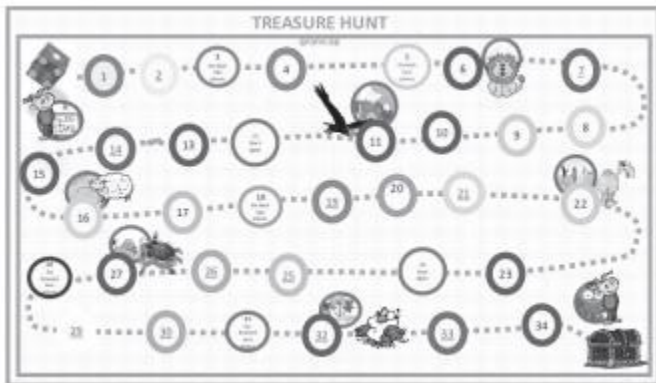


Arithmetic Progressions

Case Study Based Questions

Case Study 1

While playing a treasure hunt game, some clues (numbers) are hidden in various spots collectively forms an AP. If the number on the n th spot is $20 + 4n$, then answer the following questions to help the player in spotting the clues.



Based on the given information, solve the following questions:

Q1. Which number is on the first spot?

- a. 20
- b. 24
- c. 16
- d. 28

Q2. The common difference of the given AP, is:

- a. 20
- b. 4
- c. 16
- d. 24

Q3. Which number is on the $(n-2)$ th spot?

- a. $16+4n$
- b. $24+ 4n$
- c. $12 + 4n$
- d. $28 + 4n$

Q4. What is the sum of all the numbers on the first 10 spots?

- a. 410
- b. 420
- c. 480
- d. 430

Q5. Which spot is numbered as 116?

- a. 5th
- b. 8th
- c. 9th
- d. 24th

Solutions

1. Given, number on nth spot = $20 + 4n$

i.e.. $a_n = 20 + 4n$

.. Number on first spot, $a_1 = 20 + 4(1)$
 $= 20 + 4 = 24$

So, option (b) is correct.

2. The common difference (d) = $a_n - a_{n-1}$

$\Rightarrow d = 20 + 4n - \{20 + (n-1)\}$ [$a_n = 20 + 4n$]

$= 20 + 4n - 20 - 4n + 4 = 4$

So, option (b) is correct.

3. The number on (n-2)th spot,

$a_{n-2} = 20 + 4(n-2)$ ($a_n = 20 + 4n$)

$= 20 + 4n - 8$

$= 12 + 4n$

So, option (c) is correct.

4. The sum of first n term of an AP

$$S_n = \frac{n}{2} \{2a + (n-1)d\}$$

\therefore The sum of all the numbers on the first 10 spots,

$$\begin{aligned}
 S_{10} &= \frac{10}{2} \{2 \times 24 + (10 - 1) \times 4\} \\
 &= 5 (48 + 36) \\
 &= 5 \times 84 = 420
 \end{aligned}$$

So, option (b) is correct.

5. (d) Let nth spot is numbered as 116.

$$a_n = 116 \Rightarrow 20 + 4n = 116$$

$$4n = 96 \Rightarrow n = 24.$$

\therefore 24th spot is numbered as 116.

So, option (d) is correct.

Case Study 2

Your younger sister wants to buy an electric car and plans to take loan from a bank for her electric car. She repays her total loan of 321600 by paying every month starting with the first instalment of Rs 2000 and it increases the instalment by Rs 200 every month.



Based on the above information, solve the following questions:

Q1. Find the list of the instalment formed by the given statement.

- a. 2000, 1800, 1600....
- b. 2000, 2200, 2400....
- c. 2200, 2400, 2600....
- d. 2300, 2600, 2900,...

Q 2. The amount paid by her in 25th instalment is:

- a. 6800
- b. 3500
- c. 4800
- d. 6600

Q3. Find the difference of the amount in 4th and 6th instalment paid by younger sister.

- a. Rs200
- b. Rs400
- c. Rs600
- d. Rs800

Q4. In how many instalment, she clear her total bank loan?

- a. 1582
- b. 1580
- c. 1599
- d. 1600

Q5. Find the sum of the first seven instalments.

- a. 14000
- b. 13600
- c. 10400
- d. 12600

Solutions

1. It can be observed that these instalments are in AP having first term (instalment) as 2000 and common difference (increase instalment) as ₹200. Here, $a=2000$ and $d=200$

Therefore list of an AP is $a, a + d, a + 2d, \dots$

i.e., 2000, $2000 + 200$, $2000 + 2 \times 200$, ...

i.e., 2000, 2200, 2400, ...

