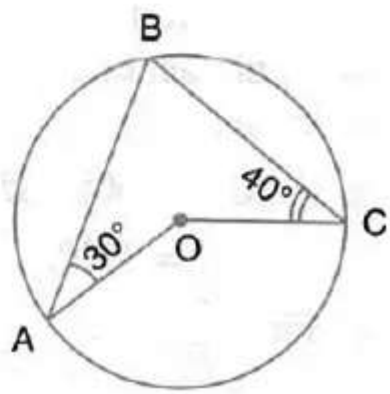
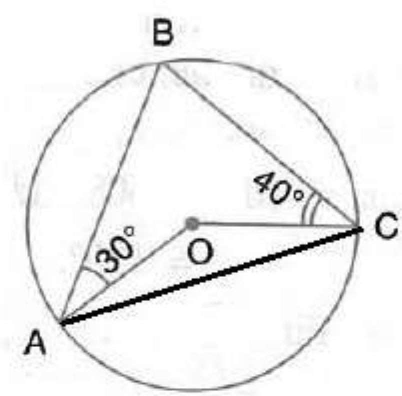
Circles

Exercise 17A

Question 1.
In the given figure, O is the centre of the circle. ∠OAB and ∠OCB are 30° and 40° respectively. Find ∠AOC. Show your steps of working.



Solution:



Join AC.

Let ∠OAC = ∠OCA = x (say)

$\therefore \quad \angle AOC = 180^\circ - 2x$

Also, $\angle BAC = 30^\circ + x$

$\angle BCA = 40^\circ + x$

In ΔABC,

$\angle ABC = 180^\circ - \angle BAC - \angle BCA$

$= 180^\circ - (30^\circ + x) - (40^\circ + x) = 110^\circ - 2x$

Now, $\angle AOC = 2\angle ABC$

$$\left(\begin{array}{l}\text{Angle at the centre is double the angle at the}\\ \text{circumference subtended by the same chord}\end{array}\right)$$

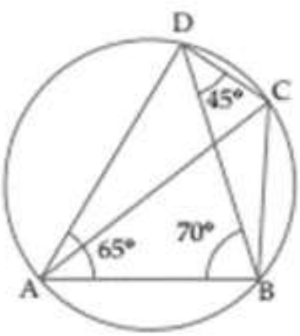
$\Rightarrow \quad 180^0 - 2x = 2\left(110^0 - 2x\right)$

$\Rightarrow \quad 2x = 40^0$

$\therefore \quad x = 20^0$

$\therefore \quad \angle AOC = 180^0 - 2 \times 20^0 = 140^0$

Question 2.
In the given figure, ∠BAD = 65°, ∠ABD = 70°, ∠BDC = 45°
(i) Prove that AC is a diameter of the circle.
(ii) Find ∠ACB.



Solution:

(i) In ΔABD,

$\angle DAB + \angle ABD + \angle ADB = 180°$

$\Rightarrow 65° + 70° + \angle ADB = 180°$

$\Rightarrow 135° + \angle ADB = 180°$

$\Rightarrow \angle ADB = 180° - 135° = 45°$

Now, $\angle ADC = \angle ADB + \angle BDC = 45° + 45° = 90°$

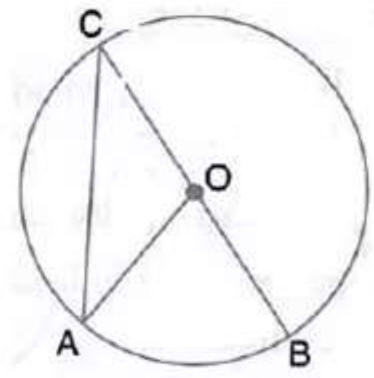
Since $\angle ADC$ is the angle of semicircle, so AC is a diameter of the circle.

(ii) $\angle ACB = \angle ADB$ ....(angles in the same segment of a circle)

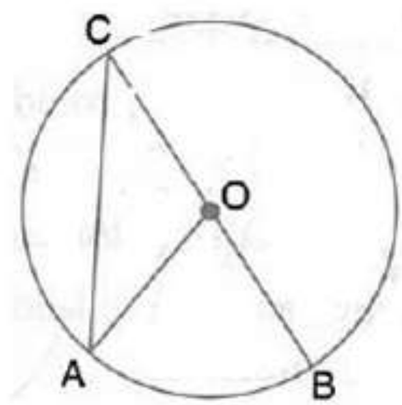
$\Rightarrow \angle ACB = 45°$

Question 3.

Given O is the centre of the circle and ∠AOB = 70°. Calculate the value of:
(i) ∠ OCA,
(ii) ∠OAC.



Solution:



Here, $\angle AOB = 2\angle ACB$

$\left(\begin{array}{l}\text{Angle at the centre is double the angle at the} \\ \text{circumference subtended by the same chord}\end{array}\right)$

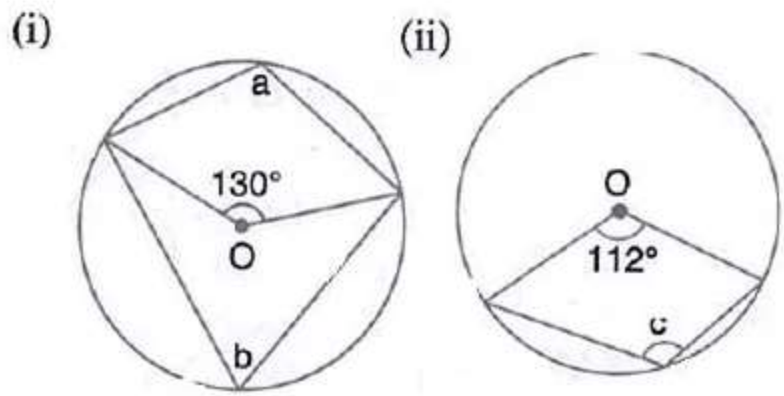
$\Rightarrow \quad \angle ACB = \dfrac{70^{\circ}}{2} = 35^{\circ}$

Now, $OC = OA$ (Radii of same circle)

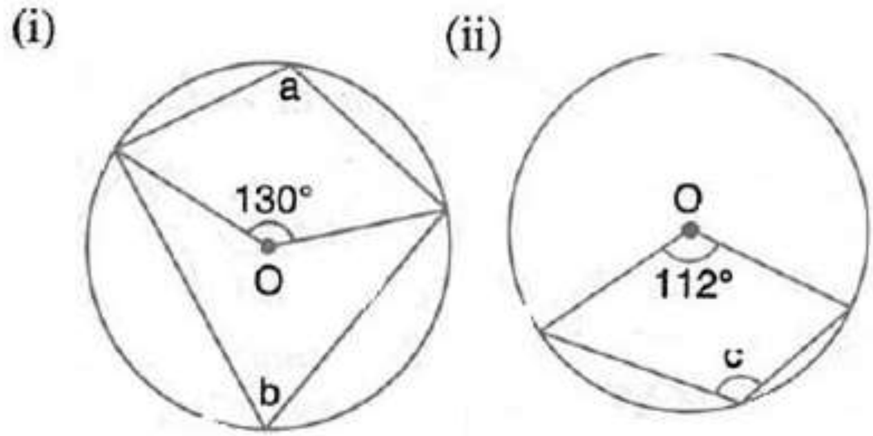
$\Rightarrow \quad \angle OCA = \angle OAC = 35^{\circ}$

Question 4.

In each of the following figures, O is the centre of the circle. Find the values of a, b, and c.



Solution:



(i) Here, $b = \frac{1}{2} \times 130^0$

$\left(\begin{array}{l}\text{Angle at the centre is double the angle at the} \\ \text{circumference subtended by the same chord}\end{array}\right)$

$\Rightarrow \quad b = 65^0$

Now, $a + b = 180^0$

(Opposite angles of a cyclic quadrilateral are suplementary)

$\Rightarrow \quad a = 180^0 - 65^0 = 115^0$

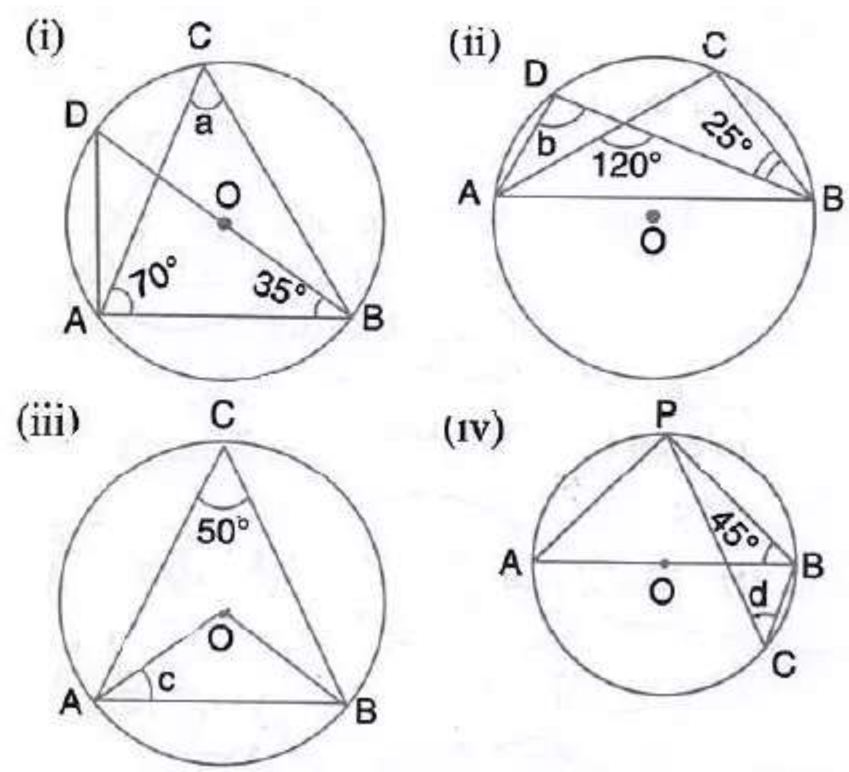
(ii) Here, $c = \frac{1}{2}\ \text{Reflex}\left(112^0\right)$

$\left(\begin{array}{l}\text{Angle at the centre is double the angle at the} \\ \text{circumference subtended by the same chord}\end{array}\right)$

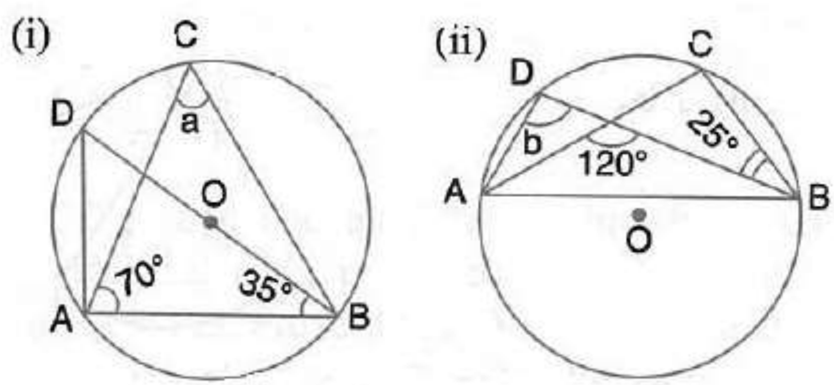
$\Rightarrow \quad c = \frac{1}{2} \times \left(360^0 - 112^0\right) = 124^0$

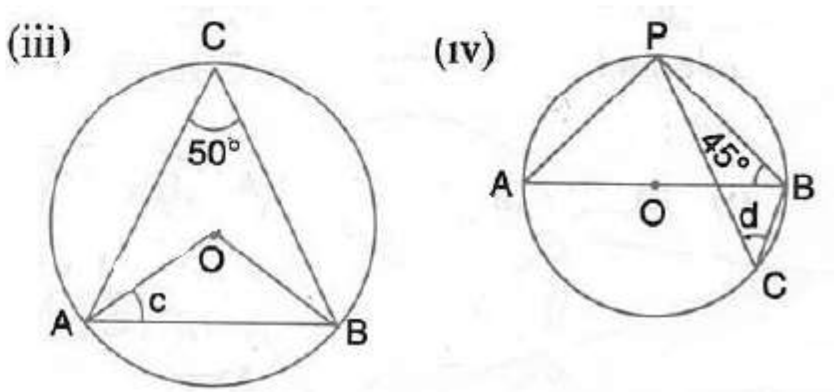
Question 5.

In each of the following figures, O is the centre of the circle. Find the value of a, b, c and d.



Solution:





(i) Here, $\angle BAD = 90^0$ (Angle in a semicircle)

$\therefore \quad \angle BDA = 90^0 - 35^0 = 55^0$

Again, $a = \angle ACB = \angle BDA = 55^0$

(Angles subtended by the same chord on the circle are equal)

(ii) Here, $\angle DAC = \angle CBD = 25^0$

(Angles subtended by the same chord on the circle are equal)

Again, $120^0 = b + 25^0$

(In a triangle, measure of exterior angle is equal to the sum of pair of opposite interior angles)

$\Rightarrow \quad b = 95^0$

(iii) $\angle AOB = 2\angle AOB = 2 \times 50^0 = 100^0$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

Also, $OA = OB$

$\Rightarrow \quad \angle OBA = \angle OAB = c$

$\therefore \quad c = \dfrac{180^0 - 100^0}{2} = 40^0$

(iv) $\angle APB = 90^0$ (Angle in a semicircle)

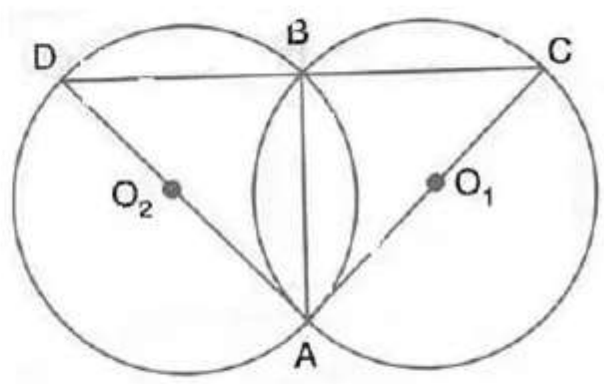
$\therefore \quad \angle BAP = 90^0 - 45^0 = 45^0$

Now, $d = \angle BCP = \angle BAP = 45^0$

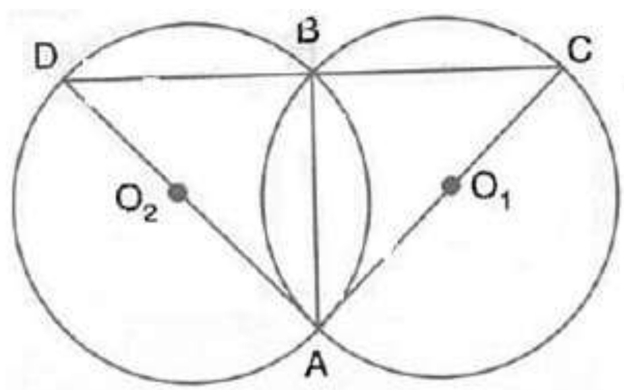
(Angles subtended by the same chord on the circle are equal)

Question 6.

In the figure, AB is common chord of the two circles. If AC and AD are diameters; prove that D, B and C are in a straight line. O_1 and O_2 are the centres of two circles.



Solution:



$\angle DBA = 90^0$ and $\angle CBA = 90^0$

(Angles in a semicircle is a right angle)

Adding both we get,

$\angle DBC = 180^0$

$\therefore$ D, B and C form a straight line.

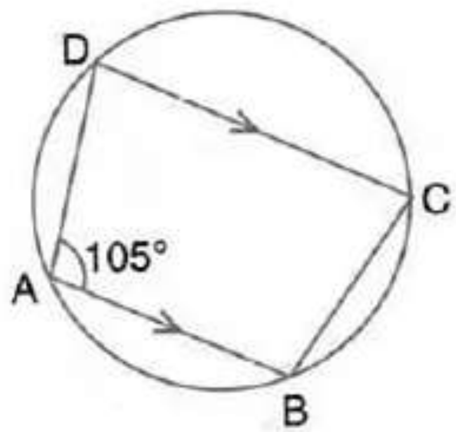
Question 7.

In the figure given below, find :

(i) ∠ BCD,

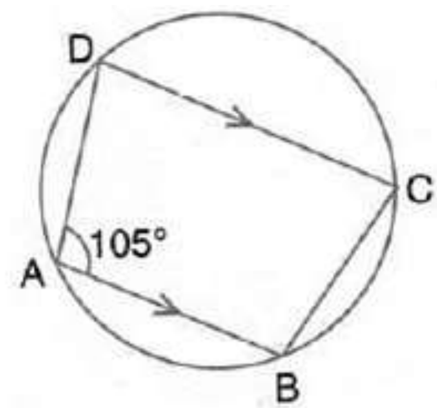
(ii) ∠ ADC,

(iii) ∠ ABC.



