Force ~~and motion~~ and Moment System :-

Force :- to define →
- → magnitude ✓
- → dirⁿ ✓
- → Point of application ✓

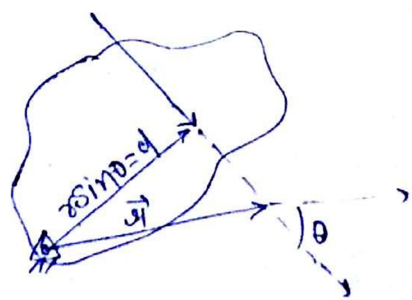
Moment of Force / Torque :-

$\vec{T} = \vec{m} = \vec{r} \times \vec{F}$

$T_A = Fd$; CW

$T_A = \vec{r} \times \vec{F}$

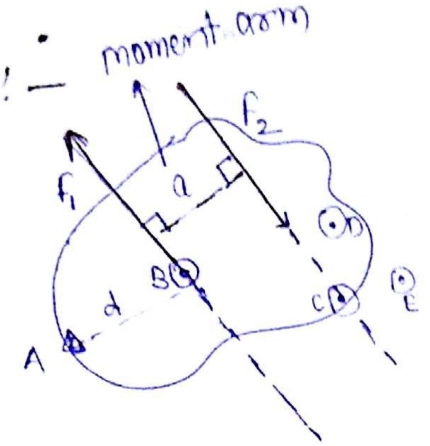
$|\vec{T_A}| = rF\sin\theta$



$T_A = Fd$

dirⁿ → ⊥ to the plane, inward.

Couple :-



$\vec{F_1} = -\vec{F_2}$

$F_1 = F_2 = F$

$\Sigma \vec{F} = 0$

<u>moment of Couple</u>

$M_A = -Fd + F(a+d) = Fa$; CW

$M_B = Fa$; CW

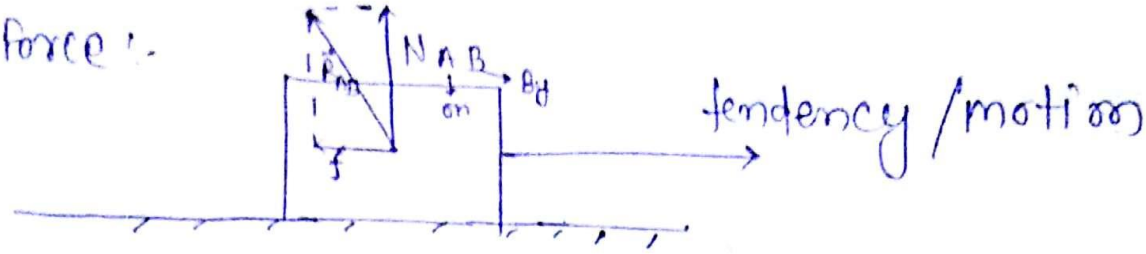
$M_C = Fa$; CW

$M_D = M_E = Fa$; CW

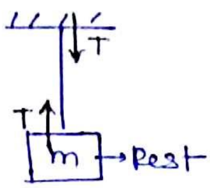
Couple is an arrangment of two equal and opposite forces acting parallel to each other whose moment remains uniform throughout on the body.

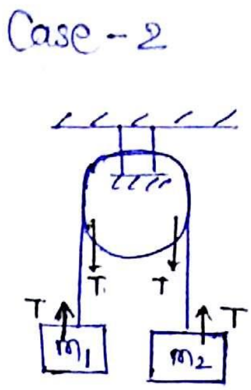
Forces
- → Gravity ($W = mg\ (\downarrow)$)
- → Contact force
  - → Normal Rxn
  - → friction force
- → tension
- → Spring force $F = kx$
- → Applied force.

