

Find the nature of the roots of the following quadratic equations:

$2x^2 + x - 1 = 0$

Answer

Nature of the quadratic equation can be found out using the following formula

$b^2 - 4ac > 0$ for real roots

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

a = 2

b = 1

c = -1

$\Longrightarrow (1)^2 - (4 \times 2 \times -1)$

$\Rightarrow 1 - (-8)$

$\Rightarrow 1 + 8$

$\Rightarrow 9$

The answer is positive and also a perfect square which implies that the quadratic equation has real and distinct roots.

1 D. Question

Find the nature of the roots of the following quadratic equations:

$x^2 - 4x + 4 = 0$

Answer

Nature of the quadratic equation can be found out using the following formula

$b^2 - 4ac > 0$ for real roots

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$a = 1$

$b = -4$

$c = 4$

$\Rightarrow (-4)^2 - (4 \times 1 \times 4)$

$\Rightarrow 16 - 16$

$\Rightarrow 0$

The answer is zero which implies that the quadratic equation has real and equal roots.

1 E. Question

Find the nature of the roots of the following quadratic equations:

$2x^2 + 5x + 5 = 0$

Answer

Nature of the quadratic equation can be found out using the following formula

$b^2 - 4ac > 0$ for real roots

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

a = 2

b = 5

c = 5

$\Rightarrow (25)^2 - (4 \times 2 \times 5)$

$\Rightarrow 25 - 40$

$\Rightarrow -15$

The answer is negative which implies that the quadratic equation does not have real roots that is they have imaginary roots.

1 F. Question

Find the nature of the roots of the following quadratic equations:

$$3x^2 - 2x + \frac{1}{3} = 0$$

Answer

Nature of the quadratic equation can be found out using the following formula

$b^2 - 4ac > 0$ for real roots

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

a = 3

b = -2

c = 1/3

$\Rightarrow (-2)^2 - (4 \times 3 \times 1/3)$

$\Rightarrow 4 - 4$

$\Rightarrow 0$

The answer is zero which implies that the quadratic equation has real and equal roots.

2 A. Question

Find that value of k in the following quadratic equation whose roots are real and equal:

$kx(x - 2) + 6 = 0$

Answer

Let us first solve the above equation,

$kx^2 - 2kx + 6 = 0$

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$a = k$

$b = -2k$

$c = 6$

Since the quadratic equations have real and equal roots,

$b^2 - 4ac = 0$ for real and equal roots

$\Rightarrow (-2k)^2 - (4 \times k \times 6) = 0$

$\Rightarrow 4k^2 - 24k = 0$

$\Rightarrow 4k(k - 6) = 0$

$\Rightarrow (k - 6) = 0$ or $4k = 0$

$\Rightarrow k = 6$ or $k = 0$

2 B. Question

Find that value of k in the following quadratic equation whose roots are real and equal:

$x^2 - 2(k + 1)x + k^2 = 0$

Answer

Let us first solve the above equation,

$x^2 - 2kx + 1x + k^2 = 0$

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$a = 1$

$b = -2(k+1)$

$c = k^2$

Since the quadratic equations have real and equal roots,

$b^2 - 4ac = 0$ for real and equal roots

$\Longrightarrow (-2(k+1))^2 - (4 \times k^2 \times 1) = 0$

$\Longrightarrow 4(k^2 + 2k + 1) - 4k^2 = 0$

$\Longrightarrow 4k^2 + 8k + 4 - 4k^2 = 0$

$\Longrightarrow 8k + 4 = 0$

$\Longrightarrow 8k = -4$

$\Longrightarrow k = -4/8$

$\Longrightarrow k = -1/2$

2 C. Question

Find that value of k in the following quadratic equation whose roots are real and equal:

$2x^2 + kx + 3 = 0$

Answer

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$a = 2$

$b = k$

$c = 3$

Since the quadratic equations have real and equal roots,

$b^2 - 4ac = 0$ for real and equal roots

$\Longrightarrow (k)^2 - (4 \times 2 \times 3) = 0$

$\Longrightarrow k^2 - 24 = 0$

$\Rightarrow k^2 = 24$

$\Rightarrow k = \sqrt{(2 \times 2 \times 6)}$

$\Rightarrow k = \pm 2\sqrt{6}$

2 D. Question

Find that value of k in the following quadratic equation whose roots are real and equal:

$(k + 1)x^2 - 2(k - 1)x + 1 = 0$

Answer

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$a = k + 1$

$b = -2(k - 1)$

$c = 1$

Since the quadratic equations have real and equal roots,

$b^2 - 4ac = 0$ for real and equal roots

$\Rightarrow (-2(k - 1))^2 - (4 \times (k + 1) \times 1) = 0$

$\Rightarrow 4(k^2 - 2k + 1) - 4k - 4 = 0$

$\Rightarrow 4k^2 - 8k + 4 - 4k - 4 = 0$

$\Rightarrow 4k^2 - 12k = 0$

$\Rightarrow 4k(k - 3) = 0$

$\Rightarrow (k - 3) = 0$ or $4k = 0$

$\Rightarrow k = 3$ or $k = 0$

2 E. Question

Find that value of k in the following quadratic equation whose roots are real and equal:

$(k + 4)x^2 + (k + 1)x + 1 = 0$

Answer

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$a = k + 4$

$b = k + 1$

$c = 1$

Since the quadratic equations have real and equal roots,

$b^2 - 4ac = 0$ for real and equal roots

$\Longrightarrow (k + 1)^2 - (4 \times (k + 4) \times 1) = 0$

$\Longrightarrow k^2 + 2k + 1 - 4k - 16 = 0$

$\Longrightarrow k^2 - 2k - 15 = 0$

$\Longrightarrow k^2 - 2k - 15 = 0$

On factorizing the above equation,

Sum = -2

Product = -15

Therefore the two numbers satisfying the above conditions are 3 and -5.