Show steps of your working.

Solution:



(i) $\angle BCD + \angle BAD = 180^0$

(Sum of opposite angles of a cyclic quadrilateral is 180^0)

$\Rightarrow \quad \angle BCD = 180^0 - 105^0 = 75^0$

(ii) Now, AB || CD

$\therefore \quad \angle BAD + \angle ADC = 180^0$

(Interior angles on same side of parallel linesis 180^0)

$\Rightarrow \quad \angle ADC = 180^0 - 105^0 = 75^0$

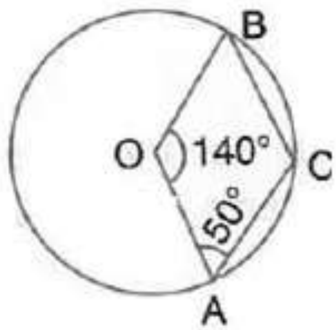
(iii) $\angle ADC + \angle ABC = 180^0$

(Sum of opposite angles of a cyclic quadrilateral is 180^0)

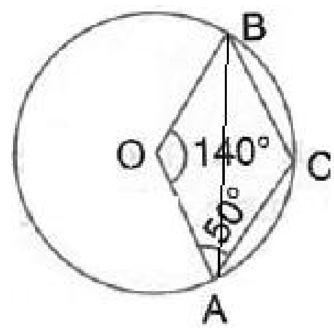
$\Rightarrow \quad \angle ABC = 180^0 - 75^0 = 105^0$

Question 8.

In the given figure, O is centre of the circle. If ∠ AOB = 140° and ∠ OAC = 50°, find :
(i) ∠ ACB,
(ii) ∠ OBC,
(iii) ∠ OAB,
(iv) ∠CBA



Solution:



Here, $\angle ACB = \frac{1}{2}\text{Reflex}(\angle AOB) = \frac{1}{2}(360^0 - 140^0) = 110^0$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

Now, OA = OB (Radii of same circle)

$\therefore \quad \angle OBA = \angle OAB = \frac{180^0 - 140^0}{2} = 20^0$

$\therefore \quad \angle CAB = 50^0 - 20^0 = 30^0$

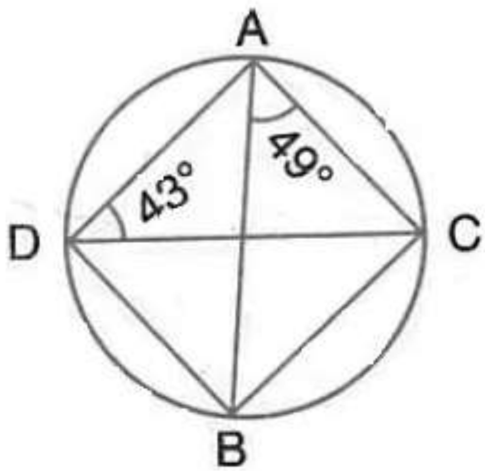
In ΔCAB,

$\angle CBA = 180^0 - 110^0 - 30^0 = 40^0$

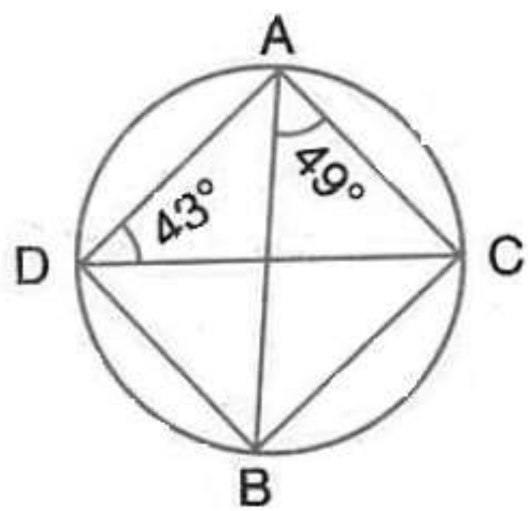
$\therefore \quad \angle OBC = \angle CBA + \angle OBA = 40^0 + 20^0 = 60^0$

Question 9.

Calculate :
(i) ∠ CDB,
(ii) ∠ ABC,
(iii) ∠ ACB.



Solution:



Here,

$\angle CDB = \angle BAC = 49^0$

$\angle ABC = \angle ADC = 43^0$

(Angles subtended by the same chord on the circle are equal)

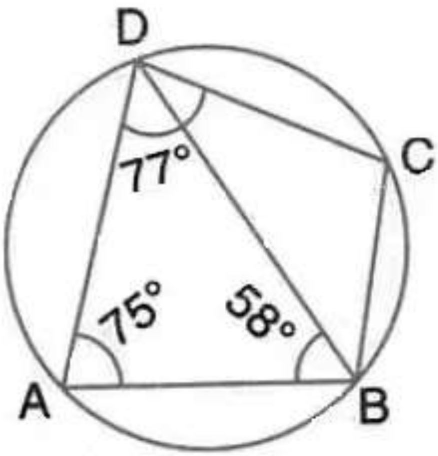
By angle – sum property of a triangle,

$\angle ACB = 180^0 - 49^0 - 43^0 = 88^0$

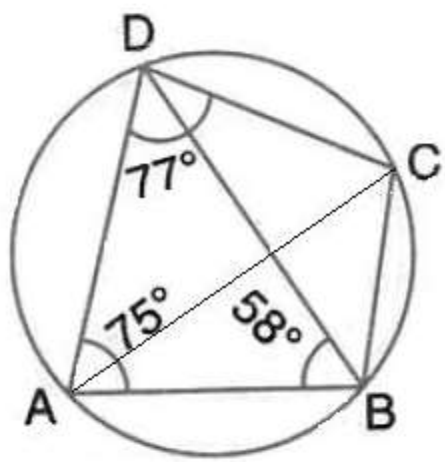
Question 10.

In the figure given below, ABCD is a cyclic quadrilateral in which ∠ BAD = 75°; ∠ ABD = 58° and ∠ADC = 77°. Find:

(i) ∠ BDC,
(ii) ∠ BCD,
(iii) ∠ BCA.



Solution:



(i) By angle – sum property of triangle ABD,

$\angle ADB = 180^0 - 75^0 - 58^0 = 47^0$

$\therefore \angle BDC = \angle ADC - \angle ADB = 77^0 - 47^0 = 30^0$

(ii) $\angle BAD + \angle BCD = 180^0$

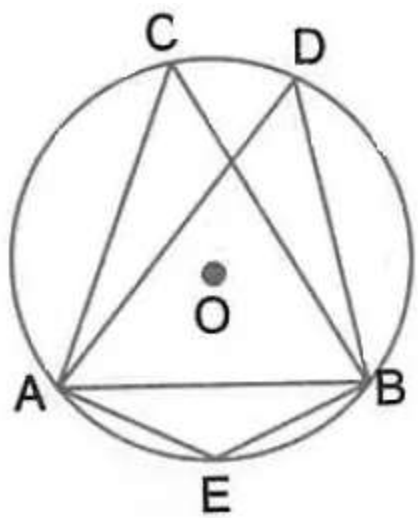
(Sum of opposite angles of a cyclic quadrilateral is 180^0)

(iii) $\angle BCA = \angle ADB = 47^{0}$

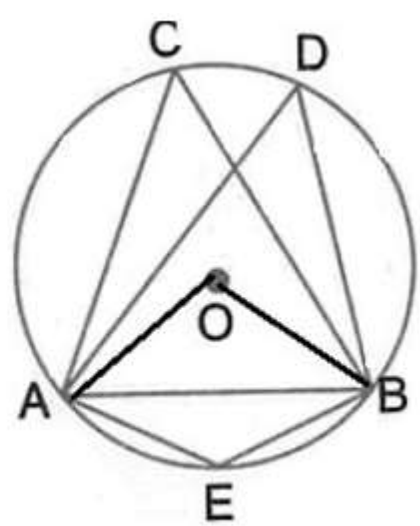
$\left(\begin{array}{l}\text{Angles subtended by the same chord on the circle} \\ \text{are equal}\end{array}\right)$

Question 11.
In the following figure, O is centre of the circle and Δ ABC is equilateral. Find :

(i) ∠ ADB
(ii) ∠ AEB



Solution:



Since ∠ACB and ∠ADB are in the same segement,

$\angle ADB = \angle ACB = 60^0$

Join OA and OB.

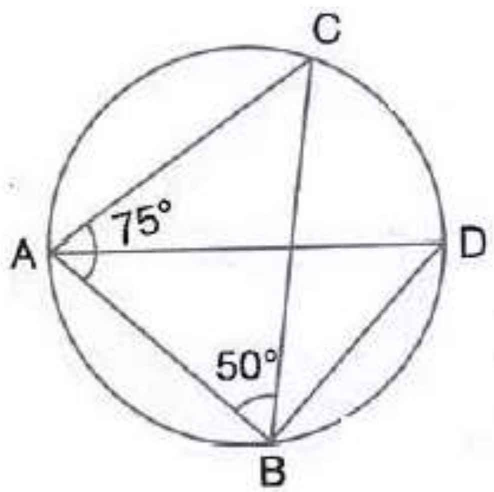
Here, $\angle AOB = 2\angle ACB = 2 \times 60^0 = 120^0$

$\angle AEB = \frac{1}{2}\text{Reflex}(\angle AOB) = \frac{1}{2}(360^0 - 120^0) = 120^0$

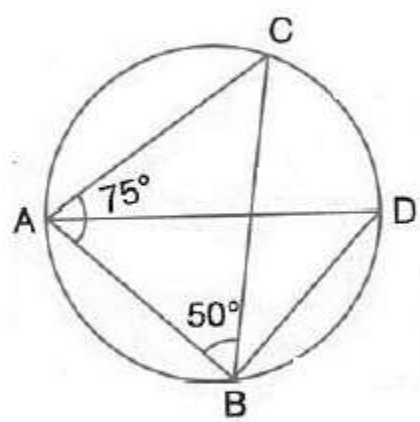
(Angle at the centre is double the angle at the circumference subtended by the same chord)

Question 12.

Given—∠ CAB = 75° and ∠ CBA = 50°. Find the value of ∠ DAB + ∠ ABD



Solution:



In ΔABC, $\angle CBA = 50^0$, $\angle CAB = 75^0$

$\angle ACB = 180^0 - (\angle CBA + \angle CAB)$

$= 180^0 - (50^0 + 75^0)$

$= 180^0 - 125^0$

$= 55^0$

But $\angle ADB = \angle ACB = 55^0$

(Angles subtended by the same chord on the circle are equal)

Now consider ΔABD,

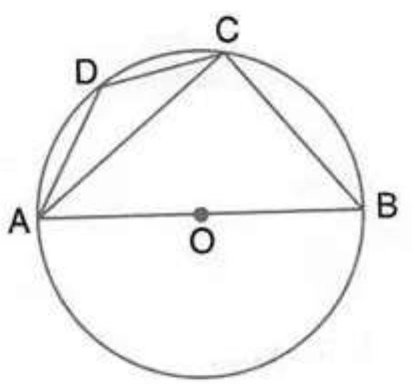
$\angle DAB + \angle ABD + \angle ADB = 180^0$

$\Rightarrow \angle DAB + \angle ABD + 55^0 = 180^0$

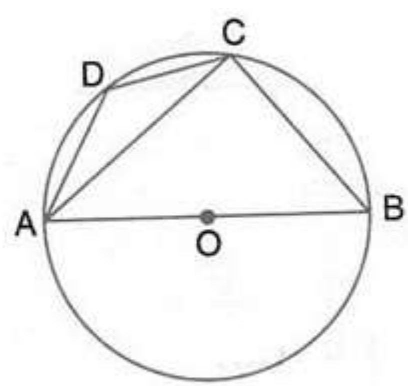
$\Rightarrow \angle DAB + \angle ABD = 180^0 - 55^0$

$\Rightarrow \angle DAB + \angle ABD = 125^0$

Question 13.
ABCD is a cyclic quadrilateral in a circle with centre O.
If ∠ ADC = 130°; find ∠ BAC.



Solution:



Here, $\angle ACB = 90^0$

(Angle in a semicircle is a right angle)

Also, $\angle ABC = 180^0 - \angle ADC = 180^0 - 130^0 = 50^0$

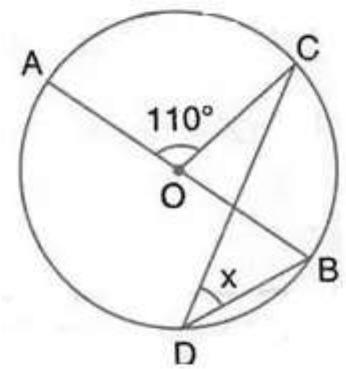
(Pair of opposite angles in a cydic quadrilateral are supplementary)

By angle sum property of right triangle ACB,

$\angle BAC = 90^0 - \angle ABC = 90^0 - 50^0 = 40^0$

Question 14.

In the figure given below, AOB is a diameter of the circle and ∠ AOC = 110°. Find ∠ BDC.



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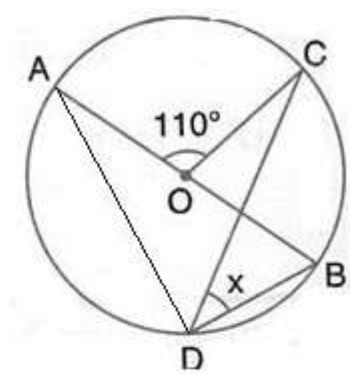


Join AD.

Here, $\angle ADC = \frac{1}{2}\angle AOC = \frac{1}{2} \times 110^0 = 55^0$

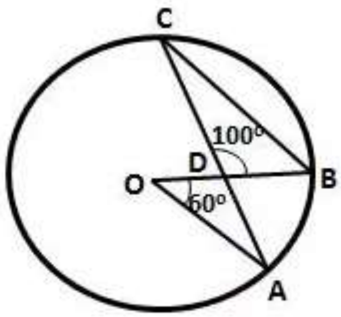
(Angle at the centre is double the angle at the circumference subtended by the same chord)

Also, $\angle ADB = 90^0$

(Angle in a semicircle is a right angle)

$\therefore \angle BDC = 90^0 - \angle ADC = 90^0 - 55^0 = 35^0$

Question 15.
In the following figure, O is centre of the circle,
∠ AOB = 60° and ∠ BDC = 100°.
Find ∠ OBC.



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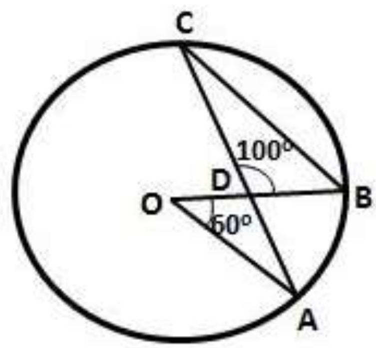


Here, $\angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 60^0 = 30^0$

$\begin{pmatrix} \text{Angle at the centre is double the angle at the} \\ \text{circumference subtended by the same chord} \end{pmatrix}$

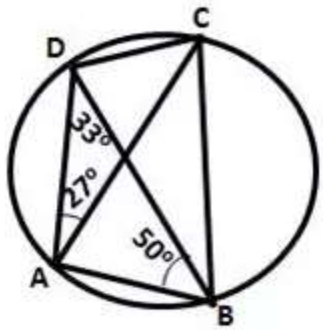
By angle sum property of ΔBDC,

$\therefore \angle DBC = 180^0 - 100^0 - 30^0 = 50^0$

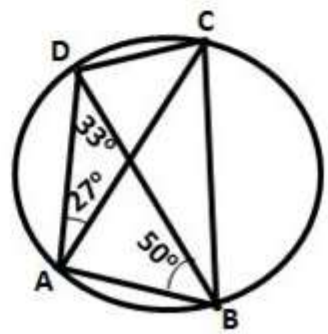
Hence, $\angle OBC = 50^0$

Question 16.
ABCD is a cyclic quadrilateral in which ∠ DAC = 27°; ∠ DBA = 50° and ∠ ADB = 33°.

Calculate :
(i) ∠ DBC,
(ii) ∠ DCB,
(iii) ∠ CAB.