Contact force :-



Tension :-





Newton's 1st law :-

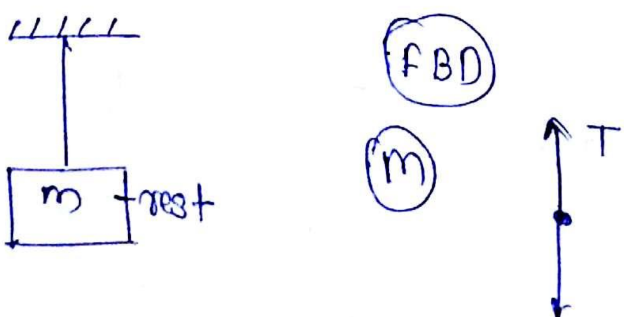
For a particle →

if $\vec{F}_R = 0$ then $\vec{a} = 0$

For a rigid body →

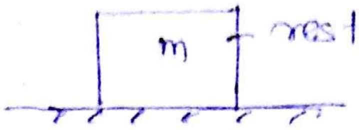
if $(\vec{F}_R)_{ext} = 0$ then $\vec{a}_{cm} = 0$

Case-1



if $\vec{a}_{cm} = 0 \Rightarrow T = mg$ → First law

Case-2



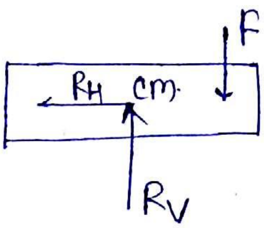
(m)

$N = mg \rightarrow 1^{st}$ law

Case-3 (massless bar)

Rxn at hing = ?

(Bar)



$\vec{a}_{cm} = 0$ (bcoz Axis of Rotation Pass through c.m.)

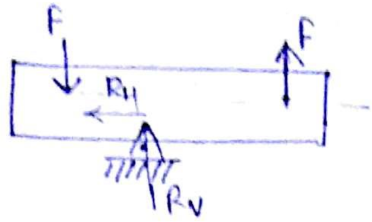
$(\vec{F}_R)_{ext} = 0$

$\Rightarrow R_V - F = 0$

$R_V = F$

$\Rightarrow R_H = 0$

Case - 4



Rxn at c.m.

$\vec{a}_{cm} = 0$

$\Sigma \vec{F}_{ext} = 0 \quad \Rightarrow -F + R_V + F = 0$

$R_V = 0$

$\Rightarrow R_H = 0$

Equilibrium :-

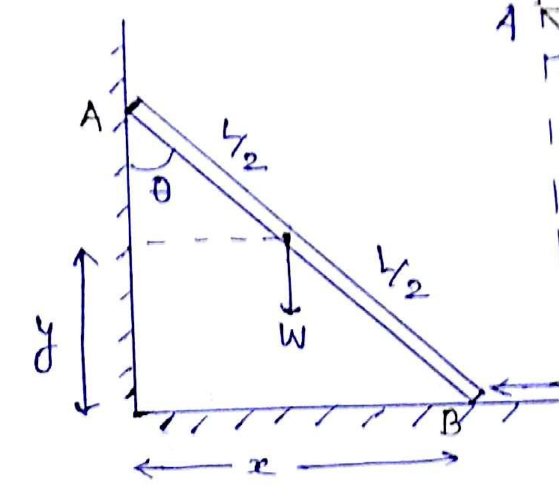
1. $\Sigma \vec{F} = 0$

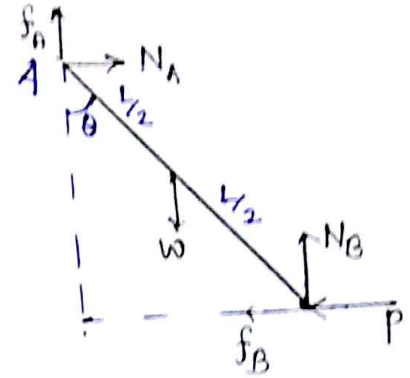
$\Sigma \vec{F_x} = \Sigma \vec{F_y} = \Sigma \vec{F_z} = 0$

2. $\Sigma \vec{M_x} = \Sigma \vec{M_y} = \Sigma \vec{M_z} = 0$

equ^m – Rest / uniform linear velocity

Queastion:- A ladder AB of weight W and length L is held in equ^m by a Horizontal force P as shown in Fig. Assuming the ladder to be idealised as a homogeneous rigid bar and the surface to be smooth, Then what is the value of P.