So, option (b) is correct.

2. It can be observed that these instalments are in an AP having first term (instalment) as Rs2000 and common difference (increase instalment) as Rs200. Here, $a = 2000$ and $d = 200$

.. The amount paid by her in 25th instalment is

$$T_{25} = a + (25-1) d$$

$$= 2000 + 24 \times 200$$

$$= 2000 + 4800 = 6800$$

So, option (a) is correct.

3. Let a and d be the first term and common difference of an AP.

Then, $a_4 = a + (4-1)d$ [:- $a_n = a + (n-1)d$]

$$= a + 3d$$

Similarly, $a_6 = a + 5d$.

.. Required difference = $a_6 - a_4$

$$= (a + 5d) - (a + 3d) = 2d$$

$$= 2 \times 200 = 400$$

So, option (b) is correct.

4. Let in n instalments, she clear her loan.

Given,

$$T_n = 321600$$

$$T_n = a + (n-1)d$$

$$.. 321600 = 2000 + (n-1)200$$

$$\Rightarrow 319600 = (n-1)200$$

$$\Rightarrow 1598 = n-1$$

$$\Rightarrow n = 1599$$

So, in 1599 instalments, she clear her bank loan.

So, option (c) is correct.

5. Here, $a = 1200$, $d = 200$

.. The sum of first seven instalments is

$$S_7 = \frac{7}{2} [2 \times 1200 + (7-1)200]$$

$$\left[\because S_n = \frac{n}{2} [2a + (n-1)d] \right]$$

$$= \frac{7}{2} [2400 + 1200]$$

$$= \frac{7}{2} [3600] = 7 \times 1800 = ₹ 12600$$

So, option (d) is correct.

Case Study 3

In an examination hall, the examiner makes students sit in such a way that no students can cheat from other student and make no student sit uncomfortably. So, the teacher

decides to mark the numbers on each chair from 1, 2, 3, There are 25 students and each student is seated at alternate position in examination room such that the sequence formed is 1, 3, 5,...



Based on the given information, solve the following questions:

Q1. What type of sequence is formed, to follow the seating arrangement of students in the examination room?

Q2. Find the seat number of the last student in the examination room.

Q3. Find the seat number of 10th vacant seat in the examination room.

OR

Which number of student will seat on the 27th seat number.

Solutions

1. Given, seating arrangement of students in the examination room is 1, 3, 5,.

Here, $a_1 = 1, a_2 = 3, a_3 = 5, \dots$

Now, $a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 5 - 3 = 2$

Here, common difference is same, so given sequence is the type of Arithmetic progression.

2. Given, $a=1, d=3-1=2$ and $n= 25$

$T_n = a + (n-1)d$

There are 25 students.

$$\therefore T_{25} = 1 + (25-1)2 = 1 + 24 \times 2 = 49$$

Hence, last student will sit on the 49th seat number.

3. The sequence of vacant seats are as follows,

2, 4, 6..... 48.

Here, $a = 2$, $d = 4 - 2 = 2$

The 10th vacant seat will be

$$T_{10} = a + (10-1)d \quad (:- T_n = a + (n-1)d)$$

$$= 2 + 9 \times 2 = 2 + 18 = 20$$

Hence, the 10th vacant seat number is 20.

OR

The sequence of seating arrangement of students in the examination room is

1, 3, 5, 7,

Let nth number of student will seat on 27th seat number.

Here,

$$a = 1, d = 3 - 1 = 2 \text{ and } T_n = 27$$

$$:- T_n = a + (n-1)d$$

$$:- 27 = 1 + (n-1)(2)$$

$$\Rightarrow n - 1 = \frac{26}{2}$$

$$\Rightarrow n = 13 + 1 = 14$$

Case Study 4

In a pathology lab, a culture test has been conducted. In the test, the number of bacteria taken into consideration in various samples is all 3-digit numbers that are divisible by 7, taken in order.



Based on the above information, solve the following questions:

Q1. How many bacteria are considered in the seventh sample?

Q2. How many samples should be taken into consideration?

Q3. Find the total number of bacteria in the first 15 samples.

OR

Find the number of samples in which sum of bacteria is 840.

Solutions

1. Here the smallest 3-digit number divisible by 6 is 102 and the largest 3-digit number divisible by 6 is 996. So, the number of bacteria taken into consideration is 102, 108, 114, 996.