Solution:



(i) $\angle DBC = \angle DAC = 27^0$

(Angles subtended by the same chord on the circle are equal)

(ii) $\angle ACB = \angle ADB = 33^0$

$\angle ACD = \angle ABD = 50^0$

(Angles subtended by the same chord on the circle are equal)

$\therefore \angle DCB = \angle ACD + \angle ACB = 50^0 + 33^0 = 83^0$

(iii) $\angle DAB + \angle DCB = 180^0$

(Pair of opposite angles in a cyclic quadrilateral are supplementary)

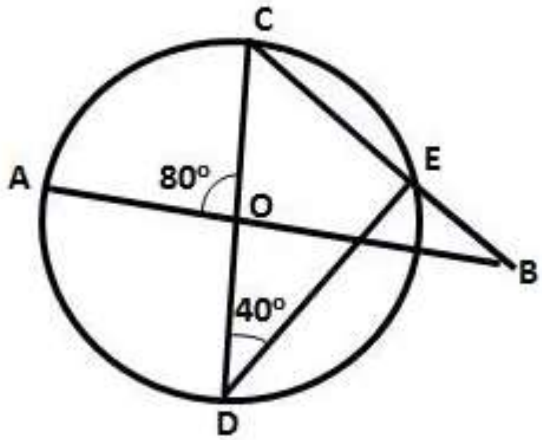
$\Rightarrow 27^0 + \angle CAB + 83^0 = 180^0$

$\Rightarrow \angle CAB = 180^0 - 110^0 = 70^0$

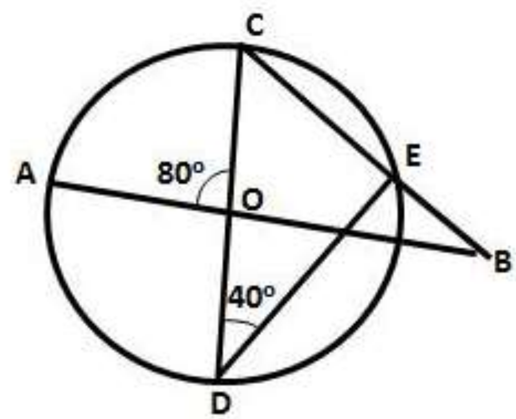
Question 17.

In the figure given alongside, AB and CD are straight lines through the centre O of a circle. If ∠AOC = 80° and ∠CDE = 40°. Find the number of degrees in:

(i) ∠DCE;
(ii) ∠ABC.



Solution:



(i) Here, $\angle CED = 90^0$

(Angle in a semicircle is a right angle)

$\therefore\ \angle DCE = 90^0 - \angle CDE = 90^0 - 40^0 = 50^0$

$\therefore\ \angle DCE = \angle OCB = 50^0$

(ii) In ΔBOC,

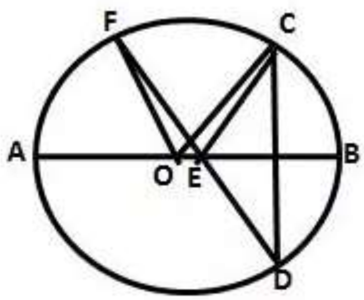
$\angle AOC = \angle OCB + \angle OBC$

(Exterior angle of a Δ is equal to the sum of pair of interior opposite angles)

$\Rightarrow \angle OBC = 80^0 - 50^0 = 30^0$ $[\angle AOC = 80^0, \text{given}]$

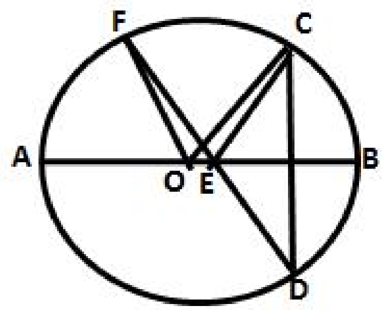
Hence, $\angle ABC = 30^0$

Question 17 (old).
In the figure given below, AB is diameter of the circle whose centre is O. Given that:
∠ ECD = ∠ EDC = 32°.



Show that ∠ COF = ∠ CEF.

Solution:



Here, $\angle COF = 2\angle CDF = 2 \times 32^0 = 64^0$ --- (i)

(Angle at the centre is double the angle at the circumference subtended by the same chord)

In ΔECD,

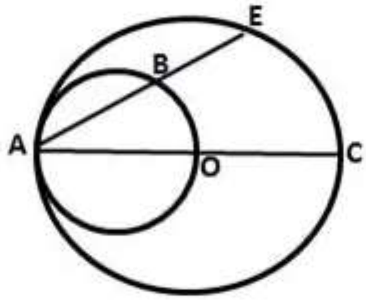
$\angle CEF = \angle ECD + \angle EDC = 32^0 + 32^0 = 64^0$ --- (ii)

(Exterior angle of a Δ is equal to the sum of pair of interior opposite angles)

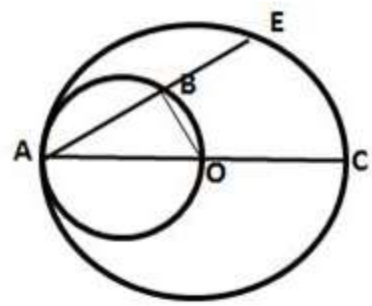
From (i) and (ii), we get

$\angle COF = \angle CEF$

Question 18.
In the figure given below, AC is a diameter of a circle, whose centre is O. A circle is described on AO as diameter. AE, a chord of the larger circle, intersects the smaller circle at B. Prove that AB = BE.



Solution:



Join OB.

Then $\angle OBA = 90^0$

(Angle in a semicircle is a right angle)

i.e. OB $\perp$ AE

We know, the perpendicular drawn from the centre to a chord bisects the chord.

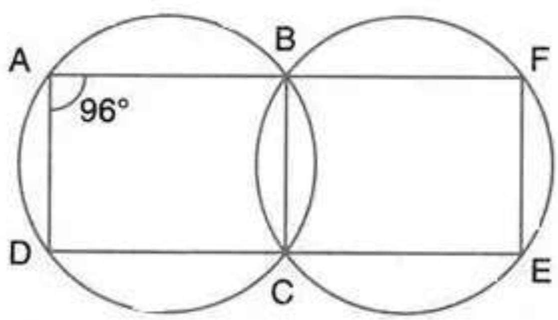
$\therefore$ AB = BE

Question 19.

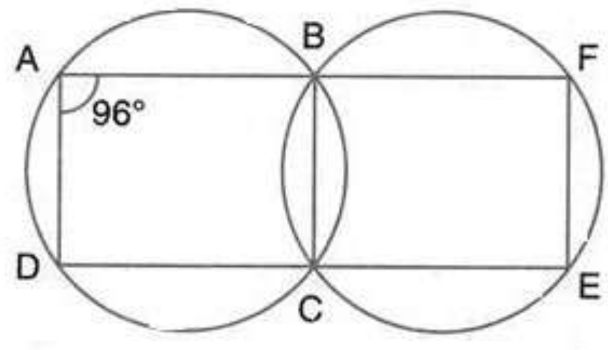
In the following figure,

(i) if ∠BAD = 96°, find BCD and

(ii) Prove that AD is parallel to FE.



Solution:



(i) ABCD is a cyclic quadrilateral

$\therefore \angle BAD + \angle BCD = 180^0$

(Pair of opposite angles in a cydic quadrilateral are supplementary)

$\Rightarrow \angle BCD = 180^0 - 96^0 = 84^0$

$\therefore \angle BCE = 180^0 - 84^0 = 96^0$

Similarly, BCEF is a cyclic quadrilateral

$\therefore \angle BCE + \angle BFE = 180^0$

(Pair of opposite angles in a cydic quadrilateral are supplementary)

$\Rightarrow \angle BFE = 180^0 - 96^0 = 84^0$

(ii) Now, $\angle BAD + \angle BFE = 96^0 + 84^0 = 180^0$

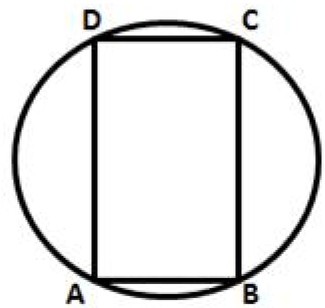
But these two are interior angles on the same side of a pair of lines AD and FE

$\therefore$ AD || FE

Question 20.
Prove that:

(i) the parallelogram, inscribed in a circle, is a rectangle.
(ii) the rhombus, inscribed in a circle, is a square.

Solution:



(i) Let ABCD be a parallelogram, inscribed in a circle.

Now, $\angle BAD = \angle BCD$

(Opposite angles of a parallelogram are equal)

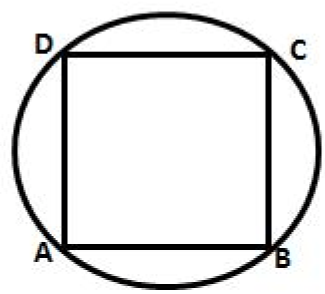
and $\angle BAD + \angle BCD = 180^0$

$\begin{pmatrix}\text{Pair of opposite angles in a cyclic quadrilateral} \\ \text{are supplementary}\end{pmatrix}$

$\therefore \angle BAD = \angle BCD = \frac{180^0}{2} = 90^0$

||ly, the other two angles are 90^0 and opposite pair of sides are equal.

$\therefore$ ABCD is a rectangle.



(ii) Let ABCD be a rhombus, inscribed in a circle.

Now, $\angle BAD = \angle BCD$

(Opposite angles of a rhombus are equal)

and $\angle BAD + \angle BCD = 180^0$

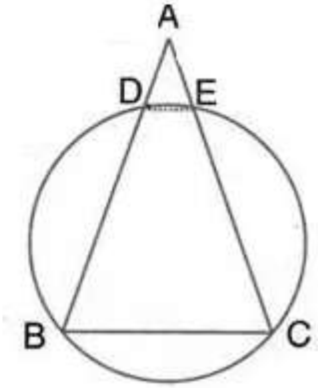
$\begin{pmatrix}\text{Pair of opposite angles in a cyclic quadrilateral} \\ \text{are supplementary}\end{pmatrix}$

$\therefore \angle BAD = \angle BCD = \frac{180^0}{2} = 90^0$

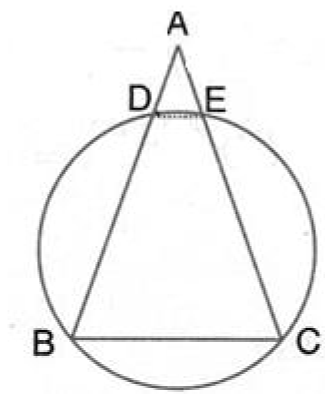
||ly, the other two angles are 90^0 and all the sides are equal.

$\therefore$ ABCD is a square.

Question 21.
In the following figure, AB = AC. Prove that DECB is an isosceles trapezium.



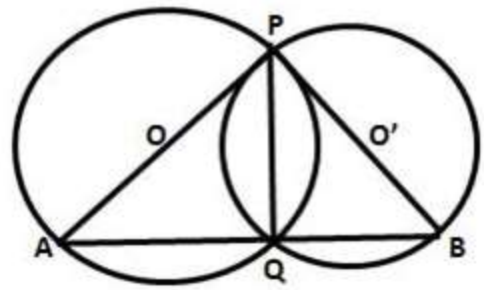
Solution:



Here, AB = AC
$\Rightarrow \angle B = \angle C$
$\therefore$ DECB is a cyclic quadrilateral.
(In a triangle, angles opposite to equal sides are equal)
Also, $\angle B + \angle DEC = 180^0$ - - - (1)
(Pair of opposite angles in a cyclic quadrilateral are supplementary)
$\Rightarrow \angle C + \angle DEC = 180^0$ [from (1)]
But this is the sum of interior angles on one side of a transversal.
$\therefore$ DE || BC
But $\angle ADE = \angle B$ and $\angle AED = \angle C$ [corresponding angles]
Thus, $\angle ADE = \angle AED$
$\Rightarrow$ AD = AE
$\Rightarrow$ AB − AD = AC − AE ($\because$ AB = AC)
$\Rightarrow$ BD = CE
Thus, we have, DE || BC and BD = CE
Hence, DECB is an isosceles trapezium.

Question 22.
Two circles intersect at P and Q. Through P diameters PA and PB of the two circles are drawn. Show that the points A, Q and B are collinear.

Solution:

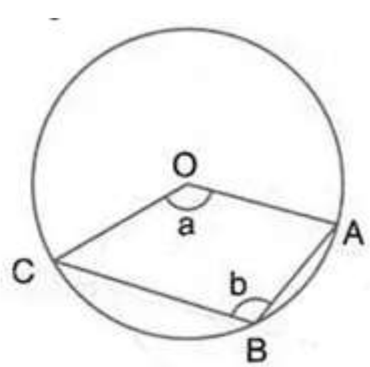


Let O and O' be the centres of two intersecting circles, where points of intersection are P and Q and PA and PB are their diameters respectively.
Join PQ, AQ and QB.

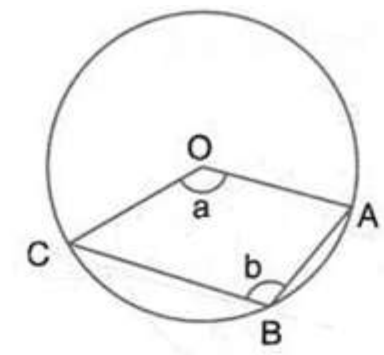
$\therefore \quad \angle AQP = 90^0$ and $\angle BQP = 90^0$
(Angle in a semicircle is a right angle)
Adding both these angles,
$\angle AQP + \angle BQP = 180^0 \quad \Rightarrow \angle AQB = 180^0$
Hence, the points A, Q and B are collinear.

Question 23.
The figure given below, shows a circle with centre O. Given: ∠ AOC = a and ∠ ABC = b.

(i) Find the relationship between a and b
(ii) Find the measure of angle OAB, if OABC is a parallelogram.



Solution:



(i) $\angle ABC = \frac{1}{2} \text{Reflex} (\angle COA)$

$\left(\begin{array}{l}\text{Angle at the centre is double the angle at the} \\ \text{circumference subtended by the same chord}\end{array}\right)$

$\Rightarrow b = \frac{1}{2}(360^0 - a)$

$\Rightarrow a + 2b = 180^0$

(ii) Since OABC is a parallelogram, so opposite angles are equal

$\therefore \quad a = b$

Using relationship in (i),

$3a = 180^0$

$\therefore \quad a = 60^0$

Also, $OC \parallel BA$

$\therefore \quad \angle COA + \angle OAB = 180^0$

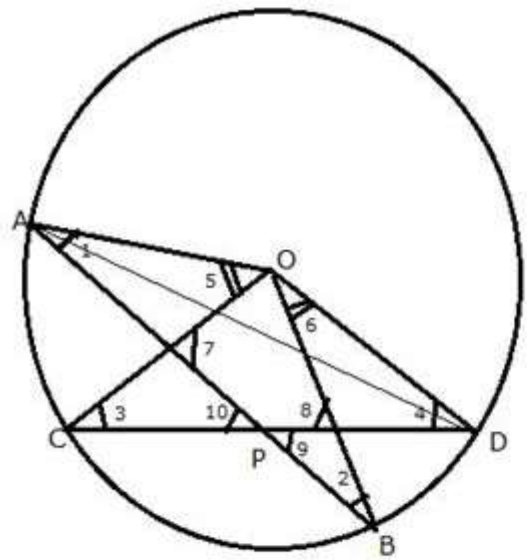
$\Rightarrow \quad 60^0 + \angle OAB = 180^0$

$\Rightarrow \quad \angle OAB = 120^0$

Question 24.

Two chords AB and CD intersect at P inside the circle. Prove that the sum of the angles subtended by the arcs AC and BD as the centre O is equal to twice the angle APC.

Solution:



Given: Two chords AB and CD intersect each other at P inside the circle. OA, OB, OC and OD are joined.

To prove: $\angle AOC + \angle BOD = 2\angle APC$

Construction: Join AD.

Proof: Arc AC subtends $\angle AOC$ at the centre and $\angle ADC$ at the remaining part of the circle.

$\angle AOC = 2\angle ADC$ ......(1)

Similarly,

$\angle BOD = 2\angle BAD$.......(2)

Adding (1) and (2),

$\angle AOC + \angle BOD = 2\angle ADC + 2\angle BAD$

$= 2(\angle ADC + \angle BAD)$.......(3)

But in ΔPAD,

Ext. $\angle APC = \angle PAD + \angle ADC$

$= \angle BAD + \angle ADC$ ....(4)

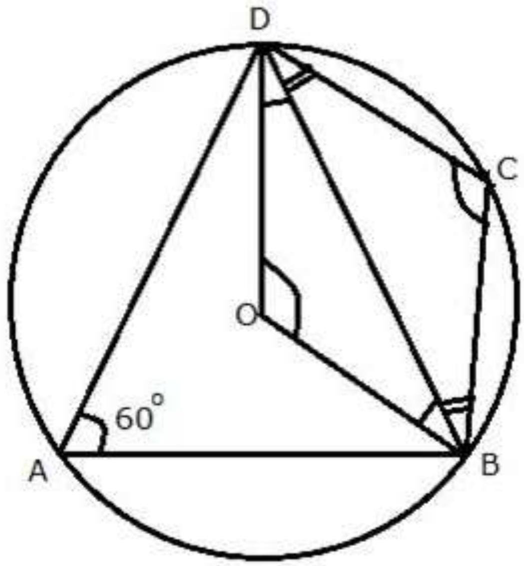
From (3) and (4),

$\angle AOC + \angle BOD = 2\angle APC$

Question 24 (old).