Sol^n)

$$\Sigma \vec{F}_H = 0 \qquad \Sigma \vec{F}_V = 0$$

$$P = N_A \quad -① \qquad W = N_B$$

moment of ACW Couple (W, N_B) = moment of CW couple (N_A, P)

$$W \times \frac{L}{2} \sin\theta = P \times L \cos\theta$$

$$P = \frac{W}{2} \tan\theta$$

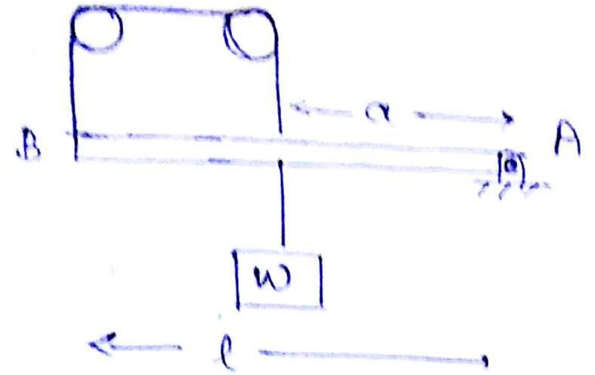
(or) body in rest So moment at any pt equal to zero

$$\Sigma M_A = 0 \Rightarrow W \frac{L}{2} \sin\theta + PL\cos\theta - N_B L \sin\theta$$

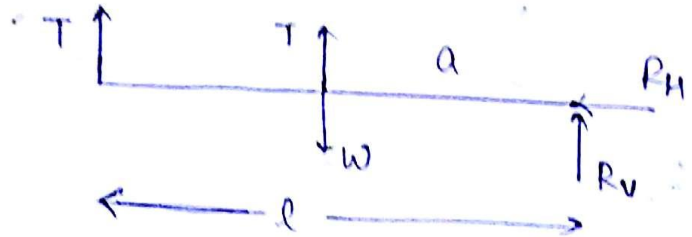
$$P = \frac{W}{2} \tan\theta$$

Question:- A weightless bar AB is supported by a hinge at A, and by a wire passing over two frictionless pulleys as shown in fig. Rxn at A is

Sol^n



Sol^n



$\Sigma \vec{F}_V = 0$

$2T + R_V = W$

$R_V = W - 2T \quad -(1)$

$\Sigma \vec{F}_H = 0$

$R_H = 0$

$\Sigma \vec{m}_A = 0$

$Tl + Ta - Wa = 0$

$T = \dfrac{Wa}{(l+a)} \quad -(2)$

from (1) & (2)

$R_V = W - 2\left[\dfrac{Wa}{l+a}\right]$

$R_V = W\left[\dfrac{l-a}{l+a}\right]$

Question: Three forces acting on a particle are $\vec{P}_1 = (3i + 6j)N$, $\vec{P}_2 = (-1.5i + 4.5j)N$, $\vec{P}_3 = (-10.5i + 1.5j)N$ if a fourth force $\vec{P}_4$ is added such that the point O is in equ^m, then P_4 will be

Sol^n

point in equ^m so

$$\vec{P}_1 + \vec{P}_2 + \vec{P}_3 + \vec{P}_4 = 0$$

$$\vec{P}_4 = -(\vec{P}_1 + \vec{P}_2 + \vec{P}_3)$$

$$= -\{(3 - 1.5 - 10.5)\hat{i} + (4.5 + 1.5)\hat{j}\}$$

$$\vec{P}_4 = 9\hat{i} - 6\hat{j}$$

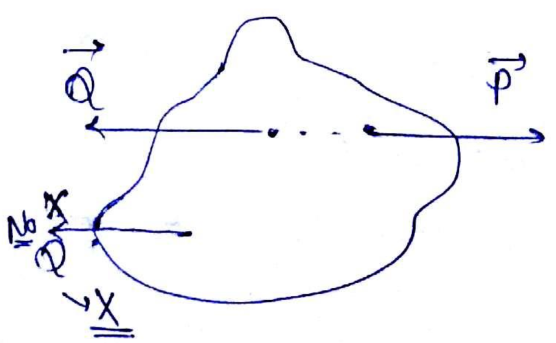
Case-1 Two Force System :-

To keep a body in equ^m under the action of two forces they must be equal in mag., opposite in dir^n, and colinear in action.