Here, first term (a) = 102, common difference (d)

$$= 108 - 102 = 6$$

and last term (a_n) = 996.

$$\therefore \text{Seventh sample, } a_7 = a + (7-1)d \quad [a_n = a + (n-1)d]$$

$$= 102 + 6 \times 6$$

$$= 102 + 36 = 138$$

2. Let n samples be taken under consideration.

Here, last term (a_n) = 996.

$$\therefore a_n = a + (n-1)d$$

$$= 996 \quad 102 + (n-1)6$$

$$\Rightarrow \quad n-1 = \frac{894}{6} = 149 \quad \Rightarrow \quad n = 150$$

3. We know that,

$$\text{Sum of first } n \text{ terms of the AP } S_n = \frac{n}{2} [2a + (n-1)d]$$

\therefore Total number of bacteria in first 15 samples.

$$S_{15} = \frac{15}{2} \{2 \times 102 + (15-1) \times 6\}$$

$$= 15 (102 + 14 \times 3) = 15 \times 144 = 2160$$

OR

Let n number of samples should be taken in which sum of bacteria is 840.

Here, $a = 102$, $d = 108 - 102 = 6$

and $5n = 840$.

$$\therefore S_n = \frac{n}{2} \{2a + (n-1)d\}$$

$$\therefore 840 = \frac{n}{2} \{2 \times 102 + (n-1)6\}$$

$$\Rightarrow 840 = n(102 + 3n - 3)$$

$$\Rightarrow 3n^2 + 99n - 840 = 0$$

$$\Rightarrow n^2 + 33n - 280 = 0$$

$$\Rightarrow n^2 + 40n - 7n - 280 = 0$$

$$\Rightarrow n(n + 40) - 7(n + 40) = 0$$

$$\Rightarrow (n + 40)(n - 7) = 0$$

$$n = -40, 7$$

But samples cannot be negative.

$$\therefore n = 7$$

So, required number of sample is 7.

Case Study 5

India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.



Based on the above information, solve the following questions: [CBSE SQP 2023-24]

Q1. In which year, the production is Rs 29,200.

Q2. Find the difference of the production during 7th year and 4th year.

Q3. Find the production during 8th year.

OR

Find the production during first 3 years.

Solutions

Given that, the production of TV sets in a factory increases uniformly by a fixed number every year i.e., production of TV sets in every year form an AP. Let the first term and common difference of this AP be 'a' and 'd' respectively. According to the question, the factory produced 16000 sets in 6th year and 22600 in 9th year.

$$\text{i.e.. } a_6 = a + (6-1)d = 16000$$

$$\Rightarrow a + 5d = 16000 \dots (1)$$

$$\text{and } a_9 = a + (9-1)d = 22600$$

$$\Rightarrow a + 8d = 22600 \dots (2)$$

Subtract eq (1) from eq (2), we get

$$(a + 8d) - (a + 5d) = 22600 - 16000$$

$$\Rightarrow 3d = 6600$$

$$\Rightarrow d = \frac{6600}{3} = 2200$$

put the value of 'd' in eq. (1), we get

$$a + 5(2200) = 16000$$

$$\Rightarrow a = 16000 - 11000$$

$$\Rightarrow a = 5000.$$

1. Let n th year, the production is,

$$a_n = 29,200$$

$$\Rightarrow a + (n-1)d = 29200$$

$$\Rightarrow 5000 + (n-1)(2200) = 29200$$

$$\Rightarrow (n-1) = \frac{24200}{2200} = 11$$

$$\therefore n = 11 + 1 = 12$$

Hence, in 12th year, the production is * 29,200.

2. Now, the production during 7th year is

$$a_7 = a + (7-1)d$$

$$= 5000 + 6(2200)$$

$$= 5000 + 13200 = 18200$$

and the production during 4th year is

$$a_4 = a + (4-1)d$$

$$= 5000 + 3(2200)$$

$$= 5000 + 6600 = 11600.$$

∴ The difference of the production during 7th year and 4th year = $18200 - 11600 - 6600$.