ABCD is a quadrilateral inscribed in a circle having $\angle A = 60°$; O is the centre of the circle.
Show that: $\angle OBD + \angle ODB = \angle CBD + \angle CDB$

Solution:



$\angle BOD = 2\angle BAD = 2\times 60^0 = 120^0$

and $\angle BCD = \frac{1}{2}\text{Reflex}(\angle BOD) = \frac{1}{2}\left(360^0 - 120^0\right) = 120^0$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

$\therefore\ \angle CBD + \angle CDB = 180^0 - 120^0 = 60^0$

(By angle sum property of triangle CBD)

Again, $\angle OBD + \angle ODB = 180^0 - 120^0 = 60^0$

(By angle sum property of triangle OBD)

$\therefore\ \angle OBD + \angle ODB = \angle CBD + \angle CDB$

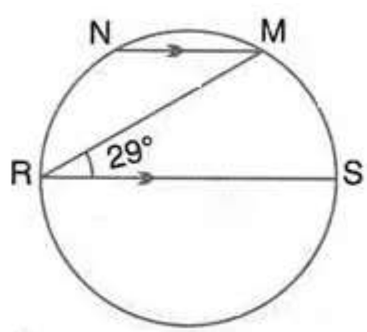
Question 25.

In the figure given RS is a diameter of the circle. NM is parallel to RS and ∠MRS = 29°

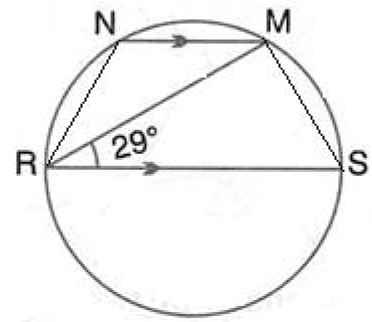
Calculate:

(i) ∠RNM;

(ii) ∠NRM.



Solution:



(i) Join RN and MS.

$\therefore \angle RMS = 90^{0}$

(Angle in a semicircle is a right angle)

$\therefore \angle RSM = 90^{0} - 29^{0} = 61^{0}$

(By angle sum property of triangle RMS)

$\therefore \angle RNM = 180^{0} - \angle RSM = 180^{0} - 61^{0} = 119^{0}$

(Pair of opposite angles in a cyclic quadrilateral are supplementary)

(ii) Also, RS || NM

$\therefore \angle NMR = \angle MRS = 29^{0}$ (Alternate angles)

$\therefore \angle NMS = 90^{0} + 29^{0} = 119^{0}$

Also, $\angle NRS + \angle NMS = 180^{0}$

(Pair of opposite angles in a cyclic quadrilateral are supplementary)

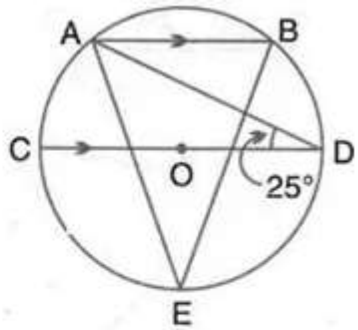
$\Rightarrow \angle NRM + 29^{0} + 119^{0} = 180^{0}$

$\Rightarrow \angle NRM = 180^{0} - 148^{0}$

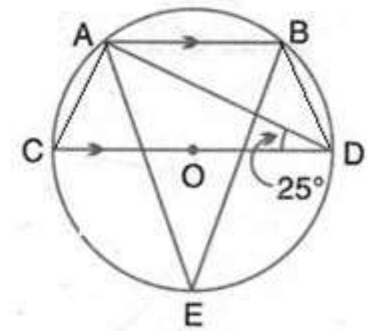
$\therefore \angle NRM = 32^{0}$

Question 26.

In the figure given alongside, AB || CD and O is the centre of the circle. If ∠ ADC = 25°; find the angle AEB. Give reasons in support of your answer.



Solution:



Join AC and BD.

$\therefore \angle CAD = 90^0$ and $\angle CBD = 90^0$

(Angle in a semicircle is a right angle)

Also, AB || CD

$\therefore \angle BAD = \angle ADC = 25^0$ (Alternate angles)

$\angle BAC = \angle BAD + \angle CAD = 25^0 + 90^0 = 115^0$

$\therefore \angle ADB = 180^0 - 25^0 - \angle BAC = 180^0 - 25^0 - 115^0 = 40^0$

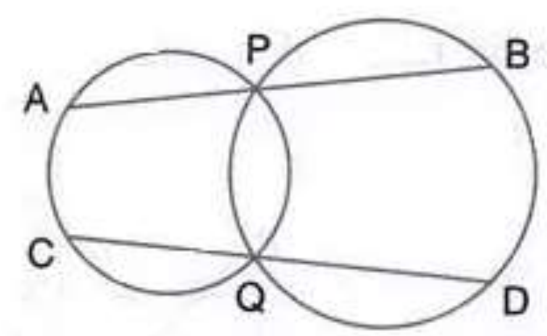
(Pair of opposite angles in a cyclic quadrilateral are supplementary)

Also, $\angle AEB = \angle ADB = 40^0$

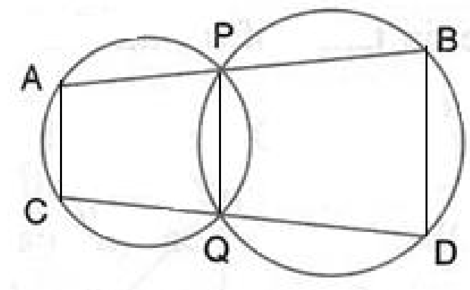
(Angles subtended by the same chord on the circle are equal)

Question 27.

Two circles intersect at P and Q. Through P, a straight line APB is drawn to meet the circles in A and B. Through Q, a straight line is drawn to meet the circles at C and D. Prove that AC is parallel to BD.



Solution:



Join AC, PQ and BD.

ACQP is a cyclic quadtrilateral

$\therefore \ \angle CAP + \angle PQC = 180^{0} \qquad ---(i)$

(Pair of opposite angles in a cyclic quadrilateral are supplementary)

PQDB is a cyclic quadrilateral

$\therefore \ \angle PQD + \angle DBP = 180^{0} \qquad ---(ii)$

(Pair of opposite angles in a cyclic quadrilateral are supplementary)

Again, $\angle PQC + \angle PQD = 180^{0} ---(iii)$

(CQD is a straight line)

Using (i), (ii) and (iii),

$\angle CAP + \angle DBP = 180^{0}$

or $\angle CAB + \angle DBA = 180^{0}$

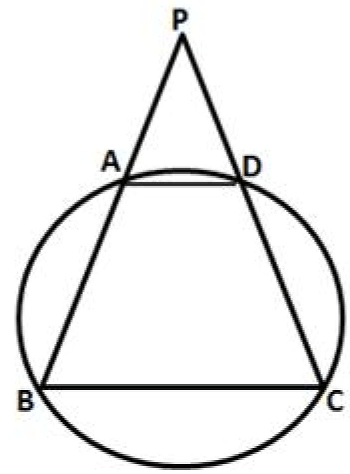
We know, if a transversal intersects two lines such that a pair of interior angles on the same side of the transversal is supplementary, then the two lines are parallel

$\therefore \quad AC \parallel BD$

Question 28.

ABCD is a cyclic quadrilateral in which AB and DC on being produced, meet at P such that PA = PD. Prove that AD is parallel to BC.

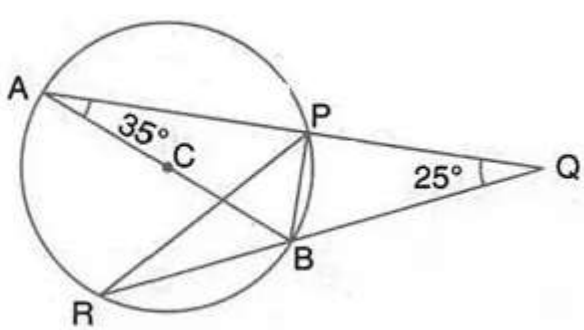
Solution:



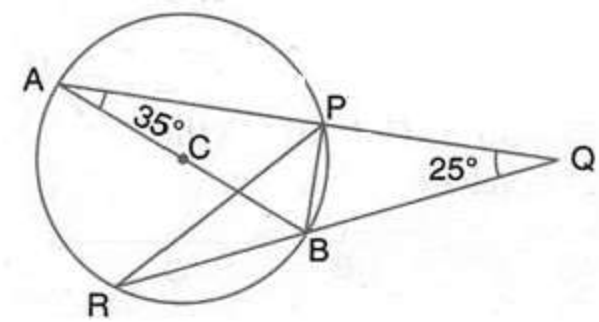
Let ABCD be the given cyclic quadrilateral.
Also, PA = PD (Given)
$\therefore$ $\angle PAD = \angle PDA$ .....(1)
$\therefore$ $\angle BAD = 180^0 - \angle PAD$
and $\angle CDA = 180^0 - \angle PDA = 180^0 - \angle PAD$ (From (1))
We know that the opposite angles of a cyclic quadrilateral are supplementary
$\therefore$ $\angle ABC = 180^0 - \angle CDA = 180^0 - (180^0 - \angle PAD) = \angle PAD$
And $\angle DCB = 180^0 - \angle BAD = 180^0 - (180^0 - \angle PAD) = \angle PAD$
$\therefore$ $\angle ABC = \angle DCB = \angle PAD = \angle PDA$
That means AD || BC.

Question 29.
AB is a diameter of the circle APBR as shown in the figure. APQ and RBQ are straight lines. Find:

(i) ∠PRB
(ii) ∠PBR
(iii) ∠BPR.



Solution:



(i) $\angle PRB = \angle PAB = 35^0$

(Angles subtended by the same chord on the circle are equal)

(ii) $\angle BPA = 90^0$

(Angle in a semicircle is a right angle)

$\therefore \angle BPQ = 90^0$

$\therefore \angle PBR = \angle BQP + \angle BPQ = 25^0 + 90^0 = 115^0$

(Exterior angle of a Δ is equal to the sum of pair of interior opposite angles)

(iii) $\angle ABP = 90^0 - \angle BAP = 90^0 - 35^0 = 55^0$

$\therefore \angle ABR = \angle PBR - \angle ABP = 115^0 - 55^0 = 60^0$

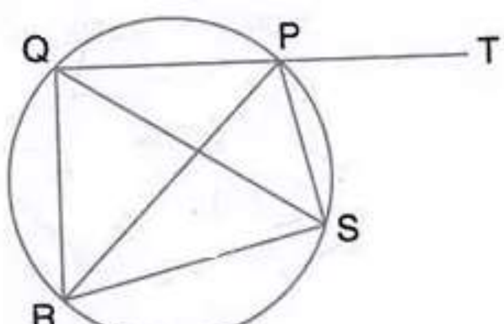
$\therefore \angle APR = \angle ABR = 60^0$

(Angles subtended by the same chord on the circle are equal)

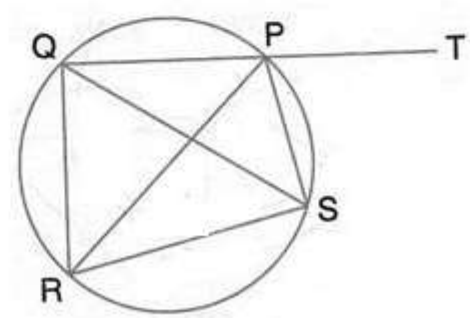
$\therefore \angle BPR = 90^0 - \angle APR = 90^0 - 60^0 = 30^0$

Question 30.

In the given figure, SP is the bisector of angle RPT and PQRS is a cyclic quadrilateral. Prove that: SQ = SR.



Solution:



PQRS is a cydic quadrilateral

$\therefore \angle QRS + \angle QPS = 180^0$ --- (i)

(Pair of opposite angles in a cyclic quadrilateral are supplementary)

Also, $\angle QPS + \angle SPT = 180^0$ --- (ii)

(Straight line QPT)

From (i) and (ii),

$\angle QRS = \angle SPT$ --- (iii)

Also, $\angle RQS = \angle RPS$ --- (iv)

(Angles subtended by the same chord on the circle are equal)

and $\angle RPS = \angle SPT$ (PS bisects $\angle RPT$) --- (v)

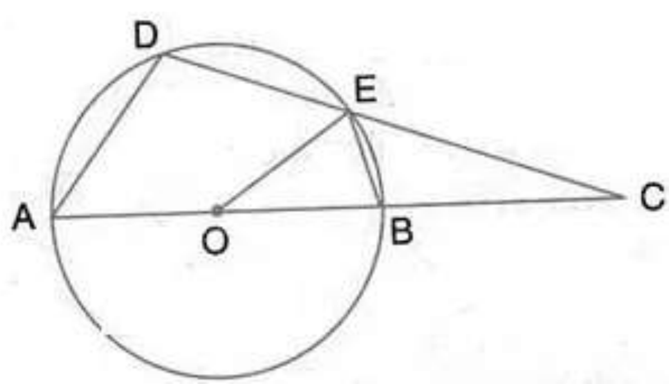
From (iii), (iv) and (v),

$\angle QRS = \angle RQS$

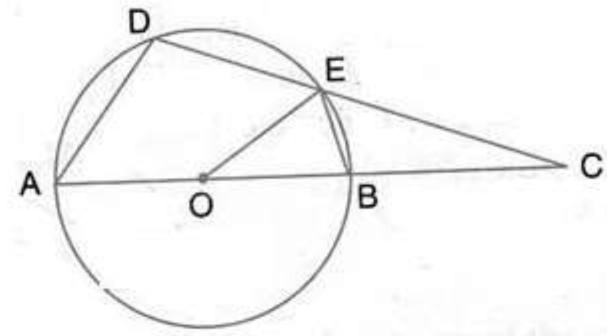
$\Rightarrow$ SQ = SR

Question 31.

In the figure, O is the centre of the circle, ∠AOE = 150°, DAO = 51°. Calculate the sizes of the angles CEB and OCE.



Solution:



$\angle ADE = \frac{1}{2} \text{Reflex}(\angle AOE) = \frac{1}{2}(360^0 - 150^0) = 105^0$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

$\angle DAB + \angle BED = 180^0$

(Pair of opposite angles in a cydic quadrilateral are supplementary)

$\Rightarrow \angle BED = 180^0 - 51^0 = 129^0$

$\therefore \angle CEB = 180^0 - \angle BED$ (Straight line)

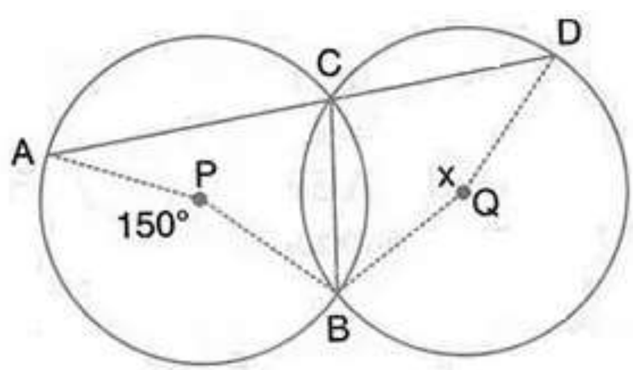
$= 180^0 - 129^0 = 51^0$

Also, by angle sum property of ΔADC,

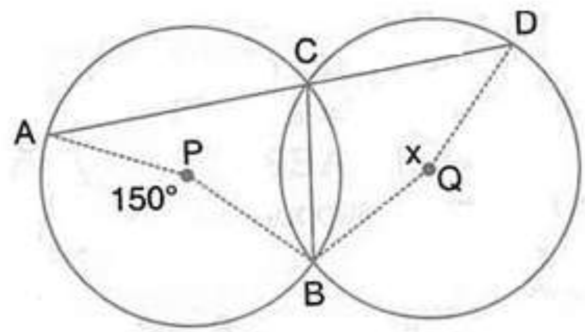
$\angle OCE = 180^0 - 51^0 - 105^0 = 24^0$

Question 32.

In the figure, P and Q are the centres of two circles intersecting at B and C. ACD is a straight line. Calculate the numerical value of x.