$\vec{P}$ & $\vec{Q}$

1. $\vec{P} + \vec{Q} = 0$

$\vec{P} = -\vec{Q}$

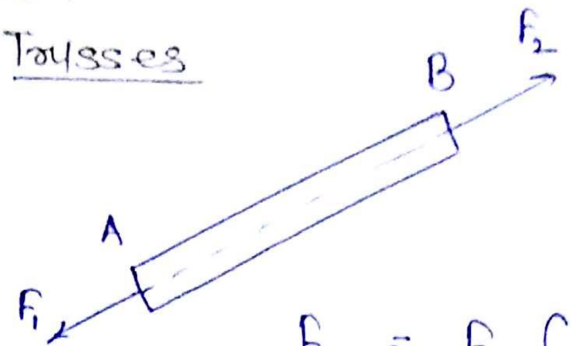


2. $\Sigma \vec{M} = 0$

Collinear

Application

Trusses

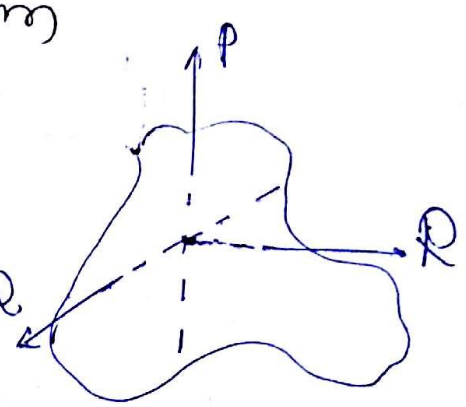


$F_{AB} = F_1$ (tensile)

$F_{CD} = F_2$ (compressive)

Case-2 Three force system

$\vec{P}, \vec{Q}$ & $\vec{R}$



(i) $\vec{P} + \vec{Q} + \vec{R}$

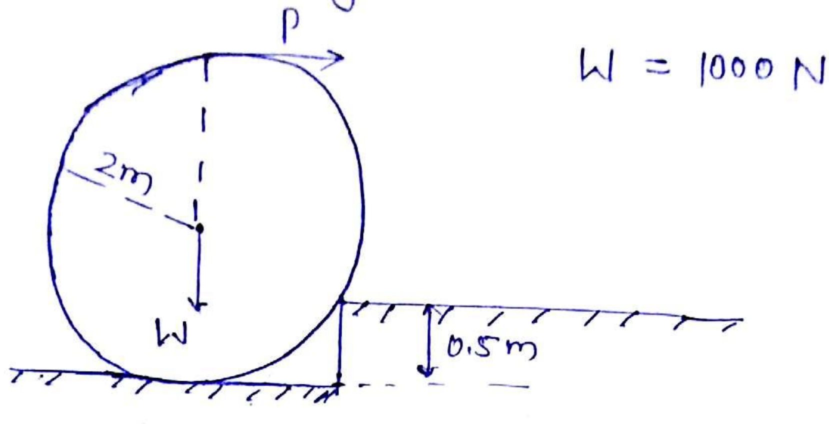
↳ Coplanes

(ii) $\Sigma \vec{M} = 0$

↳ Concurrent

To keep a body in equm they must be coplanner and concurrent

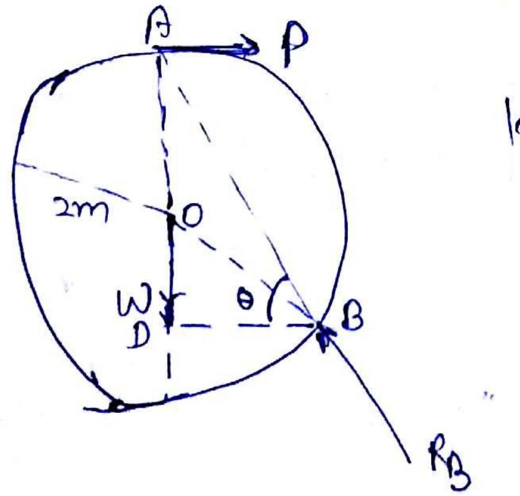
Ques:- Find the horizontal force P require to move the cylinder out of the ditch as shown in fig.