3. The production during 8th year is

$$a_8 = a + (8-1)d$$

$$= 5000 + 7 \times 2200$$

$$= 5000 + 15400$$

$$= 5000 + 15400$$

$$= 20400.$$

OR

The production during first 3 years is

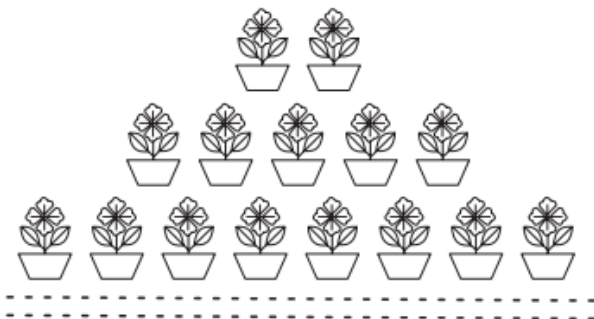
$$S_3 = \frac{3}{2} [2 \times 5000 + (3-1)(2200)]$$

$$= 3 (5000 + 2200)$$

$$= 3 \times 7200 = 21600.$$

Case Study 6

Aahana being a plant lover decides to convert her balcony into beautiful garden full of plants. She bought few plants with pots for her balcony. She placed the pots in such a way that number of pots in the first row is 2, second row is 5, third row is 8 and so on.



Based on the above information, solve the following questions: [CBSE 2023]

Q1. Find the number of pots placed in the 10th row.

Q2. Find the difference in the number of pots placed in 5th row and 2nd row.

Q3. If Aahana wants to place 100 pots in total, then find the total number of rows formed in the arrangement.

OR

If Aahana has sufficient space for 12 rows, then how many total number of pots are placed by her with the same arrangement?

Solutions

Given that, Aahana placed the pots in such a way that number of pots in the first row is 2 second row is 5, third row is 8 and so on. Such a pattern form a sequence:

2,5,8,...

Here, $5-2=8-5=3$ (Constant)

So, the above sequence form an AP.

Let the first term and common difference of this AP.

be 'a' and 'd' respectively.

Here,

$a=2$ and $d=3$

1. The number of pots placed in the 10th row is,

$$a_{10} = a + (10-1)d$$

$$= 2 + 9(3)$$

$$= 2 + 27 = 29.$$

2. The number of pots placed in 5th row is,

$$a_5 = a + (5-1)d$$

$$= 2 + 4 \times 3 = 2 + 12 = 14.$$

and the number of pots placed in 2nd row is,

$$a_2 = a + (2-1)d$$

$$= a + d = 2 + 3 = 5.$$

.. The difference in the number of pots placed in 5th row and 2nd row

$$=14-5=9.$$

3. Let 'n' number of rows formed in the arrangement

Here, $S_n=100$, $a=2$ and $d=3$

$$\therefore 100 = \frac{n}{2} \{2 \times 2 + (n-1)(3)\}$$

$$\Rightarrow 200 = n(4 + 3n - 3)$$

$$\Rightarrow 3n^2 + n - 200 = 0$$

$$\Rightarrow 3n^2 + 25n - 24n - 200 = 0$$

(By splitting the middle terms)

$$\Rightarrow n(3n + 25) - 8(3n + 25) = 0$$

$$\Rightarrow (3n + 25)(n - 8) = 0$$

$$\Rightarrow 3n + 25 = 0 \quad \text{or} \quad n - 8 = 0$$

$$\Rightarrow n = \frac{-25}{3}, 8.$$

But n cannot be negative because n is a natural number.

$$\therefore n = 8$$

So, required number of rows is 8.

OR

Given, $n = 12$, $a = 2$ and $d = 3$

$$\therefore S_n = \frac{n}{2} \{2a + (n-1)d\}$$

$$\therefore S_{12} = \frac{12}{2} \{2 \times 2 + (12-1)3\}$$

$$= 6(4 + 33) = 222$$

So, the total number of pots are 222, placed by her with the same arrangement.

Solutions for Questions 6 to 15 are Given Below

Case Study 6

Pathology Lab

In a pathology lab, a culture test has been conducted. In the test, the number of bacteria taken into consideration in various samples is all 3-digit numbers that are divisible by 7, taken in order.



On the basis of above information, answer the following questions.

- (i) How many bacteria are considered in the fifth sample?
(a) 126 (b) 140 (c) 133 (d) 149
- (ii) How many samples should be taken into consideration?
(a) 129 (b) 128 (c) 130 (d) 127
- (iii) Find the total number of bacteria in the first 10 samples.
(a) 1365 (b) 1335 (c) 1302 (d) 1540
- (iv) How many bacteria are there in the 7th sample from the last?
(a) 952 (b) 945 (c) 959 (d) 966
- (v) The number of bacteria in 50th sample is
(a) 546 (b) 553 (c) 448 (d) 496

Case Study 7

Discussion on A.P.

In a class the teacher asks every student to write an example of A.P. Two friends Geeta and Madhuri writes their progressions as $-5, -2, 1, 4, \dots$ and $187, 184, 181, \dots$ respectively. Now, the teacher asks various students of the class the following questions on these two progressions. Help students to find the answers of the questions.



