METHOD OF INTEGRATION (XII, R. S. AGGARWAL)

EXERCISE 13A (Pg.No.: 621)

Evaluate the following integrals:

Very-short Answers Questions

1. $\int (2x+9)^5\, dx$

Sol. Let $I = \int (2x+9)^5 dx$, Put $2x+9=t \Rightarrow 2dx = dt \Rightarrow dx = \frac{dt}{2}$

$$I = \int t^5 . \frac{dt}{2} \Rightarrow I = \frac{1}{2}\int t^5\, dt \Rightarrow I = \frac{1}{2}.\frac{t^6}{6}+c \quad \therefore I = \frac{(2x+9)^6}{12}+c$$

2. $\int (7-3x)^4\, dx$

Sol. Let $I = \int (7-3x)^4\, dx$, Put $7-3x=t \Rightarrow dx = -\frac{dt}{3}$

$$I = \int t^4 . \left(-\frac{dt}{3}\right) \Rightarrow I = -\frac{1}{3}\int t^4\, dt \Rightarrow I = -\frac{1}{3}\frac{t^5}{5}+c \quad \therefore I = -\frac{1}{15}(7-3x)^5+c$$

3. $\int \sqrt{3x-5}\, dx$

Sol. Let $I = \int (3x-5)^{\frac{1}{2}} dx$, Put $3x-5=t \Rightarrow 3 = \frac{dt}{dx} \Rightarrow dx = \frac{dt}{3}$

$$I = \int t^{1/2}\frac{dt}{3} \Rightarrow I = \frac{1}{3}\int t^{1/2}\, dt \Rightarrow I = \frac{1}{3}.\frac{t^{3/2}}{3/2}+c \quad \therefore I = \frac{2}{9}(3x-5)^{3/2}+c$$

4. $\int \frac{1}{\sqrt{4x+3}}\, dx$

Sol. Let $I = \int \frac{1}{(4x+3)^{1/2}} dx \Rightarrow I = \int (4x+3)^{\frac{-1}{2}} dx$, Put $4x+3=t \Rightarrow 4 = \frac{dt}{dx} \Rightarrow dx = \frac{dt}{4}$

$$I = \int t^{\frac{-1}{2}}.\frac{dt}{4} \Rightarrow I = \frac{1}{4}\int t^{\frac{-1}{2}} dt \Rightarrow I = \frac{1}{4}\frac{t^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}+c \Rightarrow I = \frac{1}{2}(4x+3)^{\frac{1}{2}}+c \quad \therefore I = \frac{1}{2}\sqrt{4x+3}+c$$

5. $\int \frac{1}{\sqrt{3-4x}}\, dx$

Sol. Let $I = \int \frac{1}{\sqrt{3-4x}} dx \Rightarrow I = \int (3-4x)^{-\frac{1}{2}} dx$, Put $3-4x=t \Rightarrow dx = -\frac{dt}{4}$

$$I = \int t^{-\frac{1}{2}}\left(\frac{dt}{-4}\right) \Rightarrow I = -\frac{1}{4}\int t^{-\frac{1}{2}} dt \quad \therefore I = -\frac{1}{2}\sqrt{3-4x}+c$$

6. $\int \frac{1}{(2x-3)^{3/2}} dx$

Sol. Let $I=\int\frac{1}{(2x-3)^{3/2}}dx \quad\Rightarrow I=\int(2x-3)^{-3/2}dx$, Put $2x-3=t\Rightarrow 2=\frac{dt}{dx}\quad\Rightarrow dx=\frac{dt}{2}$

$$I=\frac{1}{2}\int t^{-3/2}\,dt \quad\Rightarrow I=-\frac{1}{t^{1/2}}+c \quad\therefore\ I=-\frac{1}{\sqrt{2x+3}}+c$$

7. $\int e^{(2x-1)}dx$

Sol. Let $I=\int e^{(2x-1)}dx$, Put $2x-1=t\ \Rightarrow 2=\frac{dt}{dx}\quad\Rightarrow dx=\frac{dt}{2}$

$$I=\int e^{t}.\frac{dt}{2}\quad\Rightarrow I=\frac{1}{2}\int\left(e^{t}\right)+c\quad\therefore\ I=\frac{1}{2}.e^{2x-1}+c$$

8. $\int e^{(1-3x)}\,dx$

Sol. Let $I=\int e^{(1-3x)}\,dx$, Put $1-3x=t\ \Rightarrow -3=\frac{dt}{dx}\quad\Rightarrow dx=-\frac{dt}{3}$

$$I=\int e^{t}.\left(-\frac{dt}{3}\right)\ \Rightarrow I=-\frac{1}{3}\int e^{t}dt\quad\Rightarrow I=-\frac{1}{3}e^{t}+c\quad\therefore\ I=-\frac{e^{1-3x}}{3}+c$$

9. $\int 3^{2-3x}\,dx$

Sol. Let $I=\int 3^{2-3x}\,dx$, Put $2-3x=t\Rightarrow -3=\frac{dt}{dx}\quad\Rightarrow dx=-\frac{dt}{3}$

$$I=\int 3^{t}.\left(-\frac{dt}{3}\right)\quad\Rightarrow I=-\frac{1}{3}\int 3^{t}\,dt\quad\Rightarrow I=-\frac{1}{3}\int 3^{t}\,dt\quad\Rightarrow I=-\frac{1}{3}.3^{t}/\log 3$$

$$\therefore\ I=-\frac{1}{3}\frac{3^{2-3x}}{\log 3}+c$$

10. $\int\sin 3x\,dx$

Sol. Let $I=\int\sin 3x\,dx$, Put $3x=t\ \Rightarrow 3=\frac{dt}{dx}\ \Rightarrow dx=\frac{dt}{3}$

$$I=\int\sin t.\frac{dt}{3}\quad\Rightarrow I=\frac{1}{3}\int\sin t\,dt\quad\Rightarrow I=-\frac{1}{3}\cos t+c\quad\therefore\ I=-\frac{\cos(3x)}{3}+c$$

Short-Answers Questions

11. $\int\cos(5+6x)dx$

Sol. Let $I=\int\cos(5+6x)dx$, Put $5+6x=t\ \Rightarrow 6=\frac{dt}{dx}\ \Rightarrow dx=\frac{dt}{6}$

$$I=\int\cos t.\frac{dt}{6}\quad\Rightarrow I=\frac{1}{6}\int\cos t\,dt\quad\Rightarrow I=\frac{1}{6}\sin t+c\quad\therefore\ I=\frac{\sin(5+6x)}{6}+c$$

12. $\int\sin x\sqrt{1+\cos 2x}\,dx$

Sol. Let $I=\int\sin x\sqrt{1+\cos 2x}\,dx\quad\Rightarrow I=\int\sin x\sqrt{2\cos^{2}x}\,dx$

$$\Rightarrow I=\sqrt{2}\int\sin x\,\cos x.dx\text{, Put }\sin x=t\ \Rightarrow\cos x=\frac{dt}{dx}\quad\Rightarrow\cos x\,dx=dt$$

$$\Rightarrow I=\sqrt{2}\int t.\,dt\quad\Rightarrow I=\sqrt{2}.\frac{t^{2}}{2}+c\quad\therefore\ I=\frac{\sin^{2}x}{\sqrt{2}}+c$$

13. $\int \operatorname{cosec}^2(2x+5)\,dx$

Sol. Let $I=\int \operatorname{cosec}^2(2x+5)\,dx$, Put $2x+5=t \Rightarrow 2=\frac{dt}{dx} \Rightarrow dx=\frac{dt}{2}$

$$I=\int \operatorname{cosec}^2(t).\frac{dt}{2} \Rightarrow I=\frac{1}{2}\int \operatorname{cosec}^2 t.dt \Rightarrow I=\frac{1}{2}(-\cot t)+c \quad \therefore I=-\frac{\cot(2x+5)}{2}+c$$

14. $\int \sin x\cos x\,dx$

Sol. Let $I=\int \sin x.\cos x\,dx$, Put $\sin x=t \Rightarrow \cos x=\frac{dt}{dx} \Rightarrow \cos x\,dx=dt$

$$I=\int t.\,dt \Rightarrow I=\frac{t^2}{2}+c \quad \therefore I=\frac{\sin^2 x}{2}+c$$

15. $\int \sin^3 x\cos x\,dx$

Sol. Let $I=\int \sin^3 x\cos x\,dx$, Put $\sin x=t \Rightarrow \cos x=\frac{dt}{dx} \Rightarrow \cos dx=dt$

$$I=\int t^3.\,dt \Rightarrow I=\frac{t^4}{4}+c \quad \therefore I=\frac{\sin^4 x}{4}+c$$

16. $\int \sqrt{\cos x}.\sin x\,dx$

Sol. Let $I=\int \sqrt{\cos x}.\sin x\,dx$, Put $\cos x=t \Rightarrow -\sin x=\frac{dt}{dx} \Rightarrow \sin x\,dx=-dt$

$$I=\int \sqrt{t}(-dt) \Rightarrow I=-\int t^{1/2}\,dt \Rightarrow I=-\frac{t^{3/2}}{3/2}+c \quad \therefore I=-\frac{2}{3}(\cos x)^{3/2}+c$$

17. $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$

Sol. Let $I=\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$, Put $\sin^{-1}x=t \Rightarrow \frac{1}{\sqrt{1-x^2}}=\frac{dt}{dx} \Rightarrow \frac{1}{\sqrt{1-x^2}}dx=dt$

$$I=\int t.\,dt \Rightarrow I=\frac{t^2}{2}+c \quad \therefore I=\frac{(\sin^{-1}x)^2}{2}+c$$

18. $\int \frac{\sin(2\tan^{-1}x)}{1+x^2}dx$

Sol. Let $I=\int \frac{\sin(2\tan^{-1}x)}{1+x^2}dx$, Put $2\tan^{-1}x=t \Rightarrow 2.\frac{1}{1+x^2}=\frac{dt}{dx} \Rightarrow \frac{1}{1+x^2}dx=\frac{dt}{2}$

$$I=\int \sin(t).\frac{dt}{2} \Rightarrow I=\frac{1}{2}\int \sin t\,dt \Rightarrow I=-\frac{1}{2}\cos(t)+c \quad \therefore I=-\frac{1}{2}\cos(2\tan^{-1}x)+c$$

19. $\int \frac{\cos(\log x)}{x}dx$

Sol. Let $I=\int \frac{\cos(\log x)}{x}dx$, Put $\log x=t \Rightarrow \frac{1}{x}=\frac{dt}{dx} \Rightarrow \frac{1}{x}dx=dt$

$$I=\int \cos(t)dt \Rightarrow I=\sin(t)+c \quad \therefore I=\sin(\log x)+c$$

20. $\int \frac{\operatorname{cosec}^2(\log x)}{x}dx$

Sol. Let $I=\int \frac{\operatorname{cosec}^2(\log x)}{x}dx$, Put $\log x=t \Rightarrow \frac{1}{x}=\frac{dt}{dx} \Rightarrow \frac{1}{x}dx=dt$

$I=\int \operatorname{cosec}^2(t)dt \quad \Rightarrow I=-\cot(t)+c \quad \therefore\ I=-\cot(\log x)+c$

21. $\int \frac{1}{x\log x}dx$

Sol. Let $I=\int \frac{1}{x\log x}dx$, Put $\log x=t \Rightarrow \frac{1}{x}=\frac{dt}{dx} \Rightarrow \frac{1}{x}dx=dt$

$I=\int \frac{1}{t}dt \quad \Rightarrow I=\log(t)+c \quad \therefore\ I=\log(\log x)+c$

22. $\int \frac{(x+1)(x+\log x)^2}{x}dx$

Sol. Let $I=\int \frac{(x+1)(x+\log x)^2}{x}dx$, Put $x+\log x=t \Rightarrow 1+\frac{1}{x}=\frac{dt}{dx} \quad \Rightarrow \left(\frac{x+1}{x}\right)dx=dt$

$I=\int t^2.\,dt \quad \Rightarrow I=\frac{t^3}{3}+c \quad \therefore\ I=\frac{(x+\log x)^3}{3}+c$

23. $\int \frac{(\log x)^2}{x}dx$

Sol. Let $I=\int \frac{(\log x)^2}{x}dx$, Put $\log x=t \Rightarrow \frac{1}{x}=\frac{dt}{dx} \Rightarrow \frac{1}{x}dx=dt$

$I=\int t^2\,dt \quad \Rightarrow I=\frac{t^3}{3}+c \quad \therefore\ I=\frac{(\log x)^3}{3}+c$

24. $\int \frac{\cos(\sqrt{x})}{\sqrt{x}}dx$

Sol. $I=\int \frac{\cos(\sqrt{x})}{\sqrt{x}}dx$, Put $\sqrt{x}=t \Rightarrow \frac{1}{2\sqrt{x}}=\frac{dt}{dx} \Rightarrow \frac{1}{\sqrt{x}}dx=2dt$

$I=\int \cos(t).2dt \quad \Rightarrow I=2\int \cos t.dt \quad \Rightarrow I=2\sin t+c \quad \therefore\ I=2\sin(\sqrt{x})+c$

25. $\int e^{\tan x}\sec^2 x\,dx$

Sol. $I=\int e^{\tan x}.\sec^2 x\,dx$, Put $\tan x=t \Rightarrow \sec^2 x=\frac{dt}{dx} \Rightarrow \sec^2 x.dx=dt$

$I=\int e^t.dt \quad \Rightarrow I=e^t+c \quad \therefore\ I=e^{\tan x}+c$

26. $\int e^{\cos^2 x}\sin 2x\,dx$

Sol. Let $I=\int e^{\cos^2 x}\sin 2x dx$, Put $\cos^2 x=t \Rightarrow 2\cos x(-\sin x)=\frac{dt}{dx} \Rightarrow \sin 2x\,dx=-dt$

$I=\int e^t(-dt) \quad \Rightarrow I=-\int e^t dt \quad \Rightarrow I=-e^t+c \quad \therefore\ I=-e^{\cos^2 x}+c$

27. $\int \sin(ax+b)\cos(ax+b)dx$

Sol. Let $I = \int \sin(ax+b)\cos(ax+b)dx$

Put $\sin(ax+b) = t \Rightarrow \cos(ax+b).a = \frac{dt}{dx} \Rightarrow \cos(ax+b)dx = \frac{dt}{a}$

$I = \int t.\frac{dt}{a} \Rightarrow I = \frac{1}{a}\int t\,dt \Rightarrow I = \frac{1}{a}.\frac{t^2}{2}+c \quad \therefore I = \frac{\sin^2(ax+b)}{2a}+c$

28. $\int \cos^3 x dx$

Sol. Let $I = \int \cos^3 x\,dx \Rightarrow I = \int \cos^2 x \cos x\,dx \Rightarrow I = \int (1-\sin^2 x)\cos x\,dx$

Put $\sin x = t \Rightarrow \cos x = \frac{dt}{dx} \Rightarrow \cos x dx = dt$

$I = \int (1-t^2)dt \Rightarrow I = t - \frac{t^3}{3}+c \quad \therefore I = \sin x - \frac{\sin^3 x}{3}+c$

29. $\int \frac{1}{x^2} e^{-\frac{1}{x}} dx$

Sol. Let $I = \int \frac{1}{x^2} e^{-\frac{1}{x}} dx$, Put $-\frac{1}{x} = t \Rightarrow \frac{1}{x^2} = \frac{dt}{dx} \Rightarrow \frac{1}{x^2}dx = dt$

$I = \int e^t.dt \Rightarrow I = e^t + c \quad \therefore I = e^{-\frac{1}{x}} + c$

30. $\int \frac{1}{x^2}\cos\left(\frac{1}{x}\right)dx$

Sol. Let $I = \int \frac{1}{x^2}\cos\left(\frac{1}{x}\right)dx$, Put $\frac{1}{x} = t \Rightarrow -\frac{1}{x^2} = \frac{dt}{dx} \Rightarrow \frac{1}{x^2}dx = -dt$

$I = \int \cos(t)(-dt) \Rightarrow I = -\int \cos t\,dt \Rightarrow I = -\sin t + c \quad \therefore I = -\sin\left(\frac{1}{x}\right)+c$

31. $\int \frac{1}{e^x + e^{-x}}dx$

Sol. Let $I = \int \frac{1}{e^x+e^{-x}}dx \Rightarrow I = \int \frac{1}{e^x+\frac{1}{e^x}}dx \Rightarrow I = \int \frac{e^x}{(e^x)^2+1}$

Put $e^x = t \Rightarrow e^x = \frac{dt}{dx} \Rightarrow e^x dx = dt$

$I = \int \frac{dt}{t^2+1} \Rightarrow I = \tan^{-1}(t)+c \quad \therefore I = \tan^{-1}(e^x)+c$

32. $\int \frac{e^{2x}}{e^{2x}-2}dx$

Sol. Let $I = \int \frac{e^{2x}}{e^{2x}-2}dx$, Put $e^{2x}-2 = t \Rightarrow e^{2x}.2 = \frac{dt}{dx} \Rightarrow e^{2x}dx = \frac{dt}{2}$

$I = \frac{1}{2}\int \frac{dt}{t} \Rightarrow I = \frac{1}{2}\log|t|+c \quad \therefore I = \frac{1}{2}\log\left|e^{2x}-2\right|+c$

33. $\int \cot x \log(\sin x)\, dx$

Sol. Let $I = \int \cot x \log(\sin x) dx$, Put $\log(\sin x) = t \Rightarrow \frac{1}{\sin x}.\cos x = \frac{dt}{dx} \Rightarrow \cot x\, dx = dt$

$$I = \int t\, .dt \quad \Rightarrow I = \frac{t^2}{2} + c \quad \therefore\ I = \frac{(\log \sin x)^2}{2} + c$$

34. $\int \frac{\cot x}{\log(\sin x)} dx$

Sol. Let $I = \int \frac{\cot x}{\log(\sin x)} dx$, Put $\log(\sin x) = t \Rightarrow \frac{1}{\sin x}.\cos x = \frac{dt}{dx} \Rightarrow \cot\, dx = dt$

$$I = \int \frac{1}{t}.dt \quad \Rightarrow I = \log(t) + x \quad \therefore\ I = \log\left|\log(\sin x)\right| + c$$

35. $\int 2x \sin(x^2 + 1) dx$

Sol. Let $I = \int 2x \sin(x^2+1) dx$, Put $x^2 + 1 = t \Rightarrow 2x = \frac{dt}{dx} \Rightarrow 2x\, dx = dt$

$$I = \int \sin(t).dt \quad \Rightarrow I = -\cos t + c \quad \therefore\ I = -\cos(x^2+1) + c$$

36. $\int \sec x.\ \log\ (\sec x + \tan x) dx$

Sol. Let $I = \int \sec x . \log(\sec x + \tan x) dx$

Put $\log(\sec x + \tan x) = t \Rightarrow \frac{1}{\sec x + \tan x}(\sec x \tan x + \sec^2 x) = \frac{dt}{dx}$

$$\Rightarrow \frac{\sec x . \tan + \sec^2 x}{\sec x + \tan x} = \frac{dt}{dx} \Rightarrow \frac{\sec x(\sec x + \tan x)}{(\sec x + \tan x)} = \frac{dt}{dx} \Rightarrow \sec x\, dx = dt$$

$$\therefore I = \int t\, dt \quad \Rightarrow I = \frac{t^2}{2} + c \quad \therefore\ I = \frac{\{\log(\sec x + \tan x)\}^2}{2} + c$$

37. $\int \frac{\tan \sqrt{x} \sec^2 \sqrt{x}}{\sqrt{x}} dx$

Sol. Let $I = \int \frac{\tan(\sqrt{x}) \sec^2(\sqrt{x})}{\sqrt{x}} dx$

Put $\tan(\sqrt{x}) = t \Rightarrow \sec^2(\sqrt{x}).\frac{1}{2\sqrt{x}} = \frac{dt}{dx} \Rightarrow \frac{\sec^2(\sqrt{x})}{\sqrt{x}} dx = 2dt$

$$I = \int t.2dt \quad \Rightarrow I = 2.\frac{t^2}{2} + c \quad \Rightarrow I = \tan^2(\sqrt{x}) + c \quad \therefore\ I = \tan^2(\sqrt{x}) + c$$

38. $\int \frac{x \tan^{-1}(x^2)}{1 + x^4} dx$

Sol. Let $I = \int \frac{x \tan^{-1}(x^2)}{1+x^4} dx$, Put $\tan^{-1}(x^2) = t \Rightarrow \frac{1}{1+(x^2)^2}.2x = \frac{dt}{dx} \Rightarrow \frac{x}{1+x^4} dx = \frac{dt}{2}$

$$I = \int t.\frac{dt}{2} \quad \Rightarrow I = \frac{1}{2}.\frac{t^2}{2} + c \quad \Rightarrow I = \frac{t^2}{4} + c \quad \therefore\ I = \frac{\{\tan^{-1}(x^2)\}^2}{4} + c$$

39. $\int \frac{x\sin^{-1}\left(x^2\right)}{\sqrt{1-x^4}}dx$

Sol. Let $I=\int \frac{x\sin^{-1}\left(x^2\right)}{\sqrt{1-x^4}}dx$, Put $\sin^{-1}\left(x^2\right)=t \Rightarrow \frac{1}{\sqrt{1-x^4}}.2x=\frac{dt}{dx} \Rightarrow \frac{x}{\sqrt{1-x^4}}dx=\frac{dt}{2}$

$I=\int t.\frac{dt}{2} \Rightarrow I=\frac{1}{2}\int t\,dt \Rightarrow I=\frac{t^2}{4}+c \quad \therefore\ I=\frac{\left\{\sin^{-1}\left(x^2\right)\right\}^2}{4}+c$

40. $\int \frac{1}{\sqrt{1-x^2}\sin^{-1}x}dx$

Sol. Let $I=\int \frac{1}{\sqrt{1-x^2}\sin^{-1}x}dx$, Put $\sin^{-1}x=t \Rightarrow \frac{1}{\sqrt{1-x^2}}=\frac{dt}{dx} \Rightarrow \frac{1}{\sqrt{1-x^2}}dx=dt$

$I=\int \frac{1}{t}dt \Rightarrow I=\log|t|+c \quad \therefore\ I=\log\left|\sin^{-1}x\right|+c$

41. $\int \frac{\sqrt{2+\log x}}{x}dx$

Sol. Let $I=\int \frac{\sqrt{2+\log x}}{x}dx$, Put $2+\log x=t \Rightarrow \frac{1}{x}=\frac{dt}{dx} \Rightarrow \frac{1}{x}dx=dt$

$I=\int \sqrt{t}dt \Rightarrow I=\frac{t^{3/2}}{3/2}+c \quad \therefore\ I=\frac{2}{3}(2+\log x)^{3/2}+c$

42. $\int \frac{\sec^2 x}{1+\tan x}dx$

Sol. Let $I=\int \frac{\sec^2 x}{1+\tan x}dx$, Put $1+\tan x=t \Rightarrow \sec^2 x=\frac{dt}{dx} \Rightarrow \sec^2 x dx=dt$

$I=\int \frac{dt}{t} \Rightarrow I=\log|t|+c \quad \therefore\ I=\log|1+\tan x|+c$

43. $\int \frac{\sin x}{1+\cos x}dx$

Sol. Let $I=\int \frac{\sin x}{1+\cos x}dx$, Put $1+\cos x=t \Rightarrow -\sin x=\frac{dt}{dx} \Rightarrow \sin x dx=-dt$

$I=-\int \frac{dt}{t} \Rightarrow I=-\log|t|+c \quad \therefore\ I=-\log|1+\cos x|+c$

44. $\int \left(\frac{1+\tan x}{1-\tan x}\right)dx$

Sol. Let $I=\int \frac{1+\tan x}{1-\tan x}dx \Rightarrow I=\int \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}}dx \Rightarrow I=\int \frac{\cos x+\sin x}{\cos x-\sin x}dx$

Put $\cos x-\sin x=t \Rightarrow -\sin x-\cos x=\frac{dt}{dx} \Rightarrow (\sin x+\cos x)dx=-dt$

$I=-\int \frac{dt}{t} \Rightarrow I=-\log|t|+c \quad \therefore\ I=-\log|\cos x-\sin x|+c$

45. (i) $\int \frac{1+\tan x}{x+\log(\sec x)} dx$ (ii) $\int \frac{1-\sin x}{x+\cos^2 x} dx$

Sol. (i) $\int \frac{1+\tan x}{x+\log(\sec x)} dx$

Put $x+\log(\sec x)=t \Rightarrow 1+\frac{1}{\sec x}.\sec x.\tan x=\frac{dt}{dx} \Rightarrow (1+\tan x)dx=dt$

$I=\int \frac{1}{t}dt \Rightarrow I=\log|t|+c \quad \therefore\ I=\log\left|x+\log(\sec x)\right|+c$

(ii) $I=\int \frac{1-\sin 2x}{x+\cos^2 x} dx$, Put $x+\cos^2 x=t \Rightarrow 1+2\cos x(-\sin x)=\frac{dt}{dx} \Rightarrow (1-\sin 2x)dx=dt$