Solⁿ

Note-1 when this cylinder will be about to move out of the stage it will lose its contact at point C, the only contact will be at point B.

To keep cylinder in equᵐ under the action of $\vec{P}$, $\vec{W}$ & $\vec{R_B}$ they must meet at the same point that is at A.



In Δ ODB

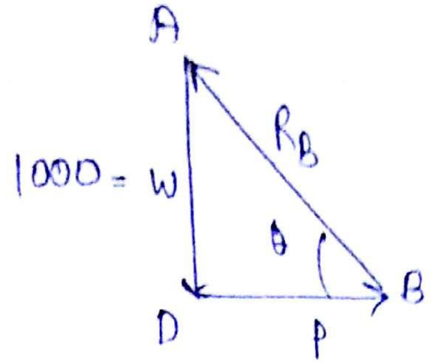
$OD^2 + DB^2 = OB^2$

$DM = \sqrt{2^2 - 1.5^2} = 1.3228$

In Δ ADB

$\tan\theta = \frac{AD}{DB} = \frac{3.5}{1.3228}$

$\theta = 69.29^\circ$



$$\frac{W}{P} = \tan\theta$$

$$P = \frac{1000}{3.5} \times 1.3228$$

$$P = 377.85 \text{ N}$$

$$\frac{W}{R_B} = \sin 69.29^\circ$$

$$R_B = \frac{1000}{\sin 69.29} = 1069.08 \text{ N}$$

OR

$$\Sigma \vec{M}_B = 0$$

$$P \times AD = W \times DB$$

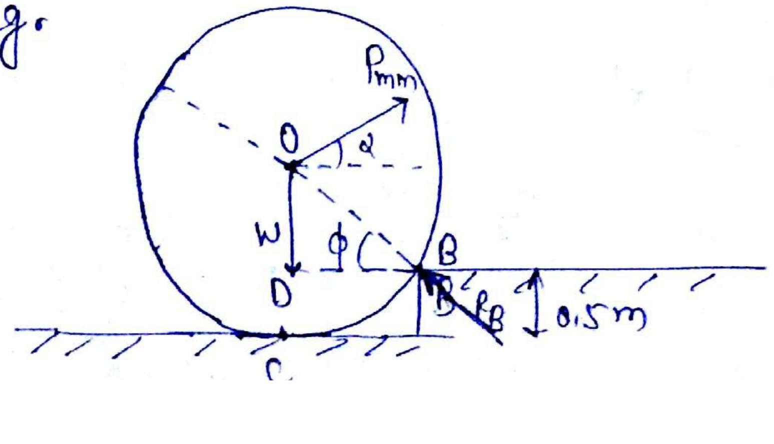
$$P = \frac{1000 \times 1.3228}{3.5}$$

$$P = 377.9 \text{ N}$$

Question:- Find the minimum force P require to move the cylinder out of ditch (out of pit) as shown in fig.

$W = 1000$ N

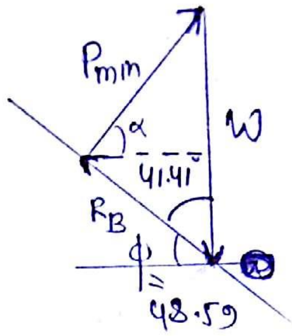
$r = 2$ m



In ΔODB

$$\sin\phi = \frac{OD}{OB} = \frac{1.5}{2}$$

$$\phi = 48.59^\circ$$

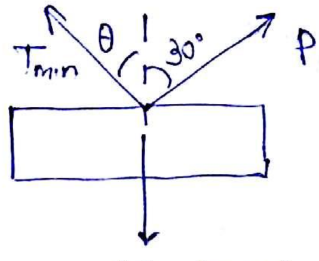


$$\frac{P_{min}}{W} = \sin 41.41^\circ$$

$$\boxed{P_{min} = 661.44\ N}$$

$$\boxed{\alpha = 41.41^\circ}$$

Ques: A block of weight 300N is supported by three forces as shown in figo. For T_{min} to be min. the value of θ is = ?