Solution:



$\angle ACB = \frac{1}{2}\angle APB = \frac{1}{2} \times 150^0 = 75^0$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

$\angle ACB + \angle BCD = 180^0$

(Straight line)

$\Rightarrow \angle BCD = 180^0 - 75^0 = 105^0$

Also, $\angle BCD = \frac{1}{2} \text{Reflex} \angle BQD = \frac{1}{2}(360^0 - x)$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

$\Rightarrow \quad 105^0 = 180^0 - \frac{x}{2}$

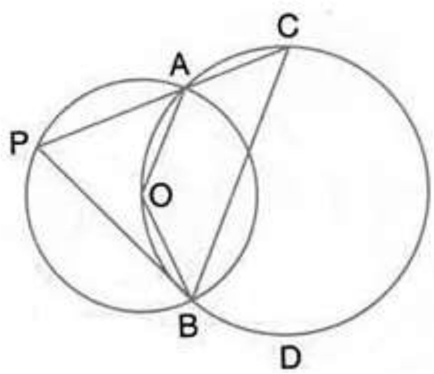
$\therefore \quad x = 2(180^0 - 105^0) = 2 \times 75^0 = 150^0$

Question 33.

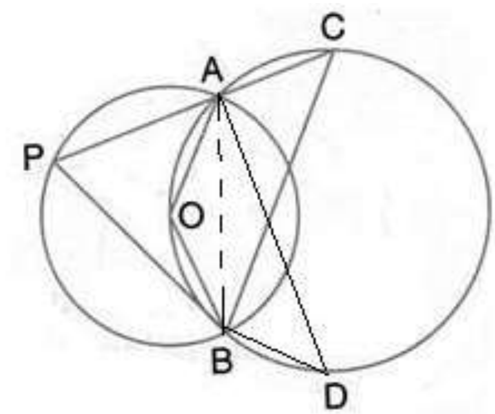
The figure shows two circles which intersect at A and B. The centre of the smaller circle is O and lies on the circumference of the larger circle. Given that ∠APB = a°. Calculate, in terms of a°, the value of:

(i) obtuse ∠AOB
(ii) ∠ACB
(iii) ∠ADB.

Give reasons for your answers clearly.



Solution:



(i) obtuse$\angle AOB = 2\angle APB = 2a^\circ$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

(ii) OABC is a cyclic quadrilateral

$\therefore \angle AOB + \angle ACB = 180^\circ$

(Pair of opposite angles in a cyclic quadrilateral are supplementary)

$\Rightarrow \angle ACB = 180^\circ - 2a^\circ$

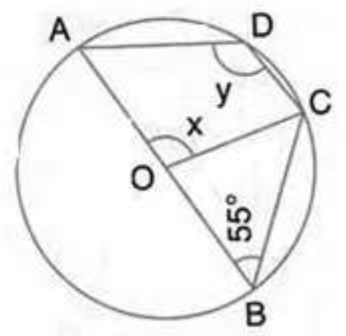
(iii) Join AB.

$\angle ADB = \angle ACB = 180^\circ - 2a^\circ$

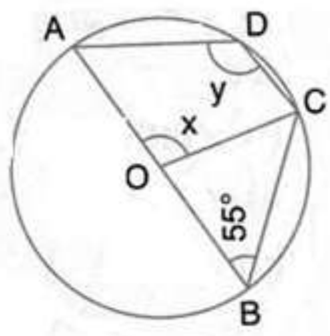
(Angles subtended by the same arc on the circle are equal)

Question 34.

In the given figure, O is the centre of the circle and ∠ ABC = 55°. Calculate the values of x and y.



Solution:



$\angle AOC = 2\angle ABC = 2 \times 55^{0}$

$\left(\begin{array}{l}\text{Angle at the centre is double the angle at the} \\ \text{circumference subtended by the same chord}\end{array}\right)$

$\therefore \quad x = 110^{0}$

ABCD is a cydic quadrilateral

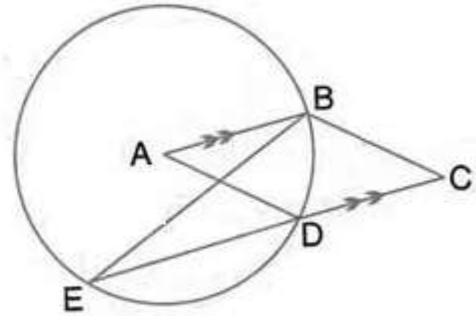
$\therefore \angle ADC + \angle ABC = 180^{0}$

$\left(\begin{array}{l}\text{Pair of opposite angles in a cydic quadrilateral} \\ \text{are supplementary}\end{array}\right)$

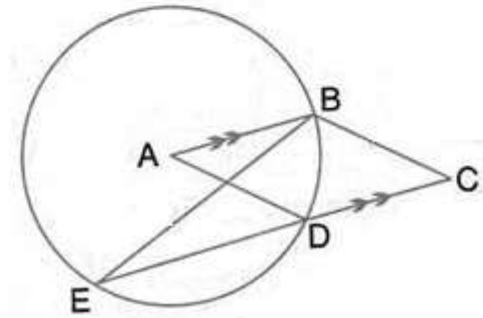
$\Rightarrow y = 180^{0} - 55^{0} = 125^{0}$

Question 35.

In the given figure, A is the centre of the circle, ABCD is a parallelogram and CDE is a straight line. Prove that ∠BCD = 2∠ABE



Solution:



$\angle BAD = 2\angle BED$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

and $\angle BED = \angle ABE$ (Alternate angles)

$\therefore \angle BAD = 2\angle ABE$ --- (i)

ABCD is a parallelogram

$\therefore \angle BAD = \angle BCD$ --- (ii)

(Opposite angles in a parallelogram are equal)

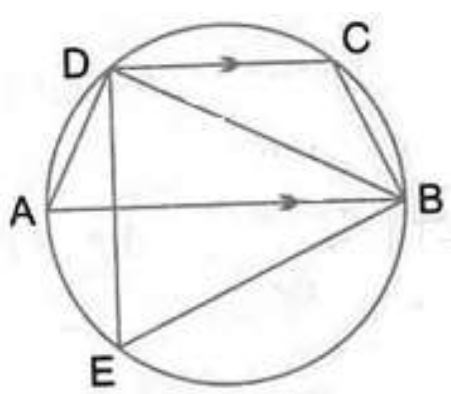
From (i) and (ii),

$\angle BCD = 2\angle ABE.$

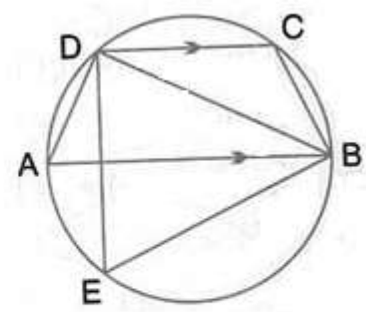
Question 36.

ABCD is a cyclic quadrilateral in which AB is parallel to DC and AB is a diameter of the circle. Given ∠BED = 65°; calculate:

(i) ∠ DAB,
(ii) ∠BDC.



Solution:



(i) $\angle DAB = \angle BED = 65^0$

(Angles subtended by the same chord on the circle are equal)

(ii) $\angle ADB = 90^0$

(Angle in a semicircle is a right angle)

$\therefore \angle ABD = 90^0 - \angle DAB = 90^0 - 65^0 = 25^0$

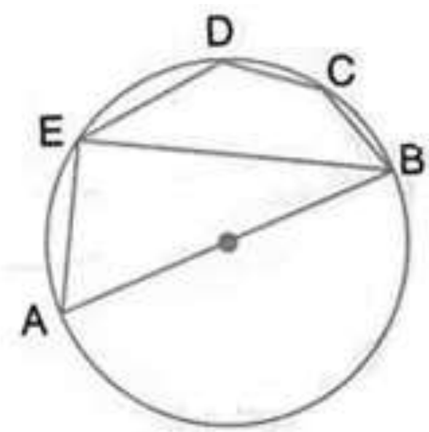
$AB \parallel DC$

$\therefore \angle BDC = \angle ABD = 25^0$ (Alternate angles)

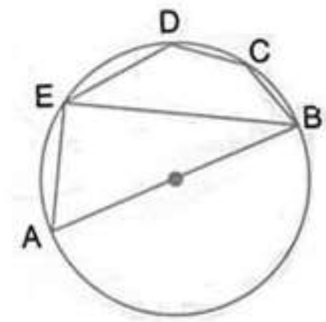
Question 37.

∠ In the given figure, AB is a diameter of the circle. Chord ED is parallel to AB and ∠ EAB = 63°; calculate:

(i) ∠EBA,
(ii) BCD.



Solution:



(i) $\angle AEB = 90^0$

(Angle in a semicircle is a right angle)

Therefore $\angle EBA = 90^0 - \angle EAB = 90^0 - 63^0 = 27^0$

(ii) AB $\parallel$ ED

Therefore $\angle DEB = EBA = 27^0$ (Alternate angles)

Therefore BCDE is a cyclic quadrilateral

Therefore $\angle DEB + \angle BCD = 180^0$

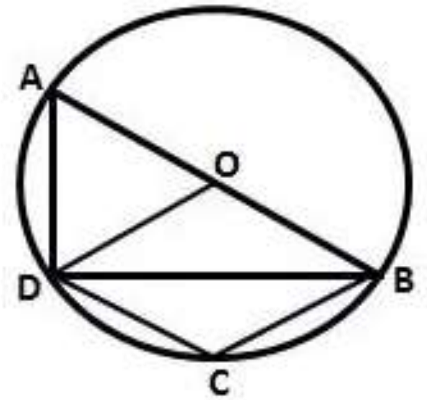
[Pair of opposite angles in a cyclic quadrilateral are supplementary]

Therefore $\angle BCD = 180^0 - 27^0 = 153^0$

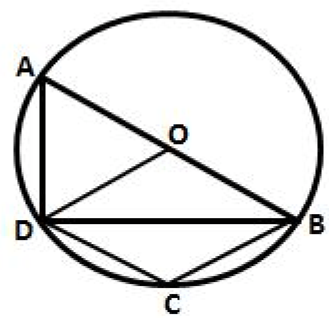
Question 38.
In the given figure, AB is a diameter of the circle with centre O. DO is parallel to CB and ∠DCB = 120°; calculate:

(i) ∠ DAB,
(ii) ∠ DBA,
(iii) ∠ DBC,
(iv) ∠ ADC.

Also, show that the ΔAOD is an equilateral triangle.



Solution:



(i) ABCD is a cyclic quadrilateral

$\therefore \angle DCB + \angle DAB = 180^0$

(Pair of opposite angles in a cyclic quadrilateral are supplementary)

$\Rightarrow \angle DAB = 180^0 - 120^0 = 60^0$

(ii) $\angle ADB = 90^0$

(Angle in a semicircle is a right angle)

$\therefore \angle DBA = 90^0 - \angle DAB = 90^0 - 60^0 = 30^0$

(iii) OD = OB

$\therefore \angle ODB = \angle OBD$

or $\angle ABD = 30^0$

Also, AB || ED

$\therefore \angle DBC = \angle ODB = 30^0$ (Alternate angles)

(iv) $\angle ABD + \angle DBC = 30^0 + 30^0 = 60^0$

$\Rightarrow \angle ABC = 60^0$

In cyclic quadrilateral ABCD,

$\angle ADC + \angle ABC = 180^0$

$\begin{pmatrix} \text{Pair of opposite angles in a cyclic quadrilateral} \\ \text{are supplementary} \end{pmatrix}$

$\Rightarrow \angle ADC = 180^0 - 60^0 = 120^0$

In ΔAOD, $OA = OD$ (radii of the same circle)

$\angle AOD = \angle DAO$ or $\angle DAB = 60^0$ [proved in (i)]

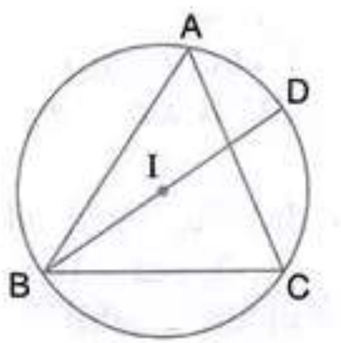
$\Rightarrow \angle AOD = 60^0$

$\angle ADO = \angle AOD = \angle DAO = 60^0$

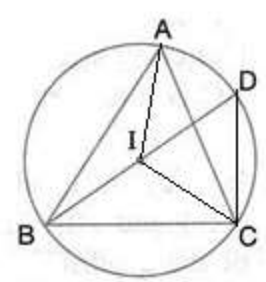
$\therefore \Delta AOD$ is an equilateral triangle.

Question 39.
In the given figure, I is the in centre of the Δ ABC. BI when produced meets the circum cirle of Δ ABC at D. Given ∠BAC = 55° and ∠ ACB = 65°, calculate:

(i) ∠DCA,
(ii) ∠ DAC,
(iii) ∠DCI,
(iv) ∠AIC.



Solution:



Join IA, IC and CD.

(i) IB is the bisector of $\angle ABC$

$\Rightarrow \angle ABD = \frac{1}{2}\angle ABC = \frac{1}{2}\left(180^0 - 65^0 - 55^0\right) = 30^0$

$\angle DCA = \angle ABD = 30^0$

(Angle in the same segment)

(ii) $\angle DAC = \angle CBD = 30^0$

(Angle in the same segment)

(iii) $\angle ACI = \frac{1}{2}\angle ACB = \frac{1}{2} \times 65^0 = 32.5^0$

(CI is the angular bisector of $\angle ACB$)

$\therefore \angle DCI = \angle DCA + \angle ACI = 30^0 + 32.5^0 = 62.5^0$

(iv) $\angle IAC = \frac{1}{2}\angle BAC = \frac{1}{2} \times 55^0 = 27.5^0$

(AI is the angular bisector of $\angle BAC$)

$\therefore \angle AIC = 180^0 - \angle IAC - \angle ICA = 180^0 - 27.5^0 - 32.5^0 = 120^0$

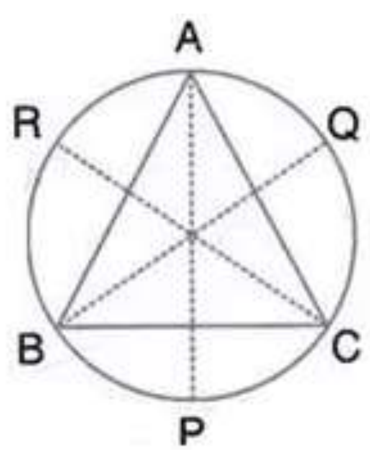
Question 40.

A triangle ABC is inscribed in a circle. The bisectors of angles BAC, ABC and ACB meet the circumcircle of the triangle at points P, Q and R respectively. Prove that:

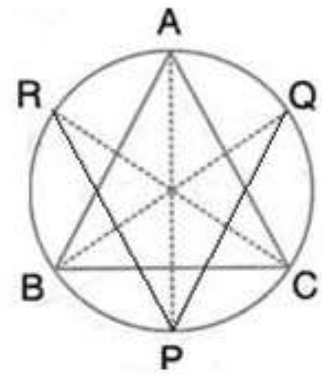
(i) $\angle ABC = 2\angle APQ$

(ii) $\angle ACB = 2\angle APR$

(iii) $\angle QPR = 90° - \frac{1}{2}BAC$



Solution:



Join PQ and PR.

(i) BQ is the bisector of $\angle ABC$

$\Rightarrow \angle ABQ = \frac{1}{2}\angle ABC$

Also, $\angle APQ = \angle ABQ$

(Angle in the same segment)

$\therefore \angle ABC = 2\angle APQ$

(ii) CR is the bisector of $\angle ACB$

$\Rightarrow \angle ACR = \frac{1}{2}\angle ACB$

Also, $\angle ACR = \angle APR$

(Angle in the same segment)

$\therefore \angle ACB = 2\angle APR$

(iii) Adding (i) and (ii),

we get

$\angle ABC + \angle ACB = 2(\angle APR + \angle APQ) = 2\angle QPR$

$\Rightarrow 180^\circ - \angle BAC = 2\angle QPR$

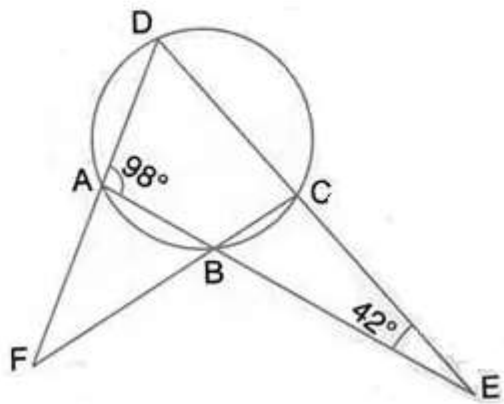
$\Rightarrow \angle QPR = 90^\circ - \frac{1}{2}\angle BAC$

Question 40 (old).