$I=\int \frac{dt}{t} \Rightarrow I=\log|t|+c \quad \therefore\ I=\log\left|x+\cos^2 x\right|+c$

46. $\int \frac{\sin 2x}{a^2+b^2\sin^2 x} dx$

Sol. Let $I=\int \frac{\sin 2x}{a^2+b^2\sin^2 x} dx$, Put $a^2+b^2\sin^2 x=t \Rightarrow b^2 2\sin x\cos x=\frac{dt}{dx} \Rightarrow \sin 2x\ dx=\frac{dt}{b^2}$

$I=\frac{1}{b^2}\int \frac{dt}{t} \Rightarrow I=\frac{1}{b^2}\log|t|+c \quad \therefore\ I=\frac{\log\left|a^2+b^2\sin^2 x\right|}{b^2}+c$

47. $\int \frac{\sin 2x}{a^2\cos^2 x+b^2\sin^2 x} dx$

Sol. Let $I=\int \frac{\sin 2x}{a^2\cos^2 x+b^2\sin^2 x} dx$

Put $a^2\cos^2 x+b^2\sin^2 x=t \Rightarrow a^2 2\cos x(-\sin x)+b^2 2\sin x\cos x=\frac{dt}{dx}$

$\Rightarrow -a^2\sin 2x+b^2\sin 2x=\frac{dt}{dx} \Rightarrow \sin 2x\left(b^2-a^2\right)=\frac{dt}{dx} \Rightarrow \sin 2x dx=\frac{dt}{b^2-a^2}$

$I=\int \frac{1}{t}.\frac{dt}{b^2-a^2} \quad \therefore\ I=\frac{1}{b^2-a^2}\log\left|a^2\cos^2 x+b^2\sin^2 x\right|+c$

48. $\int \left(\frac{2\cos x-3\sin x}{3\cos x+2\sin x}\right) dx$

Sol. Let $I=\left(\frac{2\cos x-3\sin x}{3\cos x+2\sin x}\right) dx$

Put $3\cos x+2\sin x=t \Rightarrow 3(-\sin x)+2\cos x=\frac{dt}{dx} \Rightarrow (2\cos x-3\sin x)dx=dt$

$I=\int \frac{dt}{t} \Rightarrow I=\log|t|+c \quad \therefore\ I=\log|3\cos x+2\sin x|+c$

49. $\int \frac{4x}{2x^2+3} dx$

Sol. Let $I=\int \frac{4x}{2x^2+3} dx$, Put $2x^2+3=t \Rightarrow 4x=\frac{dt}{dx} \Rightarrow 4x\ dx=dt$

$I=\int \frac{dt}{t} \Rightarrow I=\log|t|+c \quad \therefore\ I=\log\left|2x^2+3\right|+c$

50. $\int \frac{x+1}{x^2+2x-3}dx$

Sol. Let $I = \int \frac{x+1}{x^2+2x-3}dx$

Put $x^2+2x-3=t \Rightarrow (2x+2)=\frac{dt}{dx} \Rightarrow 2(x+1)dx = dt \Rightarrow (x+1)dx = \frac{dt}{2}$

$I = \frac{1}{2}\int \frac{dt}{t} \Rightarrow I = \frac{1}{2}\log|t|+c \quad \therefore I = \frac{1}{2}\log\left|x^2+2x-3\right|+c$

51. $\int \frac{4x-5}{2x^2-5x+1}dx$

Sol. Let $I = \int \frac{4x-5}{2x^2-5x+1}dx$, Put $2x^2-5x+1=t \Rightarrow 4x-5=\frac{dt}{dx} \Rightarrow (4x-5)dx = dt$

$I = \int \frac{1}{t}dt \Rightarrow I = \log|t|+c \quad \therefore I = \log\left|2x^2-5x+1\right|+c$

52. $\int \frac{9x^2-4x+5}{3x^3-2x^2+5x+1}dx$

Sol. Let $I = \int \frac{9x^2-4x+5}{3x^3-2x^2+5x+1}dx$

Put $3x^3-2x^2+5x+1=t \Rightarrow (9x^2-4x+5)=\frac{dt}{dx} \Rightarrow (9x^2-4x+5)dx = dt$

$I = \int \frac{1}{t}dt \Rightarrow I = \log|t|+c \quad \therefore I = \log\left|3x^3-2x^2+5x+1\right|+c$

53. $\int \frac{\sec x \operatorname{cosec} x}{\log(\tan x)}dx$

Sol. Let $I = \int \frac{\sec x \operatorname{coses} x}{\log(\tan x)}dx$, Put $\log(\tan x)=t \Rightarrow \frac{1}{\tan x}\sec^2 x = \frac{dt}{dx} \Rightarrow \sec x \operatorname{cosec} x dx = dt$

$I = \int \frac{1}{t}dt \Rightarrow I = \log|t|+c \quad \therefore I = \log\left|\log(\tan x)\right|+c$

54. $\int \frac{1+\cos x}{(x+\sin x)^3}dx$

Sol. Let $I = \int \frac{1+\cos x}{(x+\sin x)^3}dx$, Put $x+\sin x = t \Rightarrow 1+\cos x = \frac{dt}{dx} \Rightarrow (1+\cos x)dx = dt$

$I = \int \frac{dt}{t^3} \Rightarrow I = -\frac{1}{2t^2}+c \quad \therefore I = -\frac{1}{2(x+\sin x)^2}+c$

55. $\int \frac{\sin x}{(1+\cos x)^2}dx$

Sol. Let $I = \int \frac{\sin x}{(1+\cos x)^2}dx$, Put $1+\cos x = t \Rightarrow -\sin x = \frac{dt}{dx} \Rightarrow \sin x dx = -dt$

$I = \int -\frac{dt}{t^2} \Rightarrow I = -\int t^{-2}dt \Rightarrow I = -\frac{t^{-1}}{-1}+c \quad \therefore I = \frac{1}{1+\cos x}+c$

56. $\int \frac{2x+3}{\sqrt{x^2+3x-2}}dx$

Sol. Let $I=\int \frac{2x+3}{\sqrt{x^2+3x-2}}dx$, Put $x^2+3x-2=t \Rightarrow (2x+3)=\frac{dt}{dx} \Rightarrow (2x+3)dx=dt$

$$I=\int \frac{dt}{\sqrt{t}} \Rightarrow I=\frac{t^{1/2}}{1/2}+c \quad \therefore\ I=2\sqrt{x^2+3x-2}+c$$

57. $\int \frac{2x-1}{\sqrt{x^2-x-1}}dx$

Sol. Let $I=\int \frac{2x-1}{\sqrt{x^2-x-1}}dx$, Put $x^2-x-1=t \Rightarrow (2x-1)=\frac{dt}{dx} \Rightarrow (2x-1)dx=dt$

$$I=\int \frac{dt}{\sqrt{t}} \Rightarrow I=\frac{t^{1/2}}{1/2}+c \quad \therefore\ I=2\sqrt{x^2-x-1}+c$$

58. $\int \frac{1}{\sqrt{x+a}+\sqrt{x+b}}dx$

Sol. Let $I=\int \frac{1}{\sqrt{x+a}+\sqrt{x+b}}dx \Rightarrow I=\int \frac{1}{\sqrt{x+a}+\sqrt{x+b}}\times\frac{\sqrt{x+a}-\sqrt{x+b}}{\sqrt{x+a}-\sqrt{x+b}}dx$

$$\Rightarrow I=\int \frac{\sqrt{x+a}-\sqrt{x+b}}{\left(\sqrt{x+a}\right)^2-\left(\sqrt{x+b}\right)^2}dx \Rightarrow I=\int \frac{\sqrt{x+a}-\sqrt{x+b}}{(x+a)-(x+b)}dx \Rightarrow I=\int \frac{\sqrt{x+a}-\sqrt{x+b}}{x+a-x-b}dx$$

$$\Rightarrow I=\frac{1}{a-b}\left[\int (x+a)^{1/2}dx-\int (x+b)^{1/2}dx\right] \Rightarrow I=\frac{1}{a-b}\left[\frac{(x+a)^{3/2}}{3/2}-\frac{(x+b)^{3/2}}{3/2}\right]$$

$$\therefore\ I=\frac{2}{3(a-b)}\left[(x+a)^{3/2}-(x+b)^{3/2}\right]+c$$

59. $\int \frac{1}{\sqrt{1-3x}-\sqrt{5-3x}}dx$

Sol. Let $I=\int \frac{1}{\sqrt{1-3x}-\sqrt{5-3x}}dx \Rightarrow I=\int \frac{1}{\sqrt{1-3x}-\sqrt{5-3x}}\times\frac{\sqrt{1-3x}+\sqrt{5-3x}}{\sqrt{1-3x}+\sqrt{5-3x}}dx$

$$\Rightarrow I=\int \frac{\sqrt{1-3x}+\sqrt{5-3x}}{\left(\sqrt{1-3x}\right)^2-\left(\sqrt{5-3x}\right)^2}dx \Rightarrow I=\int \frac{\sqrt{1-3x}+\sqrt{5-3x}}{1-3x-5+3x}dx$$

$$\Rightarrow I=-\frac{1}{4}\left[\int (1-3x)^{1/2}dx+\int (5-3x)^{1/2}dx\right] \Rightarrow I=-\frac{1}{4}\left[\frac{(1-3x)^{3/2}}{3/2(-3)}+\frac{(5-3x)^{3/2}}{3/2(-3)}\right]$$

$$\Rightarrow I=\frac{2}{9.4}\left[(1-3x)^{3/2}+(5-3x)^{3/2}\right]+c \quad \therefore\ I=\frac{1}{18}\left[(1-3x)^{3/2}+(5-3x)^{3/2}\right]+c$$

60. $\int \frac{x^2}{1+x^6}dx$

Sol. Let $I=\int \frac{x^2}{1+x^6}dx \Rightarrow I=\int \frac{x^2}{1+\left(x^3\right)^2}dx$, Put $x^3=t \Rightarrow 3x^2=\frac{dt}{dx} \Rightarrow x^2dx=\frac{dt}{3}$

$$I=\frac{1}{3}\int \frac{dt}{1+t^2} \Rightarrow I=\frac{1}{3}\tan^{-1}(t)+c \quad \therefore\ I=\frac{1}{3}\tan^{-1}\left(x^3\right)+c$$

61. $\int \frac{x^3}{1+x^8}dx$

Sol. Let $I=\int \frac{x^3}{1+x^8}dx \Rightarrow I=\int \frac{x^3}{1+\left(x^4\right)^2}dx$, Put $x^4=t \Rightarrow 4x^3=\frac{dt}{dx} \Rightarrow x^3dx=\frac{dt}{4}$

$$I=\frac{1}{4}\int \frac{dt}{1+t^2} \Rightarrow I=\frac{1}{4}\tan^{-1}(t)+c \quad \therefore\ I=\frac{1}{4}\tan^{-1}\left(x^4\right)+c$$

62. $\int \frac{x}{1+x^4}dx$

Sol. Let $I=\int \frac{x}{1+x^4}dx \Rightarrow I=\int \frac{x}{1+\left(x^2\right)^2}dx$, Put $x^2=t \Rightarrow 2x=\frac{dt}{dx} \Rightarrow xdx=\frac{dt}{2}$

$$I=\frac{1}{2}\int \frac{dt}{1+t^2} \Rightarrow I=\frac{1}{2}\tan^{-1}(t)+c \quad \therefore\ I=\frac{1}{2}\tan^{-1}\left(x^2\right)+c$$

63. $\int \frac{x^5}{\sqrt{1+x^3}}dx$

Sol. Let $I=\int \frac{x^5}{\sqrt{1+x^3}}dx \Rightarrow I=\int \frac{x^3.x^2}{\sqrt{1+x^3}}dx$

Put $1+x^3=t \Rightarrow x^3=t-1 \Rightarrow 3x^2=\frac{dt}{dx} \Rightarrow x^2dx=\frac{dt}{3}$

$$I=\int \frac{t-1}{\sqrt{t}}\frac{dt}{3} \Rightarrow I=\frac{1}{3}\int \frac{t}{\sqrt{t}}dt-\frac{1}{3}\int \frac{1}{\sqrt{t}}dt \Rightarrow I=\frac{1}{3}\left[\int t^{1/2}dt-\int t^{-\frac{1}{2}}dt\right]$$

$$\Rightarrow I=\frac{1}{3}\left[\frac{t^{3/2}}{3/2}-\frac{t^{1/2}}{1/2}\right]+c \Rightarrow I=\frac{2}{3}\left[\frac{\left(1+x^3\right)^{3/2}}{3}-\left(1+x^3\right)^{1/2}\right]+c \quad \therefore I=\frac{2}{9}\left(1+x^3\right)^{3/2}-\frac{2}{3}\left(1+x^3\right)^{1/2}+c$$

64. $\int \frac{x}{\sqrt{1+x}}dx$

Sol. Let $I=\int \frac{x}{\sqrt{1+x}}dx$, Put $1+x=t \Rightarrow x=t-1 \Rightarrow dx=dt$

$$\Rightarrow I=\int \frac{t-1}{\sqrt{t}}dt \Rightarrow I=\int \sqrt{t}dt-\int \frac{1}{\sqrt{t}}dt \Rightarrow I=\frac{t^{3/2}}{3/2}-\frac{t^{1/2}}{1/2}+c \quad \therefore I=\frac{2}{3}(1+x)^{3/2}-2(1+x)^{1/2}+c$$

65. $\int \frac{1}{x\sqrt{x^4-1}}dx$

Sol. Let $I=\int \frac{1}{x\sqrt{x^4-1}}dx \Rightarrow I=\int \frac{x}{x^2\sqrt{\left(x^2\right)^2-1}}dx$

Put $x^2=t \Rightarrow 2x=\frac{dt}{dx} \Rightarrow 2x\ dx=dt \Rightarrow x\ dx=\frac{dt}{2}$

$$\Rightarrow I=\frac{1}{2}\int \frac{dt}{t\sqrt{t^2-1}} \Rightarrow I=\frac{1}{2}\sec^{-1}(t)+c \quad \therefore\ I=\frac{1}{2}\sec^{-1}\left(x^2\right)+c$$

66. $I=\int x\sqrt{x-1}\ dx$

Sol. Let $I=\int x\sqrt{x-1}\ dx$, Put $x-1=t \Rightarrow x=t+1 \Rightarrow dx=dt$

$$I=\int(t+1)\sqrt{t}\,dt \Rightarrow I=\int t\sqrt{t}\,dt+\int\sqrt{t}\,dt \Rightarrow I=\int t^{3/2}dt+\int t^{1/2}dt \Rightarrow I=\frac{t^{5/2}}{5/2}+\frac{t^{3/2}}{3/2}+c$$

$$\therefore\ I=\frac{2}{5}(x-1)^{5/2}+\frac{2}{3}(x-1)^{3/2}+c$$

67. $\int(1-x)\sqrt{1+x}\,dx$

Sol. Let $I=\int(1-x)\sqrt{1+x}\,dx$, Put $1+x=t \Rightarrow x=t-1 \Rightarrow dx=dt$

$$I=\int\{1-(t-1)\}\sqrt{t}\,dt \Rightarrow I=\int(1-t+1)\sqrt{t}\,dt \Rightarrow I=\int(2-t)\sqrt{t}\,dt$$

$$\Rightarrow I=2\int\sqrt{t}\,dt-\int t\sqrt{t}\,dt \Rightarrow I=2\int t^{1/2}dt-\int t^{3/2}dt \Rightarrow I=\frac{2t^{3/2}}{3/2}-\frac{t^{5/2}}{5/2}+c$$

$$\therefore\ I=\frac{4}{3}(1+x)^{3/2}-\frac{2}{5}(1+x)^{5/2}+c$$

68. $\int x\sqrt{x^2-1}\,dx$

Sol. Let $I=\int x\sqrt{x^2-1}\,dx$, Put $x^2-1=t \Rightarrow 2x=\frac{dt}{dx} \Rightarrow x\,dx=\frac{dt}{2}$

$$I=\int\sqrt{t}.\frac{dt}{2} \Rightarrow I=\frac{1}{2}\int t^{1/2}\,dt \Rightarrow I=\frac{1}{2}.\frac{t^{3/2}}{3/2}+c \quad \therefore\ I=\frac{1}{3}\left(x^2-1\right)^{3/2}+c$$

69. $\int x\sqrt{3x-2}\,dx$

Sol. Let $I=\int x\sqrt{3x-2}\,dx$, Put $3x-2=t \quad \therefore\ x=\frac{t+2}{3} \Rightarrow 3=\frac{dt}{dx}$

$$I=\int\left(\frac{t+2}{3}\right)\sqrt{t}.\frac{dt}{3} \Rightarrow I=\frac{1}{9}\left[\int t^{3/2}dt+2\,t^{1/2}\,dt\right] \Rightarrow I=\frac{1}{9}\left[\frac{t^{5/2}}{5/2}+\frac{2.t^{3/2}}{3/2}\right]+c$$

$$\Rightarrow I=\frac{1}{9}\left[\frac{2}{5}(3x-2)^{5/2}+\frac{4}{3}(3x-2)^{3/2}\right]+c \quad \therefore I=\frac{2}{45}(3x-2)^{5/2}+\frac{4}{27}(3x-2)^{3/2}+c$$

70. $\int\frac{dx}{x\cos^2(1+\log x)}$

Sol. Let $I=\int\frac{1}{x\cos^2(1+\log x)}dx$, Put $1+\log x=t \Rightarrow \frac{1}{x}=\frac{dt}{dx} \Rightarrow \frac{1}{x}dx=dt$

$$I=\int\frac{1}{\cos^2(t)}.dt \Rightarrow I=\int\sec^2(t)\,dt \Rightarrow I=\tan(t)+c \quad \therefore\ I=\tan(1+\log x)+c$$

71. $\int x^2\sin\left(x^3\right)dx$

Sol. Let $I=\int x^2\sin\left(x^3\right)dx$, Put $x^3=t \Rightarrow 3x^2=\frac{dt}{dx} \Rightarrow x^2dx=\frac{dt}{3}$

$$I=\int\sin(t).\frac{dt}{3} \Rightarrow I=\frac{1}{3}\int\sin t\,dt \Rightarrow I=\frac{1}{3}(-\cos t)+c \quad \therefore\ I=\frac{-\cos\left(x^3\right)}{3}+c$$

72. $\int(2x+4)\sqrt{x^2+4x+3}\,dx$

Sol. Let $I=\int(2x+4)\sqrt{x^2+4x+3}\,dx$, Put $x^2+4x+3=t \Rightarrow (2x+4)=\frac{dt}{dx} \Rightarrow (2x+4)dx=dt$

$$I=\int\sqrt{t}\,dt \Rightarrow I=\frac{t^{3/2}}{3/2}+c \quad \therefore\ I=\frac{2}{3}\left(x^2+4x+3\right)^{3/2}+c$$

73. $\int \frac{\sin x}{\sin x-\cos x}dx$

Sol. Let $I=\int \frac{\sin x}{\sin x-\cos x}dx \quad \Rightarrow I=\frac{1}{2}\int \frac{2\sin x}{\sin x-\cos x}dx$

$\Rightarrow I=\frac{1}{2}\int \frac{(\sin x+\cos x)+(\sin x-\cos x)}{(\sin x-\cos x)}dx \quad \Rightarrow I=\frac{1}{2}\int \frac{\sin x+\cos x}{\sin x-\cos x}dx+\frac{1}{2}\int \frac{\sin x-\cos x}{\sin x-\cos x}dx$

Put $\sin x-\cos x=t \quad \Rightarrow \cos x+\sin x=\frac{dt}{dx} \quad \Rightarrow (\cos x+\sin x)dx=dt$

$I=\frac{1}{2}\int \frac{dt}{t}+\frac{1}{2}\int dx \quad \Rightarrow I=\frac{1}{2}\log|\sin x-\cos x|+\frac{1}{2}.x+c \quad \therefore I=\frac{x}{2}+\frac{1}{2}\log|\sin x-\cos x|+c$

74. $\int \frac{1}{1-\tan x}dx$

Sol. Let $I=\int \frac{1}{1-\tan x}dx \Rightarrow I=\frac{1}{1-\frac{\sin x}{\cos x}} \quad \Rightarrow I=\int \frac{\cos x}{\cos x-\sin x}dx \quad \Rightarrow I=\frac{1}{2}\int \frac{2\cos x}{\cos x-\sin x}dx$

$\Rightarrow I=\frac{1}{2}\int \frac{(\cos x+\sin x)+(\cos x-\sin x)}{\cos x-\sin x}dx \quad \Rightarrow \frac{1}{2}\int \frac{\cos x+\sin x}{\cos x-\sin x}dx+\frac{1}{2}\int \frac{\cos x-\sin x}{\cos x-\sin x}dx$

Put $\cos x-\sin x=t \Rightarrow \sin x-\cos x=\frac{dt}{dx} \quad \Rightarrow (\sin x+\cos x)dx=-dt$

$I=-\frac{1}{2}\int \frac{dt}{t}+\frac{1}{2}\int dx \quad \Rightarrow I=-\frac{1}{2}\log|t|+\frac{1}{2}x+c \quad \Rightarrow I=-\frac{1}{2}\log|\cos x-\sin x|+\frac{x}{2}+c$

$\therefore \quad I=\frac{x}{2}-\frac{1}{2}\log|\cos x-\sin x|+c$ and $I=\frac{x}{2}-\frac{1}{2}\log|\sin x-\cos x|+c$

75. $\int \frac{1}{1-\cot x}dx$

Sol. Let $I=\int \frac{1}{1-\cot x}dx \quad \Rightarrow I=\int \frac{1}{1-\frac{\cos x}{\sin x}}dx \quad \Rightarrow I=\int \frac{\sin x}{\sin x-\cos x}dx$

$\Rightarrow I=\frac{1}{2}\int \frac{2\sin x}{\sin x-\cos x}dx \quad \Rightarrow I=\frac{1}{2}\int \frac{(\sin x+\cos x)+(\sin x-\cos x)}{(\sin x-\cos x)}dx$

$\Rightarrow I=\frac{1}{2}\int \frac{\sin x+\cos x}{\sin x-\cos x}dx+\frac{1}{2}\int \frac{\sin x-\cos x}{\sin x-\cos x}dx$

Put $\sin x-\cos x=t \quad \Rightarrow \cos x+\sin x=\frac{dt}{dx} \quad \Rightarrow (\cos x+\sin x)dx=dt$

$I=\frac{1}{2}\int \frac{dt}{t}+\frac{1}{2}\int dx \quad \Rightarrow I=\frac{1}{2}\log|\sin x-\cos x|+\frac{1}{2}.x+c \quad \therefore I=\frac{x}{2}+\frac{1}{2}\log|\sin x-\cos x|+c$

76. $\int \frac{\cos 2x}{(\sin x+\cos x)^2}dx$

Sol. Let $I=\int \frac{\cos 2x}{(\sin x+\cos x)^2}dx \quad \Rightarrow I=\int \frac{\cos^2 x-\sin^2 x}{(\sin x+\cos x)^2}dx$

$\Rightarrow I=\int \frac{(\cos x-\sin x)(\cos x+\sin x)}{(\cos x+\sin x)^2}dx \quad \Rightarrow I=\int \frac{\cos x-\sin x}{(\cos x+\sin x)}dx$

Put $\cos x+\sin x=t \quad \Rightarrow -\sin x+\cos x=\dfrac{dt}{dx} \quad \Rightarrow (\cos x-\sin x)dx=dt$

$$I=\int\frac{dt}{t} \quad \Rightarrow I=\log|t|+c \quad \therefore\ I=\log|\cos x+\sin x|+c$$

77. $\displaystyle\int\frac{\cos x-\sin x}{(1+\sin 2x)}dx$

Sol. Let $I=\displaystyle\int\frac{\cos x-\sin x}{1+\sin 2x}dx \quad \Rightarrow I=\int\frac{\cos x-\sin x}{(\cos x+\sin x)^2}$

Put $\cos x+\sin x=t \quad \Rightarrow -\sin x+\cos x=\dfrac{dt}{dx} \quad \Rightarrow (\cos x-\sin x)dx=dt$

$$I=\int\frac{dt}{t^2} \quad \Rightarrow I=-\frac{1}{t}+c \quad \therefore\ I=-\frac{1}{\cos x+\sin x}+c$$

78. $\displaystyle\int\frac{(x+1)(x+\log x)^2}{x}dx$

Sol. Let $I=\displaystyle\int\frac{(x+1)(x+\log x)^2}{x}dx$, Put $x+\log x=t \quad \Rightarrow 1+\dfrac{1}{x}=\dfrac{dt}{dx} \quad \Rightarrow \dfrac{x+1}{x}.dx=dt$

$$I=\int t^2.\,dt \quad \Rightarrow I=\frac{t^3}{3}+c \quad \therefore\ I=\frac{(x+\log x)^3}{3}+c$$