$k^2 - 5k + 3k - 15 = 0$

$k(k - 5) + 3(k - 5) = 0$

$(k + 3)(k - 5) = 0$

Solving first part,

$k + 3 = 0$

$k = -3$

Solving second part,

$k - 5 = 0$

$k = 5$

2 F. Question

Find that value of k in the following quadratic equation whose roots are real and equal:

$kx^2 - 5x + k = 0$

Answer

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$a = k$

$b = -5$

$c = k$

Since the quadratic equations have real and equal roots,

$b^2 - 4ac = 0$ for real and equal roots

$\Rightarrow (-5)^2 - (4 \times k \times k) = 0$

$\Rightarrow 25 - 4k^2 = 0$

$\Rightarrow 4k^2 = 25$

$\Rightarrow k^2 = 25/4$

$\Rightarrow k = \pm 5/2$

3 A. Question

Find these values of k for which the roots of the following quadratic equations are real and distinct:

$kx^2 + 2x + 1 = 0$

Answer

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$a = k$

$b = 2$

$c = 1$

Since the quadratic equations have real and distinct roots,

$b^2 - 4ac > 0$ for real and distinct roots

$\Rightarrow (2)^2 - (4 \times k \times 1) > 0$

$\Rightarrow 4 - 4k > 0$

$\Rightarrow 4k < 4$ [Dividing both sides by 4]

$\Rightarrow k < 1$

3 B. Question

Find these values of k for which the roots of the following quadratic equations are real and distinct:

$kx^2 + 6x + 1$

Answer

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$a = k$

$b = 6$

$c = 1$

Since the quadratic equations have real and distinct roots,

$b^2 - 4ac > 0$ for real and distinct roots

$\Rightarrow (6)^2 - (4 \times k \times 1) > 0$

$\Rightarrow 36 - 4k > 0$

$\Rightarrow 4k < 36$ [Dividing both sides by 4]

$\Rightarrow k < 9$

3 C. Question

Find these values of k for which the roots of the following quadratic equations are real and distinct:

$x^2 - kx + 9 = 0$

Answer

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$a = 1$

$b = -k$

$c = 9$

Since the quadratic equations have real and distinct roots,

$b^2 - 4ac > 0$ for real and distinct roots

$\Rightarrow (-k)^2 - (4 \times 9 \times 1) > 0$

$\Rightarrow k^2 - 36 > 0$

$\Rightarrow k^2 > 36$

$\Rightarrow k > \pm 6$

Which implies $k < -6$ and $k > 6$.

4. Question

Find those values of k for which the roots of the equation $x^2 + 5kx + 16 = 0$ are not real.

Answer

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$a = 1$

$b = 5k$

$c = 16$

For the quadratic equations to have imaginary [not real] roots,

$b^2 - 4ac < 0$ for imaginary [not real] roots

$\Rightarrow (5k)^2 - (4 \times 16 \times 1) > 0$

$\Rightarrow 25k^2 - 64 < 0$

$\Rightarrow 25k^2 < 64$

$\Rightarrow k^2 < 64/25$

$\Rightarrow k <\pm 8/5$

Therefore the range of k values for roots to be imaginary are:

$-8/5 < k < 8/5$

5. Question

If the roots of the quadratic equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are real and equal then prove that $2b = a + c$.

Answer

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$a = b - c$

$b = c - a$

$c = a - b$

Since the quadratic equations have real and equal roots,

$b^2 - 4ac = 0$ for real and equal roots

$\Rightarrow (c - a)^2 - [4 \times (b - c) \times (a - b)] = 0$

$\Rightarrow c^2 - 2ac + a^2 - [4 \times (ba - b^2 - ca - bc)] = 0$

$\Rightarrow c^2 - 2ac + a^2 - [4ba - 4b^2 - 4ca + 4bc] = 0$

$\Rightarrow c^2 - 2ac + a^2 - 4ba + 4b^2 + 4ca - 4bc = 0$

$\Rightarrow c^2 + a^2 - 4ba + 4b^2 + 2ac - 4bc = 0$

$\Rightarrow a^2 + 4b^2 + c^2 - 4ab - 4bc + 2ac = 0$

$\Rightarrow a^2 + (-2b)^2 + c^2 + 2 \times a(-2b) + 2 \times (-2b)c + 2 \times ac = 0$

We have the following formula:

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

So according to the formula

$\Rightarrow (a + (-2b) + c)^2 = 0$

Taking square root of both sides

$\Rightarrow a + (-2b) + c = 0$

$\therefore 2b = a + c$

Hence Proved.

Exercise 3.6

1 A. Question

Find the lowest common multiple of the following expressions:

$24x^2yz$ and $27x^4y^2z^2$

Answer

Let u = $24x^2yz$

and v = $27x^4y^2z^2$

Writing u and v in factorized form which is as follows:

u = $2 \times 2 \times 2 \times 3 \times x^2 \times y \times z$

= $2^3 \times 3 \times x^2 \times y \times z$

v = $3 \times 3 \times 3 \times x^4 \times y^2 \times z^2$

= $3^3 \times x^4 \times y^2 \times z^2$

Now selecting the common multiples from both u and v,

= $2^3 \times 3^3 \times x^4 \times y^2 \times z^2$

= 216 $x^4y^2z^2$

Therefore the LCM of u and v = 216 $x^4y^2z^2$

1 B. Question

Find the lowest common multiple of the following expressions:

$x^2 - 3x + 2$ and $x^4 + x^3 - 6x^2$

Answer

Let u(x) = $x^2 - 3x + 2$

On factorizing the above equation,

Sum = -3

Product = 2

Therefore the two numbers satisfying the above conditions are -2 and -1.

u(x) = $x^2 - x - 2x - 2$

u(x) = $x(x - 1) - 2(x - 1)$

u(x) = $(x - 2)(x - 1)$

Let v(x) = $x^4 + x^3 - 6x^2$

= $x^2(x^2 + x - 6)$

Solving the quadratic part,

$\Rightarrow x^2 + x - 6$

$\Rightarrow x^2 + 3x - 2x - 6$

$\Rightarrow x(x + 3) - 2(x + 3)$

$\Rightarrow (x - 2)(x + 3)$

Therefore $v(x) = x^2 (x - 2)(x + 3)$

By comparing u(x) and v(x),

$LCM = x^2 (x - 2)(x + 3)(x - 1)$

1 C. Question

Find the lowest common multiple of the following expressions:

(iii) $2x^2 - 8$ and $x^2 - 5x + 6$

Answer

Let $u(x) = 2x^2 - 8$

On factorizing the above equation,

$u(x) = 2(x^2 - 4)$

Let $v(x) = x^2 - 5x + 6$

On factorizing the above equation,

Sum = -5

Product = 6

Therefore the two numbers satisfying the above conditions are -2 and -3.