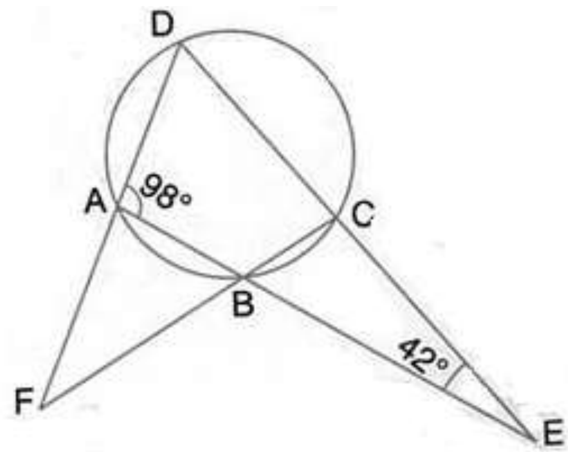
The sides AB and DC of a cyclic quadrilateral ABCD are produced to meet at E; the sides DA and CB are produced to meet at F. If ∠BEC = 42° and ∠BAD = 98°; calculate:

(i) ∠AFB,

(ii) ∠ADC.



Solution:



By angle sum property of ΔADE,

$\angle ADC = 180^0 - 98^0 - 42^0 = 40^0$

Also, $\angle ADC + \angle ABC = 180^0$

(Pair of opposite angles in a cyclic quadrilateral are supplementary)

$\therefore \angle ABC = 180^0 - 40^0 = 140^0$

Also, $\angle BAF = 180^0 - \angle BAD = 180^0 - 98^0 = 82^0$

$\therefore \angle ABC = \angle AFB + \angle BAF$

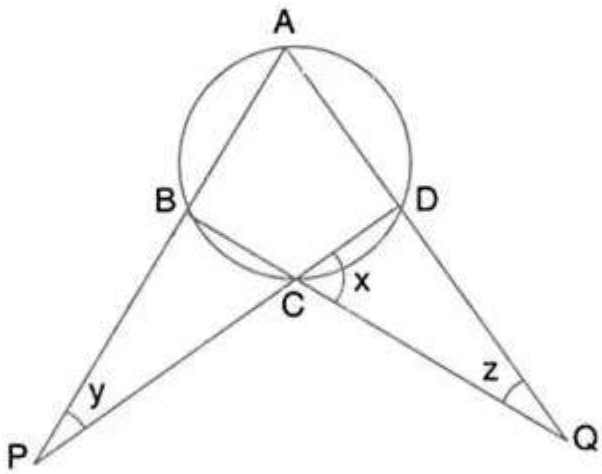
(Exterior angle of a Δ is equal to the sum of pair of interior opposite angles)

$\Rightarrow \angle AFB = 140^0 - 82^0 = 58^0$

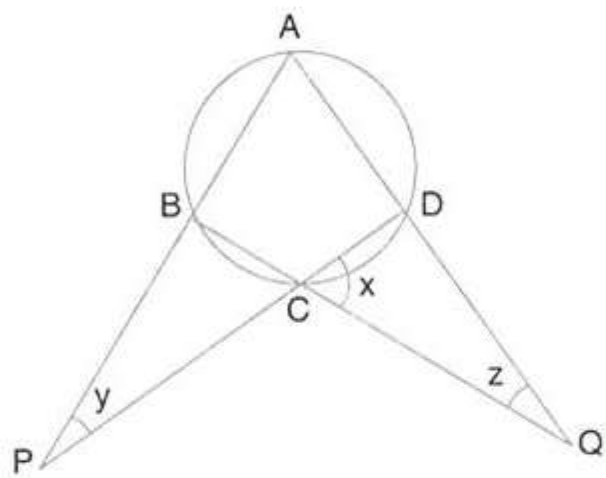
Thus, $\angle AFB = 58^0$ and $\angle ADC = 40^0$

Question 41.

Calculate the angles x, y and z if: $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$



Solution:



Let $x = 3k$, $y = 4k$ and $z = 5k$

$\angle ADB = x + z = 8k$ and $\angle ABC = x + y = 7k$

(Exterior angle of a Δ is equal to the sum of pair of interior opposite angles)

Also, $\angle ABC + \angle ADC = 180^{o}$

(Pair of opposite angles in a cyclic quadrilateral are supplementary)

$\Rightarrow \quad 8k + 7k = 180^0$

$\Rightarrow \quad 15k = 180^0$

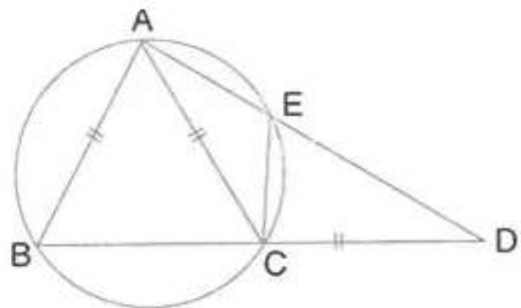
$\therefore \quad k = \frac{180^0}{15} = 12^0$

$\therefore \quad x = 3 \times 12^0 = 36^0$

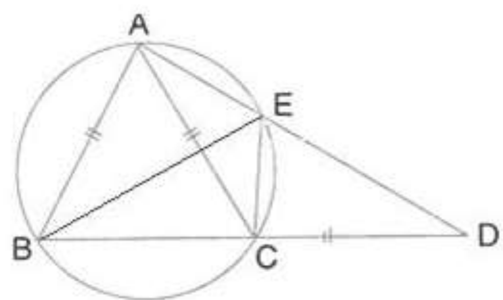
$y = 4 \times 12^0 = 48^0$

$z = 5 \times 12^0 = 60^0$

Question 42.
In the given figure, AB = AC = CD and ∠ADC = 38°. Calculate:
(i) Angle ABC
(ii) Angle BEC.



Solution:



(i) AC = CD

$\therefore \angle CAD = \angle CDA = 38^0$

$\therefore \angle ACD = 180^0 - 2 \times 38^0 = 104^0$

$\therefore \angle ACB = 180^0 - 104^0 = 76^0$ (Straight line)

Also, AB = AC

$\therefore \angle ABC = \angle ACB = 76^0$

(ii) By angle sum property,

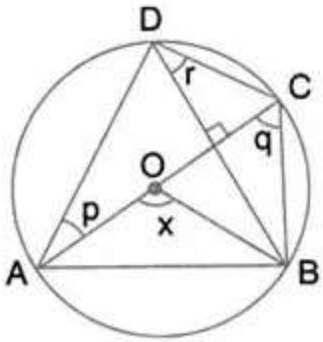
$\angle BAC = 180^0 - 2 \times 76^0 = 38^0$

$\therefore \quad \angle BEC = \angle BAC = 38^0$

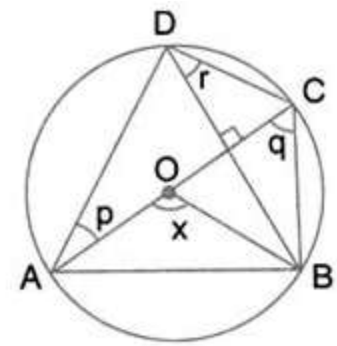
(Angles in the same chord)

Question 43.

In the given figure, AC is the diameter of circle, centre O. Chord BD is perpendicular to AC. Write down the angles p, and r in terms of x.



Solution:



$\angle AOB = 2\angle ACB = 2\angle ADB$

$\left(\begin{array}{l}\text{Angle at the centre is double the angle at the} \\ \text{circumference subtended by the same chord}\end{array}\right)$

$\Rightarrow \quad x = 2q \quad \text{and} \quad \angle ADB = \frac{x}{2}$

$\therefore \quad q = \frac{x}{2}$

Also, $\angle ADC = 90^0$

(Angle in a semicircle)

$\Rightarrow r + \frac{x}{2} = 90^0$

$\Rightarrow r = 90^0 - \frac{x}{2}$

Again, $\angle DAC = \angle DBC$

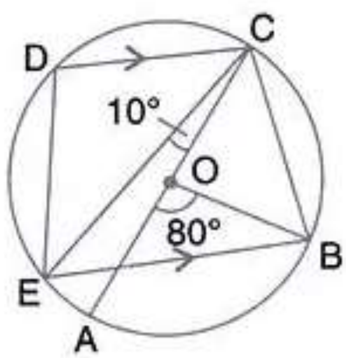
(Angle in the same segment)

$\Rightarrow p = 90^0 - q$

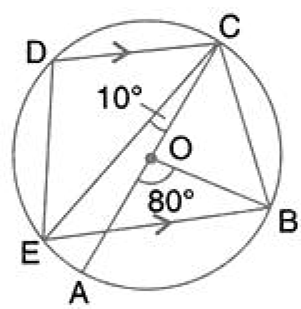
$\Rightarrow p = 90^0 - \frac{x}{2}$

Question 44.
In the given figure, AC is the diameter of circle, centre O. CD and BE are parallel. Angle AOB = 80° and angle ACE = 10°. Calculate:

(i) Angle BEC;
(ii) Angle BCD;
(iii) Angle CED.



Solution:



(i) $\angle BOC = 180^0 - 80^0 = 100^0$ (Straight line)

and $\angle BOC = 2\angle BEC$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

$\Rightarrow \quad \angle BEC = \frac{100^0}{2} = 50^0$

(ii) DC || EB

$\therefore \angle DCE = \angle BEC = 50^0$ (Alternate angles)

$\because \angle AOB = 80^0$

$\Rightarrow \angle ACB = \frac{1}{2}\angle AOB = 40^0$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

We have,

$\angle BCD = \angle ACB + \angle ACE + \angle DCE = 40^0 + 10^0 + 50^0 = 100^0$

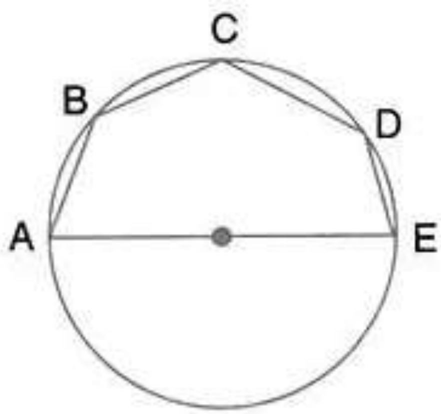
(iii) $\angle BED = 180^0 - \angle BCD = 180^0 - 100^0 = 80^0$

$\begin{pmatrix} \text{Pair of opposite angles in a cyclic quadrilateral} \\ \text{are supplementary} \end{pmatrix}$

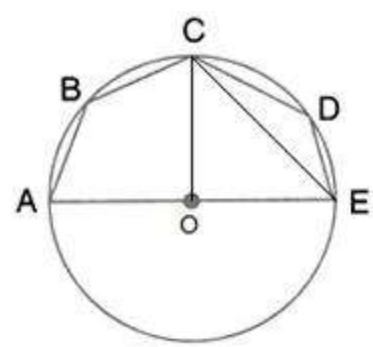
$\Rightarrow \angle CED + 50^0 = 80^0$

$\Rightarrow \angle CED = 30^0$

Question 45.
In the given figure, AE is the diameter of circle. Write down the numerical value of ∠ABC + ∠CDE. Give reasons for your answer.



Solution:



Join centre O and C and EC.

$\angle AOC = \frac{180^0}{2} = 90^0$

and $\angle AOC = 2\angle AEC$

$\begin{pmatrix} \text{Angle at the centre is double the angle at the} \\ \text{circumference subtended by the same chord} \end{pmatrix}$

$\Rightarrow \quad \angle AEC = \frac{90^0}{2} = 45^0$

Now, ABCE is a cyclic quadrilateral

$\therefore \angle ABC + \angle AEC = 180^0$

(Pair of opposite angles in a cyclic quadrilateral are supplementary)

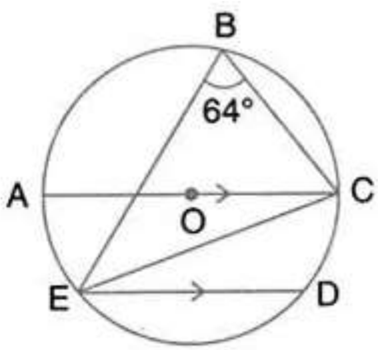
$\Rightarrow \angle ABC = 180^0 - 45^0 = 135^0$

Similarly, $\angle CDE = 135^0$

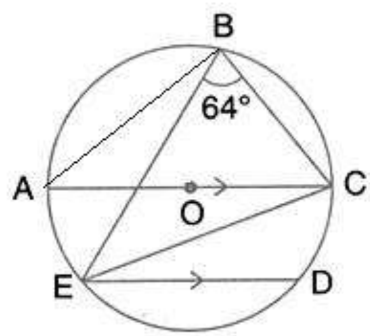
$\therefore \angle ABC + \angle CDE - 135^0 + 135^0 = 270^0$

Question 46.

In the given figure, AOC is a diameter and AC is parallel to ED. If ∠CBE = 64°, calculate ∠DEC.



Solution:



Join AB.

$\angle ABC = 90^0$

(Angle in a semi circle)

$\therefore \angle ABE = 90^0 - 64^0 = 26^0$

Now, $\angle ABE = \angle ACE = 26^0$

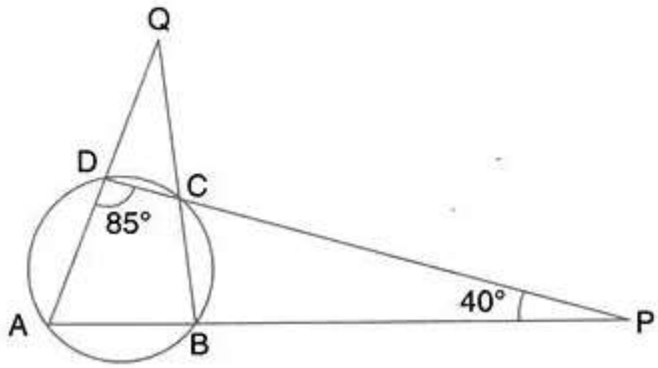
(Angle in the same segment)

Also, AC || ED

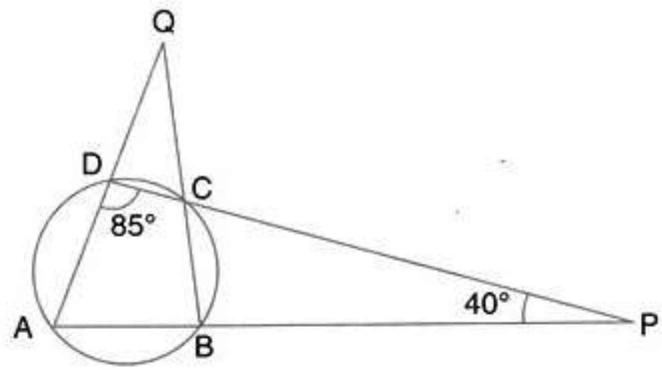
$\therefore \quad \angle DEC = \angle ACE = 26^0$ (Alternate angles)

Question 47.
Use the given figure to find

(i) ∠BAD
(ii) ∠DQB.



Solution:



(i) By angle sum property of ΔADP,

$\angle BAD = 180^0 - 85^0 - 40^0 = 55^0$

(ii) $\angle ABC = 180^0 - \angle ADC = 180^0 - 85^0 = 95^0$

(Pair of opposite angles in a cyclic quadrilateral are supplementary)

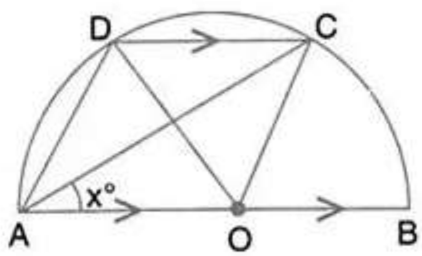
By angle sum property,

$\angle AQB = 180^0 - 95^0 - 55^0$

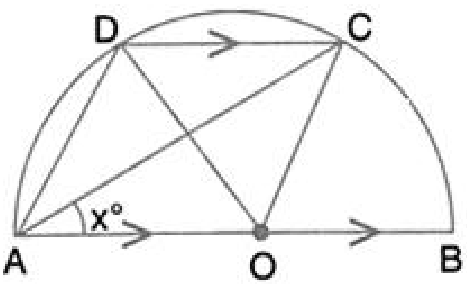
$\Rightarrow \angle DQB = 30^0$

Question 48.
In the given figure, AOB is a diameter and DC is parallel to AB. If ∠ CAB = x°; find (in terms of x) the values of:
(i) ∠COB
(ii) ∠DOC

(iii) ∠DAC
(iv) ∠ADC.