79. $\displaystyle\int x\sin^3\left(x^2\right)\cos\left(x^2\right)dx$

Sol. Let $I=\displaystyle\int x\sin^3\left(x^2\right)\cos\left(x^2\right)dx$, Put $\sin\left(x^2\right)=t \Rightarrow \cos\left(x^2\right).2x=\dfrac{dt}{dx} \quad \Rightarrow x\cos\left(x^2\right)dx=\dfrac{dt}{2}$

$$I=\int t^3.\frac{dt}{2} \quad \Rightarrow I=\frac{1}{2}\int t^3\,dt \quad \Rightarrow I=\frac{1}{2}.\frac{t^4}{4}+c \quad \therefore\ I=\frac{\sin^4\left(x^2\right)}{8}+c$$

80. $\displaystyle\int\frac{\sec^2 x}{\sqrt{1-\tan^2 x}}dx$

Sol. Let $I=\displaystyle\int\frac{\sec^2 x}{\sqrt{1-\tan^2 x}}dx$, Put $\tan x=t \quad \Rightarrow \sec^2 x=\dfrac{dt}{dx} \quad \Rightarrow \sec^2 x\,dx=dt$

$$I=\int\frac{dt}{\sqrt{1-t^2}} \quad \Rightarrow I=\sin^{-1}(t)+c \quad \therefore\ I=\sin^{-1}(\tan x)+c$$

81. $\displaystyle\int e^{-x}\operatorname{cosec}^2\left(2e^{-x}+5\right)dx$

Sol. Let $I=\displaystyle\int e^{-x}\operatorname{cosec}^2\left(2e^{-x}+5\right)dx$, Put $2e^{-x}+5=t \Rightarrow 2e^{-x}(-1)=\dfrac{dt}{dx} \quad \Rightarrow e^{-x}dx=\dfrac{dt}{-2}$

$$I=\int\operatorname{cosec}^2(t).\frac{dt}{-2} \quad \Rightarrow I=-\frac{1}{2}\int\operatorname{cosec}^2(t)dt \quad \Rightarrow I=-\frac{1}{2}(-\cot t)+c$$

$$\therefore\ I=\frac{1}{2}\cot\left(2e^{-x}+5\right)+c$$

82. $\displaystyle\int 2x\sec^3\left(x^2+3\right)\tan\left(x^2+3\right)dx$

Sol. Let $I=\displaystyle\int 2x\sec^2\left(x^2+3\right)\sec\left(x^2+3\right)\tan\left(x^2+3\right)dx$

Put $\sec\left(x^2+3\right)=t \quad \Rightarrow \sec\left(x^2+3\right)\tan\left(x^2+3\right).2x=\dfrac{dt}{dx} \quad \Rightarrow 2x\sec x\left(x^2+3\right)\tan\left(x^2+3\right)=dt$

$$I=\int t^2dt \Rightarrow I=\frac{t^3}{3}+c \quad \therefore I=\frac{\sec^3\left(x^2+3\right)}{3}+c$$

83. $\int \frac{\sin 2x}{(a+b\cos x)^2}dx$

Sol. Let $I=\int \frac{\sin 2x}{(a+b\cos x)^2}dx \Rightarrow I=\int \frac{2\sin x\cos x}{(a+b\cos x)^2}dx$

Put $a+b\cos x=t \Rightarrow \cos x=\frac{t-a}{b} \Rightarrow \sin x dx=\frac{dt}{-b} \quad \because -b\sin x=\frac{dt}{dx}$

$$I=2\int \frac{t-a}{b}.\frac{1}{(-b)}.\frac{1}{t^2}dt \Rightarrow I=\frac{2}{-b^2}\int \frac{t-a}{t^2}dt \Rightarrow I=\frac{2}{-b^2}\left[\int \frac{t}{t^2}dt-\int \frac{a}{t^2}dt\right]$$

$$\Rightarrow I=\frac{2}{-b^2}\left[\int \frac{1}{t}dt-a\int \frac{1}{t^2}dt\right] \Rightarrow I=\frac{2}{-b^2}\left[\log|t|-a\left(-\frac{1}{t}\right)+c\right]$$

$$\Rightarrow I=\frac{2}{-b^2}\left[\log|a+b\cos x|+\frac{a}{a+b\cos x}\right]+c \quad \therefore I=-\frac{2}{b^2}\left[\log|a+b\cos x|+\frac{a}{a+b\cos x}\right]+c$$

84. $\int \frac{1}{3-5x}dx$

Sol. Let $I=\int \frac{1}{3-5x}dx$, Put $3-5x=t \Rightarrow -5=\frac{dt}{dx} \Rightarrow dx=\frac{dt}{-5}$

$$I=\int \frac{1}{t}.\frac{dt}{-5} \Rightarrow I=-\frac{1}{5}\int \frac{1}{t}dt \quad \therefore I=-\frac{1}{5}\log|3-5x|+c$$

85. $\int \sqrt{1+x}dx$

Sol. Let $I=\int \sqrt{1+x}\,dx$, Put $1+x=t \Rightarrow dx=dt$

$$I=\int \sqrt{t}\,dt \Rightarrow I=\frac{t^{3/2}}{3/2}+c \quad \therefore I=\frac{2}{3}(1+x)^{3/2}+c$$

86. $\int x^2e^{x^3}\cos\left(e^{x^3}\right)dx$

Sol. Let $I=\int x^2e^{x^3}\cos\left(e^{x^3}\right)dx$, Put $e^{x^3}=t \Rightarrow e^{x^3}.3x^2=\frac{dt}{dx} \Rightarrow e^{x^3}.x^2dx=\frac{dt}{3}$

$$I=\int \cos(t).\frac{dt}{3} \Rightarrow I=\frac{1}{3}\int \cos(t)dt \Rightarrow I=\frac{1}{3}\sin(t)+c \quad \therefore I=\frac{1}{3}\sin\left(e^{x^3}\right)+c$$

87. $\int \frac{e^{m\tan^{-1}x}}{1+x^2}dx$

Sol. Let $I=\int \frac{e^{m\tan^{-1}x}}{1+x^2}dx$, Put $m\tan^{-1}x=t \Rightarrow m.\frac{1}{1+x^2}=\frac{dt}{dx} \Rightarrow \frac{1}{1+x^2}dx=\frac{dt}{m}$

$$I=\int e^t.\frac{dt}{m} \Rightarrow I=\frac{1}{m}.e^t+c \quad \therefore I=\frac{1}{m}e^{m\tan^{-1}x}+c$$

88. $\int \frac{(x+1)e^x}{\cos^2\left(xe^x\right)}dx$

Sol. Let $I=\int \frac{(x+1)e^x}{\cos^2\left(xe^x\right)}dx$, Put $xe^x=t \Rightarrow xe^x+e^x.1=\frac{dt}{dx} \Rightarrow e^x(x+1)=\frac{dt}{dx} \Rightarrow (x+1)e^xdx=dt$

$$I=\int\frac{dt}{\cos^2(t)} \Rightarrow I=\int\sec^2(t)\,dt \Rightarrow I=\tan(t)+c \quad \therefore\ I=\tan\left(xe^x\right)+c$$

89. $\int\frac{e^{\sqrt{x}}\cos\left(e^{\sqrt{x}}\right)}{\sqrt{x}}dx$

Sol. Let $I=\int\frac{e^{\sqrt{x}}\cos\left(e^{\sqrt{x}}\right)}{\sqrt{x}}dx$, Put $e^{\sqrt{x}}=t \Rightarrow e^{\sqrt{x}}.\frac{1}{2\sqrt{x}}=\frac{dt}{dx} \Rightarrow \frac{e^{\sqrt{x}}}{\sqrt{x}}dx=2\,dt$

$$I=\int\cos(t)2\,dt \Rightarrow I=2\int\cos(t)\,dt \Rightarrow I=2\sin(t)+c \quad \therefore\ I=2\sin\left(e^{\sqrt{x}}\right)+c$$

90. $\int\sqrt{e^x-1}\,dx$

Sol. Let $I=\int\sqrt{e^x-1}\,dx$, Put $e^x-1=t^2 \Rightarrow e^x=t^2+1 \Rightarrow e^x=2t\frac{dt}{dx} \Rightarrow dx=\frac{2t}{e^x}dt \Rightarrow dx=\frac{2t}{t^2+1}dt$

$$I=\int\sqrt{t^2}.\frac{2t}{t^2+1}dt \Rightarrow I=\int\frac{2t^2}{t^2+1}dt=2\int\frac{t^2+1-1}{t^2+1}dt=2\int\left(1-\frac{1}{t^2+1}\right)dt$$

$$\Rightarrow I=2\left[t-\tan^{-1}t\right]+c=2\left[\sqrt{e^x-1}-\tan^{-1}\sqrt{e^x-1}\right]+c$$

91. $I=\int\frac{1}{x-\sqrt{x}}dx$

Sol. Let $I=\int\frac{1}{x-\sqrt{x}}dx \Rightarrow I=\int\frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)}dx$, Put $\sqrt{x}-1=t \Rightarrow \frac{1}{2\sqrt{x}}=\frac{dt}{dx} \Rightarrow \frac{1}{\sqrt{x}}dx=2dt$

$$I=\int\frac{1}{t}\frac{dt}{2} \quad \therefore\ I=\frac{1}{2}\log|t|+c=\frac{1}{2}\log\left(\sqrt{x}-1\right)+c$$

92. $\int\frac{\sec^2\left(2\tan^{-1}x\right)}{1+x^2}dx$

Sol. Let $I=\int\frac{\sec^2\left(2\tan^{-1}x\right)}{1+x^2}dx$, Put $2\tan^{-1}x=t \Rightarrow 2\frac{1}{1+x^2}=\frac{dt}{dx} \Rightarrow \frac{1}{1+x^2}dx=\frac{dt}{2}$

$$I=\int\sec^2(t).\frac{dt}{2} \Rightarrow I=\frac{1}{2}\tan(t)+c \quad \therefore\ I=\frac{1}{2}\tan\left(2\tan^{-1}x\right)+c$$

93. $\int\frac{1+\sin 2x}{x+\sin^2 x}dx$

Sol. Let $I=\int\frac{1+\sin 2x}{x+\sin^2 x}dx$, Put $x+\sin^2 x=t \Rightarrow 1+2\sin x.\cos x=\frac{dt}{dx} \Rightarrow (1+\sin 2x)dx=dt$

$$I=\int\frac{dt}{t} \Rightarrow I=\log|t|+c \quad \therefore\ I=\log\left|x+\sin^2 x\right|+c$$

94. $\int\frac{1-\tan x}{x+\log(\cos x)}dx$

Sol. Let $I=\int\frac{1-\tan x}{x+\log(\cos x)}dx$, Put $x+\log(\cos x)=t \Rightarrow 1+\frac{1}{\cos x}(-\sin x)=\frac{dt}{dx} \Rightarrow (1-\tan x)dx=dt$

$$I=\int\frac{1}{t}.dt \Rightarrow I=\log|t|+c \quad \therefore\ I=\log\left|x+\log(\cos x)\right|+c$$

95. $\int \frac{1+\cot x}{x+\log \sin x} dx$

Sol. Let $I = \int \frac{1+\cot x}{x+\log(\sin x)} dx$, Put $x+\log(\sin x) = t \Rightarrow 1+\frac{1}{\sin x}.\cos x = \frac{dt}{dx} \Rightarrow (1+\cot x)dx = dt$

$I = \int \frac{dt}{t} \quad \Rightarrow I = \log|t|+c \quad \therefore I = \log\left|x+\log(\sin x)\right|+c$

96. $\int \frac{\tan x.\sec^2 x}{1-\tan^2 x} dx$

Sol. Let $I = \int \frac{\tan x.\sec^2 x}{1-\tan^2 x} dx$, Put $1-\tan^2 x = t \Rightarrow 0-2\tan x.\sec^2 x = \frac{dt}{dx} \Rightarrow \tan x.\sec^2 x dx = \frac{dt}{-2}$

$I = \int \frac{1}{t}.\left(\frac{dt}{-2}\right) \quad \Rightarrow I = -\frac{1}{2}\log|t|+c \quad \therefore I = -\frac{1}{2}\log\left|1-\tan^2 x\right|+c$

97. $\int \frac{\sin(2\tan^{-1} x)}{1+x^2} dx$

Sol. Let $I = \int \frac{\sin(2\tan^{-1} x)}{1+x^2} dx$, Put $2\tan^{-1} x = t \Rightarrow 2.\frac{1}{1+x^2} = \frac{dt}{dx} \Rightarrow \frac{1}{1+x^2} dx = \frac{dt}{2}$

$I = \int \sin(t).\frac{dt}{2} \quad \Rightarrow I = -\frac{1}{2}\cos(t)+c \quad \therefore I = -\frac{1}{2}\cos(2\tan^{-1} x)+c$

98. $\int \frac{1}{x^{1/2}+x^{1/3}} dx$

Sol. Let $I = \int \frac{1}{x^{1/2}+x^{1/3}} dx$

LCM of 2 and 3 is 6. Then put $x = t^6 \Rightarrow dx = 6t^5 dt$

$I = \int \frac{6t^5}{t^3+t^2} dt \quad \Rightarrow I = \int \frac{6t^5}{t^2(1+t)} dt \quad \Rightarrow I = 6\int \frac{t^3}{t+1} dt \quad \Rightarrow I = 6\int \frac{t^3+1-1}{t+1} dt$

$\Rightarrow I = 6\left[\int \frac{(t+1)(t^2-t+1)}{t+1} dt - \int \frac{1}{t+1} dt\right] \quad \Rightarrow I = 6\left[\frac{t^3}{3}-\frac{t^2}{2}+t-\log|t+1|\right]+c$

$\Rightarrow I = \left[2t^3-3t^2+6t-6\log|t+1|\right]+c \quad \therefore I = 2\sqrt{x}-3x^{1/3}+6x^{1/6}-6\log\left|1+x^{1/6}\right|+c$

99. $\int (\sin^{-1} x)^2 dx$

Sol. Let $I = \int (\sin^{-1} x)^2 dx$, Put $\sin^{-1} x = t \quad \therefore x = \sin t$

$\frac{1}{\sqrt{1-x^2}} = \frac{dt}{dx} \Rightarrow dx = \sqrt{1-x^2}\, dt \quad \Rightarrow dx = \sqrt{1-\sin^2 t}\, dt \quad \Rightarrow dx = \cos t\, dt$

$\Rightarrow I = \int t^2 \cos t\, dt \quad \Rightarrow I = t^2 \int \cos t\, dt - \int\left[\frac{d(t^2)}{dt}\int \cos t\, dt\right] dt$

$\Rightarrow I = t^2 \sin t - \int 2t.\sin t\, dt \quad \Rightarrow t^2 \sin t - 2\left\{t\int \sin t\, dt - \int\left[\frac{d(t)}{dt}\int \sin t\, dt\right] dt\right\}$

$\Rightarrow I = t^2 \sin t - 2\left[-t\cos t + \int 1.\cos t\, dt\right] \quad \Rightarrow I = t^2 \sin t + 2t\cos t - 2\sin t + c$

$\therefore \quad I = x\left(\sin^{-1} x\right)^2 + 2\sqrt{1-x^2}\,\sin^{-1} x - 2x + c$

100. $\int \frac{2x\tan^{-1} x^2}{1+x^4}\,dx$

Sol. Let $I = \int \frac{2x\tan^{-1}\left(x^2\right)}{1+x^4}\,dx$, Put $\tan^{-1}\left(x^2\right) = t \;\Rightarrow \frac{1}{1+\left(x^2\right)^2}.2x = \frac{dt}{dx} \;\Rightarrow \frac{2x}{1+x^4}\,dx = dt$

$$I = \int t.dt \quad \Rightarrow I = 1.\frac{t^2}{2} + c \quad \Rightarrow I = \frac{t^2}{2} + c \qquad \therefore\; I = \frac{\left\{\tan^{-1}\left(x^2\right)\right\}^2}{2} + c$$

101. $\int \frac{x^2+1}{x^4+1}\,dx$

Sol. Let $I = \int \frac{x^2+1}{x^4+1}\,dx$

Divide Numerator and Denominator by x^2, $I = \int \frac{\left(1+\frac{1}{x^2}\right)}{x^2+\frac{1}{x^2}+2-2}\,dx \quad \Rightarrow I = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}\,dx$

$$\Rightarrow I = \int \frac{\left(1+\frac{1}{x^2}\right)}{(x)^2 - 2.x.\frac{1}{x} + \left(\frac{1}{x}\right)^2 + 2}\,dx \quad \Rightarrow I = \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2 + \left(\sqrt{2}\right)^2}\,dx$$

Put $t = x - \frac{1}{x} \;\Rightarrow dt = \left(1+\frac{1}{x^2}\right)dx$, $\; I = \int \frac{dt}{(t)^2 + \left(\sqrt{2}\right)^2} \;\Rightarrow I = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right) + c$

$$\Rightarrow I = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right) + c \quad \therefore\; I = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + c$$

102. $\int \frac{(\sin x + \cos x)}{\sqrt{\sin 2x}}$

Sol. By given integral we get

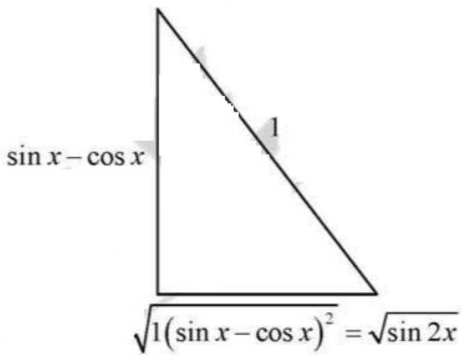
$\int \frac{dt}{1-t^2} = \sin^{-1} t \;\; = \sin^{-1}(\sin x - \cos x)$

$= \tan^{-1}\left(\frac{\sin x - \cos x}{\sqrt{\sin 2x}}\right) \;\; = \tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x}}\right)$

Put $\sin x - \cos x = t \qquad \ldots$ (i)

$\Rightarrow 1 - \sin 2x = t^2 \;\; \Rightarrow \sin 2x = 1 - t^2$

and (i) $\Rightarrow (xosx + \sin x)\,dx = dt$



EXERCISE 13B (Pg. No.: 658)

Evaluate the following integrals:

1. (i) $\int \sin^2 x\, dx$ (ii) $\int \cos^2 x\, dx$

Sol. (i) Let $I=\int \sin^2 x\, dx \Rightarrow I=\int \frac{1-\cos 2x}{2}dx \Rightarrow I=\frac{1}{2}\int(1-\cos 2x)dx$

$\Rightarrow I=\frac{1}{2}\left[x-\frac{\sin 2x}{2}\right]+c \quad \therefore\ I=\frac{x}{2}-\frac{\sin 2x}{4}+c$

(ii) Let $I=\int \cos^2 x\, dx \Rightarrow I=\int \frac{1+\cos 2x}{2}dx \Rightarrow I=\frac{1}{2}\int(1+\cos 2x)dx$

$\Rightarrow I=\frac{1}{2}\left[x+\frac{\sin 2x}{2}\right]+c \quad \therefore\ I=\frac{x}{2}+\frac{\sin 2x}{4}+c$

2. (i) $\int \cos^2\left(\frac{x}{2}\right)dx$ (ii) $\int \cot^2\left(\frac{x}{2}\right)dx$

Sol. (i) Let $I=\int \cos^2\left(\frac{x}{2}\right)dx \Rightarrow I=\int\left(\frac{1+\cos x}{2}\right)dx \Rightarrow I=\frac{1}{2}\int(1+\cos x)dx$

$\Rightarrow I=\frac{1}{2}(x+\sin x)+c \quad \therefore\ I=\frac{x}{2}+\frac{\sin x}{2}+c$

(ii) Let $I=\int \cot^2\left(\frac{x}{2}\right)dx \Rightarrow I=\int\left\{\operatorname{cosec}^2\left(\frac{x}{2}\right)-1\right\}dx \Rightarrow I=\int \operatorname{cosec}^2\left(\frac{x}{2}\right)dx-\int dx$

$\Rightarrow I=\frac{-\cot\frac{x}{2}}{1/2}-x+c \quad \therefore\ I=-2\cot\left(\frac{x}{2}\right)-x+c$

3. (i) $\int \sin^2 nx\, dx$ (ii) $\int \sin^5 x\, dx$

Sol. (i) Let $I=\int \sin^2 nx\, dx \Rightarrow I=\int \frac{1-\cos 2nx}{2}dx \Rightarrow I=\frac{1}{2}\int(1-\cos 2nx)dx$

$\Rightarrow I=\frac{1}{2}\left[x-\frac{\sin 2nx}{2n}\right]+c \quad \therefore\ I=\frac{x}{2}-\frac{\sin 2nx}{4n}+c$

(ii) Let $I=\int \sin^5 x\, dx \Rightarrow I=\int(\sin^2 x)^2.\sin x\, dx \Rightarrow I=\int(1-\cos^2 x)^2.\sin x\, dx$

Put $\cos x=t \Rightarrow -\sin x=\frac{dt}{dx} \Rightarrow \sin x\, dx=-dt$

$I=\int(1-t^2)^2(-dt) \Rightarrow I=\int(1-2t^2+t^4)(-dt) \Rightarrow I=-\left[t-2\frac{t^3}{3}+\frac{t^5}{5}\right]+c$

$\Rightarrow I=-\cos x+\frac{2}{3}\cos^3 x-\frac{1}{5}\cos^5 x+c \quad \therefore\ I=\frac{2}{3}\cos^3 x-\cos x-\frac{1}{5}\cos^5 x+c$

4. $\int \cos^3(3x+5)dx$

Sol. Let $I=\int \cos^3(3x+5)dx \Rightarrow I=\int \cos^2(3x+5).\cos(3x+5)dx$

$\Rightarrow I=\int\{1-\sin^2(3x+5)\}\cos(3x+5)dx$

Put $\sin(3x+5)=t \Rightarrow \cos(3x+5).3=\frac{dt}{dx} \Rightarrow \cos(3x+5)dx=\frac{dt}{3}$

$$I=\int\left(1-t^2\right)\frac{dt}{3} \Rightarrow I=\frac{1}{3}\int\left(1-t^2\right)dt \Rightarrow I=\frac{1}{3}\left[t-\frac{t^3}{3}\right]+c$$

$$\therefore\ I=\frac{1}{3}\sin(3x+5)-\frac{\sin^3(3x+5)}{9}+c$$

5. $\int\sin^7(3-2x)dx$

Sol. Let $I=\int\sin^7(3-2x)dx$

$$\Rightarrow I=\int\left\{\sin^2(3-2x)\right\}^3.\sin(3-2x)\,dx \Rightarrow \int\left\{1-\cos^2(3-2x)\right\}^3.\sin(3-2x)\,dx$$

Put $\cos(3-2x)=t \Rightarrow -\sin(3-2x)(-2)=\frac{dt}{dx} \Rightarrow \sin(3-2x)dx=\frac{dt}{2}$

$$I=\int\left(1-t^2\right)^3\frac{dt}{2} \Rightarrow I=\frac{1}{2}\int\left\{(1)^3-\left(t^2\right)^3-3(1)^2t^2+3\left(t^2\right)^2.1\right\}dt$$

$$\Rightarrow I=\frac{1}{2}\int\left(1-t^6-3t^2+3t^4\right)dt \Rightarrow I=\frac{1}{2}\left[t-\frac{t^7}{7}-\frac{3t^3}{3}+\frac{3t^5}{5}\right]+c$$

$$\therefore\ I=\frac{1}{2}\cos(3-2x)-\frac{1}{14}\cos^7(3-2x)-\frac{1}{2}\cos^3(3-2x)+\frac{3}{10}\cos^5(3-2x)+c$$

6. (i) $\int\frac{1-\cos 2x}{1+\cos 2x}dx$ (ii) $I=\int\frac{1+\cos 2x}{1-\cos 2x}dx$

Sol. (i) Let $I=\int\frac{1-\cos 2x}{1+\cos 2x}dx \Rightarrow I=\int\frac{2\sin^2 x}{2\cos^2 x}dx \Rightarrow I=\int\tan^2 x\,dx$

$$\Rightarrow I=\int\left(\sec^2 x-1\right)dx \quad \therefore\ I=\tan x-x+c$$

(ii) Let $I=\int\frac{1+\cos 2x}{1-\cos 2x}dx \Rightarrow I=\int\frac{2\cos^2 x}{2\sin^2 x}dx \Rightarrow I=\int\cot^2 x\,dx$

$$\Rightarrow I=\int\left(\operatorname{cosec}^2 x-1\right)dx \quad \therefore\ I=-\cot x-x+c$$

7. (i) $\int\frac{1-\cos x}{1+\cos x}dx$ (ii) $\int\frac{1+\cos x}{1-\cos x}dx$

Sol. (i) Let $I=\int\frac{1-\cos x}{1+\cos x}dx \Rightarrow I=\int\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}dx$

$$\Rightarrow I=\int\tan^2\left(\frac{x}{2}\right)dx \Rightarrow I=\int\left(\sec^2\frac{x}{2}-1\right)dx \quad \therefore\ I=2\tan\frac{x}{2}-x+c$$