$v(x) = x^2 - 3x - 2x + 6$

$v(x) = x(x - 3) - 2(x - 3)$

$v(x) = (x - 2)(x - 3)$

By comparing u(x) and v(x),

$LCM = 2(x^2 - 4)(x - 3)$

1 D. Question

Find the lowest common multiple of the following expressions:

$x^2 - 1$; $(x^2 + 1)(x + 1)$ and $x^2 + x - 1$

Answer

Let $u(x) = (x^2 - 1) = (x + 1)(x - 1)$

Let $v(x) = (x^2 + 1)(x + 1)$

Let $w(x) = x^2 + x - 1$

By comparing u(x), v(x) and w(x),

The LCM is $= (x + 1)(x - 1)(x^2 + 1)(x^2 + x - 1)$

$= (x^2 - 1)(x^2 + 1)(x^2 + x - 1)$

$= (x^4 - 1)(x^2 + x - 1)$

1 E. Question

Find the lowest common multiple of the following expressions:

$18(6x^4 + x^3 - x^2)$ and $45(2x^6 + 3x^5 + x^4)$

Answer

Let $u(x) = 18(6x^4 + x^3 - x^2)$

$= 18\,x^2(6x^2 + x - 1)$

Let us solve the inner quadratic equation,

$6x^2 + x - 1 = 0$

$6x^2 + x - 1 = 0$

$6x^2 + 3x - 2x - 1 = 0$

$3x(2x + 1) - 1(2x + 1) = 0$

$(3x - 1)(2x + 1) = 0$

So $u(x) = 18\,x^2(3x - 1)(2x + 1)$

Let $v(x) = 45(2x^6 + 3x^5 + x^4)$

$= 45\,x^4(2x^2 + 3x + 1)$

Let us solve the inner quadratic equation,

$2x^2 + 3x + 1 = 0$

$2x^2 + 2x + x + 1 = 0$

$2x(x + 1) + 1(x + 1) = 0$

$(2x + 1)(x + 1) = 0$

So $v(x) = 45x^4(2x + 1)(x + 1)$

By comparing the above equation, we get

LCM = $90x^4(2x + 1)(x + 1)(3x - 1)$

2. Question

Find the highest common factor of the following expressions:

(i) a^3b^4, ab^5, a^2b^8

(ii) $16x^2y^2$, $48x^4z$

(iii) $x^2 - 7x + 12$; $x^2 - 10x + 21$ and $x^2 + 2x - 15$

(iv) $(x + 3)^2(x - 2)$ and $(x + 3)(x - 2)^2$

(v) $24(6x^4 - x^3 - 2x^2)$ and $20(6x^6 + 3x^5 + x^4)$

Answer

(i) Let $u(x) = a^3b^4$

Let $v(x) = ab^5$

Let $w(x) = a^2b^8$

By comparing all the above equations, we get,

HCF = ab^4 (Least power of a and b)

(ii) Let $u(x) = 2^4 \times x^2 \times y^2$

Let $v(x) = 2^4 \times 3 \times x^4 \times z$

By comparing all the above equations, we get,

HCF = $2^4 \times x^2$

= $16x^2$ (Least power of x, y and z and also common terms of u(x) and v(x))

(iii) Let $u(x) = x^2 - 7x + 12$

$= x^2 - 4x - 3x + 12$

$= x(x - 4) - 3(x - 4)$

$= (x - 3)(x - 4)$

Let $v(x) = x^2 - 10x + 21$

$= x^2 - 7x - 3x + 21$

$= x(x - 7) - 3(x - 7)$

$= (x - 7)(x - 3)$

Let $w(x) = x^2 + 2x - 15$

$= x^2 + 5x - 3x - 15$

$= x(x + 5) - 3(x + 5)$

$= (x - 3)(x + 5)$

By comparing all the above equations, we get,

HCF = $(x + 3)$ [only common term from u(x), v(x) and w(x)].

(iv) Let $u(x) = (x + 3)^2 (x - 2)$

Let $v(x) = (x + 3)(x - 2)^2$

By comparing all the above equations, we get,

HCF = $(x + 3)(x - 2)$ [Least power and common term from u(x) and v(x)].

(v) Let $u(x) = (8 \times 3)(6x^4 - x^3 - 2x^2)$

$= (8 \times 3) x^2 (6x^2 - x - 2)$

$= (8 \times 3) x^2 (6x2 - 4x + 3x - 2)$

$= (8 \times 3) x^2 (2x(3x - 2) + 1(3x - 2))$

$= (8 \times 3) x^2 (2x + 1)(3x - 2)$

Let $v(x) = 20(6x^6 + 3x^5 + x^4)$

$= (4 \times 5) x^4(6x^2 + 3x + 1)$

$= (4 \times 5) x^4(6x^2 + 3x + 1)$

By comparing all the above equations, we get,

HCF = $4 x^2 (2x + 1)$ [Least power and common term from u(x) and v(x)].

3. Question

If $u(x) = (x - 1)^2$ and $v(x) = (x^2 - 1)$ the check the true of the relation LCM × HCF = u(x) × v(x).