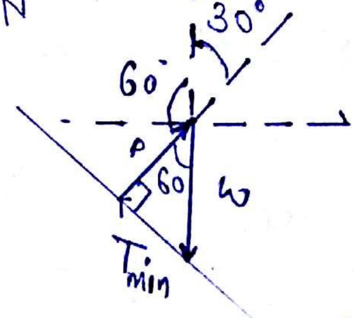


Sol^n

For T_{min}

P should be ⊥ to T

use lemi's thm also

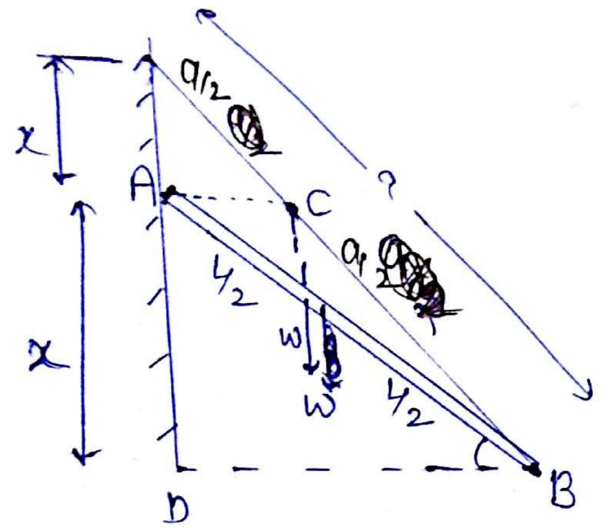


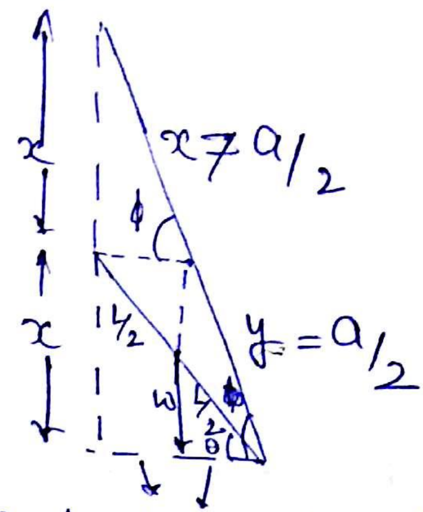
θ = 60°

Question:- A bar AB of length L is supported against the smooth verticle wall and by a wire of length 'a' as shown in fig. For the bar to be in equ^m the value of x is = ?

a) $\sqrt{\frac{l^2-a^2}{3}}$ b) $\sqrt{\frac{l^2+a^2}{3}}$ c) $\sqrt{\frac{a^2-l^2}{3}}$ d) $\frac{l+a}{3}$

Soln





$x\cos\phi = \frac{L}{2}\cos\phi = y\cos\phi$

$x = y = a/2$

$DB = 2AC$

$\sqrt{l^2-x^2} = 2\left(\sqrt{\frac{a^2}{4}-x^2}\right)$

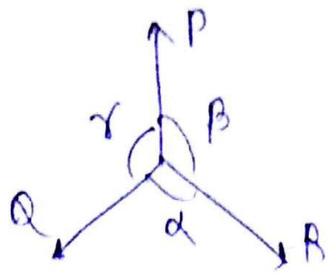
$l^2 - x^2 = 4\left(\frac{a^2}{4} - x^2\right)$