(ii) Let $I=\int\frac{1+\cos x}{1-\cos x}dx \Rightarrow I=\int\frac{2\cos^2\frac{x}{2}}{2\sin^2\frac{x}{2}}dx \Rightarrow I=\int\cot^2\frac{x}{2}dx$

$$I=\int\left(\operatorname{cosec}^2\frac{x}{2}-1\right)dx \Rightarrow I=\frac{-\cot\frac{x}{2}}{1/2}-x+c \quad \therefore\ I=-2\cot\frac{x}{2}-x+c$$

8. $\int\sin 3x.\sin 4x\,dx$

Sol. Let $I=\int \sin 3x.\sin 4x\,dx \quad \Rightarrow I=\frac{1}{2}\int 2\sin 3x\sin 4x\,dx \quad \Rightarrow I=\frac{1}{2}\int\{\cos(3x-4x)-\cos(3x+4x)\}dx$

$\Rightarrow I=\frac{1}{2}\int(\cos x-\cos 7x)dx \quad \Rightarrow I=\frac{1}{2}\left[\sin x-\frac{\sin 7x}{7}\right]+c \quad \therefore I=\frac{1}{2}\sin x-\frac{1}{14}\sin 7x+c$

9. $\int \cos 4x\cos 3x dx$

Sol. Let $I=\int \cos 4x.\cos 3x\,dx \quad \Rightarrow I=\frac{1}{2}\int\{\cos(4x+3x)+\cos(4x-3x)\}dx$

$I=\frac{1}{2}\int\{\cos 7x+\cos x\}dx \quad \Rightarrow I=\frac{1}{2}\left[\frac{\sin 7x}{7}+\sin x\right]+c$

10. $\int \sin 4x\sin 8x\,dx$

Sol. Let $I=\int \sin 4x\sin 8x\,dx \quad \Rightarrow I=\frac{1}{2}\int 2\sin 4x\sin 8x\,dx$

$I=\frac{1}{2}\int\{\cos(4x-8x)-\cos(4x+8x)\}dx \quad \Rightarrow I=\frac{1}{2}\int\{\cos(-4x)-\cos(12x)\}dx$

$\Rightarrow I=\frac{1}{2}\int(\cos 4x-\cos 12x)dx \quad \Rightarrow I=\frac{1}{2}\left[\frac{\sin 4x}{4}-\frac{\sin 12x}{12}\right]+c \quad \therefore I=\frac{\sin 4x}{8}-\frac{\sin 12x}{24}+c$

11. $\int \sin 6x\cos x\,dx$

Sol. Let $I=\int \sin 6x\cos x\,dx \quad \Rightarrow I=\frac{1}{2}\int 2\sin 6x\cos x\,dx$

$\Rightarrow I=\frac{1}{2}\int\{\sin(6x+x)+\sin(6x-x)\}dx \quad \Rightarrow I=\frac{1}{2}\int\{\sin 7x+\sin 5x\}dx$

$\Rightarrow I=\frac{1}{2}\left[-\frac{\cos 7x}{7}-\frac{\cos 5x}{5}\right]+c \quad \therefore I=-\frac{\cos 7x}{14}-\frac{\cos 5x}{10}+c$

12. $\int \sin x\sqrt{1+\cos 2x}\,dx$

Sol. Let $I=\int \sin x\sqrt{1+\cos 2x}\,dx \quad \Rightarrow I=\int \sin x\sqrt{2\cos^2 x}\,dx$

$\Rightarrow I=\int \sin x\sqrt{2}\cos x\,dx \quad \Rightarrow I=\sqrt{2}\int \sin x\cos x\,dx$

Put $\sin x=t \quad \Rightarrow \cos x=\frac{dt}{dx} \quad \Rightarrow \cos x\,dx=dt$

$I=\sqrt{2}\int t.\,dt \quad \Rightarrow I=\sqrt{2}.\frac{t^2}{2}+c \quad \therefore I=\frac{\sin^2 x}{\sqrt{2}}+c$

13. $\int \cos^4 x\,dx$

Sol. Let $I=\int \cos^4 x\,dx \quad \Rightarrow I=\int \cos^2 x.\cos^2 x\,dx \quad \Rightarrow I=\int\left(\frac{1+\cos 2x}{2}\right)^2 dx$

$\Rightarrow \frac{1}{4}\int(1+2\cos 2x+\cos^2 2x)dx \quad \Rightarrow I=\frac{1}{4}\left[\int dx+2\int \cos 2x\,dx+\int \cos^2 2x\,dx\right]$

$\Rightarrow I=\frac{1}{4}\left[x+2\frac{\sin 2x}{2}+\int\left(\frac{1+\cos 4x}{2}\right)\right]dx \quad \Rightarrow I=\frac{1}{4}\left[x+\sin 2x+\frac{1}{2}\left(x+\frac{\sin 4x}{4}\right)\right]+c$

$\Rightarrow I=\frac{1}{4}\left[\frac{3x}{2}+\sin 2x+\frac{\sin 4x}{8}\right]+c \quad \therefore I=\frac{3}{8}x+\frac{1}{4}\sin 2x+\frac{1}{32}\sin 4x+c$

14. $\int \cos 2x \cos 4x \cos 6x \, dx$

Sol. Let $I = \int \cos 2x . \cos 4x . \cos 6x \, dx \quad \Rightarrow I = \frac{1}{2}\int 2\cos 2x . \cos 4x . \cos 6x \, dx$

$\Rightarrow I = \frac{1}{2}\int \cos 2x \left(2\cos 4x \cos 6x\right) dx \quad \Rightarrow I = \frac{1}{2}\int \cos 2x \left\{\cos\left(4x+6x\right)+\cos\left(4x-6x\right)\right\} dx$

$\Rightarrow I = \frac{1}{2}\int \cos 2x \left(\cos 10x + \cos 2x\right) dx \quad \Rightarrow I = \frac{1}{2}\int \cos 2x \cos 10x \, dx + \frac{1}{2}\int \cos^2 2x \, dx$

$\Rightarrow I = \frac{1}{2}.\frac{1}{2}\int 2\cos 2x . \cos 10x \, dx + \frac{1}{2}\int \frac{1+\cos 4x}{2} dx$

$\Rightarrow I = \frac{1}{4}\int \left\{\cos\left(2x+10x\right)+\cos\left(2x-10x\right)\right\} dx + \frac{1}{4}\int \left(1+\cos 4x\right) dx$

$\Rightarrow I = \frac{1}{4}\int \left\{\cos 12x + \cos 8x\right\} dx + \frac{1}{4}\left(x + \frac{\sin 4x}{4}\right)$

$\Rightarrow I = \frac{1}{4}\left[\frac{\sin 12x}{12}+\frac{\sin 8x}{8}\right] + \frac{1}{4}x + \frac{\sin 4x}{16} + c \quad \Rightarrow I = \frac{\sin 12x}{48} + \frac{\sin 8x}{32} + \frac{x}{4} + \frac{\sin 4x}{16} + c$

$\therefore \; I = \frac{x}{4} + \frac{\sin 4x}{16} + \frac{\sin 8x}{32} + \frac{\sin 12x}{48} + c$

15. $\int \sin^3 x \cos x \, dx$

Sol. Let $I = \int \sin^3 x . \cos x \, dx$, Put $\sin x = t \;\Rightarrow\; \cos x = \frac{dt}{dx} \;\Rightarrow\; \cos x \, dx = dt$

$I = \int t^3 \, dt \quad \Rightarrow I = \frac{t^4}{4} + c \quad \therefore \; I = \frac{\sin^4 x}{4} + c$

16. $\int \sec^4 x \, dx$

Sol. Let $I = \int \sec^4 x \, dx \quad \Rightarrow I = \int \sec^2 x . \sec^2 x \, dx \quad \Rightarrow I = \int \left(1+\tan^2 x\right)\sec^2 x \, dx$

Put $\tan x = t \;\Rightarrow\; \sec^2 x = \frac{dt}{dx} \;\Rightarrow\; \sec^2 x \, dx = dt$

$I = \int \left(1+t^2\right) dt \quad \Rightarrow I = t + \frac{t^3}{3} + c \quad \therefore \; I = \tan x + \frac{\tan^3 x}{3} + c$

17. $\int \cos^3 x . \sin^4 x \, dx$

Sol. Let $I = \int \cos^3 x . \sin^4 x \, dx \quad \Rightarrow I = \int \sin^4 x . \cos^2 x . \cos x \, dx \quad \Rightarrow I = \int \sin^4 x \left(1-\sin^2 x\right) . \cos x \, dx$

Put $\sin x = t \quad \Rightarrow \cos x = \frac{dt}{dx} \quad \Rightarrow \cos x \, dx = dt$

$I = \int t^4\left(1-t^2\right) dt \quad \Rightarrow I = \int t^4 dt - \int t^6 \, dt \quad \Rightarrow I = \frac{t^5}{5} - \frac{t^7}{7} + c \quad \therefore \; I = \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + c$

18. $\int \cos^4 x \sin^3 x \, dx$

Sol. Let $I = \int \cos^4 x \sin^3 x \, dx \quad \Rightarrow I = \int \cos^4 x . \sin^2 x . \sin x \, dx \quad \Rightarrow I = \int \cos^4 x . \left(1-\cos^2 x\right)\sin x \, dx$

Put $\cos x = t \;\Rightarrow\; -\sin x = \frac{dt}{dx} \;\Rightarrow\; \sin x \, dx = -dt$

$I = \int t^4\left(1-t^2\right)\left(-dt\right) \quad \Rightarrow I = \int t^4\left(t^2-1\right) dt \quad \Rightarrow I = \frac{t^7}{7} - \frac{t^5}{5} + c$

$\therefore \quad I=\frac{\cos^7 x}{7}-\frac{\cos^5 x}{5}+c$

19. $\int \sin^{2/3} x \cos^3 x \, dx$

Sol. Let $I=\int \sin^{2/3} x \cos^3 x \, dx \quad \Rightarrow I=\int \sin^{2/3} x . \cos^2 x \cos x \, dx \quad \Rightarrow I=\int \sin^{2/3} x\left(1-\sin^2 x\right)\cos x \, dx$

Put $\sin x=t \quad \Rightarrow \cos x=\frac{dt}{dx} \quad \Rightarrow \cos x \, dx=dt$

$I=\int t^{2/3}\left(1-t^2\right)dt \quad \Rightarrow I=\int t^{2/3}dt-\int t^{2/3}.t^2 \, dt \quad \Rightarrow I=\int t^{2/3}dt-\int t^{8/3}dt$

$\Rightarrow I=\frac{t^{5/3}}{5/3}-\frac{t^{11/3}}{11/3}+c \quad \therefore \ I=\frac{3}{5}\sin^{5/3} x-\frac{3}{11}\sin^{11/3} x+c$

20. $\int \cos^{3/5} x \sin^3 x \, dx$

Sol. Let $I=\int \cos^{3/5} x \sin^3 x \, dx$

$\Rightarrow I=\int \cos^{3/5} x \sin^2 x . \sin x \, dx \quad \Rightarrow I=\int \cos^{3/5} x\left(1-\cos^2 x\right).\sin x \, dx$

Put $\cos x=t \quad \Rightarrow -\sin x=\frac{dt}{dx} \quad \Rightarrow \sin dx=-dt$

$I=\int t^{3/5}\left(1-t^2\right)(-dt) \quad \Rightarrow \int t^{3/5}\left(t^2-1\right)dt \quad \Rightarrow I=\int t^{3/5}t^2 dt-\int t^{3/5}dt$

$\Rightarrow I=\int t^{13/5}dt-\int t^{3/5}dt \quad \Rightarrow I=\frac{t^{18/5}}{18/5}-\frac{t^{8/5}}{8/5}+c \quad \therefore \ I=\frac{5}{18}\cos^{18/5} x-\frac{5}{8}\cos^{8/5} x+c$

21. $\int \operatorname{cosec}^4(2x)dx$

Sol. Let $I=\int \operatorname{cosec}^4(2x)dx$

$\Rightarrow I=\int\left\{\operatorname{cosec}^2(2x)\right\}^2 dx \quad \Rightarrow I=\int\left\{1+\cot^2(2x)\right\}\operatorname{cosec}^2(2x)dx$

Put $\cot(2x)=t \quad \Rightarrow -\operatorname{cosec}^2(2x).2=\frac{dt}{dx} \quad \Rightarrow \operatorname{cosec}^2(2x)dx=\frac{dt}{-2}$

$I=\int\left(1+t^2\right)\left(-\frac{dt}{2}\right) \quad \Rightarrow I=-\frac{1}{2}\int\left(1+t^2\right)dt \quad \Rightarrow I=-\frac{1}{2}\left[t+\frac{t^3}{3}\right]+c$

$\therefore \quad I=-\frac{1}{2}\cot(2x)-\frac{1}{6}\cot^3(2x)+c$

22. $\int \frac{\cos 2x}{\cos x}dx$

Sol. Let $I=\int \frac{\cos 2x}{\cos x}dx \quad \Rightarrow I=\int \frac{2\cos^2 x-1}{\cos x}dx \quad \Rightarrow I=2\int \cos x \, dx-\int \sec x \, dx$

$\therefore \quad I=2\sin x-\log\left|\sec x+\tan x\right|+c$

23. $\int \frac{\cos(x)}{\cos(x+\alpha)}dx$

Sol. Let $I=\int \frac{\cos x}{\cos(x+\alpha)}dx$, Put $x+\alpha=t \quad \Rightarrow x=t-\alpha \Rightarrow 1=\frac{dt}{dx} \quad \Rightarrow dx=dt$

$\Rightarrow I=\int \frac{\cos(t-\alpha)}{\cos t}dt \quad \Rightarrow I=\int \frac{\cos t.\cos\alpha+\sin t.\sin\alpha}{\cos t}dt$

$$\Rightarrow I = \cos\alpha \int dt + \sin\alpha \int \tan t\, dt \qquad \Rightarrow I = \cos\alpha . t + \sin\alpha\left(-\log \cos t\right) + c$$

$$\therefore \; I = \left(x+\alpha\right)\cos\alpha - \sin\alpha \log \cos \left| x+\alpha \right| + c$$

$$I = x\cos\alpha - \sin\alpha \log\cos\left| x+\alpha \right| + c$$

24. $\int \cos^3 x . \sin 2x\, dx$

Sol. Let $I = \int \cos^3 x .\, \sin 2x\, dx \quad \Rightarrow I = \int \cos^3 x . 2\sin x . \cos x\, dx \quad \Rightarrow I = 2\int \cos^4 x\, \sin x\, dx$

Put $\cos x = t \quad \Rightarrow -\sin x = \dfrac{dt}{dx} \quad \Rightarrow \sin x\, dx = -dt$

$$\Rightarrow I = 2\int t^4 .\left(-dt\right) \;=\; -2\frac{t^5}{5} + c \quad \therefore\; I = -\frac{2}{5}\cos^5 x + c$$

25. $\int \dfrac{\cos^9 x}{\sin x} dx$

Sol. Let $I = \int \dfrac{\cos^9 x}{\sin x} dx \;\Rightarrow I = \int \dfrac{\left(\cos^2 x\right)^4 . \cos x}{\sin x} dx \;\Rightarrow I = \int \dfrac{\left(1-\sin^2 x\right)^4}{\sin x} . \cos x\, dx$

Put $\sin x = t \quad \Rightarrow \cos x = \dfrac{dt}{dx} \quad \Rightarrow \cos x\;\; dx = dt$

$$I = \int \frac{\left(1-t^2\right)^4}{t} dt \quad \Rightarrow I = \int\left(\frac{1-4t^2+6t^4-4t^6+t^8}{t}\right)dt$$

$$\Rightarrow I = \int \frac{1}{t}dt - 4\int t\, dt + 6\int t^3 dt - 4\int t^5 dt + \int t^7\, dt \quad \Rightarrow I = \log\left|t\right| - 4\frac{t^2}{2} + 6\frac{t^4}{4} - 4\frac{t^6}{6} + \frac{t^8}{8} + c$$

$$\therefore\; I = \log\left|\sin x\right| - 2\sin^2 x + \frac{3}{2}\sin^4 x - \frac{2}{3}\sin^6 x + \frac{1}{8}\sin^8 x + c$$

26. $\int \cos^4\left(2x\right)dx$

Sol. Let $I = \int \cos^4\left(2x\right)dx \quad \Rightarrow I = \int \cos^2\left(2x\right).\cos^2\left(2x\right)dx$

$$\Rightarrow I = \int\left(\frac{1+\cos 4x}{2}\right)\left(\frac{1+\cos 4x}{2}\right)dx \quad \Rightarrow I = \frac{1}{4}\int\left(1+2\cos 4x+\cos^2 4x\right)dx$$

$$\Rightarrow I = \frac{1}{4}\left[x + \frac{2\sin 4x}{4} + \int\left(\frac{1+\cos 8x}{2}\right)dx\right] \quad \Rightarrow I = \frac{1}{4}\left[x + \frac{\sin 4x}{2} + \frac{1}{2}\left(x + \frac{\sin 8x}{8}\right)\right] + c$$

$$\Rightarrow I = \frac{1}{4}\left[\frac{3x}{2} + \frac{\sin 4x}{2} + \frac{\sin 8x}{16}\right] + c \quad \therefore\; I = \frac{3}{8}x + \frac{\sin 4x}{8} + \frac{\sin 8x}{64} + c$$

27. $\int \dfrac{\sin^2 x}{\left(1+\cos x\right)^2} dx$

Sol. Let $I = \int \dfrac{\sin^2 x}{\left(1+\cos x\right)^2} dx \quad \Rightarrow I = \int \dfrac{1-\cos^2 x}{\left(1+\cos x\right)^2} dx \quad \Rightarrow I = \int \dfrac{\left(1+\cos x\right)\left(1-\cos x\right)}{\left(1+\cos x\right)^2} dx$

$$\Rightarrow I = \int \frac{\left(1-\cos x\right)}{\left(1+\cos x\right)} dx \quad \Rightarrow I = \int \frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}} dx \quad \Rightarrow I = \int \tan^2 \frac{x}{2} dx$$

$$\Rightarrow I=\int\left(\sec^2\frac{x}{2}-1\right)dx \quad \Rightarrow I=\frac{\tan\frac{x}{2}}{1/2}-x+c \quad \therefore\ I=2\tan\frac{x}{2}-x+c$$

28. $\int\frac{dx}{3\cos x+4\sin x}$

Sol. Let $I=\int\frac{1}{3\cos x+4\sin x}dx$, Put $3=r\sin\theta$ ...(i) & $4=r\cos\theta$ ...(ii)

Square both side and adding equation (i) & (ii)

$$9+16=r^2\cos^2\theta+r^2\sin^2\theta \quad \Rightarrow 25=r^2\left(\cos^2\theta+\sin^2\theta\right) \quad \Rightarrow r^2=25 \quad \therefore\ r=5$$

Diving equation (i) by (ii), $\frac{r\sin\theta}{r\cos\theta}=\frac{3}{4} \quad \Rightarrow \tan\theta=\frac{3}{4} \quad \therefore\ \theta=\tan^{-1}\left(\frac{3}{4}\right)$

$$I=\int\frac{1}{r\sin\theta\cos x+r\cos\theta\sin x} \quad \Rightarrow I=\int\frac{1}{r\left(\cos x\sin\theta+\sin x\cos\theta\right)}dx$$

$$\Rightarrow I=\int\frac{1}{r\sin\left(x+\theta\right)}dx \quad \Rightarrow I=\frac{1}{r}\int\operatorname{cosec}\left(x+\theta\right)dx \quad \Rightarrow I=\frac{1}{5}\log\left|\tan\left(\frac{x+\theta}{2}\right)\right|+c$$

$$\therefore\ I=\frac{1}{5}\log\left|\tan\left(\frac{x}{2}+\frac{1}{2}\tan^{-1}\frac{3}{4}\right)\right|+c$$

29. $\int\frac{1}{\left(a\cos x+b\sin x\right)^2}dx$

Sol. Let $I=\int\frac{1}{\left(a\cos x+b\sin x\right)^2}dx$, Put $a=r\sin\theta$ ...(i) & $b=r\cos\theta$ ...(ii)

Square both side and adding equation (i) & (ii), $a^2+b^2=r^2\sin^2\theta+r^2\cos\theta$

$$\Rightarrow a^2+b^2=r^2\left(\sin^2\theta+\cos^2\theta\right) \quad \Rightarrow a^2+b^2=r^2 \quad \therefore\ r=\sqrt{a^2+b^2}$$

Dividing equation (i) by (ii), $\frac{r\sin\theta}{r\cos\theta}=\frac{a}{b} \quad \Rightarrow \tan\theta=\frac{a}{b} \quad \therefore\ \theta=\tan^{-1}\left(\frac{a}{b}\right)$

$$I=\int\frac{1}{\left(r\sin\theta\cos x+r\cos\theta\sin x\right)^2} \quad \Rightarrow I=\int\frac{1}{r^2\left(\cos x\sin\theta+\sin x\cos\theta\right)^2}dx$$

$$\Rightarrow I=\int\frac{1}{r^2\sin^2\left(x+\theta\right)}dx \quad \Rightarrow I=\frac{1}{r^2}\int\operatorname{cosec}^2\left(x+\theta\right)dx \quad \Rightarrow I=\frac{1}{r^2}\left\{-\cot\left(x+\theta\right)\right\}+c$$

$$\therefore\ I=-\frac{\cot\left(x+\tan^{-1}\left(\frac{a}{b}\right)\right)}{a^2+b^2}+c$$

30. $\int\frac{1}{\left(\cos x-\sin x\right)}dx$

Sol. Let $I=\int\frac{1}{\cos x-\sin x}dx$, Put $1=r\sin\theta$ ...(i) & $1=r\cos\theta$ ...(ii)

Square both side and adding equation (i) & (ii)

$$1+1=r^2\sin\theta+r^2\cos^2\theta \quad \Rightarrow 2=r^2\left(\sin^2\theta+\cos^2\theta\right) \quad \Rightarrow r^2=2 \quad \therefore\ r=\sqrt{2}$$

Dividing equation (i) by (ii), $\frac{r\sin\theta}{r\cos\theta}=\frac{1}{1}$ $\Rightarrow \tan\theta=1$ $\therefore\ \theta=\frac{\pi}{4}$

$$I=\int\frac{1}{r\sin\theta\cos x-r\cos\theta\sin x}dx \Rightarrow I=\int\frac{1}{r\sin(\theta-x)}dx \Rightarrow I=\frac{1}{r}\int\operatorname{cosec}(\theta-x)\,dx$$

$$\Rightarrow I=-\frac{1}{r}\log\tan\left(\frac{\theta-x}{2}\right)+c \Rightarrow I=-\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{1}{2}.\frac{\pi}{4}-\frac{x}{2}\right)\right|+c$$

$$\therefore\ I=-\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{\pi}{8}-\frac{x}{2}\right)\right|+c$$

31. $I=\int(2\tan x-3\cot x)^2\,dx$

Sol. Let $I=\int(2\tan x-3\cot x)^2\,dx \Rightarrow I=\int\left(4\tan^2 x-2.2\tan x.3\cot x+9\cot^2 x\right)dx$

$$\Rightarrow I=4\int\left(\sec^2 x-1\right)dx-12\int dx+9\int\left(\operatorname{cosec}^2 x-1\right)dx$$

$$\Rightarrow I=4(\tan x-x)-12x+9(-\cot x-x)+c \quad \therefore\ I=4\tan x-9\cot x-25x+c$$

32. $\int\sin x.\sin 2x.\sin 3x\,dx$

Sol. Let $I=\int\sin x.\sin 2x.\sin 3x\,dx \Rightarrow I=\frac{1}{2}\int 2\sin x.\sin 2x.\sin 3x dx$

$$\Rightarrow I=\frac{1}{2}\int\sin x.(2\sin 2x\sin 3x)dx \Rightarrow I=\frac{1}{2}\int\sin x\{\cos(2x-3x)-\cos(2x+3x)\}dx$$

$$\Rightarrow I=\frac{1}{2}\int\sin x\{\cos x-\cos 5x\}dx \Rightarrow I=\frac{1}{2}\int\sin x\cos x\,dx-\frac{1}{2}\int\sin x.\cos 5x\,dx$$

$$\Rightarrow I=\frac{1}{2}.\frac{1}{2}\int 2\sin x\cos x\,dx-\frac{1}{2}.\frac{1}{2}\int 2\sin x\cos 5x\,dx$$

$$\Rightarrow I=\frac{1}{4}\int\sin 2x\,dx-\frac{1}{4}\int\{\sin(x+5x)+\sin(x-5x)\}dx$$

$$\Rightarrow I=\frac{-1}{4}\left(\frac{\cos 2x}{2}\right)-\left[\frac{1}{4}\int(\sin 6x-\sin 4x)dx\right]$$

$$\Rightarrow I=-\frac{1}{8}\cos 2x+\frac{1}{4}\frac{\cos 6x}{6}-\frac{1}{4}.\frac{\cos 4x}{4}+c \qquad \therefore\ I=\frac{\cos 6x}{24}-\frac{\cos 2x}{8}-\frac{\cos 4x}{16}+c$$