Answer

$u(x) = (x - 1)^2$

$= (x - 1)(x - 1)$

$v(x) = (x^2 - 1)$

$= (x + 1)(x - 1)$

LCM of u(x) and v(x) = $(x - 1)^2 (x + 1)$

HCF of u(x) and v(x) = $(x - 1)$

$u(x) \times v(x) = (x - 1)(x - 1) \times (x^2 - 1)$

$= (x^2 - 2x + 1) \times (x^2 - 1)$

$= x^4 - 2x^3 + x^2 - x^2 + 2x - 1$

$= x^4 - 2x^3 + 2x - 1$

HCF × LCM = $(x - 1)^2 (x + 1) \times (x - 1)$

$= (x^2 - 2x + 1)(x^2 - 1)$

$= x^4 - 2x^3 + x^2 - x^2 + 2x - 1$

$= x^4 - 2x^3 + 2x - 1$

So it is observed that HCF × LCM = u(x) × v(x).

Hence Proved.

4. Question

The product of two expressions is $(x - 7)(x^2 + 8x + 12)$. If the HCF of these expressions is $(x + 6)$ then find their LCM.

Answer

Product = $(x - 7)(x^2 + 8x + 12)$

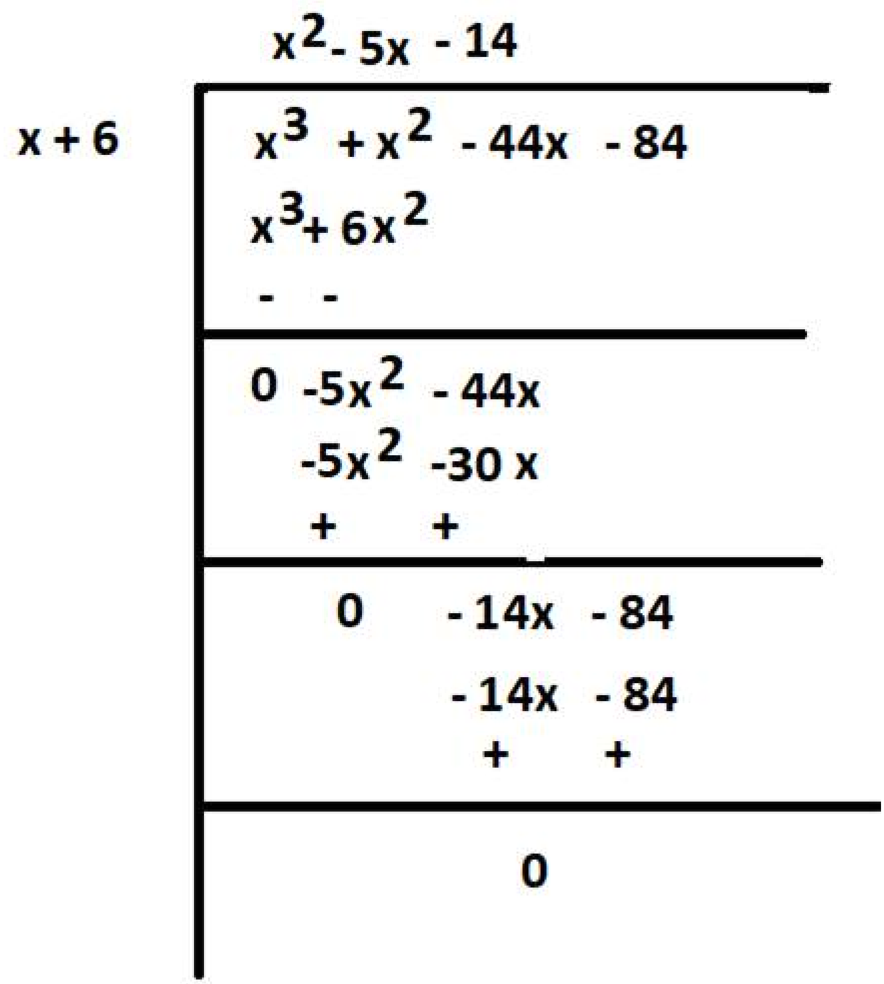
$= x^3 + 8x^2 + 12x - 7x^2 - 56x - 84$

$= x^3 + x^2 - 44x - 84$

The LCM = Product / HCF

$= (x^3 + x^2 - 44x - 84) / (x + 6)$

The division is as follows:



5. Question

The HCF and LCM of two quadratic expressions are respectively (x – 5) and $x^3 - 19x - 30$, then find both the expressions.

Answer

HCF = (x – 5)

LCM = $x^3 - 19x - 30$

$= x^3 - 19x - 38 + 8$

$= x^3 + 8 - 19x - 38$

$= x^3 + 2^3 - 19(x + 2)$

$x^3 + 2^3 = (x + 2)(x^2 - 2x + 4)$ [Using $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$]

$= (x + 2)(x^2 - 2x + 4) - 19(x + 2)$

$= (x + 2)(x^2 - 2x + 4 - 19)$

$= (x + 2)(x^2 - 2x - 15)$

$= (x + 2)(x^2 - 5x + 3x - 15)$

$= (x + 2)(x(x - 5) + 3(x - 5))$

$= (x + 2)(x - 5)(x + 3)$

Since HCF = x – 5 it will belong to both polynomials.

So $u(x) = (x - 5)(x + 3)$

$= x^2 - 2x - 15$

So $v(x) = (x - 5)(x + 2)$

$= x^2 - 3x - 10$

Miscellaneous Exercise 3

1. Question

If one zero of the polynomial $f(x) = 5x^2 + 13x + k$ is reciprocal of the other than the value of k will be:

A. 0

B. 1/5

C. 5

D. 6

Answer

Let the first root be a.

So the second root as per the question is 1/a.

Product of zeroes = $a \times 1/a$

= 1

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$\therefore a = 5, b = 13, c = k$

Product of zeroes = c / a

$k/5 = 1$

Therefore k = 5

So the correct answer is C [5].