- (i) Find the 34th term of the progression written by Madhuri.
 (a) 286 (b) 88 (c) -99 (d) 190
- (ii) Find the sum of common difference of the two progressions.
 (a) 6 (b) -6 (c) 1 (d) 0
- (iii) Find the 19th term of the progression written by Geeta.
 (a) 49 (b) 59 (c) 52 (d) 62
- (iv) Find the sum of first 10 terms of the progression written by Geeta.
 (a) 85 (b) 95 (c) 110 (d) 200
- (v) Which term of the two progressions will have the same value?
 (a) 31 (b) 33 (c) 32 (d) 30

Case Study 8

Shoe Shop

Meena's mother start a new shoe shop. To display the shoes, she put 3 pairs of shoes in 1st row, 5 pairs in 2nd row, 7 pairs in 3rd row and so on.



On the basis of above information, answer the following questions.

- (i) If she puts a total of 120 pairs of shoes, then the number of rows required are
 (a) 5 (b) 6 (c) 7 (d) 10
- (ii) Difference of pairs of shoes in 17th row and 10th row is
 (a) 7 (b) 14 (c) 21 (d) 28
- (iii) On next day, she arranges x pairs of shoes in 15 rows, then $x =$
 (a) 21 (b) 26 (c) 31 (d) 42

(iv) Find the pairs of shoes in 30th row.

- (a) 61 (b) 67 (c) 56 (d) 59

(v) The total number of pairs of shoes in 5th and 8th row is

- (a) 7 (b) 14 (c) 28 (d) 56

Case Study 9

Pocket Money

Anuj gets pocket money from his father everyday. Out of the pocket money, he saves ₹ 2.75 on first day, ₹ 3 on second day, ₹ 3.25 on third day and so on.



On the basis of above information, answer the following questions.

(i) What is the amount saved by Anuj on 14th day?

- (a) ₹ 6.25 (b) ₹ 6 (c) ₹ 6.50 (d) ₹ 6.75

(ii) What is the total amount saved by Anuj in 8 days?

- (a) ₹ 18 (b) ₹ 33 (c) ₹ 24 (d) ₹ 29

(iii) What is the amount saved by Anuj on 30th day?

- (a) ₹ 10 (b) ₹ 12.75 (c) ₹ 10.25 (d) ₹ 9.75

(iv) What is the total amount saved by him in the month of June, if he starts savings from 1st June?

- (a) ₹ 191 (b) ₹ 191.25 (c) ₹ 192 (d) ₹ 192.5

(v) On which day, he save tens times as much as he saved on day-1?

- (a) 9th (b) 99th (c) 10th (d) 100th

Case Study 10

Board Game

In a board game, the number of sea shells in various cells forms an A.P. If the number of sea shells in the 3rd and 11th cell together is 68 and number of shells in 11th cell is 24 more than that of 3rd cell, then answer the following questions based on this data.

(i) What is the difference between the number of sea shells in the 19th and 20th cells?

- (a) 2 (b) 3 (c) 8 (d) 7

(ii) How many sea shells are there in the first cell?

- (a) 52 (b) 18 (c) 16 (d) 54

(iii) How many total sea shells are there in first 13 cells?

(a) 442

(b) 221

(c) 204

(d) Can't be determined

(iv) Altogether, how many sea shells are there in the first 5 cells?

(a) 220

(b) 125

(c) 96

(d) 110

(v) What is the sum of number of sea shells in the 7th and 9th cell?

(a) 42

(b) 32

(c) 74

(d) 80

Case Study 11

Number Cards Game

Amit was playing a number card game. In the game, some number cards (having both +ve or -ve numbers) are arranged in a row such that they are following an arithmetic progression. On his first turn, Amit picks up 6th and 14th card and finds their sum to be -76. On the second turn he picks up 8th and 16th card and finds their sum to be -96.

Based on the above information, answer the following questions.

(i) What is the difference between the numbers on any two consecutive cards?

(a) 7

(b) -5

(c) 11

(d) -3

(ii) The number on first card is,

(a) 12

(b) 3

(c) 5

(d) 7

(iii) What is the number on the 19th card?

(a) -88

(b) -83

(c) -92

(d) -102

(iv) What is the number on the 23rd card?

(a) -103

(b) -122

(c) -108

(d) -117

(v) The sum of numbers on the first 15 cards is

(a) -840

(b) -945

(c) -427

(d) -420



Case Study 12

Treasure Hunt Game

While playing a treasure hunt game, some clues (numbers) are hidden in various spots collectively forms an A.P. If the number on the n^{th} spot is $20 + 4n$, then answer the following questions to help the player in spotting the clues.