33. $\int\frac{1-\cot x}{1+\cot x}dx$

Sol. Let $I=\int\frac{1-\cot x}{1+\cot x}dx \Rightarrow I=\int\frac{1-\frac{\cos x}{\sin x}}{1+\frac{\cos x}{\sin x}}dx \Rightarrow I=\int\frac{\sin x-\cos x}{\sin x+\cos x}dx$

Put $\sin x+\cos x=t \Rightarrow \cos x-\sin x=\frac{dt}{dx} \Rightarrow -(\sin x-\cos x)dx=dt \Rightarrow (\sin x-\cos x)dx=-dt$

$$I=-\int\frac{dt}{t} \Rightarrow I=-\log|t|+c \quad \therefore\ I=-\log|\sin x+\cos x|+c$$

34. $\int \frac{dx}{2\sin x + \cos x + 3}$

Sol. Let $I = \int \frac{1}{2\sin x + \cos x + 3} dx$, Put $\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$, $\cos = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$I = \int \frac{1}{2\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right) + \left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right) + 3} dx \Rightarrow I = \int \frac{1+\tan^2\frac{x}{2}}{4\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2} + 3\left(1+\tan^2\frac{x}{2}\right)} dx$$

$$\Rightarrow I = \int \frac{\sec^2\frac{x}{2}}{4\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2} + 3 + 3\tan^2\frac{x}{2}} dx \Rightarrow I = \int \frac{\sec^2\frac{x}{2}}{2\tan^2\frac{x}{2} + 4\tan\frac{x}{2} + 4} dx$$

Put $\tan\frac{x}{2} = t \Rightarrow \sec^2\frac{x}{2}.\frac{1}{2} = \frac{dt}{dx} \Rightarrow \sec^2\frac{x}{2} dx = 2dt$

$$I = 2\int \frac{dt}{2t^2 + 4t + 4} \Rightarrow I = \frac{2}{2}\int \frac{1}{t^2 + 2t + 2} dt \Rightarrow I = \int \frac{1}{(t)^2 + 2.t.1 + (1)^2 + (1)^2 - (1)^2 + 2}$$

$$\Rightarrow I = \int \frac{1}{(t+1)^2 + (1)^2} dt \Rightarrow I = \tan^{-1}\left(\frac{t+1}{1}\right) + c \quad \therefore I = \tan^{-1}\left(\tan\frac{x}{2} + 1\right) + c$$

EXERCISE 13C (Pg.no.: 678)

Evaluate the following integrals :

1. $\int x\, e^x\, dx$

Sol. Let $I=\int x\, e^x\, dx \Rightarrow I = x\int e^x dx - \int\left[\frac{d(x)}{dx}\int e^x\, dx\right]dx \Rightarrow I = xe^x - \int 1.e^x\, dx$

$\Rightarrow I = x\,e^x - \int e^x\, dx \Rightarrow I = x\,e^x - e^x + c \quad \therefore\ I = e^x(x-1)+c$

2. $\int x\, \cos x\, dx$

Sol. Let $I=\int x\, \cos x\, dx \Rightarrow I = x\int \cos x\, dx - \int\left[\frac{d(x)}{dx}\int \cos x\, dx\right]dx$

$\Rightarrow I = x\sin x - \int 1.\sin x\, dx \Rightarrow I = x\sin x - \int \sin x\, dx \quad \therefore\ I = x\sin x + \cos x + c$

3. $\int x e^{2x} dx$

Sol. Let $I=\int x\, e^{2x} dx \Rightarrow I = x\int e^{2x} dx - \int\left[\frac{d(x)}{dx}\int e^{2x} dx\right]dx \Rightarrow I = x.\frac{e^{2x}}{2} - \int 1.\frac{e^{2x}}{2} dx$

$\Rightarrow I = \frac{x}{2}e^{2x} - \frac{1}{2}\int e^{2x} dx \Rightarrow I = \frac{x}{2}e^{2x} - \frac{1}{2}.\frac{e^{2x}}{2} + c \quad \therefore I = \frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + c$

4. $\int x \sin 3x\, dx$

Sol. Let $I=\int x \sin 3x\, dx \Rightarrow I = x\int \sin 3x\, dx - \int\left[\frac{d(x)}{dx}\int \sin 3x\, dx\right]dx$

$\Rightarrow I = x\left(-\frac{\cos 3x}{3}\right) - \int 1.\left(-\frac{\cos 3x}{3}\right)dx \Rightarrow I = -\frac{x}{3}\cos 3x + \frac{1}{3}\int \cos 3x\, dx$

$\Rightarrow I = -\frac{x}{3}\cos 3x + \frac{1}{3}.\frac{\sin 3x}{3} + c \quad \therefore\ I = -\frac{x\cos 3x}{3} + \frac{\sin 3x}{9} + c$

5. $\int x\, \cos 2x\, dx$

Sol. Let $I=\int x\, \cos 2x\, dx \Rightarrow I = x\int \cos 2x\, dx - \int\left[\frac{d(x)}{dx}\int \cos 2x\, dx\right]dx$

$\Rightarrow I = \frac{x\sin 2x}{2} - \int 1.\frac{\sin 2x}{2}\, dx \Rightarrow I - \frac{x}{2}\sin 2x - \frac{1}{2}\left(-\frac{\cos 2x}{2}\right) + c$

$\therefore\ I = \frac{x}{2}\sin 2x + \frac{\cos 2x}{4} + c$

6. $I = \int x \log 2x\, dx$

Sol. Let $I = \log 2x\int x\, dx - \int\left[\frac{d(\log 2x)}{dx}\int x\, dx\right]dx$

$\Rightarrow I = \log 2x.\frac{x^2}{2} - \int \frac{1}{2x}.2.\frac{x^2}{2}\, dx \Rightarrow I = \frac{x^2}{2}\log 2x - \frac{1}{2}\int x\, dx$

$\Rightarrow I = \frac{x^2}{2}\log 2x - \frac{1}{2}.\frac{x^2}{2} + c \quad \therefore\ I = \frac{x^2}{2}\log 2x - \frac{x^2}{4} + c$

7. $\int x \operatorname{cosec}^2 x \, dx$

Sol. Let $I = \int x \operatorname{cosec}^2 x \, dx \quad \Rightarrow I = x\int \operatorname{cosec}^2 x \, dx - \int\left[\frac{d(x)}{dx}\int \operatorname{cosec}^2 x \, dx\right]dx$

$\Rightarrow I = x(-\cot x) - \int 1.(-\cot x)\, dx \quad \Rightarrow I = -x \cot x + \int \cot x \, dx$

$\therefore \; I = -x \cot x + \log|\sin x| + c$

8. $\int x^2 \cos x \, dx$

Sol. Let $I = \int x^2 \cos x \, dx \quad \Rightarrow I = x^2\int \cos x \, dx - \int\left[\frac{d(x^2)}{dx}\int \cos x \, dx\right]dx$

$\Rightarrow I = x^2 \sin x - \int 2x.\sin x \, dx \quad \Rightarrow I = x^2 \sin x - 2\left[x\int \sin x \, dx - \int\left[\frac{d(x)}{dx}\int \sin x \, dx\right]dx\right]$

$\Rightarrow I = x^2 \sin x - 2\left[-x\cos x - \int 1.(-\cos x)dx\right] \quad \Rightarrow I = x^2 \sin x - 2\left[-x\cos x + \int \cos x \, dx\right]$

$\Rightarrow I = x^2 \sin x - 2[-x\cos x + \sin x] + c \quad \therefore \; I = x^2 \sin x + 2x\cos x - 2\sin x + c$

9. $\int x \sin^2 x \, dx$

Sol. Let $I = \int x\sin^2 x\, dx \quad \Rightarrow I = \int x\left(\frac{1-\cos 2x}{2}\right)dx = \frac{1}{2}\int(x - x\cos 2x)dx$

$\Rightarrow I = \frac{1}{2}\int x\, dx - \frac{1}{2}\int x\cos 2x\, dx = \frac{1}{2}\times\frac{x^2}{2} - \frac{1}{2}\left[x\int \cos 2x\, dx - \int\left(\frac{dx}{dx}\right)\cos 2x\, dx\right]dx$

$\Rightarrow I = \frac{x^2}{4} - \frac{1}{2}\left[x\frac{\sin 2x}{2} - \int\frac{\sin 2x}{2}dx\right] = \frac{x^2}{4} - \frac{1}{2}\left[\frac{x}{2}\sin 2x + \frac{\cos 2x}{4}\right] + c = \frac{x^2}{4} - \frac{x}{4}\sin 2x - \frac{1}{8}\cos 2x + c$

10. $\int x \tan^2 x \, dx$

Sol. Let $I = \int x \tan^2 x \, dx \quad \Rightarrow I = x\int \tan^2 x \, dx - \int\left[\frac{d(x)}{dx}\int \tan^2 x \, dx\right]dx$

$\Rightarrow I = x\int(\sec^2 x - 1)dx - \int\left[1.\int(\sec^2 x - 1)dx\right]dx$

$\Rightarrow I = x(\tan x - x) - \int(\tan x - x)dx \quad \Rightarrow I = x \tan x - x^2 - \int \tan x \, dx + \int x \, dx$

$\Rightarrow I = x \tan x - x^2 + \log|\cos x| + \frac{x^2}{2} + c \quad \therefore I = x\tan x - \frac{x^2}{2} + \log|\cos x| + c$

11. $\int x^2 e^x \, dx$

Sol. Let $I = \int x^2 e^x \, dx \quad \Rightarrow I = x^2\int e^x \, dx - \int\left[\frac{d(x^2)}{dx}\int e^x \, dx\right]dx$

$\Rightarrow I = x^2.e^x - \int 2x.e^x \, dx \quad \Rightarrow I = x^2.e^x - 2\left[x\int e^x \, dx - \int\left[\frac{d(x)}{dx}\int e^x dx\right]dx\right]$

$\Rightarrow I = x^2 e^x - 2\left[xe^x - e^x\right] + c \quad \Rightarrow I = x^2 e^x - 2xe^x + 2e^x + c \quad \therefore \; I = e^x\left(x^2 - 2x + 2\right) + c$

12. $\int x^2 \cos^3 x\, dx$

Sol. Let $I = \int x^2 \cos x\, dx \quad \Rightarrow I = x^2 \int \cos x\, dx - \int \left[\frac{d(x^2)}{dx} \int \cos x\, dx \right] dx$

$\Rightarrow I = x^2 \sin x - \int 2x(\sin x) dx \quad \Rightarrow I = x^2 \sin x - 2\left[x \int \sin x\, dx - \int \left[\frac{d(x)}{dx} \int \sin x\, dx \right] dx \right]$

$\Rightarrow I = x^2 \sin x - 2\left[-x\cos x - \int 1.(-\cos x) dx \right] \quad \Rightarrow I = x^2 \sin x - 2(-x\cos x + \sin x) + c$

$\therefore \quad I = x^2 \sin x + 2x\cos x - 2\sin x + c \Rightarrow \frac{2x}{9}\cos 3x + \frac{\sin 3x}{36}(4 + 3x^2)$

13. $\int x^2 e^{3x} dx$

Sol. Let $I = \int x^2 e^{3x} dx \quad \Rightarrow I = x^2 \int e^{3x} dx - \int \left[\frac{d(x^2)}{dx} \int e^{3x} dx \right] dx \quad \Rightarrow I = x^2 . \frac{e^{3x}}{3} - \int \left(2x . \frac{e^{3x}}{3} \right) dx$

$\Rightarrow I = \frac{x^2}{3} e^{3x} - \frac{2}{3} \int x\, e^{3x} dx \quad \Rightarrow I = \frac{x^2}{3} . e^{3x} - \frac{2}{3} \left[x \int e^{3x} dx - \int \left[\frac{d(x)}{dx} . \int e^{3x} dx \right] dx \right]$

$\Rightarrow I = \frac{x^2}{3} . e^{3x} - \frac{2}{3} \left[x . \frac{e^{3x}}{3} - \int 1 . \frac{e^{3x}}{3} dx \right] \quad \Rightarrow I = \frac{x^2}{3} . e^{3x} - \frac{2}{3} \left[\frac{x . e^{3x}}{3} - \frac{1}{3} . \frac{e^{3x}}{3} \right] + c$

$\Rightarrow I = \frac{x^2}{3} e^{3x} - \frac{2x}{9} e^{3x} + \frac{2}{27} e^{3x} + c \quad \therefore \; I = e^{3x} \left(\frac{x^2}{3} - \frac{2x}{9} + \frac{2}{27} \right) + c$

14. $\int x^2 \sin^2 x\, dx$

Sol. Let $I = \int x^2 \sin^2 x\, dx \quad \Rightarrow I = x^2 \int \sin^2 x\, dx - \int \left[\frac{d(x^2)}{dx} \int \sin^2 x\, dx \right] dx$

$\Rightarrow I = x^2 \int \frac{1 - \cos 2x}{2} dx - \int \left[2x . \int \frac{1 - \cos 2x}{2}\, dx \right] dx$

$\Rightarrow I = \frac{x^2}{2} \left(x - \frac{\sin 2x}{2} \right) - \int x \left(x - \frac{\sin 2x}{2} \right) dx \quad \Rightarrow I = \frac{x^3}{2} - \frac{x^2 \sin 2x}{4} - \int x^2 dx + \frac{1}{2} \int x \sin 2x\, dx$

$\Rightarrow I = \frac{x^3}{2} - \frac{x^2 \sin 2x}{4} - \frac{x^3}{3} + \frac{1}{2} \left[x \int \sin 2x\, dx - \int \left[\frac{d(x)}{dx} \int \sin 2x\, dx \right] dx \right]$

$\Rightarrow I = \frac{x^3}{2} - \frac{x^2 \sin 2x}{4} - \frac{x^3}{3} + \frac{1}{2} \left[-\frac{x \cos 2x}{2} - \int 1 \left(-\frac{\cos 2x}{2} \right) dx \right]$

$\Rightarrow I = \frac{x^3}{6} - \frac{x^2 \sin 2x}{4} + \frac{1}{2} \left[-\frac{x}{2} \cos 2x + \frac{1}{2} . \frac{\sin 2x}{2} \right] + c$

$\therefore \quad I = \frac{1}{6} x^3 - \frac{1}{4} x^2 \sin 2x - \frac{1}{4} x \cos 2x + \frac{1}{8} \sin 2x + c$

15. $\int x^3 \log 2x\, dx$

Sol. Let $I = \int x^3 \log 2x\, dx \quad \Rightarrow I = \log 2x \int x^3 dx - \int \left[\frac{d(\log 2x)}{dx} \int x^3 dx \right] dx$

$$\Rightarrow I = \log 2x.\frac{x^4}{4} - \int \frac{1}{2x}.2.\frac{x^4}{4}dx \quad \Rightarrow I = \frac{x^4}{4}\log 2x - \frac{1}{4}\int x^3 dx$$

$$\Rightarrow I = \frac{x^4}{4}\log 2x - \frac{1}{4}.\frac{x^4}{4} + c \quad \therefore I = \frac{x^4}{4}\log 2x - \frac{x^4}{16} + c$$

16. $\int (1+x)\log x \, dx$

Sol. Let $I = \int (1+x)\log x \, dx \quad \Rightarrow I = \log x \int (1+x)dx - \int \left[\frac{d(\log x)}{dx}\int (1+x)dx\right]dx$

$$\Rightarrow I = \log x\left(x + \frac{x^2}{2}\right) - \int \frac{1}{x}\left(x + \frac{x^2}{2}\right)dx \quad \Rightarrow I = \left(\frac{x^2}{2} + x\right)\log x - \int dx - \frac{1}{2}\int x\,dx$$

$$\Rightarrow I = \left(\frac{x^2}{2} + x\right)\log x - x - \frac{1}{2}.\frac{x^2}{2} + c \quad \therefore I = \left(\frac{x^2}{2} + x\right)\log x - \left(x + \frac{x^2}{4}\right) + c$$

17. $\int \frac{\log x}{x^n} dx$

Sol. Let $I = \int \frac{\log x}{x^n} dx \quad \Rightarrow I = \int x^{-n}\log x \, dx \quad \Rightarrow I = \log x \int x^{-n}dx - \int \left[\left(\frac{d(\log x)}{dx}\right)\left(x^{-n}dx\right)\right]dx$

$$\Rightarrow I = \log x.\frac{x^{-n+1}}{-n+1} - \int \frac{1}{x}.\frac{x^{-n+1}}{(-n+1)}dx \quad \Rightarrow I = \frac{x^{-n+1}.\log x}{-n+1} - \int \frac{x^{-n}.x}{x.(-n+1)}dx$$

$$\Rightarrow I = \frac{x^{-n+1}\log x}{(-n+1)} - \frac{1}{(-n+1)}.\frac{x^{-n+1}}{(-n+1)} + c \quad \therefore I = \frac{x^{1-n}.\log x}{(1-n)} - \frac{x^{1-n}}{(1-n)^2} + c$$

18. $\int 2x^3 e^{x^2} dx$

Sol. Let $I = \int 2x^3 e^{x^2} dx \quad \Rightarrow I = \int 2x.x^2 \, e^{x^2} dx$, Put $x^2 = t \quad \Rightarrow 2x = \frac{dt}{dx} \quad \Rightarrow 2x \, dx = dt$

$$I = \int t.e^t \, dt \quad \Rightarrow I = \int t.e^t \, dt \quad \Rightarrow I = t\int e^t dt - \int \left[\frac{d(t)}{dt}\int e^t \, dt\right]dt$$

$$\Rightarrow I = t.e^t - \int 1.e^t \, dt \quad \Rightarrow I = t\,e^t - e^t + c \quad \Rightarrow I = e^t(t-1) + c \quad \therefore I = e^{x^2}\left(x^2 - 1\right) + c$$

19. $\int x \sin^3 x \, dx$

Sol. $I = \int x \sin^3 x \, dx \quad \Rightarrow I = x\int \sin^3 x \, dx - \int \left[\frac{d(x)}{dx}\int \sin^3 x \, dx\right]dx$

$$\Rightarrow I = x\int \left(\frac{3\sin x - \sin 3x}{4}\right)dx - \int \left[1.\int \left(\frac{3\sin x - \sin 3x}{4}\right)\right]dx$$

$$\Rightarrow I = \frac{x}{4}\int (3\sin x - \sin 3x)dx - \frac{1}{4}\int \left(-3\cos x + \frac{\cos 3x}{3}\right)dx$$

$$\Rightarrow I = \frac{x}{4}\left(-3\cos x + \frac{\cos 3x}{3}\right) - \frac{1}{4}\left(-3\sin x + \frac{\sin 3x}{3.3}\right) + c$$

$$\therefore I = -\frac{3x}{4}\cos x + \frac{x\cos 3x}{12} + \frac{3}{4}\sin x - \frac{\sin 3x}{36} + c$$

20. $\int x\cos^3 x\,dx$

Sol. Let $I=\int x\cos^3 x\,dx \quad \Rightarrow I = x\int \cos^3 x\,dx - \int\left[\frac{d(x)}{dx}\int \cos^3 x\,dx\right]dx$

$\Rightarrow I = x\int\left(\frac{\cos 3x+3\cos x}{4}\right)dx - \int\left[1.\int\left(\frac{\cos 3x+3\cos x}{4}\right)dx\right]dx$

$\Rightarrow I = \frac{x}{4}\int(\cos 3x+3\cos x)dx - \frac{1}{4}\int\left(\frac{\sin 3x}{3}+3\sin x\right)dx$

$\Rightarrow I = \frac{x}{4}\left(\frac{\sin 3x}{3}+3\sin x\right) - \frac{1}{4}\left(-\frac{\cos 3x}{9}-3\cos x\right)+c \quad \therefore\ I = \frac{x\sin 3x}{12}+\frac{3x}{4}\sin x+\frac{\cos 3x}{36}+\frac{3\cos x}{4}+c$

21. $\int x^3\cos\left(x^2\right)dx$

Sol. Let $I=\int x^2.x\cos\left(x^2\right)dx$, Put $x^2=t \quad \Rightarrow 2x=\frac{dt}{dx} \quad \Rightarrow x\,dx=\frac{dt}{2}$

$I=\int 1.\cos(t)\frac{dt}{2} \quad \Rightarrow I=\frac{1}{2}\int t\cos(t)\,dt \quad \Rightarrow I=\frac{1}{2}\left[t\int\cos t\,dt-\int\left[\frac{d(t)}{dt}\int\cos t\,dt\right]dt\right]$

$\Rightarrow I=\frac{1}{2}\left[t\sin t-\int 1.\sin t\,dt\right] \quad \Rightarrow I=\frac{1}{2}[t\sin t+\cos t]+c \quad \therefore\ I=\frac{x^2\sin\left(x^2\right)}{2}+\frac{\cos\left(x^2\right)}{2}+c$

22. $\int \sin x\log(\cos x)dx$

Sol. Let $I=\int\sin x\log(\cos x)dx$, Put $\cos x=t \quad \Rightarrow -\sin x=\frac{dt}{dx} \quad \Rightarrow \sin x dx=-dt$

$I=\int\log(t)(-dt) \quad \Rightarrow I=-\int\log(t)\,dt \quad \Rightarrow I=-\left[\log t\int dt-\int\left(\frac{d(\log t)}{dt}\int dt\right)dt\right]$

$\Rightarrow I=-\left[\log t.t-\int\frac{1}{t}.t\,dt\right] \quad \Rightarrow I=-\left[t\log t-\int dt\right] \quad \Rightarrow I=-[t\log t-t]+c$

$\therefore\ I=-\cos x\log(\cos x)+\cos x+\mathrm{c}$

23. $\int x\sin x\cos x\,dx$

Sol. Let $I=\int x\sin x\cos x\,dx \quad \Rightarrow I=\frac{1}{2}\int x.2\sin x\cos x\,dx \quad \Rightarrow I=\frac{1}{2}\int x\sin 2x\,dx$

$\Rightarrow I=\frac{1}{2}\left[x\int\sin 2x\,dx-\int\left(\frac{d(x)}{dx}\int\sin 2x\,dx\right)dx\right]$

$\Rightarrow I=\frac{1}{2}\left[-\frac{x\cos 2x}{2}-\int 1.\left(-\frac{\cos 2x}{2}\right)dx\right] \quad \Rightarrow I=\frac{1}{2}\left[-\frac{x\cos 2x}{2}+\int\frac{\cos 2x}{2}\,dx\right]$

$\Rightarrow I=\frac{1}{2}\left[-\frac{x\cos 2x}{2}+\frac{1}{2}.\frac{\sin 2x}{2}\right]+c \quad \therefore\ I=-\frac{x\cos 2x}{4}+\frac{\sin 2x}{8}+c$

24. $\int\cos\left(\sqrt{x}\right)dx$

Sol. Let $I=\int\cos\left(\sqrt{x}\right)dx$, Put $\left(\sqrt{x}\right)=t \quad \Rightarrow \frac{1}{2\sqrt{x}}=\frac{dt}{dx} \quad \Rightarrow dx=2\sqrt{x}\,dt \quad \Rightarrow dx=2t\,dt$

$$I=\int\cos(t).2t\,dt \quad\Rightarrow I=2\int t.\cos t\,dt \quad\Rightarrow I=2\left[t\int\cos t\,dt-\int\left(\frac{d(t)}{dt}\int\cos t\,dt\right)dt\right]$$

$$\Rightarrow I=2\left[t\sin t-\int 1.\sin t\,dt\right] \quad\Rightarrow I=2\left[t\sin t-\int\sin t\,dt\right] \quad\Rightarrow I=2[t\sin t+\cos t]+c$$

$$\therefore\ I=2\left[\sqrt{x}.\sin\left(\sqrt{x}\right)+\cos\left(\sqrt{x}\right)\right]+c$$

25. $\int\operatorname{cosec}^3 x\,dx$

Sol. Let $I=\int\operatorname{cosec}^3 x\,dx \quad\Rightarrow I=\int\operatorname{cosec}^2 x.\operatorname{cosec} x\,dx$

$$\Rightarrow I=\operatorname{cosec} x\int\operatorname{cosec}^2 x\,dx-\int\left[\frac{d(\operatorname{cosec} x)}{dx}\int\operatorname{cosec}^2 x\,dx\right]dx$$

$$\Rightarrow I=\operatorname{cosec} x(-\cot x)-\int(-\operatorname{cosec} x\cot x)(-\cot x)\,dx$$

$$\Rightarrow I=-\operatorname{cosec} x\cot x-\int\operatorname{cosec} x\cot^2 x\,dx \quad\Rightarrow I=-\operatorname{cosec} x\cot x-\int\operatorname{cosec} x\left(\operatorname{cosec}^2 x-1\right)dx$$

$$\Rightarrow I=-\operatorname{cosec} x\cot x-\int\operatorname{cosec}^3 x+\int\operatorname{cosec} x\,dx \quad\Rightarrow I=-\operatorname{cosec} x\cot x-I+\log\left|\tan\frac{x}{2}\right|+c$$

$$\Rightarrow 2I=-\operatorname{cosec} x\cot x+\log\left|\tan\frac{x}{2}\right|+c \quad\therefore\ I=-\frac{1}{2}\operatorname{cosec} x\cot x+\frac{1}{2}\log\left|\tan\frac{x}{2}\right|+c$$