2. Question

The zeroes of the polynomials $x^2 - x - 6$ are:

A. 1, 6

B. 2, −3

C. 3, −2

D. 1, −6

Answer

Solving the quadratic part,

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow x^2 - 3x + 2x - 6 = 0$

$\Rightarrow x(x - 3) + 2(x - 3) = 0$

$\Rightarrow (x + 2)(x - 3) = 0$

Solving first part,

$x + 2 = 0$

$x = -2$

Solving second part,

$x - 3 = 0$

$x = 3$

So the correct answer is C [3, -2].

3. Question

If one zero of the polynomial $2x^2 + x + k$ is 3 then the value of k will be:

A. 12

B. 21

C. 24

D. −21

Answer

Given one zero = 3

Let second zero a.

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

∴ a = 2, b = 1, c = k

Sum of zeroes = -b/a

3 + a = -1/2

a = -1/2 – 3

a = -7/2

Product of zeroes = 3 × (-7/2)

= -21/2

Product of zeroes = c/a

k/2 = -21/2

k = -21

So the correct answer is D [-21].

4. Question

If α, β, are the zeroes of the polynomial $x^2 - p(x + 1) - c$ such that (α + 1) (β + 1) = 0 then the value of c will be:

A. 0

B. –1

C. 1

D. 2

Answer

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

∴ a = 1

b = -p

c = -p-c

Sum of zeroes = -b/a

$= - (-p) / 1$

$\alpha + \beta = p$ …………………………... (i)

Product of Zeroes = c/a

$\alpha\beta = - (p + c)$ …………………… (ii)

Now from LHS we have,

$(\alpha + 1)(\beta + 1) = \alpha\beta + \alpha + \beta + 1$

From (i) and (ii) we have the values of $\alpha\beta$ and $(\alpha + \beta)$

$(\alpha + 1)(\beta + 1) = - (p + c) + p + 1$

$= - p – c + p + 1$

$= c + 1$

Given $(\alpha + 1) (\beta + 1) = 0$

$c + 1 = 0$

$c = -1$

So the correct answer is B [-1].

5. Question

If the roots of the quadratic equation $x^2 – kx + 4 = 0$ are equal then the value of k will be:

A. 2

B. 1

C. 4

D. 3

Answer

$x^2 – kx + 4 = 0$

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$a = 1$

$b = -k$

$c = 4$

Since the quadratic equations have real and equal roots,

$b^2 - 4ac = 0$ for real and equal roots

$\Rightarrow (-k)^2 - (4 \times 1 \times 4) = 0$

$\Rightarrow k^2 - 16 = 0$

$\Rightarrow k^2 = 16$

$\Rightarrow k = \pm 4$

So the correct answer is C [4].

6. Question

If x = 1 is a common root of the equations $ax^2 + ax + 3 = 0$ and $x^2 + x + b = 0$ then the value of ab will be:

A. 1

B. 3.5

C. 6

D. 3

Answer

Put x = 1 in the first equation,

a + a + 3 = 0

2a = -3

a = -3/2

Put x = 1 in the second equation,

1 + 1 + b = 0

b + 2 = 0

b = -2

Therefore the value of ab = -3/2 × -2

= 3

So the correct answer is D [3].

7. Question

The discriminant of the quadratic equation $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$ will be:

A. 10

B. 64

C. 46

D. 30

Answer

The discriminant of the equation can be found out using the following formula:

$\Rightarrow b^2 - 4ac$

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$\therefore a = 3\sqrt{3}$

$b = 10$

$c = \sqrt{3}$

$\Rightarrow b^2 - 4ac$

$\Rightarrow (10)^2 - (4 \times 3\sqrt{3} \times \sqrt{3})$

$\Rightarrow (10)^2 - (4 \times 9)$

$\Rightarrow 100 - 36$

$= 64$

So the correct answer is B [64].

8. Question

The nature of roots of the quadratic equation $4x^2 - 12x - 9 = 0$ is :

A. real and equal

B. real and distinct

C. imaginary and equal

D. imaginary and distinct

Answer

The nature of roots of the equation can be found out using the following formula:

$\Rightarrow b^2 - 4ac$

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$\therefore a = 4$

$b = -12$

$c = -9$

$\Rightarrow b^2 - 4ac$

$\Rightarrow (-12)^2 - (4 \times -9 \times 4)$

$\Rightarrow 144 - (-144)$

$\Rightarrow 144 + 144$

$= 288$

Since $b^2 - 4ac>0$, the roots will be real and distinct.

So the correct answer is B [real and distinct].

9. Question

The HCF of expression $8a^2b^2c$ and $20ab^3c^2$ is:

A. $4ab^2c$

B. 4abc

C. $40a^2b^3c^2$

D. 40abc

Answer

Let us write both the expressions in factorized form.

Let $u(x) = 2^3 \times a^2 \times b^2 \times c$

Let $v(x) = 2^2 \times 5 \times a \times b^3 \times c^2$

So by comparing u(x) and v(x), HCF = $4ab^2c$ [Least power of variables and common terms]

So the correct answer is A [$4ab^2c$].

10. Question

The LCM of expressions $x^2 - 1$ and $x^2 + 2x + 1$ is :

A. $x + 1$

B. $(x^2 - 1)(x + 1)$

C. $(x - 1)(x + 1)^2$

D. $(x^2 - 1)(x + 1)^2$

Answer

Let us factorize $x^2 + 2x + 1$

But $x^2 + 2x + 1$ is a perfect square and it can be expressed as $(x + 1)^2$

$u(x) = x^2 + 2x + 1$

$u(x) = x^2 + x + x + 1$

$u(x) = x(x + 1) + 1(x + 1)$

$u(x) = (x + 1)(x + 1)$

Let $v(x) = x^2 - 1$

$= (x + 1)(x - 1)$

So by comparing both polynomials, LCM is = $(x1)(x+1)^2$

So the correct answer is C $[(x - 1)(x + 1)^2]$.