Solution:



(i) $\angle COB = 2\angle CAB = 2x$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

(ii) $\angle OCD = \angle COB = 2x$ (Alternate angles)

In ΔOCD, $OC = OD$

$\therefore \angle ODC = \angle OCD = 2x$

By angle sum property of ΔOCD,

$\angle DOC = 180^0 - 2x - 2x = 180^0 - 4x$

(iii) $\angle DAC = \frac{1}{2}\angle DOC = \frac{1}{2}\left(180^0 - 4x\right) = 90^0 - 2x$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

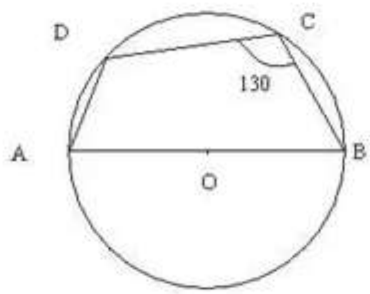
(iv) $DC \parallel AO$

$\therefore \angle ACD = \angle OAC = x$ (Alternate angles)

By angle sum property,

$\angle ADC = 180^0 - \angle DAC - \angle ACD = 180^0 - \left(90^0 - 2x\right) - x = 90^0 + x$

Question 49.
In the given figure, AB is the diameter of a circle with centre O. ∠BCD = 130°. Find:
(i) ∠DAB
(ii) ∠DBA



Solution:

i. ABCD is a cyclic quadrilateral

m∠DAB = 180° - ∠DCB

= 180° - 130°

= 50°

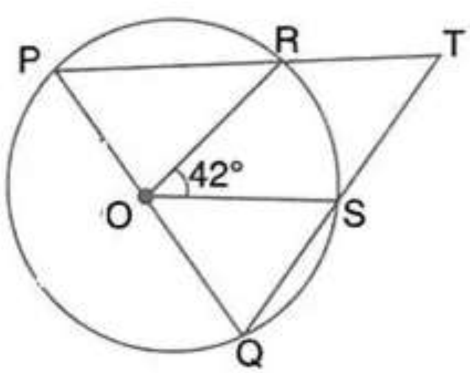
ii. In ΔADB,

m∠DAB + m∠ADB + m∠DBA = 180°

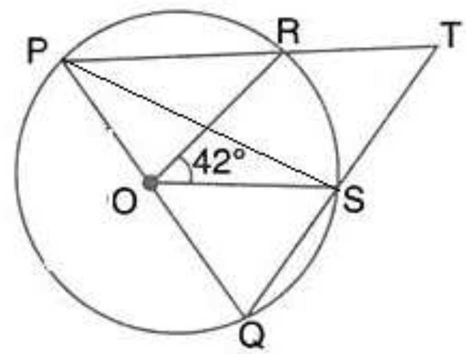
⇒50° + 90° + m∠DBA = 180°

⇒m∠DBA = 40°

Question 50.
In the given figure, PQ is the diameter of the circle whose centre is O. Given ∠ROS = 42°; calculate ∠RTS.



Solution:



Join PS.

$\angle PSQ = 90^0$

(Angle in a semicircle)

Also, $\angle SPR = \frac{1}{2}\angle ROS$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

$\Rightarrow \quad \angle SPT = \frac{1}{2} \times 42^0 = 21^0$

$\therefore$ In right triangle PST,

$\angle PTS = 90^0 - \angle SPT$

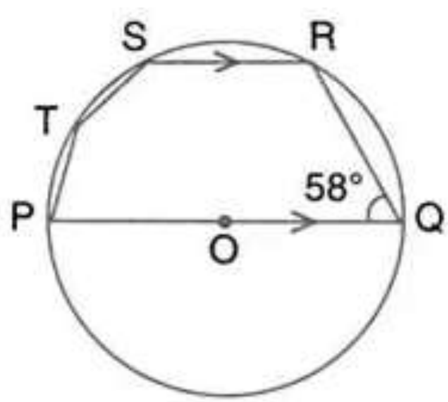
$\Rightarrow \quad \angle RTS = 90^0 - 21^0 = 69^0$

Question 51.

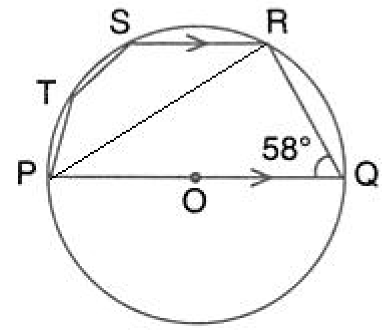
In the given figure, PQ is a diameter. Chord SR is parallel to PQ. Given that ∠PQR = 58°; calculate

(i) ∠RPQ

(ii) ∠STP.



Solution:



Join PR.

(i) $\angle PRQ = 90^{0}$

(Angle in a semicircle)

$\therefore$ In right triangle PQR,

$$\angle RPQ = 90^{0} - \angle PQR = 90^{0} - 58^{0} = 32^{0}$$

(ii) Also, $SR \parallel PQ$

$\therefore \angle PRS = \angle RPQ = 32^{0}$ (Alternate angles)

In cyclic quadrilateral PRST,

$$\angle STP = 180^{0} - \angle PRS = 180^{0} - 32^{0} = 148^{0}$$

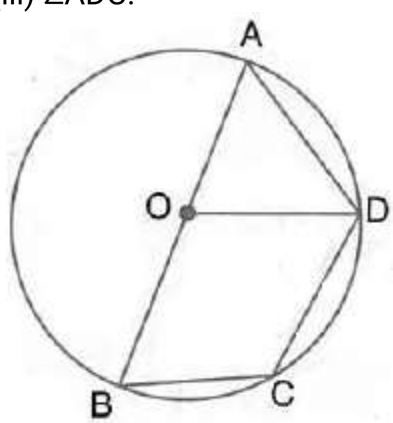
(Pair of opposite angles in a cyclic quadrilateral are supplementary)

Question 52.

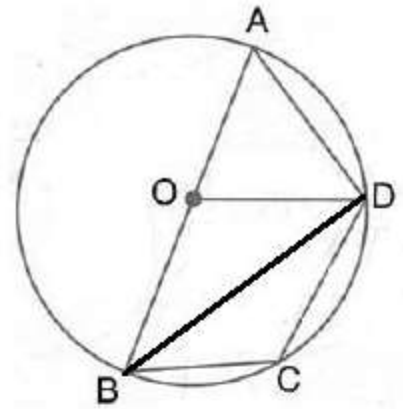
AOD = 60°; calculate the numerical values of:

AB is the diameter of the circle with centre O. OD is parallel to BC and ∠AOD = 60°; calculate the numerical values of:

(i) ∠ABD,
(ii) ∠DBC,
(iii) ∠ADC.



Solution:



Join BD.

(i) $\angle ABD = \frac{1}{2}\angle AOD = \frac{1}{2} \times 60^0 = 30^0$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

(ii) $\angle BDA = 90^0$

(Angle in a semicircle)

Also, ΔOAD is equilateral $\left(\because \angle OAD = 60^0\right)$

$\therefore \angle ODB = 90^0 - \angle ODA = 90^0 - 60^0 = 30^0$

Also, $OD \parallel BC$

$\therefore \angle DBC = \angle ODB = 30^0$ (Alternate angles)

(iii) $\angle ABC = \angle ABD + \angle DBC = 30^0 + 30^0 = 60^0$

In cyclic quadrilateral ABCD,

$\angle ADC = 180^0 - \angle ABC = 180^0 - 60^0 = 120^0$

(Pair of opposite angles in a cyclic quadrilateral are supplementary)

Question 53.

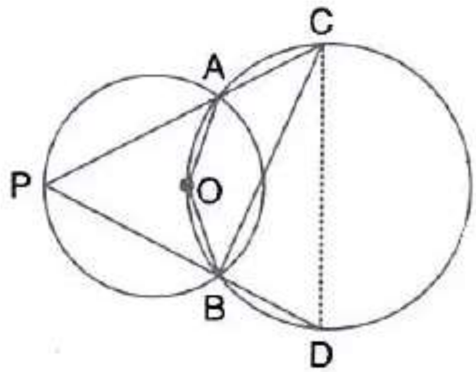
In the given figure, the centre of the small circle lies on the circumference of the bigger circle. If ∠APB = 75° and ∠BCD = 40"; find:

(i) ∠AOB,

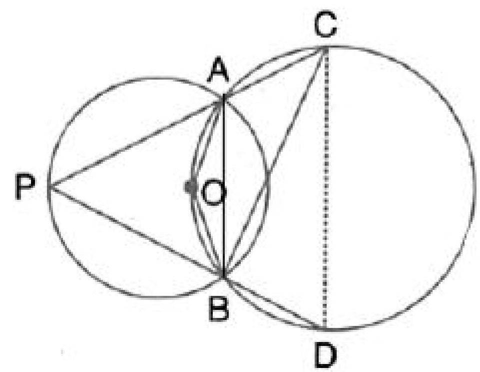
(ii) ∠ACB,

(iii) ∠ABD,

(iv) ∠ADB.



Solution:



Join AB and AD.

(i) $\angle AOB = 2\angle APB = 2 \times 75^\circ = 150^\circ$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

(ii) In cyclic quadrilateral AOBC,

$\angle ACB = 180^\circ - \angle AOB = 180^\circ - 150^\circ = 30^\circ$

(Pair of opposite angles in a cyclic quadrilateral are supplementary)

(iii) In cyclic quadrilateral ABDC,

$\angle ABD = 180^\circ - \angle ACD = 180^\circ - \left(40^\circ + 30^\circ\right) = 110^\circ$

(Pair of opposite angles in a cyclic quadrilateral are supplementary)

(iv) In cyclic quadrilateral AOBD,

$\angle ADB = 180^\circ - \angle AOB = 180^\circ - 150^\circ = 30^\circ$

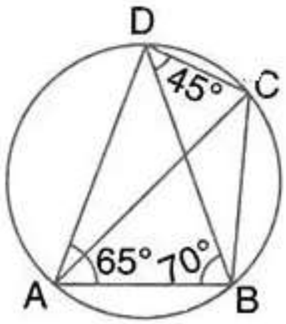
(Pair of opposite angles in a cyclic quadrilateral are supplementary)

Question 54.

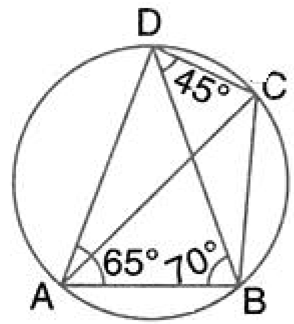
In the given figure, ∠BAD = 65°, ∠ABD = 70° and ∠BDC = 45°; find:

(i) ∠BCD,

(ii) ∠ACB.
Hence, show that AC is a diameter.



Solution:



(i) In cyclic quadrilateral ABCD,

$\angle BCD = 180^0 - \angle BAD = 180^0 - 65^0 = 115^0$

(Pair of opposite angles in a cyclic quadrilateral are supplementary)

(ii) By angle sum property of ΔABD,

$\angle ADB = 180^0 - 65^0 - 70^0 = 45^0$

Again, $\angle ACB = \angle ADB = 45^0$

(Angle in the same segment)

$\therefore \angle ADC = \angle ADB + \angle BDC = 45^0 + 45^0 = 90^0$

Hence, AC is a semicircle.

(Since angle in a semicircle is a right angle)

Question 55.

In a cyclic quadrilateral ABCD, ∠A : ∠C = 3 : 1 and ∠B : ∠D = 1 : 5; find each angle of the quadrilateral.

Solution:

Let $\angle A$ and $\angle C$ be $3x$ and x respectively.

In cyclic quadrilateral ABCD,

$\angle A + \angle C = 180^0$

(Pair of opposite angles in a cyclic quadrilateral are supplementary)

$\Rightarrow 3x + x = 180^0$

$\Rightarrow \quad x = \frac{180^0}{4} = 45^0$

$\therefore \angle A = 135^0$ and $\angle C = 45^0$

Let the measure of $\angle B$ and $\angle D$ be y and $5y$ respectively.

In cyclic quadrilateral ABCD,

$\angle B + \angle D = 180^0$

(Pair of opposite angles in a cyclic quadrilateral are supplementary)

$\Rightarrow y + 5y = 180^0$

$\Rightarrow \quad y = \frac{180^0}{6} = 30^0$

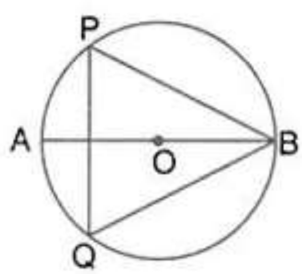
$\therefore \angle B = 30^0$ and $\angle D = 150^0$

Question 56.

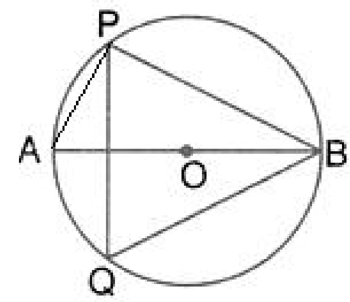
The given figure shows a circle with centre O and ∠ABP = 42°. Calculate the measure of

(i) ∠PQB

(ii) ∠QPB + ∠PBQ



Solution:



Join AP.

(i) $\angle APB = 90^0$

(Angle in a semicircle)

$\therefore \angle BAP = 90^0 - \angle ABP = 90^0 - 42^0 = 48^0$

Now, $\angle PQB = \angle BAP = 48^0$

(Angle in the same segment)

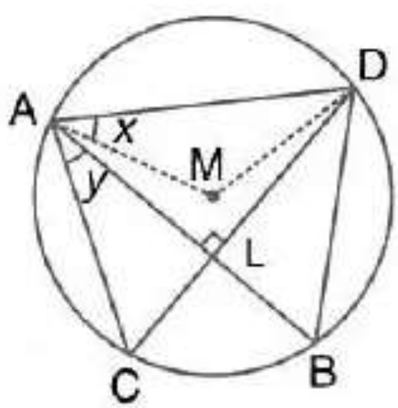
(ii) By angle sum property of ΔBPQ,

$\angle QPB + \angle PBQ = 180^0 - \angle PQB = 180^0 - 48^0 = 132^0$

Question 57.

In the given figure, M is the centre of the circle. Chords AB and CD are perpendicular to each other. If ∠ MAD =x and ∠BAC = y.

(i) express ∠AMD in terms of x.
(ii) express ∠ABD in terms of y.
(iii) prove that : x = y



Solution:

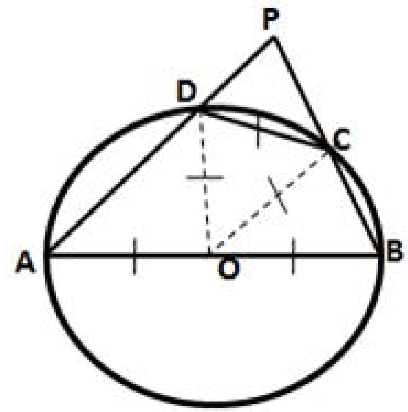
In the figure, M is the centre of the circle.
Chords AB and CD are perpendicular to each other at L.
$\angle MAD = x$ and $\angle BAC = y$

(i) In ΔAMD,
$MA = MD$
$\therefore \angle MAD = \angle MDA = x$
But in ΔAMD,
$\angle MAD + \angle MDA + \angle AMD = 180^0$
$\Rightarrow x + x + \angle AMD = 180^0$
$\Rightarrow 2x + \angle AMD = 180^0$
$\Rightarrow \angle AMD = 180^0 - 2x$
(ii) $\therefore$ Arc AD $\angle AMD$ at the centre and $\angle ABD$ at the remaining
(Angle in the same segment)
(Angle at the centre is double the angle at the circumference subtended by the same chord)
$\Rightarrow \angle AMD = 2\angle ABD$
$\Rightarrow \angle ABD = \frac{1}{2}\angle AMD$
$\Rightarrow \angle ABD = \frac{1}{2}\left(180^0 - 2x\right)$
$\Rightarrow \angle ABD = 90^0 - x$
$AB \perp CD, \angle ALC = 90^0$
In ΔALC,
$\therefore \angle LAC + \angle LCA = 90^0$
$\Rightarrow \angle BAC + \angle DAC = 90^0$
$\Rightarrow y + \angle DAC = 90^0$
$\therefore \angle DAC = 90^0 - y$
We have, $\angle DAC = \angle ABD$ [angles in the same segment]
$\therefore \angle ABD = 90^0 - y$
(iii) We have, $\angle ABD = 90^0 - y$ and $\angle ABD = 90^0 - x$ [proved]
$\therefore 90^0 - x = 90^0 - y$
$\Rightarrow x = y$

Question 61 (old).
In a circle, with centre O, a cyclic quadrilateral ABCD is drawn with AB as a diameter of the circle and CD equal to radius of the circle. If AD and BC produced meet at point P;

show that ∠APB = 60°.