26. $\int x\sin^3 x\cos x\,dx$

Sol. Let $I=\int x\sin^3 x\cos x\,dx \quad\Rightarrow I=\dfrac{x\sin^4 x}{4}-\dfrac{1}{4}\int\sin^4 x\,dx$

$$\Rightarrow I=\frac{x}{4}\sin^4 x-\frac{1}{4}\int\sin^2 x.\sin^2 x\,dx \quad\Rightarrow I=\frac{x}{4}\sin^4 x-\frac{1}{4}\int\left(\frac{1-\cos 2x}{2}\right).\left(\frac{1-\cos 2x}{2}\right)dx$$

$$\Rightarrow I=\frac{x}{4}\sin^4 x-\frac{1}{16}\int\left(1-2\cos 2x+\cos^2 2x\right)dx$$

$$\Rightarrow I=\frac{x}{4}\sin^4 x-\frac{1}{16}\int dx+\frac{1}{16}.2\int\cos 2x\,dx-\frac{1}{16}\int\cos^2 2x\,dx$$

$$\Rightarrow I=\frac{x}{4}\sin^4 x-\frac{1}{16}x+\frac{1}{8}.\frac{\sin 2x}{2}-\frac{1}{16}\int\left(\frac{1+\cos 4x}{2}\right)dx$$

$$\Rightarrow I=\frac{x}{4}\sin^4 x-\frac{x}{16}+\frac{\sin 2x}{16}-\frac{1}{32}\left(x+\frac{\sin 4x}{4}\right)+c \quad\therefore I=\frac{x}{4}\sin^4 x-\frac{3x}{32}+\frac{\sin 2x}{16}-\frac{\sin 4x}{128}+c$$

27. $\int\sin x.\log(\cos x)dx$

Sol. Let $I=\int\sin x\log(\cos x)dx$, Put $\cos x=t \quad\Rightarrow-\sin x=\dfrac{dt}{dx} \quad\Rightarrow \sin x\,dx=-dt$

$$I=\int\log(t)(-dt) \quad\Rightarrow I=-\int\log(t)\,dt \quad\Rightarrow I=-\left[\log t\int dt-\int\left(\frac{d(\log t)}{dt}\int dt\right)dt\right]$$

$$\Rightarrow I=-\left[\log t.t-\int\frac{1}{t}.t\,dt\right] \quad\Rightarrow I=-\left[t\log t-\int dt\right] \quad\Rightarrow I=-[t\log t-t]+c,$$

$\Rightarrow I=-t\log t+t+c$, $\Rightarrow I=-\cos x\log\cos x+\cos x+c$, where $t=\cos x$

28. $\int \frac{\log(\log x)}{x}\, dx$

Sol. Let $I = \int \frac{\log(\log x)}{x}\, dx$, Put $\log x = t \Rightarrow \frac{1}{x} = \frac{dt}{dx} \Rightarrow \frac{1}{x}\, dx = dt$

$$I = \int \log(t)\, dt \Rightarrow I = \log t \int dt - \int \left[\frac{d(\log t)}{dt} \int dt \right] dt \Rightarrow I = \log t . t - \int \frac{1}{t} . t\, dt$$

$$\Rightarrow I = t \log t - \int dt \Rightarrow I = t \log t - t + c \Rightarrow I = \log x \log(\log x) - \log(x) + c$$

$$\therefore I = \log x \left[\log(\log x) - 1 \right] + c$$

29. $\int \log(2 + x^2)\, dx$

Sol. Let $I = \int \log(2 + x^2)\, dx \Rightarrow I = \log(2 + x^2) \int dx - \int \left[\frac{d\{\log(2 + x^2)\}}{dx} \int dx \right] dx$

$$\Rightarrow I = \log(2 + x^2) . x - \int \frac{1}{2 + x^2}(2x)\, x\, dx \Rightarrow I = x \log(2 + x^2) - 2 \int \frac{x^2}{x^2 + 2} dx$$

$$\Rightarrow I = x \log(2 + x^2) - 2 \int \frac{(x^2 + 2) - 2}{x^2 + 2}\, dx \Rightarrow I = x \log(2 + x^2) - 2 \int \frac{x^2 + 2}{x^2 + 2} dx + 4 \int \frac{1}{x^2 + 2} dx$$

$$\Rightarrow I = x \log(2 + x^2) - 2 \int dx + 4 \int \frac{1}{(x)^2 + (\sqrt{2})^2} dx$$

$$\Rightarrow I = x \log(2 + x^2) - 2x + 4 . \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}} \right) + c \quad \therefore I = x \log(2 + x^2) - 2x + 2\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}} \right) + c$$

30. $\int \frac{x}{1 + \sin x}\, dx$

Sol. Let $I = \int \frac{x}{1 + \sin x} dx \Rightarrow I = \int \frac{x}{1 + \sin x} \times \frac{1 - \sin x}{1 - \sin x} dx \Rightarrow I = \int \frac{x(1 - \sin x)}{(1 - \sin^2 x)} dx$

$$\Rightarrow I = \int \frac{x(1 - \sin x)}{\cos^2 x} dx \Rightarrow I = \int x(\sec^2 x - \tan x . \sec x)\, dx$$

$$\Rightarrow I = x \int (\sec^2 x - \tan x . \sec x)\, dx - \int \left[\frac{d(x)}{dx} \int (\sec^2 x - \sec x . \tan x)\, dx \right]$$

$$\Rightarrow I = x(\tan x - \sec x) - \int 1 . \left[(\tan x - \sec x)\, dx \right]$$

$$\Rightarrow I = x(\tan x - \sec x) - \int \tan x\, dx + \int \sec x\, dx$$

$$\therefore I = x(\tan x - \sec x) + \log |\cos x| + \log |\sec x + \tan x| + c$$

31. $I = \int \left\{ \frac{1}{\log x} - \frac{1}{(\log x)^2} \right\} dx$

Sol. Let $I=\int\left(\frac{1}{(\log x)}-\frac{1}{(\log x)^2}\right)dx$. Put $t=\log x$, $e^t=e^{\log x}=x \Rightarrow \frac{dx}{dt}=e^t \Rightarrow dx=e^t dt$

$$I=\int\left(\frac{1}{t}-\frac{1}{t^2}\right)e^t dt=e^t.\frac{1}{t}=\frac{x}{\log x}+c \qquad \therefore \int e^x\left(f(x)+f'(x)\right)dx=e^x f(x)$$

32. $I=\int e^{-x}\cos 2x\cos 4x\,dx$

Sol. Let $I=\frac{1}{2}\int e^{-x}\,2\cos 2x\cos 4x\,dx \Rightarrow I=\frac{1}{2}\int e^{-x}\{\cos(2x+4x)+\cos(2x-4x)\}dx$

$$\Rightarrow I=\frac{1}{2}\int e^{-x}(\cos 6x+\cos 2x)\,dx \Rightarrow I=\frac{1}{2}\int e^{-x}\cos 6x\,dx+\frac{1}{2}\int e^{-x}\cos 2x\,dx$$

$$\Rightarrow I=\frac{1}{2}I_1+\frac{1}{2}I_2 \qquad \ldots(1)$$

$$I_1=\int e^{-x}\cos 6x\,dx \Rightarrow I_1=e^{-x}\int\cos 6x\,dx-\int\left[\frac{d(e^{-x})}{dx}\int\cos 6x\,dx\right]dx$$

$$\Rightarrow I_1=e^{-x}\frac{\sin 6x}{6}-\int e^{-x}(-1)\frac{\sin 6x}{6}dx \Rightarrow I_1=\frac{1}{6}e^{-x}\sin 6x+\frac{1}{6}\int e^{-x}\sin 6x\,dx$$

$$\Rightarrow I_1=\frac{1}{6}e^{-x}\sin 6x+\frac{1}{6}\left[e^{-x}\int\sin 6x\,dx-\int\left[\frac{d(e^{-x})}{dx}\int\sin 6x\,dx\right]dx\right]$$

$$\Rightarrow I_1=\frac{1}{6}e^{-x}\sin 6x+\frac{1}{6}\left[e^{-x}\left(-\frac{\cos 6x}{6}\right)-\int(-e^{-x})\left(\frac{-\cos 6x}{6}\right)dx\right]$$

$$\Rightarrow I_1=\frac{1}{6}e^{-x}\sin 6x-\frac{e^{-x}\cos 6x}{36}-\frac{1}{36}\int e^{-x}\cos 6x\,dx$$

$$\Rightarrow I_1=\frac{1}{6}e^{-x}\sin 6x-\frac{1}{36}e^{-x}\cos 6x-\frac{1}{36}I_1 \Rightarrow \frac{37}{36}I_1=\frac{1}{6}e^{-x}\sin 6x-\frac{1}{36}e^{-x}\cos 6x+C_1$$

$$\Rightarrow I_1=\frac{6}{37}e^{-x}\sin 6x-\frac{1}{37}e^{-x}\cos 6x+C_1 \quad \therefore I_1=\frac{e^{-x}}{37}(6\sin 6x-\cos 6x)+c_1$$

$$I_2=\int e^{-x}\cos 2x\,dx \Rightarrow I_2=e^{-x}\int\cos 2x\,dx-\int\left[\frac{d(e^{-x})}{dx}\int\cos 2x\,dx\right]dx$$

$$\Rightarrow I_2=e^{-x}\frac{\sin 2x}{2}+\int e^{-x}\frac{\sin 2x}{2}dx \Rightarrow I_2=\frac{1}{2}e^{-x}\sin 2x+\frac{1}{2}\int e^{-x}\sin 2x\,dx$$

$$\Rightarrow I_2=\frac{1}{2}e^{-x}\sin 2x+\frac{1}{2}\left[e^{-x}\int\sin 2x\,dx-\int\left[\frac{d(e^{-x})}{dx}\int\sin 2x\,dx\right]dx\right]$$

$$\Rightarrow I_2=\frac{e^{-x}\sin 2x}{2}+\frac{1}{2}\left[-e^{-x}\frac{\cos 2x}{2}-\int e^{-x}\frac{\cos 2x}{2}dx\right]$$

$$\Rightarrow I_2=\frac{1}{2}e^{-x}\sin 2x-\frac{1}{4}e^{-x}\cos 2x-\frac{1}{4}\int e^{-x}\cos 2x\,dx \Rightarrow I_2=\frac{1}{2}e^{-x}\sin 2x-\frac{1}{4}e^{-x}\cos 2x-\frac{1}{4}I_2$$

$$\Rightarrow I_2+\frac{1}{4}I_2=\frac{1}{2}e^{-x}\sin 2x-\frac{1}{4}e^{-x}\cos 2x+c_2 \Rightarrow \frac{5}{4}I_2=\frac{1}{4}e^{-x}(2\sin 2x-\cos 2x)+c_2$$

$$\therefore I_2=\frac{e^{-x}}{5}(2\sin 2x-\cos 2x)+c_2$$

Putting the value of I_1 & I_2 equation (1), $I = e^{-x}\left[\frac{1}{74}(6\sin 6x - \cos 6x) + \frac{1}{10}(2\sin 2x - \cos 2x)\right] + c$

33. $\int e^{\sqrt{x}} dx$

Sol. Let $I = \int e^{\sqrt{x}}\, dx$, Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} = \frac{dt}{dx} \Rightarrow dx = 2\sqrt{x}\, dt \Rightarrow dx = 2t\, dt$

$$I = 2\int t.\, e^t\, dt \Rightarrow I = 2\left[t\int e^t\, dt - \int\left[\frac{d(t)}{dt}\int e^t\, dt\right]dt\right] \Rightarrow I = 2\left[t.e^t - \int 1.e^t\, dt\right]$$

$$\Rightarrow I = 2\left[t.e^t - e^t\right] + c \Rightarrow I = e^t.2(t-1) + c \quad \therefore\ I = 2\, e^{\sqrt{x}}\left(\sqrt{x} - 1\right) + c$$

34. $\int e^{\sin x} \sin 2x\, dx$

Sol. Let $I = \int e^{\sin x}.\sin 2x\, dx$, Put $\sin x = t \Rightarrow \cos x = \frac{dt}{dx} \Rightarrow \cos x\, dx = dt$

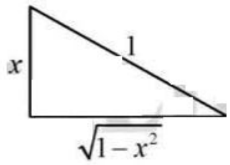
$$I = \int e^{\sin x}.2\sin x \cos x\, dx \Rightarrow I = 2\int e^t.t\, dt$$

$$\Rightarrow I = 2\left[t\int e^t\, dt - \int\left[\frac{d(t)}{dt}\int e^t\, dt\right]dt\right] \Rightarrow I = 2\left[t.e^t - \int 1.e^t\, dt\right]$$

$$\Rightarrow I = 2\left[t.e^t - e^t\right] + c \Rightarrow I = e^t.2(t-1) + c \quad \therefore I = 2\, e^{\sin x}(\sin x - 1) + c$$

35. $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$

Sol. Let $I = \int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$



Put $\sin^{-1} x = t \Rightarrow x = \sin t$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} = \frac{dt}{dx} \Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$$

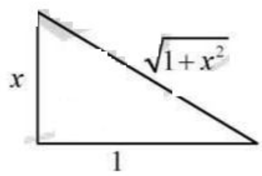
$$I = \int \sin t.\, t\, dt \Rightarrow I = t\int \sin t\, dt - \int\left[\frac{d(t)}{dt}\int \sin t\, dt\right]dt$$

$$\Rightarrow I = -t\cos t + \int 1.\cos t\, dt \Rightarrow I = -t\cos t + \sin t + c$$

$$\Rightarrow I = -\sin^{-1} x \cos\left(\sin^{-1} x\right) + \sin\left(\sin^{-1} x\right) + c \quad \therefore I = -\sqrt{1-x^2}\ \sin^{-1} x + x + c$$

36. $\int \frac{x^2 \tan^{-1} x}{\left(1+x^2\right)} dx$

Sol. Let $I = \int \frac{x^2 \tan^{-1} x}{1+x^2} dx$



Put $\tan^{-1} x = t \Rightarrow x = \tan t$

$$\frac{1}{1+x^2} = \frac{dt}{dx} \Rightarrow \frac{1}{1+x^2} dx = dt$$

$$I = \int \tan^2 t.t\, dt \Rightarrow I = t\int \tan^2 t\, dt - \int\left[\frac{d(t)}{dt}\int \tan^2 t\, dt\right]dt$$

$$\Rightarrow I = t\int\left(\sec^2 t - 1\right)dt - \int\left[1.\int\left(\sec^2 t - 1\right)dt\right]dt$$

$$\Rightarrow I = t(\tan t - t) - \int(\tan t - t)\, dt \quad \Rightarrow I = t\ \tan t - t^2 + \log(\cos t) + \frac{t^2}{2} + c$$

$$x + \tan^{-1} x - \frac{1}{x}\log(1+x^2) - \frac{1}{2}(\tan^{-1} x)^2 + c$$

$$\therefore\quad I = x\ \tan^{-1} x^2 - \frac{1}{2}\log(1+x^2) - \frac{(\tan^{-2} x)^2}{2} + c$$

37. $\int \frac{\log(x+2)}{(x+2)^2} dx$

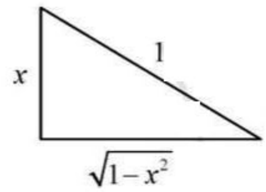
Sol. Let $I = \int \frac{\log(x+2)}{(x+2)^2} dx$, Put $x+2=t \quad \Rightarrow dx = dt$

$$I = \int \frac{\log t}{t^2}\, dt \quad \Rightarrow I = \int \log t. t^{-2}\, dt \quad \Rightarrow I = \log t \int t^{-2}\, dt - \int\left[\frac{d(\log t)}{dt}\int t^{-2}\, dt\right] dt$$

$$I = \log t . \frac{t^{-1}}{(-1)} - \int \frac{1}{t} . \frac{t^{-1}}{(-1)}\, dt \quad \Rightarrow I = -\frac{\log t}{t} - \frac{1}{t} + c \quad \therefore\ I = -\frac{\log(x+2)+1}{(x+2)} + c$$

38. $\int x \sin^{-1} x\, dx$

Sol. Let $I = \int x \sin^{-1} x\, dx$



Put $t = \sin^{-1} x$, $\frac{x}{1} = \sin t$

$$\Rightarrow \frac{dt}{dx} = \frac{1}{\sqrt{1-x^2}} \quad \Rightarrow \frac{dt}{dx} = \frac{1}{\sqrt{1-\sin^2 t}} = \cos t dt = dx$$

$$I = \int \sin t . t . \cos t\, dt = \frac{1}{2}\int t . 2\sin t . \cos t\, dt$$

$$= \frac{1}{2}\int t . \sin 2t\, dt = \frac{1}{2}\left[t\int \sin 2t\, dt - \int\left(\frac{dt}{dt}\int \sin 2t\, dt\right) dt\right]$$

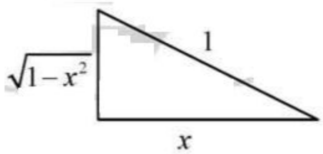
$$= \frac{1}{2}\left[t\left(-\frac{\cos 2t}{2}\right) + \int \frac{\cos 2t}{2}\, dt\right] = \frac{1}{2}\left[-\frac{t}{2}\cos 2t + \frac{\sin 2t}{4}\right] + c$$

$$= \frac{1}{4}\left[-\left(-t . \cos 2t + \frac{2\sin t . \cos t}{2}\right)\right] + c = \frac{1}{4}\left[-\sin^{-1} x(1 - 2\sin^2 t) + x\sqrt{1-x^2}\right] + c$$

$$= \frac{1}{4}\left[-\sin^{-1} x(1-2x^2) + x\sqrt{1-x^2}\right] + c$$

39. $\int x \cos^{-1} x\, dx$

Sol. Let $I = \int x \cos^{-1} x\, dx$



Put $t = \cos^{-1} x \quad \Rightarrow \frac{x}{1} = \cos t$

$$\Rightarrow dt = \frac{-1}{\sqrt{1-x^2}} dx \quad \Rightarrow dt = \frac{-1}{\sqrt{1-\cos^2 t}} dx \quad \Rightarrow -\sin t\, dt = dx$$

$$I = \int \cos t . t . (-\sin t\, dt) = \frac{-1}{2}\int t . 2\sin t . \cos t\, dt = \frac{-1}{2}\int t \sin 2t\, dt$$

$$= \frac{-1}{2}\left[t\int \sin 2t\,.dt - \left(\frac{dt}{dt}\int \sin 2t\,dt\right)dt\right] = \frac{-1}{2}\left[t.\left(\frac{-\cos 2t}{2}\right) + \int \frac{\cos 2t}{2}dt\right]$$

$$= \frac{1}{4}\left[-t\cos 2t + \frac{\sin 2t}{2}\right] + c = \frac{-1}{4}\left[-\cos^{-1}x\left(2\cos^2 t - 1\right) + \frac{2\sin t\,.\cos t}{2}\right] + c$$

$$= \frac{-1}{4}\left[-\cos^{-1}x\left(2x^2 - 1\right) + \sqrt{1-x^2}\,.x\right] + c = \frac{-1}{4}\left[-\left(2x^2-1\right)\cos^{-1}x + x\sqrt{1-x^2}\right] + c$$

40. $\int \cot^{-1} x\,dx$

Sol. Let $I = \int \cot^{-1} x\,dx \quad \Rightarrow I = \cot^{-1} x\int dx - \int\left[\frac{d\left(\cot^{-1}x\right)}{dx}\int dx\right]dx$

$\Rightarrow I = \cot^{-1} x.x + \int \frac{1}{1+x^2}.x\,dx \quad \Rightarrow I = x\cot^{-1} x + \int \frac{x}{1+x^2}dx$

Put $1+x^2 = t \quad \Rightarrow 2x = \frac{dt}{dx} \quad \Rightarrow x\,dx = \frac{dt}{2}$

$I = x\cot^{-1} x + \frac{1}{2}\int \frac{dt}{t} \quad \therefore I = x\cot^{-1} x + \frac{1}{2}\log\left(1+x^2\right) + c$

41. $\int x\cot^{-1} x\,dx$

Sol. Let $I = \int x\cot^{-1}(x)\,dx \quad \Rightarrow I = \cot^{-1} x\int x\,dx - \int\left[\frac{d\left(\cot^{-1}x\right)}{dx}\int x\,dx\right]dx$

$\Rightarrow I = \cot^{-1} x.\frac{x^2}{2} + \int \frac{1}{1+x^2}.\frac{x^2}{2}dx \quad \Rightarrow I = \frac{x^2}{2}\cot^{-1} x + \frac{1}{2}\int \frac{x^2}{x^2+1}dx$

$\Rightarrow I = \frac{x^2}{2}\cot^{-1} + \frac{1}{2}\int \frac{\left(x^2+1\right)-1}{x^2+1}dx \quad \Rightarrow I = \frac{x^2}{2}\cot^{-1} x + \frac{1}{2}\int dx - \frac{1}{2}\int \frac{1}{1+x^2}dx$

$\therefore \quad I = \frac{x^2}{2}\cot^{-1} x + \frac{1}{2}.x - \frac{1}{2}\tan^{-1} x + c$

42. $\int x^2\cot^{-1} x\,dx$

Sol. Let $I = \int x^2\cot^{-1} x\,dx \quad \Rightarrow I = \cot^{-1} x\int x^2\,dx - \int\left[\frac{d\left(\cot^{-1}x\right)}{dx}\int x^2\,dx\right]dx$

$\Rightarrow I = \cot^{-1} x.\frac{x^3}{3} + \int \frac{1}{1+x^2}.\frac{x^3}{3}dx \quad \Rightarrow I = \frac{x^3}{3}\cot^{-1} x + \frac{1}{3}\int \frac{x^3}{x^2+1}dx$

$\Rightarrow I = \frac{x^3}{3}\cot^{-1} x + \frac{1}{3}\int\left(x - \frac{x}{x^2+1}\right)dx \quad \Rightarrow I = \frac{x^3}{3}\cot^{-1} x + \frac{1}{3}\int x\,dx - \frac{1}{3}\int \frac{x}{x^2+1}dx$

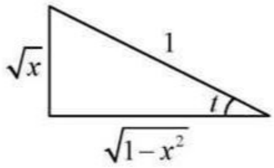
Put $x^2+1 = t \quad \Rightarrow 2x = \frac{dt}{dx} \quad \Rightarrow x\,dx = \frac{dt}{2}$

$I = \frac{x^3}{3}\cot^{-1} x + \frac{1}{3}.\frac{x^2}{2} - \frac{1}{3}.\frac{1}{2}\int \frac{1}{t}.dt \quad \Rightarrow I = \frac{x^3}{3}\cot^{-1} x + \frac{x^2}{6} - \frac{1}{6}\log|t| + c$

$\therefore \quad I = \frac{x^3}{3}\cot^{-1} x + \frac{x^2}{6} - \frac{1}{6}\log\left|x^2+1\right| + c$

43. $\int \sin^{-1}\left(\sqrt{x}\right)dx$

Sol. Let $I=\int \sin^{-1}\left(\sqrt{x}\right)dx$



Put $t=\sin^{-1}\sqrt{x} \quad \Rightarrow \sqrt{x}=\sin t \quad \Rightarrow x=\sin^2 t$

$\Rightarrow \frac{dt}{dx}=\frac{d\sin^{-1}\sqrt{x}}{d\sqrt{x}}\times\frac{d\sqrt{x}}{dx} \quad \Rightarrow \frac{dt}{dx}=\frac{1}{\sqrt{1-x}}\cdot\frac{1}{2\sqrt{x}}$

$\Rightarrow \frac{dt}{dx}=\frac{1}{\sqrt{1-\sin^2 t}}\cdot\frac{1}{2\sin t} \quad \Rightarrow \frac{dt}{dx}=\frac{1}{2\sin t\cos t} \quad \Rightarrow \sin 2t\,dt=dx$

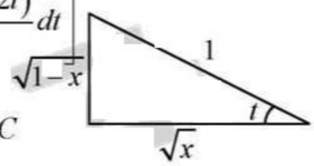
$\Rightarrow \int t.\sin 2t\,dt=t\int\sin 2t\,dt-\int\left(\frac{dt}{dt}\right)\sin 2t\,dt=-t\frac{\cos 2t}{2}+\int\frac{\cos 2t\,dt}{2}=\frac{-t}{2}\cos 2t+\frac{\sin 2t}{4}+c$

$=\frac{-1}{2}\sin^{-1}\sqrt{x}\left(1-2\sin^2 t\right)+\frac{2\sin t.\cos t}{4}+c=\frac{-1}{2}(1-2x)\sin^{-1}\sqrt{x}+\frac{1}{2}\sqrt{x}\sqrt{1-x}+c$