11. Question

If LCM of expressions $6x^2y^4$ and $10xy^2$ is $30x^2y^4$ then HCF will be:

A. $6x^2y^2$

B. $2xy^2$

C. $10x^2y^4$

D. $60x^3y^6$

Answer

Let $u(x) = 2 \times 3 \times x^2 \times y^4$

Let $v(x) = 2 \times 5\ x \times x \times y^2$

So by comparing u(x) and v(x), HCF = $2xy^2$ [Least power of variables and common terms]

So the correct answer is B [$2xy^2$].

12. Question

Write Shridharacharya Formula of finding the roots of the quadratic equation $ax^2 + bx + c = 0$.

Answer

The generalized formula for the quadratic equation is as follows:

$ax^2 + bx + c = 0$

There is one formula developed by Shridharacharya to determine the roots of a quadratic equation which is as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

13. Question

Writing the general form of the discriminant of the equation $ax^2 + by + c = 0$ explain the nature of the roots.

Answer

The generalized formula for the quadratic equation is as follows:

$ax^2 + bx + c = 0$

The discriminant of the equation can be found out using the following formula:

$\Rightarrow b^2 - 4ac$

If $b^2 - 4ac > 0$, then the roots will be real and distinct.

If $b^2 - 4ac = 0$, then the roots will be real and equal.

If $b^2 - 4ac < 0$, then the roots will be imaginary.

14. Question

Find the zeroes of the quadratic polynomial $2x^2 - 8x + 6$ and examine the truth of the relationship between the zeroes and the coefficients.

Answer

To find the zeros of the polynomial let us first solve the polynomial by equating it to zero. Factorizing the given polynomial

$2x^2 - 8x + 6 = 0$

To factorize the polynomial we have,

Sum of the value should be equal = -8

Product should be equal to = 2 × 6

= 12

So two numbers are -2, -6

$2x^2 - 2x - 6x + 6 = 0$

$2x(x - 1) - 6(x - 1) = 0$

$(2x-6)(x - 1) = 0$

$2x-6 = 0$ or $x-1 = 0$

Solving first part,

$2x-6 = 0$

$2x = 6$

$x = 3$

Solving second part,

$x - 1 = 0$

$x = 1$

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

∴ a = 2, b = -8, c = 6

Sum of zeroes = -b / a

= - (-8) / 2

= 4

Product of zeroes = c / a

= 6 / 2

= 3

Zeroes obtained are 3 and 1 and their sum is 4 and product is 3 which is matching to the answer obtained through the ratio of coefficients.

15. Question

If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - px + q$ then find the values of the following:

(i) $\alpha^2 + \beta^2$ (ii) $\frac{1}{\alpha} + \frac{1}{\beta}$

Answer

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$\therefore a = 1$

$b = -p$

$c = q$

Sum of zeroes = -b / a

= - (-p) / 1

$\alpha + \beta = p$

Product of zeroes = c / a

= q / 1

$\alpha\beta = q$

(i) $\alpha^2 + \beta^2$

$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (p)^2 - 2q$

$= p^2 - 2q$

(ii)

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$$

$$\therefore \frac{1}{\alpha} + \frac{1}{\beta} = \frac{p}{q}$$

16. Question

If the polynomial $x^4 – 6x^3 + 16x + 10$ is divided by another polynomial $x^2 – 2x + k$ and the remainder obtained is $(x + a)$, then find the values of k and a.

Answer

The division is as follows:

$$x^2 - 2x + k \,)\, x^4 - 6x^3 + 16x^2 - 25x + 10 \,(x^2 - 4x + (8 - k)$$

$$x^4 - 2x^3 + kx^2$$

$$-4x^3 + (16 - k)x^2 - 25x$$

$$-4x^3 + 8x^2 - 4kx$$

$$(8 - k)x^2 + (4k - 25)x + 10$$

$$(8 - k)x^2 - 2(8 - k)x + k(8 - k)$$

$$(4k - 25 + 16 - 2k)x + [10 - k(8 - k)]$$

There is an important relation between dividend, divisor, quotient and remainder which is as follows:

Dividend = Divisor × Quotient + Remainder

Dividend – Remainder = Divisor as the when remainder is subtracted from dividend, the result obtained is completely divisible by divisor.

$(-9 + 2k)\, x + (10 – 8k + k^2) = (x + a)$

On comparing both sides of coefficients of x:

$2k – 9 = 1$

$2k = 10$

$k = 5$

On comparing both sides:

$10 – 8k + k^2 = a$

$10 - (8 × 5) + 5^2 = a$

$10 – 40 + 25 = a$

$35 – 40 = a$

$-5 = a$

Hence $a = -5$ and $k = 5$.

17. Question

The area of a rectangular plot is 528 m^2. The length of the plot (in m) is 1 more than double of breadth. Representing by the required quadratic equation find the length and breadth of the plot.

Answer

Let the length be l and breadth be b.

Area = l × b

Given Length = 2b + 1

(2b + 1) × b = 528

$2b^2 + b = 528$

$2b^2 + b - 528 = 0$

$2b^2 + 33b - 32b - 528 = 0$

2b(b – 16) + 33(b – 16) = 0

(2b + 33) (b – 16) = 0

Solving the first part,

2b + 33 = 0

2b = -33

b = -33/2

Solving second part,

b – 16 = 0

b = 16

Breadth cannot be negative.

So breadth = 16 m

Length = 2 × 16 + 1

= 33 m

18. Question

Solve the quadratic equation $x^2 + 4x - 5 = 0$ by the method of completing the square.

Answer

$x^2 + 4x - 5 = 0$

$x^2 + 4x = 5$

Now by taking half of the coefficient of x and then squaring it and adding on both LHS and RHS sides.