- (i) Which number is on the first spot?
 (a) 20 (b) 24 (c) 16 (d) 28
- (ii) Which number is on the $(n - 2)^{\text{th}}$ spot?
 (a) $16 + 4n$ (b) $24 + 4n$ (c) $12 + 4n$ (d) $28 + 4n$
- (iii) Which number is on the 34^{th} spot?
 (a) 156 (b) 116 (c) 120 (d) 160
- (iv) What is the sum of all the numbers on the first 10 spots?
 (a) 410 (b) 420 (c) 480 (d) 410
- (v) Which spot is numbered as 116?
 (a) 5^{th} (b) 8^{th} (c) 9^{th} (d) 24^{th}

Case Study 13

A sequence is an ordered list of numbers. A sequence of numbers such that the difference between the consecutive terms is constant is said to be an arithmetic progression (A.P.).

On the basis of above information, answer the following questions.

- (i) Which of the following sequence is an A.P.?
 (a) 10, 24, 39, 52, (b) 11, 24, 39, 52, ... (c) 10, 24, 38, 52, ... (d) 10, 38, 52, 66,
- (ii) If x, y and z are in A.P., then
 (a) $x + z = y$ (b) $x - z = y$ (c) $x + z = 2y$ (d) None of these
- (iii) If $a_1, a_2, a_3, \dots, a_n$ are in A.P., then which of the following is true?
 (a) $a_1 + k, a_2 + k, a_3 + k, \dots, a_n + k$ are in A.P., where k is a constant.
 (b) $k - a_1, k - a_2, k - a_3, \dots, k - a_n$ are in A.P., where k is a constant.
 (c) $ka_1, ka_2, ka_3, \dots, ka_n$ are in A.P., where k is a constant.
 (d) All of these
- (iv) If the n^{th} term ($n > 1$) of an A.P. is smaller than the first term, then nature of its common difference (d) is
 (a) $d > 0$ (b) $d < 0$
 (c) $d = 0$ (d) Can't be determined
- (v) Which of the following is incorrect about A.P.?
 (a) All the terms of constant A.P. are same.
 (b) Some terms of an A.P. can be negative.
 (c) All the terms of an A.P. can never be negative.
 (d) None of these.

Case Study 14

Assessment on A.P.

Jack is much worried about his upcoming assessment on A.P. He was vigorously practicing for the exam but unable to solve some questions. One of these questions is as shown below.

If the 3^{rd} and the 9^{th} terms of an A.P. are 4 and -8 respectively, then help Jack in solving the problem.



- (i) What is the common difference?
 (a) 2 (b) -1 (c) -2 (d) 4
- (ii) What is the first term?
 (a) 6 (b) 2 (c) -2 (d) 8
- (iii) Which term of the A.P. is -160?
 (a) 80^{th} (b) 85^{th} (c) 81^{th} (d) 84^{th}
- (iv) Which of the following is not a term of the given A.P.?
 (a) -123 (b) -100 (c) 0 (d) -200
- (v) What is the 75^{th} term of the A.P.?
 (a) -140 (b) -102 (c) -150 (d) -158

Case Study 15

Application of A.P. in Day to Day Life

Do you know, we can find A.P. in many situations in our day-to-day life. One such example is a tissue paper roll, in which the first term is the diameter of the core of the roll and twice the thickness of the paper is the common difference. If the sum of first n rolls of tissue on a roll is $S_n = 0.1n^2 + 7.9n$, then answer the following questions.



- (i) Find S_{n-1} .
 (a) $0.1n^2 - 0.2n - 7.8$ (b) $0.1n^2 - 7.9n$
 (c) $0.1n^2 + 7.7n - 7.8$ (d) None of these
- (ii) Find the radius of the core.
 (a) 8 cm (b) 4 cm
 (c) 16 cm (d) Can't be determined
- (iii) $S_2 =$
 (a) 16.2 (b) 8.2 (c) 2.8 (d) 4.8
- (iv) What is the diameter of roll when one tissue sheet is rolled over it?
 (a) 7.6 cm (b) 7.9 cm (c) 8.1 cm (d) 8.2 cm
- (v) Find the thickness of each tissue sheet.
 (a) 2 cm (b) 1 cm (c) 1 mm (d) 2 mm

HINTS & EXPLANATIONS

6. Here the smallest 3-digit number divisible by 7 is 105. So, the number of bacteria taken into consideration is 105, 112, 119, ..., 994

So, first term (a) = 105, $d = 7$ and last term = 994

(i) (c): $t_5 = a + 4d = 105 + 28 = 133$

(ii) (b): Let n samples be taken under consideration.