$=\frac{-1}{2}(1-2x)\sin^{-1}\sqrt{x}+\frac{1}{2}\sqrt{x(1-x)}+c$

44. $\int \cos^{-1}\left(\sqrt{x}\right)dx$

Sol. $I=-\int t\sin 2t\,dt=-\left[t\int\sin 2t\,dt-\int\left(\frac{dt}{dt}\cdot\int\sin 2t\right)dt\right]=-\left[\frac{t(-\cos 2t)}{2}-\int\frac{(-\cos 2t)}{2}dt\right]$

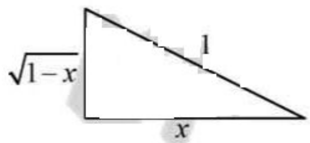


$=-\left[\frac{-t\cos 2t}{2}+\frac{\sin 2t}{4}\right]+C=\frac{t\cos 2t}{2}-\frac{\sin 2t}{4}+C=\frac{t\left(2\cos^2 t-1\right)}{2}-\frac{\sin t\cos t}{2}+C$

$=\frac{\cos^{-1}\sqrt{x}\left(2\left(\sqrt{x}\right)^2-1\right)}{2}-\frac{\sqrt{1-x}\times\sqrt{x}}{2}+C=\frac{1}{2}(2x-1)\cos^{-1}\sqrt{x}-\frac{1}{2}\sqrt{x(1-x)}+C$

45. $\int \cos^{-1}\left(4x^3-3x\right)dx$

Sol. Let $I=\int\cos^{-1}\left(4x^3-3x\right)dx$



Put $x=\cos\theta \quad \therefore \theta=\cos^{-1}x \quad \Rightarrow dx=-\sin\theta\,d\theta$

$I=\int\cos^{-1}\left(4\cos^3\theta-3\cos\theta\right)(-\sin\theta)d\theta$

$\Rightarrow I=\int\cos^{-1}(\cos 3\theta)\sin\theta\,d\theta \quad \Rightarrow I=-\int 3\theta\sin\theta\,d\theta$

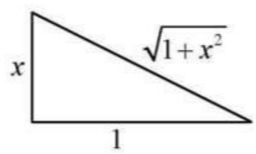
$\Rightarrow I=-3\left[\theta\int\sin\theta\,d\theta-\int\left[\frac{d(\theta)}{d\theta}\int\sin\theta\,d\theta\right]d\theta\right]$

$\Rightarrow I=-3\left[-\theta\cos\theta+\int 1.\cos\theta\,d\theta\right] \quad \Rightarrow I=3\,\theta\cos\theta-3\sin\theta+c$

$\Rightarrow I=3\left(\cos^{-1}x\right)x-3\sqrt{1-x^2}+c \quad \therefore I=3x\cos^{-1}x-3\sqrt{1-x^2}+c$

46. $\int\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$

Sol. Let $I=\int\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$



Put $x=\tan\theta \quad \therefore \theta=\tan^{-1}(x) \quad \Rightarrow x=\sec^2\theta\,.\,d\theta$

$$I=\int \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right).\sec^2\theta\, d\theta \quad \Rightarrow I=\int \cos^{-1}(\cos 2\theta).\sec^2\theta\, d\theta$$

$$\Rightarrow I=\int 2\theta \sec^2\theta\, d\theta \quad \Rightarrow I=2\int \theta \sec^2\theta\, d\theta$$

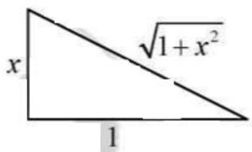
$$\Rightarrow I=2\left[\theta\int \sec^2\theta\, d\theta-\int\left[\frac{d\theta}{d\theta}\int \sec^2\theta\, d\theta\right]d\theta\right] \quad \Rightarrow I=2\left[\theta \tan\theta-\int 1.\tan\theta\, d\theta\right]$$

$$\Rightarrow I=2\left[\theta \tan\theta+\log(\cos\theta)\right]+c \quad \Rightarrow I=2\theta \tan\theta+2\log(\cos\theta)+c$$

$$\Rightarrow I=2x \tan^{-1}x-\frac{1}{2}.2\log\left(1+x^2\right)+c \quad \therefore\ I=2x \tan^{-1}x-\log\left(1+x^2\right)+c$$

47. $\int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$

Sol. Let $I=\int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$



Put $x=\tan\theta \quad \therefore\ \theta=\tan^{-1}(x) \quad \Rightarrow dx=\sec^2\theta\, d\theta$

$$I=\int \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)\sec^2\theta\, d\theta \quad \Rightarrow I=\int \tan^{-1}(\tan 2\theta)\sec^2\theta\, d\theta$$

$$\Rightarrow I=2\int \theta \sec^2\theta\, d\theta \quad \Rightarrow I=2\left[\theta \tan\theta-\int 1.\tan\theta\, d\theta\right] \quad \Rightarrow I=2\left[\theta \tan\theta+\log(\cos\theta)\right]+c$$

$$\Rightarrow I=2\theta \tan\theta+2\log(\cos\theta)+c \quad \Rightarrow I=2x \tan^{-1}x-\frac{1}{2}.2\log\left(1+x^2\right)+c$$

$$\therefore\ I=2x \tan^{-1}x-\log\left(1+x^2\right)+c$$

48. $\int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)dx$

Sol. Let $I=\int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)dx$, Put $x=\tan\theta \quad \therefore\ \theta=\tan^{-1}(x) \quad \Rightarrow dx=\sec^2\theta$

$$I=\int \tan^{-1}\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right).\sec^2\theta\, d\theta \quad \Rightarrow I=\int 3\theta \sec^2\theta\, d\theta \quad \Rightarrow I=3\int \theta \sec^2\theta\, d\theta$$

$$\Rightarrow I=3\left[\theta\int \sec^2\theta\, d\theta-\int\left[\frac{d\theta}{d\theta}\int \sec^2\theta\, d\theta\right]d\theta\right] \quad \Rightarrow I=3\left[\theta \tan\theta-\int 1.\tan\theta\, d\theta\right]$$

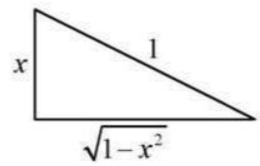
$$\Rightarrow I=3\left[\theta \tan\theta+\log(\cos\theta)\right]+C \quad \Rightarrow I=3\theta \tan\theta+3\log(\cos\theta)+C$$

$$\Rightarrow I=3\tan^{-1}x\tan\left(\tan^{-1}x\right)+3\log\left|\cos\left(\tan^{-1}x\right)\right|+C$$

$$\Rightarrow I=3x \tan^{-1}x-\frac{1}{2}.3\log\left(1+x^2\right)+C \quad \therefore\ I=3x \tan^{-1}x-\frac{3}{2}\log\left(1+x^2\right)+C$$

49. $\int \frac{\sin^{-1}x}{x^2}dx$

Sol. Let $I=\int \frac{\sin^{-1}x}{x^2}dx$



Put $x=\sin\theta \quad \therefore\ \theta=\sin^{-1}x \quad \Rightarrow dx=\cos\theta\, d\theta$

$$I=\int \frac{\sin^{-1}(\sin\theta)}{\sin^2\theta}\cos\theta\ d\theta \quad \Rightarrow I=\int \theta.\text{cosec}^2\theta\ \cos\theta\ d\theta$$

$$\Rightarrow I=\int \theta.\ \cot\theta\ \text{cosec}\theta\ d\theta \quad \Rightarrow I=\theta\int \text{cosec}\theta\ \cot\theta\ d\theta-\int\left[\frac{d(\theta)}{d\theta}\int \text{cosec}\ \theta\ \cot\ \theta\ d\theta\right]d\theta$$

$$\Rightarrow I=-\theta\cos ec\theta+\int 1.\cos ec\theta\, d\theta \quad \Rightarrow I=-\theta\,\text{cosec}\theta+\log\left|\ \text{cosec}\theta-\cot\theta\ \right|+c$$

$$\therefore\ I=-\frac{\sin^{-1}x}{x}+\log\left|\frac{1}{x}-\frac{\sqrt{1-x^2}}{x}\right|+c$$

50. $\int \frac{\tan x.\sec^2 x}{1-\tan^2 x}dx$

Sol. Let $I=\int \frac{\tan x.\sec^2 x}{1-\tan^2 x}dx$, Put $\tan x=t \quad \Rightarrow \sec^2 x=\frac{dt}{dx} \quad \Rightarrow \sec^2 x\ dx=dt$

$$I=\int \frac{t}{1-t^2}dt,\ \text{Put } 1-t^2=z \quad \Rightarrow -2t=\frac{dz}{dt} \quad \Rightarrow t\ dt=\frac{dz}{-2}$$

$$I=-\frac{1}{2}\int \frac{1}{z}dz \quad \Rightarrow I=-\frac{1}{2}\log|z|+c \quad \Rightarrow I=-\frac{1}{2}\log\left|1-t^2\right|+c$$

$$\therefore\ I=-\frac{1}{2}\log\left|1-\tan^2 x\right|+c$$

51. $\int e^{3x}\sin 4x\ dx$

Sol. Let $I=\int e^{3x}\sin 4x\ dx \quad \Rightarrow I=\sin 4x\int e^{3x}\ dx-\int\left[\frac{d(\sin\ 4x)}{dx}\int e^{3x}\ dx\right]dx$

$$\Rightarrow I=\sin 4x.\frac{e^{3x}}{3}-\int \cos 4x.4.\frac{e^{3x}}{3}dx \quad \Rightarrow I=\frac{1}{3}\sin 4x.\ e^{3x}-\frac{4}{3}\int e^{3x}\ \cos 4x\ dx$$

$$\Rightarrow I=\frac{1}{3}e^{3x}\sin 4x-\frac{4}{3}\left[\cos\ 4x\int e^{3x}dx-\int \frac{d(\cos 4x)}{dx}\int e^{3x}\ dx\right]$$

$$\Rightarrow I=\frac{1}{3}e^{3x}\ \sin 4x-\frac{4}{3}\left[\cos 4x.\frac{e^{3x}}{3}+\int \sin 4x.4.\frac{e^{3x}}{3}\ dx\right]$$

$$\Rightarrow I=\frac{1}{3}e^{3x}\ \sin 4x-\frac{4}{9}e^{3x}\cos\ 4x-\frac{16}{9}\int e^{3x}\sin 4x\ dx \quad \Rightarrow \frac{25}{9}I=\frac{1}{3}e^{3x}\sin 4x-\frac{4}{9}e^{3x}\ \cos\ 4x+C$$

$$\Rightarrow I=\frac{3}{25}e^{3x}\sin 4x-\frac{4}{25}e^{3x}\ \cos\ 4x+c \quad \therefore\ I=\frac{e^{3x}}{25}(3\ \sin 4x-4\cos 4x)+c$$

52. $\int e^{2x}\sin x\ dx$

Sol. Let $I=\int e^{2x}\sin x dx \quad \Rightarrow I=\sin x\int e^{2x}\ dx-\int\left[\frac{d(\sin x)}{dx}\int e^{2x}dx\right]dx$

$$\Rightarrow I=\sin x.\frac{e^{2x}}{2}-\int \cos x.\frac{e^{2x}}{2}dx \quad \Rightarrow I=\frac{1}{2}e^{2x}\sin x-\frac{1}{2}\int e^{2x}\cos x\ dx$$

$$\Rightarrow I=\frac{1}{2}e^{2x}\sin x-\frac{1}{2}\left[\cos x\int e^{2x}dx-\int\left[\frac{d(\cos x)}{dx}\int e^{2x}dx\right]dx\right]$$

$$\Rightarrow I=\frac{1}{2}e^{2x}\sin x-\frac{1}{2}\left[\cos x.\frac{e^{2x}}{2}+\int\sin x.\frac{e^{2x}}{2}dx\right] \quad \Rightarrow I=\frac{1}{2}e^{2x}\sin x-\frac{1}{4}e^{2x}\cos x-\frac{1}{4}\int e^{2x}\sin x\,dx$$

$$\Rightarrow I=\frac{1}{2}e^{2x}\cdot\sin x-\frac{1}{4}e^{2x}\cos x-\frac{1}{4}I \Rightarrow \frac{5I}{4}=\frac{1}{2}e^{2x}\sin x-\frac{1}{4}e^{2x}\cos x+C$$

$$\therefore \quad I=\frac{e^{2x}}{5}(2\sin x-\cos x)+c$$

53. $\int e^{2x}\sin x\cos x\ dx$

Sol. Let $I=\int e^{2x}\sin x\cos x\ dx \quad \Rightarrow I=\frac{1}{2}\int e^{2x}(2\sin x\cos x)\ dx$

$$\Rightarrow I=\frac{1}{e}\int e^{2x}\sin 2x\ dx \quad \Rightarrow I=\frac{1}{2}\left[\sin 2x\int e^{2x}dx-\int\left[\frac{d(\sin 2x)}{dx}\int e^{2x}dx\right]dx\right]$$

$$\Rightarrow I=\frac{1}{2}\left[\sin 2x.\frac{e^{2x}}{2}-\int\cos 2x.2.\frac{e^{2x}}{2}dx\right] \quad \Rightarrow I=\frac{1}{2}\left[\frac{e^{2x}}{2}\sin 2x-\int e^{2x}\cos 2x\ dx\right]$$

$$\Rightarrow I=\frac{e^{2x}}{4}\sin 2x-\frac{1}{2}\int e^{2x}\cos 2x\ dx$$

$$\Rightarrow I=\frac{e^{2x}}{4}\sin 2x-\frac{1}{2}\left[\cos 2x\int e^{2x}dx-\int\left[\frac{d(\cos 2x)}{dx}\int e^{2x}dx\right]dx\right]$$

$$\Rightarrow I=\frac{e^{2x}}{4}\sin 2x-\frac{1}{2}\left[\cos 2x.\frac{e^{2x}}{2}+\int\sin 2x.2.\frac{e^{2x}}{2}dx\right]$$

$$\Rightarrow I=\frac{e^{2x}}{4}\sin 2x-\frac{e^{2x}\cos 2x}{4}-\frac{1}{2}\int e^{2x}\sin 2x\ dx=\frac{e^{2x}\sin 2x}{4}-\frac{e^{2x}\cos 2x}{4}=I$$

$$\Rightarrow 2I=\frac{e^{2x}}{4}\sin 2x-\frac{e^{2x}}{4}\cos 2x+c \qquad \therefore \ I=\frac{e^{2x}}{8}(\sin 2x-\cos 2x)+c$$

54. $\int e^{2x}\cos(3x+4)dx$

Sol. Let $I=\int e^{2x}\cos(3x+4)dx \quad \Rightarrow I=\cos(3x+4)\int e^{2x}dx-\int\left[\frac{d\{\cos(3x+4)\}}{dx}\int e^{2x}dx\right]dx$

$$\Rightarrow I=\cos(3x+4).\frac{e^{2x}}{2}+\int\sin(3x+4).3\frac{e^{2x}}{2}dx$$

$$\Rightarrow I=\frac{e^{2x}}{2}\cos(3x+4)+\frac{3}{2}\int e^{2x}\sin(3x+4)dx$$

$$\Rightarrow I=\frac{e^{2x}}{2}\cos(3x+4)+\frac{3}{2}\left[\sin(3x+4)\int e^{2x}dx-\int\left[\frac{d\sin(3x+4)}{dx}.\int e^{2x}dx\right]dx\right]$$

$$\Rightarrow I=\frac{e^{2x}}{2}\cos(3x+4)+\frac{3}{2}\sin(3x+4).\frac{e^{2x}}{2}-\int\cos(3x+4).3\frac{e^{2x}}{2}dx$$

$$\Rightarrow I=\frac{e^{2x}}{2}\cos(3x+4)+\frac{3}{4}e^{2x}\sin(3x+4)-\frac{9}{4}\int e^{2x}\cos(3x+4)dx$$

$$\Rightarrow \frac{13}{4}I=\frac{e^{2x}}{4}\{2\cos(3x+4)+3\sin(3x+4)\} \quad \therefore \ I=\frac{e^{2x}}{13}\{2\cos(3x+4)+3\sin(3x+4)\}$$

55. $\int e^{-x}\cos x\,dx$

Sol. Let $I=\int e^{-x}\cos x\,dx \quad \Rightarrow I=\cos x\int e^{-x}dx-\int\left[\frac{d(\cos x)}{dx}\int e^{-x}dx\right]dx$

$\Rightarrow I=\cos x.\frac{e^{-x}}{-1}+\int\sin x.\frac{e^{-x}}{-1}dx \quad \Rightarrow I=-e^{-x}\cos x-\int e^{-x}\sin x\,dx$

$\Rightarrow I=-e^{-x}\cos x-\left[\sin x\int e^{-x}dx-\int\left[\frac{d(\sin x)}{dx}\int e^{-x}dx\right]dx\right]$

$\Rightarrow I=-e^{-x}\cos x-\left[\sin x\frac{e^{-x}}{-1}-\int\cos x.\frac{e^{-x}}{-1}dx\right] \quad \Rightarrow I=-e^{-x}\cos x+e^{-x}\sin x-\int e^{-x}\cos x\,dx$

$\Rightarrow 2I=-e^{-x}\cos x+e^{-x}\sin x+c \quad \therefore I=\frac{e^{-x}}{2}(\sin x-\cos x)+c$

56. $\int e^{x}(\sin x+\cos x)dx$

Sol. Let $I=\int e^{x}(\sin x+\cos x)dx$, where $f(x)=\sin x,\ f'(x)=\cos x$

$\Rightarrow I=\int e^{x}\{f(x)+f'(x)\}dx \quad \Rightarrow I=e^{x}.f(x)+c \quad \therefore I=e^{x}\sin x+c$

57. $\int e^{x}(\cot x-\operatorname{cosec}^2 x)dx$

Sol. Let $I=\int e^{x}(\cot x-\operatorname{cosec}^2 x)\,dx$, where $f(x)=\cot x,\ f'(x)=-\operatorname{cosec}^2 x$

$\Rightarrow I=\int e^{x}\{f(x)+f'(x)\}\,dx \quad \Rightarrow I=e^{x}.f(x)+c \quad \therefore I=e^{x}\cot x+c$

58. $\int e^{x}(\sec x+\sec x.\tan x)dx$

Sol. Let $I=\int e^{x}(\sec x+\sec x\tan x)dx$, where $f(x)=\sec x,\ f'(x)=\sec x\tan x$

$\Rightarrow I=\int e^{x}[f(x)+f'(x)]dx \quad \Rightarrow I=e^{x}f(x)+c \quad \therefore I=e^{x}\sec x+c$

59. $\int e^{x}\left(\tan^{-1}x+\frac{1}{1+x^2}\right)dx$

Sol. Let $I=\int e^{x}\left(\tan^{-1}x+\frac{1}{1+x^2}\right)dx$, where $f(x)=\tan^{-1}x,\ f'(x)=\frac{1}{1+x^2}$

$\Rightarrow I=\int e^{x}[f(x)+f'(x)]dx \quad \Rightarrow I=e^{x}.f(x)+c \quad \therefore I=e^{x}\tan^{-1}x+c$

60. $\int e^{x}\{\cot x+\log(\sin x)\}dx$

Sol. Let $I=\int e^{x}\{\cot x+\log(\sin x)\}dx$. where $f(x)=\log(\sin x), f'(x)=\cot x$

$\Rightarrow I=\int e^{x}[f(x)+f'(x)]dx \quad \Rightarrow I=e^{x}f(x)+c \quad \therefore I=e^{x}\log(\sin x)+c$

61. $\int e^{x}(\tan x-\log(\cos x))dx$

Sol. Let $I=\int e^{x}(\tan x-\log(\cos x))dx$, where $f(x)=-\log(\cos x), f'(x)=\tan x$

$\Rightarrow I=\int e^{x}[f(x)+f'(x)]dx \quad \Rightarrow I=e^{x}f(x)+C \quad \therefore I=-e^{x}\{\log(\cos x)\}+C$

62. $\int e^{x}[\sec x+\log(\sec x+\tan x)]dx$

Sol. Let $I=\int e^{x}[\sec x+\log(\sec x+\tan x)]dx$, where $f(x)=\log(\sec x+\tan x), f'(x)=\sec x$

$$\Rightarrow I=\int e^x\left[f(x)+f'(x)\right]dx \quad \Rightarrow I=e^x f(x)+c \quad \therefore\ e^x\log(\sec x+\tan x)+c$$

63. $\int e^x\left(\frac{1+\sin x\cos x}{\cos^2 x}\right)dx$

Sol. Let $I=\int e^x\left(\frac{1+\sin x\cos x}{\cos^2 x}\right)dx \quad \Rightarrow I=\int e^x\left(\frac{1}{\cos^2 x}+\frac{\sin x.\cos x}{\cos^2 x}\right)dx$

$\Rightarrow I=\int e^x\left(\sec^2 x+\tan x\right)dx$, where $f(x)=\tan x,\ f'(x)=\sec^2 x$

$\Rightarrow I=\int e^x\{f(x)+f'(x)\}dx \quad \Rightarrow I=e^x f(x)+c \quad \therefore\ I=e^x\tan x+c$

64. $\int e^x\left(\frac{\sin x\cos x-1}{\sin^2 x}\right)dx$

Sol. Let $I=\int e^x\left(\frac{\sin x\cos x-1}{\sin^2 x}\right)dx \quad \Rightarrow I=\int e^x\left(\frac{\sin x.\cos x}{\sin^2 x}-\frac{1}{\sin^2 x}\right)dx$

$\Rightarrow I=\int e^x\left(\cot x-\operatorname{cosec}^2 x\right)dx$, where $f(x)=\cot x,\ f'(x)=-\operatorname{cosec}^2 x$

$\Rightarrow I=\int e^x\{f(x)+f'(x)\}dx \quad \Rightarrow I=e^x f(x)+c \quad \therefore\ I=e^x\cot x+c$

65. $\int e^x\left(\frac{\cos x+\sin x}{\cos^2 x}\right)dx$

Sol. Let $I=\int e^x\left(\frac{\cos x+\sin x}{\cos^2 x}\right)dx \quad \Rightarrow I=\int e^x\left(\frac{\cos x}{\cos^2 x}+\frac{\sin x}{\cos^2 x}\right)dx$

$\Rightarrow I=\int e^x\left(\sec x+\tan x\sec x\right)dx$, where $f(x)=\sec x,\ f'(x)=\sec x\tan x$

$\Rightarrow I=\int e^x\left[f(x)+f'(x)\right]dx \quad \Rightarrow I=e^x.f(x)+c \quad \therefore I=e^x.\sec x+c$

66. $\int e^x\left(\frac{2-\sin 2x}{1-\cos 2x}\right)dx$

Sol. Let $I=\int e^x\left(\frac{2-\sin 2x}{1-\cos 2x}\right)dx \quad \Rightarrow I=\int e^x\left(\frac{2-\sin 2x}{2\sin^2 x}\right)dx \quad \Rightarrow I=\int e^x\left(\frac{2}{2\sin^2 x}-\frac{2\sin x\cos x}{2\sin^2 x}\right)dx$

$\Rightarrow I=\int e^x\left(\operatorname{cosec}^2 x-\cot x\right)dx$, where $f(x)=-\cot x,\ f'(x)=\operatorname{cosec}^2 x$

$\Rightarrow I=\int e^x\{f(x)+f'(x)\}dx \quad \Rightarrow I=e^x f(x)+c \quad \therefore\ I=-e^x\cot x+c$

67. $\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$

Sol. Let $I=\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$

$\Rightarrow I=\int e^x\left(\frac{1+\sin x}{2\cos^2\frac{x}{2}}\right)dx \quad \Rightarrow I=\int e^x\left(\frac{1}{2\cos^2\frac{x}{2}}+\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)dx$

$\Rightarrow I=\int e^x\left(\frac{1}{2}\sec^2\frac{x}{2}+\tan\frac{x}{2}\right)dx$, where $f(x)=\tan\frac{x}{2}, f'(x)=\sec^2\frac{x}{2}.\frac{1}{2}$

$\Rightarrow I=\int e^x\left[f(x)+f'(x)\right]dx \quad \Rightarrow I=e^x f(x)+c \quad \therefore\ I=e^x\tan\frac{x}{2}+c$

68. $\int e^x\left(\frac{\sin 4x-4}{1-\cos 4x}\right)dx$

Sol. Let $I=\int e^x\left(\frac{\sin 4x-4}{1-\cos 4x}\right)dx \quad \Rightarrow I=\int e^x\left(\frac{\sin 4x-4}{2\sin^2 2x}\right)dx$

$\Rightarrow I=\int e^x\left(\frac{\sin 4x}{2\sin^2 2x}-\frac{4}{2\sin^2 2x}\right)dx \quad \Rightarrow I=\int e^x\left(\frac{2\sin 2x\cos 2x}{2\sin^2 2x}-\frac{2}{\sin^2 2x}\right)dx$

$\Rightarrow \; I=\int e^x\left(\cot 2x-2\operatorname{cosec}^2 2x\right)dx$, where $f(x)=\cot 2x,\; f'(x)=-2\operatorname{cosec}^2 2x$