$l^2 - x^2 = a^2 - 4x^2$

$x = \sqrt{\frac{a^2-l^2}{3}}$

Lami's theorem :-

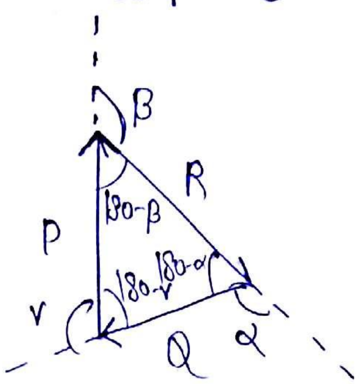
In a particle is in equ^m under $\vec{P}, \vec{Q}$ & $\vec{R}$



in equ^m $\vec{P} + \vec{Q} + \vec{R} = 0$

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

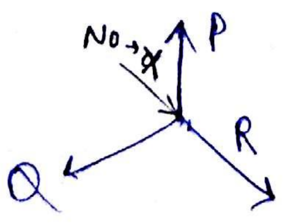
* For a particle to be in equ^m only one condition $\Sigma \vec{F} = 0$ {moment by default zero}

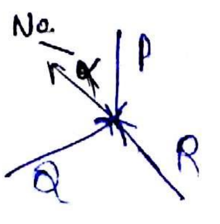


$$\frac{P}{\sin(180-\alpha)} = \frac{Q}{\sin(180-\beta)} = \frac{R}{\sin(180-\gamma)}$$

$$\Rightarrow \boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$$

Limitations :- Converging & diverging only.

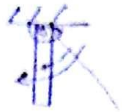




Types of equm :-

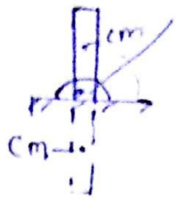
i) Stable equm

V = minimum



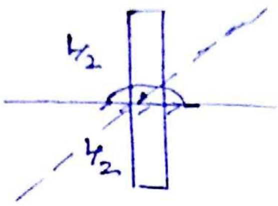
ii) Unstable equm

V = maximum



iii) Neutral equm

V = Constant



total potential Energy = V

→ GPE

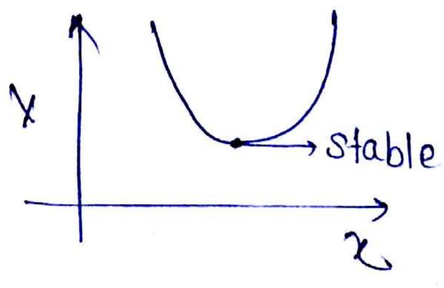
→ Elastic P.E.

Case-1 Single DOF system →

no. of independent = 1 (say x)

V = f(x)