\therefore Last term = 994

$$\Rightarrow a + (n-1)d = 994 \Rightarrow 105 + (n-1)7 = 994 \Rightarrow n = 128$$

(iii) (a): Total number of bacteria in first 10 samples

$$= S_{10} = \frac{10}{2}[2(105) + 9(7)] = 1365$$

(iv) (a): t_7 from end = $(128 - 7 + 1)^{\text{th}}$ term from beginning = 122^{th} term = $105 + 121(7) = 952$

(v) (c): $t_{50} = 105 + 49 \times 7 = 448$

7. Geeta's A.P. is -5, -2, 1, 4, ...

Here, first term (a_1) = -5 and common difference (d_1) = -2 + 5 = 3

Similarly, Madhuri's A.P. is 187, 184, 181, ...

Here first term (a_2) = 187 and common difference (d_2) = 184 - 187 = -3

(i) (b): $t_{34} = a_2 + 33d_2 = 187 + 33(-3) = 88$

(ii) (d): Required sum = $3 + (-3) = 0$

(iii) (a): $t_{19} = a_1 + 18d_1 = (-5) + 18(3) = 49$

(iv) (a): $S_{10} = \frac{n}{2}[2a_1 + (n-1)d_1] = \frac{10}{2}[2(-5) + 9(3)] = 85$

(v) (b): Let n^{th} terms of the two A.P.'s be equal.

$$\therefore -5 + (n-1)3 = 187 + (n-1)(-3)$$

$$\Rightarrow 6(n-1) = 192 \Rightarrow n = 33$$

8. Number of pairs of shoes in 1st, 2nd, 3rd row, ... are 3, 5, 7, ...

So, it forms an A.P. with first term $a = 3$, $d = 5 - 3 = 2$

(i) (d): Let n be the number of rows required.

$$\therefore S_n = 120$$

$$\Rightarrow \frac{n}{2}[2(3) + (n-1)2] = 120$$

$$\Rightarrow n^2 + 2n - 120 = 0 \Rightarrow n^2 + 12n - 10n - 120 = 0$$

$$\Rightarrow (n+12)(n-10) = 0 \Rightarrow n = 10$$

So, 10 rows required to put 120 pairs.

(ii) (b): No. of pairs in 17th row = $t_{17} = 3 + 16(2) = 35$

No. of pairs in 10th row = $t_{10} = 3 + 9(2) = 21$

$$\therefore \text{Required difference} = 35 - 21 = 14$$

(iii) (c): Here $n = 15$

$$\therefore t_{15} = 3 + 14(2) = 3 + 28 = 31$$

(iv) (a): No. of pairs in 30th row = $t_{30} = 3 + 29(2) = 61$

(v) (c): No. of pairs in 5th row = $t_5 = 3 + 4(2) = 11$

No. of pairs in 8th row = $t_8 = 3 + 7(2) = 17$

$$\therefore \text{Required sum} = 11 + 17 = 28$$

9. Here the savings form an A.P. i.e., ₹ 2.75, ₹ 3, ₹ 3.25, ...

$$\text{So, } a = 2.75, d = 3 - 2.75 = 0.25$$

(i) (b): Amount saved by Anuj on 14th day

$$= t_{14} = a + 13d = 2.75 + 13(0.25) = ₹ 6$$

(ii) (d): Total amount saved by Anuj in 8 days

$$= S_8 = \frac{8}{2}[2(2.75) + 7(0.25)] = ₹ 29$$

(iii) (a): Amount saved by Anuj on 30th day

$$= t_{30} = a + 29d = 2.75 + 29(0.25) = ₹ 10$$

(iv) (b): Number of days in June = 30

$$\therefore S_{30} = \frac{30}{2}[2(2.75) + 29(0.25)] = ₹ 191.25$$

(v) (d): Let on n^{th} day, he saves 10 times as he saves on 1st day.