$\Rightarrow I=\int e^x\left[f(x)+f'(x)\right]dx \quad \Rightarrow I=e^x.f(x)+c \quad \therefore\; I=e^x\cot 2x+c$

69. $\int \frac{e^x\left[\sqrt{1-x^2}\sin^{-1}x+1\right]}{\sqrt{1-x^2}}dx$

Sol. Let $I=\int e^x\left[\frac{\sqrt{1-x^2}\sin^{-1}x+1}{\sqrt{1-x^2}}\right]dx \quad \Rightarrow I=\int e^x\left[\frac{\sqrt{1-x^2}\sin^{-1}x}{\sqrt{1-x^2}}+\frac{1}{\sqrt{1-x^2}}\right]dx$

$\Rightarrow I=\int e^x\left[\sin^{-1}x+\frac{1}{\sqrt{1-x^2}}\right]dx$, where $f(x)=\sin^{-1}x,\; f'(x)=\frac{1}{\sqrt{1-x^2}}$

$\Rightarrow I=\int e^x\left[f(x)+f'(x)\right]dx \quad \Rightarrow I=e^x f(x)+c \quad \therefore\; I=e^x.\sin^{-1}x+c$

70. $\int e^x\left(\frac{1+x\log x}{x}\right)dx$

Sol. Let $I=\int e^x\left(\frac{1+x\log x}{x}\right)dx \quad \Rightarrow I=\int e^x\left(\frac{1}{x}+\frac{x\log x}{x}\right)dx$

$\Rightarrow I=\int e^x\left(\frac{1}{x}+\log x\right)dx$, where $f(x)=\log x,\; f'(x)=\frac{1}{x}$

$\Rightarrow I=\int e^x\left[f(x)+f'(x)\right]dx \quad \Rightarrow I=e^x f(x)+c \quad \therefore\; I=e^x.\log x+c$

71. $\int e^x\left\{\frac{x}{(1+x)^2}\right\}dx$

Sol. Let $I=\int e^x\left\{\frac{x}{(x+1)^2}\right\}dx \quad \Rightarrow I=\int e^x\left\{\frac{(x+1)-1}{(x+1)^2}\right\}dx \quad \Rightarrow I=\int e^x\left\{\frac{1}{x+1}-\frac{1}{(x+1)^2}\right\}dx$

$\Rightarrow I=\int e^x\left\{\frac{1}{x+1}-\frac{1}{(x+1)^2}\right\}dx$, where $f(x)=\frac{1}{x+1}, f'(x)=-\frac{1}{(x+1)^2}$

$\Rightarrow I=\int e^x\left[f(x)+f'(x)\right]dx \quad \Rightarrow I=e^x f(x)+c \quad \Rightarrow I=e^x.\frac{1}{x+1}+c \quad \therefore\; I=\frac{e^x}{x+1}+c$

72. $\int e^x\left\{\frac{x-1}{(x+1)^3}\right\}dx$

Sol. Let $I=\int e^x\left\{\frac{x-1}{(x+1)^3}\right\}dx \quad \Rightarrow I=\int e^x\left\{\frac{(x+1)-2}{(x+1)^3}\right\}dx \quad \Rightarrow I=\int e^x\left\{\frac{x+1}{(x+1)^3}-\frac{2}{(x+1)^3}\right\}dx$

$$\Rightarrow I=\int e^x\left\{\frac{1}{(x+1)^2}-\frac{2}{(x+1)^3}\right\}dx,\text{ where } f(x)=\frac{1}{(x+1)^2},\ f'(x)=-\frac{2}{(x+1)^3}$$

$$\Rightarrow I=\int e^x\left[f(x)+f'(x)\right]dx \quad \Rightarrow I=e^x.f(x)+c \quad \therefore\ I=e^x.\frac{1}{(x+1)^2}+c$$

73. $\int e^x\left\{\frac{2-x}{(1-x)^2}\right\}dx$

Sol. Let $I=\int e^x\left\{\frac{2-x}{(1-x)^2}\right\}dx \quad \Rightarrow I=\int e^x\left\{\frac{(1-x)+1}{(1-x)^2}\right\}dx \quad \Rightarrow I=\int e^x\left\{\frac{(1-x)}{(1-x)^2}+\frac{1}{(1-x)^2}\right\}dx$

$$\Rightarrow I=\int e^x\left\{\frac{1}{1-x}+\frac{1}{(1-x)^2}\right\}dx,\text{ where } f(x)=\frac{1}{(1-x)},\ f'(x)=\frac{1}{(1-x)^2}$$

$$\Rightarrow I=\int e^x\left[f(x)+f'(x)\right]dx \quad \Rightarrow I=e^x f(x)+c \quad \therefore\ I=e^x.\frac{1}{1-x}+c$$

74. $\int e^x\left\{\frac{x-3}{(x-1)^3}\right\}dx$

Sol. Let $I=\int e^x\left\{\frac{x-3}{(x-1)^3}\right\}dx \quad \Rightarrow I=\int e^x\left\{\frac{x-1-2}{(x-1)^3}\right\}dx \quad \Rightarrow I=\int e^x\left\{\frac{x-1}{(x-1)^3}-\frac{2}{(x-1)^3}\right\}dx$

$$\Rightarrow I=\int e^x\left\{\frac{1}{(x-1)^2}-\frac{2}{(x-1)^3}\right\}dx,\text{ where } f(x)=\frac{1}{(x-1)^2},\ f'(x)=-\frac{2}{(x-1)^3}$$

$$\Rightarrow I=\int e^x\left[f(x)+f'(x)\right]dx \quad \Rightarrow I=e^x.f(x)+c \quad \therefore\ I=e^x.\frac{1}{(x-1)^2}+c$$

75. $\int e^{3x}\left(\frac{3x-1}{9x^2}\right)dx$

Sol. Let $I=\int e^{3x}\left(\frac{3x-1}{9x^2}\right)dx$, Put $3x=t \ \Rightarrow dx=\frac{dt}{3} \ \Rightarrow 3=\frac{dt}{dx}$

$$I=\int e^t\left(\frac{t-1}{t^2}\right)\frac{dt}{3} \quad \Rightarrow I=\frac{1}{3}\int e^t\left(\frac{1}{t}-\frac{1}{t^2}\right)dt,\text{ where } f(t)=\frac{1}{t},\ f'(t)=-\frac{1}{t^2}$$

$$\Rightarrow I=\int e^t\left[f(t)+f'(t)\right]dx$$

$$\Rightarrow I=\frac{1}{3}e^t.f(t)+c \quad \Rightarrow I=\frac{1}{3}e^t.\frac{1}{t}+c \quad \Rightarrow I=\frac{1}{3}.\frac{e^t}{3x}+c$$

$$\therefore\ I=\frac{e^{3x}}{9x}+c$$

76. $\int e^x\left\{\frac{x+1}{(x+2)^2}\right\}dx$

Sol. Let $I=\int e^x\left\{\frac{x+1}{(x+2)^2}\right\}dx \quad \Rightarrow I=\int e^x\left\{\frac{x+2-1}{(x+2)^2}\right\}dx \quad \Rightarrow I=\int e^x\left\{\frac{x+2}{(x+2)^2}-\frac{1}{(x+2)^2}\right\}dx$

$$\Rightarrow I=\int e^{x}\left\{\frac{1}{x+2}-\frac{1}{(x+2)^{2}}\right\}dx, \text{ where } f(x)=\frac{1}{x+2}, f'(x)=-\frac{1}{(x+2)^{2}}$$

$$\Rightarrow I=\int e^{x}\left[f(x)+f'(x)\right]dx \quad \Rightarrow I=e^{x}.f(x)+c \quad \therefore\ I=e^{x}.\frac{1}{x+2}+c$$

77. $\int\frac{x\,e^{2x}}{(1+2x)^{2}}dx$

Sol. Let $I=\int e^{2x}.\frac{x}{(1+2x)^{2}}dx$, Put $2x=t \ \Rightarrow 2=\frac{dt}{dx} \ \Rightarrow dx=\frac{dt}{2}$

$$\Rightarrow I=\int e^{t}.\frac{t}{2(1+t)^{2}}.\frac{dt}{2} \quad \Rightarrow I=\frac{1}{4}\int e^{t}\left\{\frac{t}{(t+1)^{2}}\right\}dt$$

$$\Rightarrow I=\frac{1}{4}\int e^{t}\left\{\frac{(t+1)-1}{(t+1)^{2}}\right\}dt \quad \Rightarrow I=\frac{1}{4}\int e^{t}\left\{\frac{t+1}{(t+1)^{2}}-\frac{1}{(t+1)^{2}}\right\}dt$$

$$\Rightarrow I=\frac{1}{4}\int e^{t}\left\{\frac{1}{(t+1)}-\frac{1}{(t+1)^{2}}\right\}dt, \text{ where } f(x)=\frac{1}{t+1}, f'(x)=-\frac{1}{(t+1)^{2}}$$

$$\Rightarrow I=\int e^{x}\left[f(x)+f'(x)\right]dx \quad \Rightarrow I=e^{x}.f(x)+c \quad \Rightarrow I=\frac{1}{4}.e^{t}.\frac{1}{t+1}+c \quad \therefore\ I=\frac{e^{2x}}{4(2x+1)}+c$$

78. $\int e^{2x}\left(\frac{2x-1}{4x^{2}}\right)$

Sol. Let $I=\int e^{2x}\left(\frac{2x-1}{4x^{2}}\right)dx$, Put $2x=t \ \Rightarrow 2=\frac{dt}{dx} \quad \Rightarrow dx=\frac{dt}{2}$

$$I=\frac{1}{2}\int e^{t}\left(\frac{t-1}{t^{2}}\right)dt \quad \Rightarrow I=\frac{1}{2}\int e^{t}\left(\frac{t}{t^{2}}-\frac{1}{t^{2}}\right)dt$$

$$\Rightarrow I=\frac{1}{2}\int e^{t}\left(\frac{1}{t}-\frac{1}{t^{2}}\right)dt, \text{ where } f(x)=\frac{1}{t}, f'(x)=-\frac{1}{t^{2}}$$

$$\Rightarrow I=\int e^{x}\left[f(x)+f'(x)\right]dx \quad \Rightarrow I=e^{x}.f(x)+c \quad \Rightarrow I=\frac{1}{2}.e^{t}.\frac{1}{t}+c \quad \Rightarrow I=\frac{e^{2x}}{2.2x}+c$$

$$\therefore\ I=\frac{e^{2x}}{4x}+c$$

79. $\int e^{x}\left(\log x+\frac{1}{x^{2}}\right)dx$

Sol. Let $I=\int e^{x}\left(\log x+\frac{1}{x^{2}}\right)dx \quad \Rightarrow I=\int e^{x}\left[\left(\log x+\frac{1}{x}\right)+\left\{-\frac{1}{x}+\frac{1}{x^{2}}\right\}\right]dx$

$$\Rightarrow I=\int e^{x}\left(\log x+\frac{1}{x}\right)dx-\int e^{x}\left(\frac{1}{x}-\frac{1}{x^{2}}\right)dx,$$

$$\Rightarrow I=\int e^{x}\left[f_{1}(x)+f_{1}'(x)\right]dx-\int e^{x}\left[f_{2}(x)+f_{2}'(x)\right]dx$$

where $f_{1}(x)=\log x,\ f_{1}'(x)=\frac{1}{x},\ \ f_{2}(x)=\frac{1}{x}, f_{2}'(x)=-\frac{1}{x^{2}}$

$$\Rightarrow I = e^x f_1(x) - e^x f_2(x) + c \quad \Rightarrow I = e^x.\log x - e^x.\frac{1}{x} + c \quad \therefore\ I = e^x\left(\log x - \frac{1}{x}\right) + c$$

80. $\displaystyle\int \frac{\log x}{(1+\log x)^2}dx$

Sol. Let $I = \displaystyle\int \frac{\log x}{(1+\log x)^2}dx$, Put $\log x = t \ \therefore\ x = e^t \Rightarrow \frac{1}{x} = \frac{dt}{dx} \quad \Rightarrow dx = x\ dt \quad \Rightarrow dx = e^t\ dt$

$$I = \int e^t.\frac{t}{(1+t)^2}dt \quad \Rightarrow I = \int e^t\left\{\frac{t+1-1}{(t+1)^2}\right\}dt \quad \Rightarrow I = \int e^t\left\{\frac{t+1}{(t+1)^2} - \frac{1}{(t+1)^2}\right\}dt$$

$$\Rightarrow I = \int e^t\left\{\frac{1}{t+1} - \frac{1}{(t+1)^2}\right\}dt \quad \Rightarrow I = e^t.\frac{1}{t+1} + c \quad \therefore\ I = \frac{x}{\log x + 1} + c$$

81. $\displaystyle\int\{\sin(\log x) + \cos(\log x)\}dx$

Sol. Let $I = \displaystyle\int\{\sin(\log x) + \cos(\log x)\}dx$

Put $\log x = t \quad \therefore\ x = e^t \quad \Rightarrow \frac{1}{x} = \frac{dt}{dx} \quad \Rightarrow dx = x\ dt \quad \Rightarrow dx = e^t\ dt$

$I = \displaystyle\int e^t(\sin t + \cos t)dt$, where $f(x) = \sin t,\ f'(x) = \cos t$

$$\Rightarrow I = \int e^x[f(x) + f'(x)]dx \quad \Rightarrow I = e^x.f(x) + c \quad \Rightarrow I = e^t\sin t + c \quad \therefore\ I = x\sin(\log x) + c$$

82. $\displaystyle\int\left\{\frac{1}{\log x} - \frac{1}{(\log x)^2}\right\}dx$

Sol. Let $I = \displaystyle\int\left\{\frac{1}{\log x} - \frac{1}{(\log x)^2}\right\}dx$, Put $\log x = t \quad \therefore\ x = e^t$

$$\frac{1}{x} = \frac{dt}{dx} \quad \Rightarrow \frac{1}{x}dx = dt \quad \Rightarrow dx = x\ dt \quad \Rightarrow dx = e^t\ dt$$

$I = \displaystyle\int\left(\frac{1}{t} - \frac{1}{t^2}\right)e^t\ dt$, where $f(x) = \frac{1}{t},\ f'(x) = -\frac{1}{t^2}$

$$\Rightarrow I = \int e^x[f(x) + f'(x)]dx \quad \Rightarrow I = e^x.f(x) + c \quad \Rightarrow I = e^t.\frac{1}{t} + c \quad \therefore I = \frac{x}{\log x} + c$$

83. $\displaystyle\int\left\{\log(\log x) + \frac{1}{(\log x)^2}\right\}dx$

Sol. Let $I = \displaystyle\int\left\{\log(\log x) + \frac{1}{(\log x)^2}\right\}dx$

Put $\log x = t \quad \therefore\ x = e^t \quad \Rightarrow \frac{1}{x} = \frac{dt}{dx} \quad \Rightarrow dx = x\ dt \quad \Rightarrow dx = e^t\ dt$

$$I = \int e^t\left(\log t + \frac{1}{t^2}\right)dt \quad \Rightarrow I = \int e^t\left[\left(\log t + \frac{1}{t}\right) + \left\{-\frac{1}{t} + \frac{1}{t^2}\right\}\right]dt$$

$$\Rightarrow I = \int e^t\left(\log t + \frac{1}{t}\right)dt - \int e^t\left(\frac{1}{t} - \frac{1}{t^2}\right)dt,$$

$$\Rightarrow I=\int e^t\left[f_1(t)+f_1'(t)\right]dt-\int e^t\left[f_2(t)+f_2'(t)\right]dt \quad \Rightarrow I=e^t f_1(t)-e^t f_2(t)+c$$

where $f_1(t)=\log t,\ f_1'(t)=\frac{1}{t},\ f_2(t)=\frac{1}{t}, f_2'(t)=-\frac{1}{t^2}$

$$\Rightarrow I=e^t.\log t-e^t.\frac{1}{t}+c \quad \Rightarrow I=e^t\left(\log t-\frac{1}{t}\right)+c \quad \therefore I=x\left(\log(\log x)-\frac{1}{\log x}\right)+c$$

84. $\int\frac{\sin^{-1}(\sqrt{x})-\cos^{-1}(\sqrt{x})}{\sin^{-1}(\sqrt{x})+\cos^{-1}(\sqrt{x})}dx$

Sol. Let $I=\int\frac{\sin^{-1}(\sqrt{x})-\cos^{-1}(\sqrt{x})}{\sin^{-1}(\sqrt{x})+\cos^{-1}(\sqrt{x})}$ $\quad [\because \sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}]$

$$\Rightarrow I=\int\frac{\sin^{-1}(\sqrt{x})-\cos^{-1}(\sqrt{x})}{\pi/2}dx \quad \Rightarrow I=\frac{2}{\pi}\int\left\{\sin^{-1}(\sqrt{x})-\left[\frac{\pi}{2}-\sin^{-1}(\sqrt{x})\right]\right\}dx$$

$$\Rightarrow I=\frac{2}{\pi}\int\left\{\sin^{-1}(\sqrt{x})-\left[\frac{\pi}{2}-\sin^{-1}(\sqrt{x})\right]\right\}dx \quad \Rightarrow I=\frac{2}{\pi}\int\left\{2\sin^{-1}(\sqrt{x})-\frac{\pi}{2}\right\}dx$$

$$\Rightarrow I=\frac{2}{\pi}\int\left\{2\sin^{-1}(\sqrt{x})-\frac{\pi}{2}\right\}dx \quad \Rightarrow I=\frac{4}{\pi}\int\sin^{-1}(\sqrt{x})dx-\int dx \quad \Rightarrow I=\frac{4}{\pi}I_1-x \qquad \ldots(1)$$

$$I_1=\int\sin^{-1}(\sqrt{x})\,dx,\ \text{Put } \sqrt{x}=t \quad \Rightarrow \frac{1}{2\sqrt{x}}=\frac{dt}{dx} \quad \Rightarrow dx=2\sqrt{x}\ dt \quad \Rightarrow dx=2t\ dt$$

$$\Rightarrow I_1=\int\sin^{-1}(t).2t\ dt \quad \Rightarrow I_1=2\int t\ \sin^{-1}(t)dt$$

$$\Rightarrow I_1=2\sin^{-1}t\int t\,dt-\int\left[\frac{d(\sin^{-1}t)}{dt}\int t\ dt\right]dt \quad \Rightarrow I_1=2\left[\sin^{-1}t.\frac{t^2}{2}-\int\frac{1}{\sqrt{1-t^2}}.\frac{t^2}{2}\ dt\right]$$

$$\Rightarrow I_1=2\left[\frac{t^2}{2}\sin^{-1}t+\frac{1}{2}\int-\frac{t^2}{\sqrt{1-t^2}}\ dt\right] \quad \Rightarrow I_1=2\left[\frac{t^2}{2}\sin^{-1}t+\frac{1}{2}\int\frac{(1-t^2)-1}{\sqrt{1-t^2}}\ dt\right]$$

$$\Rightarrow I_1=2\left[\frac{t^2}{2}\sin^{-1}t+\frac{1}{2}\int\frac{1-t^2}{\sqrt{1-t^2}}\ dt-\frac{1}{2}\int\frac{1}{\sqrt{1-t^2}}dt\right]$$

$$\Rightarrow I_1=2\left[\frac{t^2}{2}\sin^{-1}t+\frac{1}{2}\int\sqrt{1-t^2}\ dt-\frac{1}{2}\sin^{-1}t\right]+c$$

$$\Rightarrow I_1=2\left[\frac{t^2}{2}\sin^{-1}t+\frac{t}{4}\sqrt{1-t^2}+\frac{1}{4}\sin^{-1}t-\frac{1}{2}\sin^{-1}t\right]+c$$

$$\Rightarrow I_1=2\left[\frac{t^2}{2}\sin^{-1}t+\frac{t}{4}\sqrt{1-t^2}-\frac{1}{4}\sin^{-1}t\right]+c \quad \Rightarrow I_1=\left[x\sin^{-1}(\sqrt{x})+\frac{\sqrt{x}}{2}\sqrt{1-x}-\frac{1}{2}\sin^{-1}\sqrt{x}\right]+c$$

Put the value of I_1 in equations (1), $\therefore\ I=\frac{4}{\pi}\left[x\sin^{-1}(\sqrt{x})+\frac{\sqrt{x}}{2}\sqrt{1-x}-\frac{1}{2}\sin^{-1}\sqrt{x}\right]-x+c$

85. $\int 5^{5^{5^x}}.5^{5^x}.5^x\,dx$

Sol. Let $I=\int 5^{5^{5^x}}.5^{5^x}.5^x\,dx$, Put $t=5^{5^x}$ $\Rightarrow \log t=5^x\log 5$ $\Rightarrow \frac{1}{t}dt=5^x\log 5.\log 5\,dx$

$$\Rightarrow dt = t.5^x(\log 5)^2 dx \ \Rightarrow dt = 5^{5^x}.5^x(\log 5)^2 dx \ \Rightarrow \frac{dt}{(\log 5)^2} = 5^{5^x}.5^x dx$$

$$\therefore \quad I = \int 5^t . \frac{dt}{(\log 5)^2} = \frac{1}{(\log 5)^2} . \frac{5^t}{\log 5} + c = \frac{5^t}{(\log 5)^3} + c = \frac{5^{5^{5^x}}}{(\log 5)^3} + c$$

86. $\int e^{2x}\left(\frac{1+\sin 2x}{1+\cos 2x}\right)dx$

Sol. Let $I = \int e^{2x}\left(\frac{1+\sin 2x}{1+\cos 2x}\right)dx$

Put $t = 2x \ \Rightarrow \frac{dt}{2} = dx$, $\int e^t\left(\frac{1+\sin t}{1+\cos t}\right)\frac{dt}{2} = \frac{1}{2}\int e^t\left(\frac{1}{1+\cos t} + \frac{\sin t}{1+\cos t}\right)dt$

$$= \frac{1}{2}\int e^t\left(\frac{1}{2\cos^2\frac{t}{2}} + \frac{2\sin\frac{t}{2}.\cos\frac{t}{2}}{2\cos^2\frac{t}{2}}\right)dt = \frac{1}{2}\int e^t\left(\frac{1}{2}\sec^2\frac{t}{2} + \tan\frac{t}{2}\right)dt$$

Here $f(t) = \tan\frac{t}{2} \ \Rightarrow f'(t) = \frac{1}{2}\sec^2\frac{t}{2}$

$$\int e^t\left(\frac{1+\sin t}{1+\cos t}\right)\frac{dt}{2} = \frac{1}{2}\int e^t\{f(t) + f'(t)\}dt = \frac{1}{2}e^t.f(t) + c = \frac{1}{2}e^{2x}.\tan\frac{t}{2} + c$$

$$= \frac{1}{2}e^{2x}\tan\left(\frac{2x}{2}\right) + c = \frac{1}{2}e^{2x}\tan x + c$$

87. $\int e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$

Sol. Let $I = \int e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$

Again Let $2x = t$

Diffı on the both side w.r. to x, we get $\frac{d}{dx}(2x) = \frac{d}{dx}(t)$

$$2 = \frac{dt}{dx}$$

$$dx = \frac{dt}{2}$$

$$I = \int e^t\left(\frac{1-\sin t}{1-\cos t}\right)\frac{dt}{2}$$

$$I = \frac{1}{2}\int e^t\left(\frac{1-\sin t}{1-\cos t}\right)dt$$

$$I = \frac{1}{2}\int e^t\left(\frac{1-2\sin\left(\frac{t}{2}\right)\cos\left(\frac{t}{2}\right)}{2\sin^2\left(\frac{t}{2}\right)}\right)dt$$

$$I = \frac{1}{2}\int e^t\left(\frac{1}{2\sin^2(t/2)} - \frac{2\sin(t/2)\cos(t/2)}{2\sin^2(t/2)}\right)dt$$

$$I = \frac{1}{2}\int e^t \left(\frac{1}{2\sin^2(t/2)} - \frac{\cos(t/2)}{\sin(t/2)} \right) dt$$

$$I = \frac{1}{2}\int e^t \left(\frac{1}{2}\operatorname{cosec}^2(t/2) - \cot(t/2) \right) dt$$

$$I = -\frac{1}{2}\int e^t \left[\cot(t/2) + \left(-\frac{1}{2}\operatorname{cosec}^2(t/2) \right) \right] dt$$

Let $F(t) = \cot(t/2)$

Diffination on the both side w.r. to t we get

$$\frac{d}{dt}\left[F(t)\right] = \frac{d}{dt}\left[\cot(t/2)\right]$$

$$F'(t) = -\operatorname{cosec}^2(t/2) \times \frac{1}{2}$$

$$F'(t) = -\frac{1}{2}\operatorname{cosec}^2(t/2)$$

Using $\int e^t \left[F(t) + F'(t)\right] dt = e^t F(t) + c$

$$I = -\frac{1}{2}F(t) + c$$

$$I = -\frac{1}{2}e^t \cot(t/2) + c$$

$$I = -\frac{1}{2}e^{2x} \cot\left(\frac{2x}{2}\right) + c$$

$$I = -\frac{1}{2}e^{2x} \cot x + c$$