Coefficient of x = 4

Half of 4 = 2

Squaring the half of 4 = 4

$x^2 + 4x + 4 = 5 + 4$

$(x + 2)^2 = 9$

On simplifying both RHS and LHS we get an equation of following form,

$(x \pm A)^2 = k^2$

Taking square root of both sides,

$(x + 2) = \pm 3$

Solving first with positive sign of 3,

$x + 2 = 3$

$x = 3 – 2$

$x = 1$

Solving with negative sign of 3,

$x + 2 = -3$

$x = -3 – 2$

$x = -5$

19 A. Question

Solve the following equation by factorization method:

$$\frac{1}{x} = \frac{1}{x-2} = 3,\ x \neq 0,\ 2$$

Answer

Let us solve the above equation by equating it to zero,

$$\frac{1}{x} - \frac{1}{x-2} = 3$$

$$\frac{x - 2 - x}{x\,(x - 2)} = 3$$

$$\frac{-2}{x^2 - 2x} = 3$$

$-2 = 3(x^2 - 2x)$

$3x^2 - 6x + 2 = 0$

These problems cannot be solved by normal factorization as it do not contain normal factors.

There is one formula developed by Shridharacharya to determine the roots of a quadratic equation which is as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Before putting the values in the formula let us check the nature of roots by $b^2 - 4ac > 0$

$\Longrightarrow (-6)^2 - (4 \times 3 \times 2)$

$\Longrightarrow 36 - 24$

$\Longrightarrow 12$

Since $b^2 - 4ac = 12$ the roots are real and distinct.

Now let us put the values in the above formula

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - (4 \times 3 \times 3)}}{2 \times 3}$$

$$x = \frac{6 \pm \sqrt{36 - 24}}{6}$$

$$x = \frac{6 \pm \sqrt{12}}{6}$$

$$x = \frac{2\,(3 \pm \sqrt{3})}{6}$$

$$x = \frac{3 \pm \sqrt{3})}{3}$$

19 B. Question

Solve the following equation by factorization method:

$$\frac{1}{x-1}-\frac{1}{x+5}=\frac{6}{7}, x \neq 1, -5$$

Answer

$$\frac{1}{x-1} - \frac{1}{x+5} = \frac{6}{7}$$

$$\frac{(x+5) - (x-1)}{(x-1) \times (x+5)} = \frac{6}{7}$$

$$\frac{6}{x^2 - x + 5x - 5} = \frac{6}{7}$$

$$\frac{6}{x^2 - 4x - 5} = \frac{6}{7}$$

Cross multiplying,

$7 \times 6 = 6 \times (x^2 - 4x - 5)$

$42 = 6x^2 - 24x - 30$

$6x^2 - 24x - -30 - 42 = 0$

$6x^2 - 24x - 72 = 0$

$6(x^2 - 4x - 12) = 0$

$x^2 - 4x - 12 = 0$

On factorizing,

$x^2 - 6x + 2x - 12 = 0$

$x(x - 6) + 2(x - 6) = 0$

$(x + 2)(x - 6) = 0$

Solving first part,

$x + 2 = 0$

$x = -2$

Solving second part,

$x - 6 = 0$

$x = 6$

19 C. Question

Solve the following equation by factorization method:

$$x - \frac{1}{x} = 3,\ x \neq 0$$

Answer

Let us solve the above equation by equating it to zero,

$$x - \frac{1}{x} = 3$$

$$\frac{x^2 - 1}{x} = 3$$

$x^2 - 1 = 3x$

$x^2 - 3x - 1 = 0$

These problems cannot be solved by normal factorization as it do not contain normal factors.

There is one formula developed by Shridharacharya to determine the roots of a quadratic equation which is as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Before putting the values in the formula let us check the nature of roots by $b^2 - 4ac > 0$

$\Rightarrow (-3)^2 - (4 \times 1 \times -1)$

$\Rightarrow 9 + 4$

$\Rightarrow 13$

Since $b^2 - 4ac = 13$ the roots are real and distinct.

Now let us put the values in the above formula

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - (4 \times 1 \times -1)}}{2 \times 1}$$

$$x = \frac{3 \pm \sqrt{9 + 4}}{2}$$

$$x = \frac{3 \pm \sqrt{13}}{2}$$

19 D. Question

Solve the following equation by factorization method:

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30} \quad x \neq -4, 7$$

Answer

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$$

$$\frac{(x-7) - (x+4)}{(x-7) \times (x+4)} = \frac{11}{30}$$

$$\frac{-11}{x^2 + 4x - 7x - 28} = \frac{11}{30}$$

$$\frac{-11}{x^2 - 3x - 28} = \frac{11}{30}$$

Cross Multiplying,

$30 \times (-11) = 11 \times (x^2 - 3x - 28)$

$-330 = 11x^2 - 33x - 308$

$11x^2 - 33x - 308 + 330 = 0$

$11x^2 - 33x + 22 = 0$

$11(x^2 - 3x + 2) = 0$

$x^2 - 3x + 2 = 0$

On factorizing,

$x^2 - 2x - x + 2 = 0$

$x (x - 2) - 1(x - 2) = 0$

$(x - 2) (x - 1) = 0$

Solving first part,

$x - 2 = 0$

$x = 2$

Solving Second part,

$x - 1 = 0$

$x = 1$

20. Question

If one root of a quadratic equation $2x^2 + px - 15 = 0$ is -5 and the root of the quadratic equation $p(x^2 + x) + k = 0$ are equal then find the value of k.