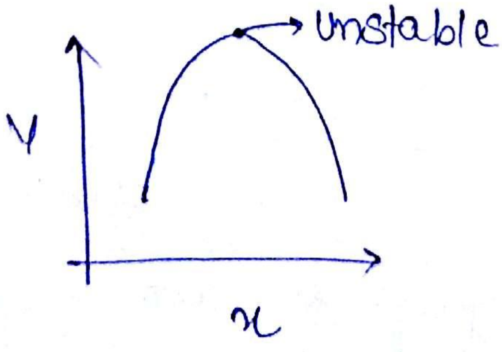
i) Stable



$$\frac{dV}{dx} = 0$$

$$\frac{d^2V}{dx^2} > 0$$

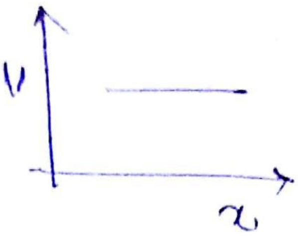
ii) Unstable



$$\frac{dV}{dx} = 0$$

$$\frac{d^2V}{dx^2} < 0$$

iii) Neutral



$$\frac{dV}{dx} = \frac{d^2V}{dx^2} = \frac{d^3V}{dx^3} = \cdots = 0$$

<u>Case-2</u> If multiple DOF system

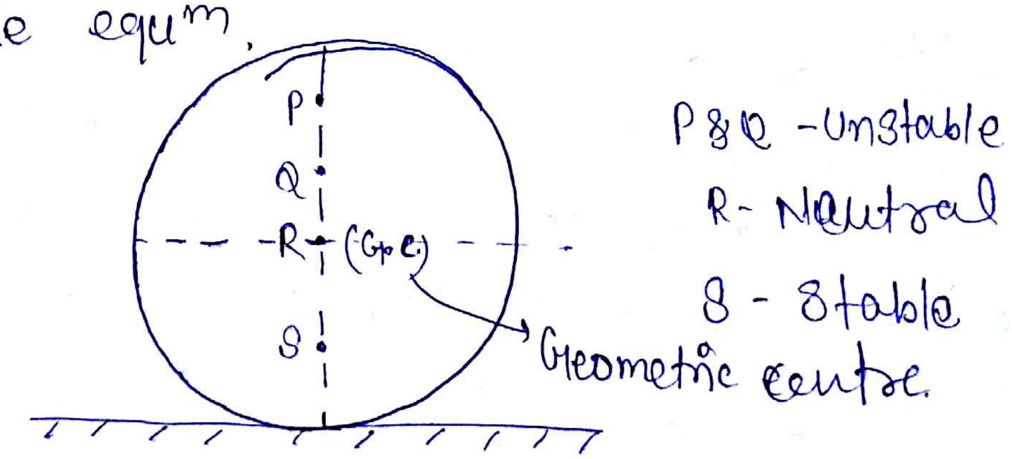
no. of independent variable > 1 (x, y, z)

$V = f(x, y, z)$

⇒ For stable, unstable & Neutral equ^n

$$\frac{\partial V}{\partial x} = \frac{\partial V}{\partial y} = \frac{\partial z}{\partial z} \cdots\cdots = 0$$

<u>Question:</u>– For the sphere shown in Fig the C.G. should lie at what point to keep the sphere in stable equ^m.



Total potential energy at S is minimum

S - Stable.

7

(i) Stable

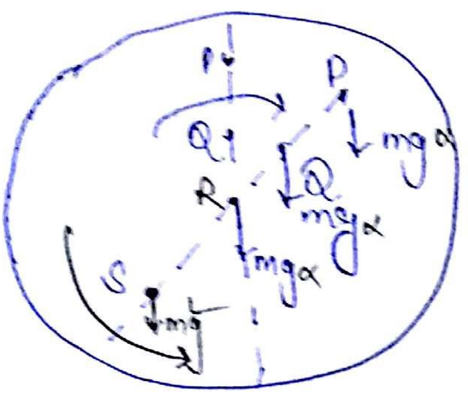
Restoring moments > Distrubing moments

(ii) Unstable

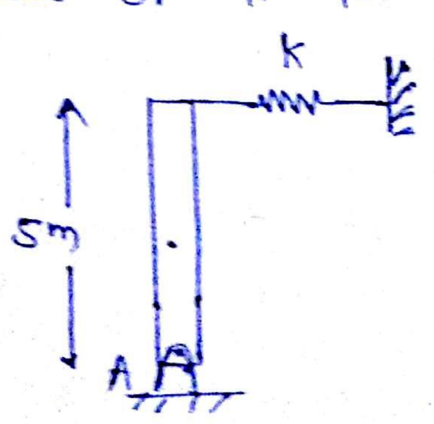
R.M. < D.M.

(iii) ~~Stab~~ Neutral

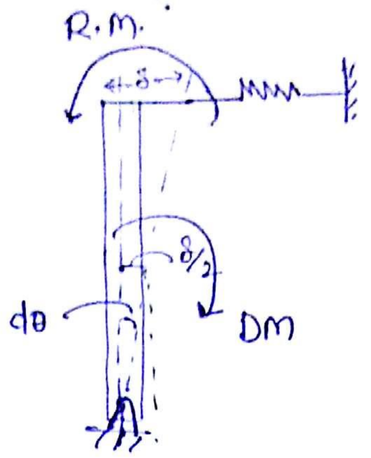
R.M. = D.M.



Ques. A column of 6KN and length 5 m is hinged at point A and supported by a spring of stiffness (K). As shown in fig. The value of k to keep the system in equm



Sol^n



For stable equ^m

R.M. > D.M.

$$(k\delta)5 > 6 \times \frac{\delta}{2}$$

$$k > 0.6 \text{ kN/m}$$

if $k < 0.6$ kN/m → unstable

$k = 0.6$ kN/m → Neutra.

Ques for the system as shown below the value of k to keep it in stable equ^m is ?

(H.W.)