$$t_n = 10(2.75) \Rightarrow a + (n-1)d = 27.5 \Rightarrow n = 100$$

10. Let the number of sea shells in the cells be of the form $a, a + d, a + 2d, \dots$

According to question, we have

$$(a + 2d) + (a + 10d) = 68$$

$$\Rightarrow 2a + 12d = 68 \Rightarrow a + 6d = 34 \quad \dots(1)$$

$$\text{Also, } (a + 10d) - (a + 2d) = 24 \Rightarrow d = 3$$

$$\text{From (1), we get } a + 6(3) = 34 \Rightarrow a = 16$$

(i) (b): Required difference, $d = 3$

(ii) (c): Number of sea shells in the first cell (a) = 16

(iii) (a): Total number of sea shells in 13 cells = S_{13}

$$= \frac{13}{2}[2(16) + 12(3)] = 6.5(68) = 442$$

(iv) (d): $S_5 = \frac{5}{2}[2(16) + 4(3)] = 110$

(v) (c): Required sum = $t_7 + t_9 = (a + 6d) + (a + 8d)$
 $= 2(16) + 14(3) = 74$

11. Let the numbers on the cards be $a, a + d, a + 2d, \dots$
 According to question, We have $(a + 5d) + (a + 13d) = -76$

$$\Rightarrow 2a + 18d = -76 \Rightarrow a + 9d = -38 \quad \dots(1)$$

$$\text{And } (a + 7d) + (a + 15d) = -96$$

$$\Rightarrow 2a + 22d = -96 \Rightarrow a + 11d = -48 \quad \dots(2)$$

From (1) and (2), we get

$$2d = -10 \Rightarrow d = -5$$

$$\text{From (1), } a + 9(-5) = -38 \Rightarrow a = 7$$

(i) (b): The difference between the numbers on any two consecutive cards = common difference of the A.P. = -5

(ii) (d): Number on first card = $a = 7$

(iii) (b): Number on 19th card = $a + 18d = 7 + 18(-5) = -83$

(iv) (a): Number on 23rd card = $a + 22d = 7 + 22(-5) = -103$

$$(v) (d): S_{15} = \frac{15}{2}[2(7) + 14(-5)] = -420$$

12. Number on n^{th} spot = $20 + 4n$
i.e., $t_n = 20 + 4n$

(i) (b): Number on 1st spot = $t_1 = 20 + 4(1) = 24$

(ii) (c): Number on $(n - 2)^{\text{th}}$ spot = t_{n-2}
= $20 + 4(n - 2)$
= $20 + 4n - 8 = 12 + 4n$

(iii) (a): Number on 34th spot = t_{34}
= $20 + 4(34) = 156$

(iv) (b): Here $a = t_1 = 24$
Now, $t_2 = 20 + 4(2) = 20 + 8 = 28$

$$\therefore d = t_2 - t_1 = 4$$

$$\text{So, required sum} = S_{10} = \frac{10}{2}[2(24) + 9(4)] = 420$$

(v) (d): Let n^{th} spot is numbered as 116.

$$\therefore t_n = 116$$

$$\Rightarrow 20 + 4n = 116 \Rightarrow 4n = 96 \Rightarrow n = 24$$

13. (i) (c) (ii) (c) (iii) (d)

(iv) (b) (v) (c)

14. We have, 3rd term = 4 and 9th term = -8

$$\text{i.e., } a + 2d = 4 \quad \dots(1)$$

$$\text{and } a + 8d = -8 \quad \dots(2)$$

Solving (1) and (2), we get

$$d = -2, a = 8$$

(i) (c) (ii) (d)

(iii) (b): Let $t_n = -160 \Rightarrow a + (n - 1)d = -160$
 $\Rightarrow 8 + (n - 1)(-2) = -160 \Rightarrow (n - 1)(-2) = -168$
 $\Rightarrow n - 1 = 84 \Rightarrow n = 85$

$$\text{So, } t_{85} = -160$$

(iv) (a)

$$(v) (a): t_{75} = a + 74d = 8 + 74(-2) = -140$$

15. Here $S_n = 0.1n^2 + 7.9n$

(i) (c): $S_{n-1} = 0.1(n - 1)^2 + 7.9(n - 1)$
= $0.1n^2 + 7.7n - 7.8$

(ii) (b): $S_1 = t_1 = a = 0.1(1)^2 + 7.9(1) = 8 \text{ cm}$
= Diameter of core

So, radius of the core = 4 cm

$$(iii) (a): S_2 = 0.1(2)^2 + 7.9(2) = 16.2$$

(iv) (d): Required diameter = $t_2 = S_2 - S_1 = 16.2 - 8$
= 8.2 cm

(v) (c): As $d = t_2 - t_1 = 8.2 - 8 = 0.2 \text{ cm}$

So, thickness of tissue = $0.2 \div 2 = 0.1 \text{ cm} = 1 \text{ mm}$