Answer

Since -5 is the zero of the first equation,

Put x = -5 in first equation

$2(-5)^2 + p(-5) - 15 = 0$

$2\times 25 - 5p - 15 = 0$

$5p = 50 - 15$

$5p = 35$

$p = 7$

If the roots are equal, then

$b^2 - 4ac = 0$

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$\therefore a = p = 7$

$b = p = 7$

$c = k$

$(7)^2 - (4 \times 7 \times k) = 0$

$(7)^2 - 28k = 0$

$28k = 49$

$k = 49 / 28$

$k = 7 / 4$

21 A. Question

Solve the following quadratic equations by using Shridharacharya Quadratic Formula:

$p^2x^2 + (p^2 - q^2)x - q^2 = 0$

Answer

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

$a = p^2$

$b = (p^2 – q^2)$

$c = – q^2$

There is one formula developed by Shridharacharya to determine the roots of a quadratic equation which is as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Before putting the values in the formula let us check the nature of roots by $b^2 – 4ac >0$

$\Longrightarrow ((p^2 – q^2))^2 – (4 \times p \times – q^2)$

$\Longrightarrow (p^4 –2 p^2 q^2 + q^4) - (-4p^2q^2)$

$\Longrightarrow p^4 –2 p^2 q^2 + q^4 + 4p^2q^2$

$\Longrightarrow (p^4 + 2 p^2 q^2 + q^4)$

$\Longrightarrow ((p^2 + q^2))^2$

Now let us put the values in the above formula

$$x = \frac{-(p^2 - q^2) \pm \sqrt{(p^2 + q^2)^2}}{2 \times p^2}$$

$$x = \frac{-(p^2 - q^2) \pm (p^2 - q^2)}{2p^2}$$

Solving with positive value first,

$$x = \frac{-p^2 + q^2 + p^2 + q^2}{2p^2}$$

$$x = \frac{2q^2}{2p^2}$$

$x = q^2 / p^2$

Solving with negative value second,

$$x = \frac{-p^2 + q^2 - p^2 - q^2}{2p^2}$$

$$x = \frac{-2p^2}{2p^2}$$

x = -1

21 B. Question

Solve the following quadratic equations by using Shridharacharya Quadratic Formula:

$9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$

Answer

When we compare the above quadratic equation with the generalized one we get,

$ax^2 + bx + c = 0$

a = 9

$b = -9(a + b)$

$c = (2a^2 + 5ab + 2b^2)$

There is one formula developed by Shridharacharya to determine the roots of a quadratic equation which is as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Before putting the values in the formula let us check the nature of roots by $b^2 - 4ac > 0$

$\Rightarrow (-9(a + b))^2 - (4 \times 9 \times (2a^2 + 5ab + 2b^2))$

$\Rightarrow 81(a^2 + 2ab + b^2) - (36(2a^2 + 5ab + 2b^2))$

$\Rightarrow 81a^2 + 162ab + 81b^2 - 72a^2 - 180ab - 72b^2$

$\Rightarrow 9a^2 - 18ab + 9b^2$

$\Rightarrow 9(a^2 - 2ab + b^2)$

$\Rightarrow 3^2(a - b)^2$

Now let us put the values in the above formula

$$x = \frac{-(-9(a+b)) \pm \sqrt{3^2(a-b)^2}}{2 \times 9}$$

$$x = \frac{9(a+b) \pm 3(a-b)}{18}$$

Solving with positive value first,

$$x = \frac{9a + 9b + 3a - 3b}{18}$$

$$x = \frac{12a + 6b}{18}$$

$$x = \frac{6(2a + b)}{18}$$

$$x = \frac{2a + b}{3}$$

Solving with negative value second,

$$x = \frac{9a + 9b - 3a + 3b}{18}$$

$$x = \frac{12b + 6a}{18}$$

$$x = \frac{6(2b + a)}{18}$$

$$x = \frac{2b + a}{3}$$

22. Question

The LCM and HCF of two quadratic expressions are respectively $x^3 - 7x + 6$ and $(x - 1)$. Find the expressions.

Answer

Given:

HCF = $x - 1$

Let the two polynomials be $u(x)$ and $v(x)$

So $(x - 1)$ is common to both $u(x)$ and $v(x)$

LCM = $x^3 - 7x + 6$

= $x^3 - 7x + 6$

$= x^3 – (1 + 6) x + 6$

$= x^3 – x - 6x + 6$

$= x(x^2 – 1) - 6(x - 1)$

$(x^2 – 1) = (x + 1) (x – 1)$

$a^2 – b^2 = (a + b) (a – b)$

Here a = x and b = 1

$= x(x + 1) (x – 1) - 6(x - 1)$

$= (x-1) [x(x + 1) - 6]$

Now solving the inner quadratic equation,

$x^2 + x - 6 = 0$

$x^2 + 3x - 2x - 6 = 0$

$x(x + 3) - 2(x + 3) = 0$

$(x + 3) (x – 2) = 0$

$\therefore$ LCM = (x-1) (x + 3) (x – 2)

Since HCF = (x-1), which implies that both u(x) and v(x) contains (x – 1)

Therefore u(x) = (x – 1) (x + 3)

$= x^2 + 2x - 3$

v (x) = (x – 1) (x – 2)

$= x^2 – 3x + 2$

Therefore the polynomial are $x^2 + 2x – 3$ and $x^2 – 3x + 2$.

23. Question

The LCM of two polynomials in $x^3 – 6x^2 + 3x + 10$ and HCF is (x + 1). If one polynomial is $x^2 – 4x – 5$ then find the other polynomial.

Answer

LCM = $x^3 – 6x^2 + 3x + 10$

$= x^3 – 5x^2 - x^2 + 3x + 10$

$= x^2(x – 5) – (x2 – 3x – 10)$

$= x^2(x - 5) - (x2 - 5x + 2x - 10)$

$= x^2(x - 5) - (x(x - 5) + 2(x - 5))$

$= x^2(x - 5) - (x + 2)(x - 5)$

$= (x - 5)[x^2 - x - 2]$

$= (x - 5)[x^2 - 2x + x - 2]$

$= (x - 5)[x(x - 2) + 1(x - 2)]$

$= (x - 5)(x - 2)(x + 1)$

Since HCF = $(x + 1)$ it belongs to both the polynomials.

So $u(x) = (x + 1)(x - 5)$

$= x^2 - 4x - 5$

So $v(x) = (x + 1)(x - 2)$

$= x^2 - x - 2$