Solution:



Join OD and OC.

In ΔOCD, OD = OD = CD

$\therefore$ ΔOCD is an equilateral triangle

$\therefore \angle ODC = 60^0$

Also, in cyclic quadrilateral ABCD,

$\angle ADC + \angle ABC = 180^0$

(Pair of opposite angles in a cyclic quadrilateral are supplementary)

$\Rightarrow \angle ODA + 60^0 + \angle ABP = 180^0$

$\Rightarrow \angle OAD + \angle ABP = 120^0$ $(\because OA = OD)$

$\Rightarrow \angle PAB + \angle ABP = 120^0$

By angle sum property of ΔPAB,

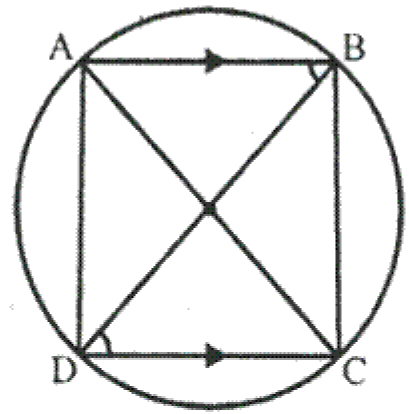
$\therefore \angle APB = 180^0 - \angle PAB - \angle ABP = 180^0 - 120^0 = 60^0$

Exercise 17 B

Question 1.

In a cyclic-trapezium, the non-parallel sides are equal and the diagonals are also equal. Prove it.

Solution:



A cyclic trapezium ABCD in which AB || DC and AC and BD are joined.

To prove –

(i) $AD = BC$

(ii) $AC = BD$

Proof:

$\because$ *Chord AD subtends* $\angle ABD$ *and chord BC subtends* $\angle BDC$ *at the circumference of the circle.*

But $\angle ABD = \angle BDC$ [*proved*]

Chord $AD =$ *Chord* BC

$\Rightarrow AD = BC$

Now in ΔADC *and* ΔBCD

$DC = DC$ [*common*]

$\angle CAD = \angle CBD$ [*angles in the same segment*]

and $AD = BC$ [*proved*]

By Side – Angle – Side criterion of congrunce, we have

$\therefore \Delta ADC \cong \Delta BCD$ [*SAS axion*]

The corresponding parts of the congruent triangles are congruent.

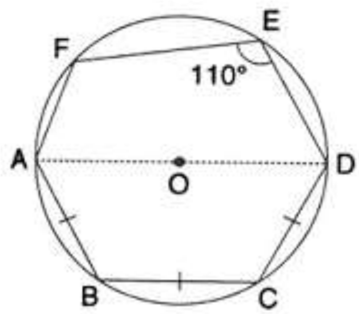
$\therefore AC = BD$ [*c.p.c.t*]

Question 2.

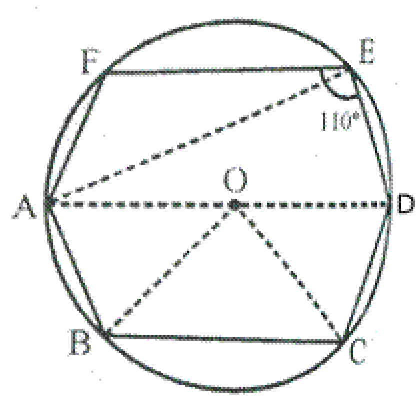
In the following figure, AD is the diameter of the circle with centre 0. chords AB, BC and CD are equal. If ∠DEF = 110°, calculate:

(i) ∠ AFE,

(ii) ∠FAB.



Solution:



Join AE, OB and OC.

(i) $\because$ AOD is the diameter,

$\therefore \angle AED = 90°$ [Angle in a semi − circle]

But $\angle DEF = 110°$ [given]

$\therefore \angle AEF = \angle DEF - \angle AED$

$= 110° - 90° = 20°$

(ii) $\because$ Chord AB = Chord BC = Chord CD [given]

$\therefore \angle AOB = \angle BOC = \angle COD$ (Equal chords subtends equal angles at the cenre)

But $\angle AOB + \angle BOC + \angle COD = 180°$ [AOD is a straigth line]

$\therefore \angle AOB = \angle BOC = \angle COD = 60°$

In ΔOAB, $OA = OB$

$\therefore \angle OAB = \angle OBA$ [Radii of the same circle]

But $\angle OAB + \angle OBA = 180° - \angle AOB$

$= 180° - 60°$

$= 120°$

$\therefore \angle OAB = \angle OBA = 60°$

In cyclic quadrilateral ADEF,

$\angle DEF + \angle DAF = 180°$

$\Rightarrow \angle DAF = 180° - \angle DEF$

$= 180° - 110°$

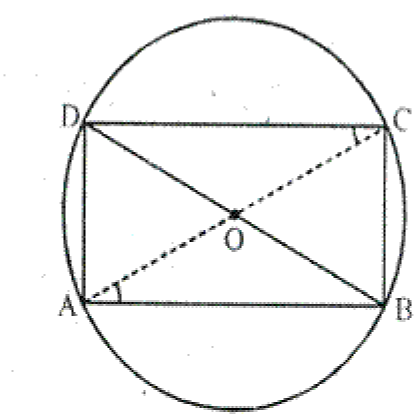
$= 70°$

Now, $\angle FAB = \angle DAF + \angle OAB$

$= 70° + 60° = 130°$

Question 3.
If two sides of a cycle-quadrilateral are parallel; prove that:
(i) its other two side are equal.
(ii) its diagonals are equal.

Solution:



Given –

ABCD is a cyclic quadriteral in which AB ∥ DC. AC and BD are its diagonals.

To prove –

(i) $AD = BC$

(ii) $AC = BD$

Proof –

(i) $AB \parallel DC \Rightarrow \angle DCA = \angle CAB$ [alternate angles]

Now, chord AD subtends $\angle DCA$ *and chord BC subtends* $\angle CAB$ *at the circumference of the circle.*

$\therefore \angle DCA = \angle CAB$ [proved]

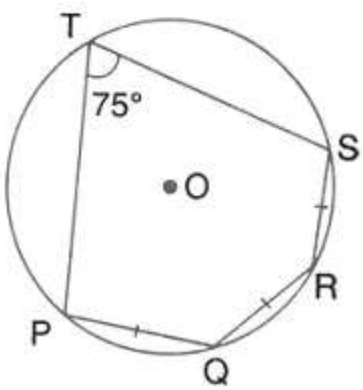
$\therefore$ *Chord AD = Chord BC or AD = BC*

(ii) *Now in* ΔABC *and* ΔADB,

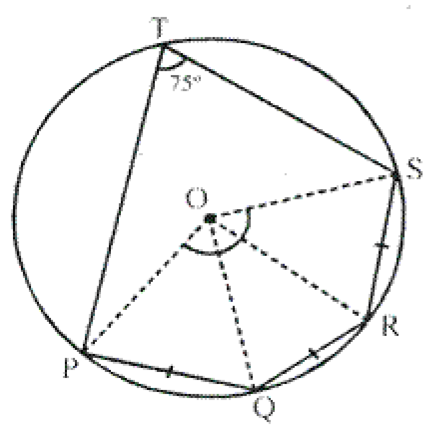
$AB = AB$ [common]
$\angle ACB = \angle ADB$ [Angles in the same segment]
$BC = AD$ [proved]
By Side − Angle − Side criterion of congruence, we have
$\Delta ACB \cong \Delta ADB$ [SAS postulate]
The corresponding parts of the congruent triangles are congruent.
$\therefore AC = BD$ [c.p.c.t]

Question 4.
The given figure show a circle with centre O. also, PQ = QR = RS and ∠PTS = 75°.
Calculate:

(i) ∠POS,
(ii) ∠ QOR,
(iii) ∠PQR.



Solution:



Join OP, OQ, OR and OS.

$\because PQ = QR = RS,$

$\angle POQ = \angle QOR = \angle ROS$ [Equal chords subtends equal angles at the centre]

Arc PQRS subtends $\angle POS$ at the center and $\angle PTS$ at the remaining part of the circle.

$\therefore \angle POS = 2\angle PTS = 2 \times 75° = 150°$

$\Rightarrow \angle POQ + \angle QOR + \angle ROS = 150°$

$\Rightarrow \angle POQ = \angle QOR = \angle ROS = \frac{150°}{3} = 50°$

In ΔOPQ, $OP = OQ$ [radii of the same circle]

$\therefore \angle OPQ = \angle OQP$

But $\angle OPQ + \angle OQP + \angle POQ = 180°$

$\therefore \angle OPQ + \angle OQP = 50° = 180°$

$\Rightarrow \angle OPQ + \angle OQP = 180° - 50°$

$\Rightarrow \angle OPQ + \angle OPQ = 130°$

$\Rightarrow 2\angle OPQ = 130°$

$\Rightarrow \angle OPQ = \angle OQP = \frac{130°}{2} = 65°$

Similarly we can prove that

In $\Delta OQR, \angle OQR = \angle ORQ = 65°$

and in $\Delta ORS, \angle ORS = \angle OSR = 65°$

(i) Now $\angle POS = 150°$

(ii) $\angle QOR = 50°$ and

(iii) $\angle PQR = \angle PQO + \angle OQR = 65° + 65° = 130°$

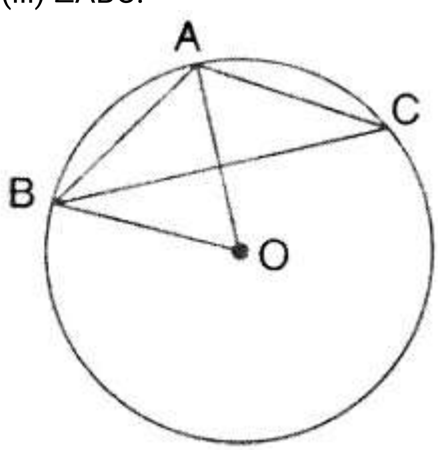
Question 5.

In the given figure, AB is a side of a regular six-sided polygon and AC is a side of a regular eight-sided polygon inscribed in the circle with centre O. calculate the sizes of:

(i) ∠ AOB,

(ii) ∠ ACB,

(iii) ∠ABC.



Solution:

(i) Arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at the remaining part of the circle.

$\therefore \angle ACB = \frac{1}{2}\angle AOB$

Since AB is the side of a regular hexagon,

$\angle AOB = 60°$

(ii) $\angle AOB = 60° \Rightarrow \angle ACB = \frac{1}{2} \times 60° = 30°$

(iii) Since AC is the side of a regulare octagon,

$\angle AOC = \frac{360}{8} = 45°$

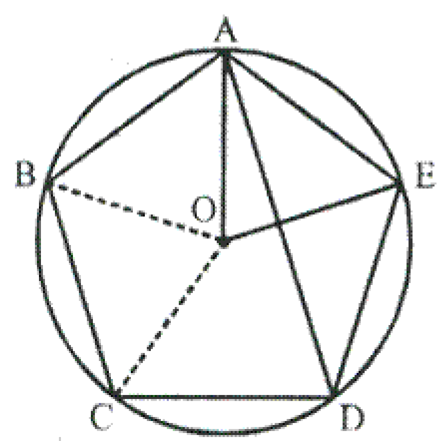
Again, Arc AC subtends $\angle AOC$ at the centre and $\angle ABC$ at the remaining part of the circle.

$\Rightarrow \angle ABC = \frac{1}{2}\angle AOC$

$\Rightarrow \angle ABC = \frac{45°}{2} = 22.5°$

Question 6.
In a regular pentagon ABCDE, inscribed in a circle; find ratio between angle EDA and angel ADC.

Solution:



Arc AE subtends $\angle AOE$ at the centre and

$\angle ADE$ at the remaining part of the circle.

$\therefore \angle ADE = \frac{1}{2} \angle AOE$

$= \frac{1}{2} \times 72°$

$= 36°$ [central angle of a regular pentagon at O]

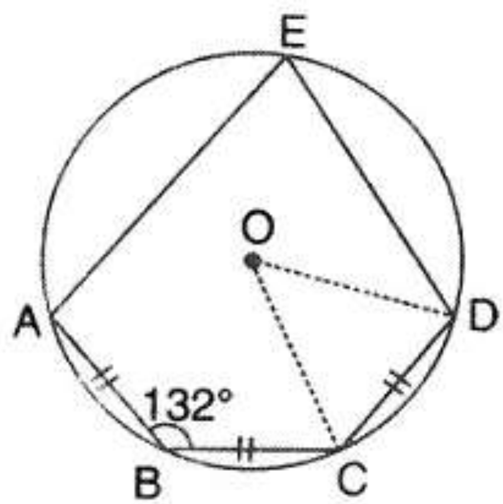
$\angle ADC = \angle ADB + \angle BDC$

$= 36° + 36° + 72°$

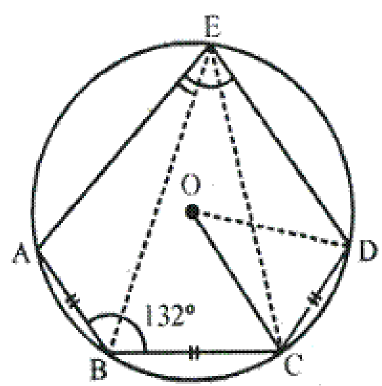
$\therefore \angle ADE : \angle ADC = 36° : 72° = 1:2$

Question 7.
In the given figure. AB = BC = CD and ∠ABC = 132°, calculate:

(i) ∠AEB,
(ii) ∠ AED,
(iii) ∠COD.

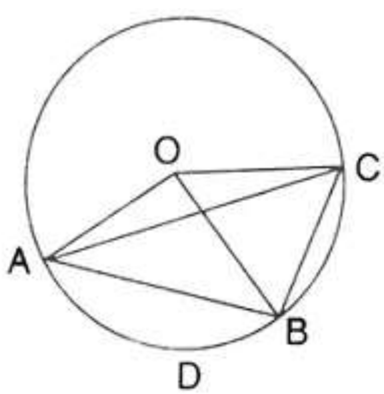


Solution:

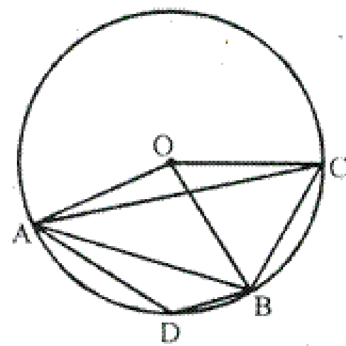


In the figure, O is the centre of circle, with $AB = BC = CD$.
Also, $\angle ABC = 132°$.
(i) In cyclic quadrilateral ABCE
$\angle ABC + \angle AEC = 180°$ [sum of opposite angles]
$\Rightarrow 132° + \angle AEC = 180°$
$\Rightarrow \angle AEC = 180° - 132°$
$\Rightarrow \angle AEC = 48°$
Since $AB = BC$, $\angle AEB = \angle BEC$ [equal chords subtends equal angles]
$\therefore \angle AEB = \frac{1}{2}\angle AEC$
$= \frac{1}{2} \times 48°$
$= 24°$
(ii) Similarly, $AB = BC = CD$
$\angle AEB = \angle BEC = \angle CED = 24°$
$\angle AED = \angle AEB + \angle BEC + \angle CED$
$= 24° + 24° + 24° = 72°$
(iii) Arc CD subtends $\angle COD$ at the centre and
$\angle CED$ at the remaining part of the circle.
$\therefore \angle COD = 2\angle CED$
$= 2 \times 24°$
$= 48°$

Question 8.
In the figure, O is the centre of the circle and the length of arc AB is twice the length of arc BC. If angle AOB = 108°, find:
(i) ∠ CAB,
(ii) ∠ADB.



Solution:



(i)Join AD and DB.

Arc AB = 2 arc BC and $\angle AOB = 180°$

$$\therefore \angle BOC = \frac{1}{2}\angle AOB$$

$$= \frac{1}{2} \times 108°$$

$$= 54°$$

Now, Arc BC subtends $\angle BOC$ *at the centre and* $\angle CAB$ *at the remaining part of the circle.*

$$\therefore \angle CAB = \frac{1}{2}\angle BOC$$

$$= \frac{1}{2} \times 54°$$

$$= 27°$$

(ii) Again, Arc AB subtends $\angle AOB$ *at the centre and* $\angle ACB$ *at the remaining part of the circle.*

$$\therefore \angle ACB = \frac{1}{2}\angle AOB$$

$$= \frac{1}{2} \times 108°$$

$$= 54°$$

In cyclic quadrilateral ADBC

$\angle ADB + \angle ACB = 180°$ [*sum of opposite angles*]

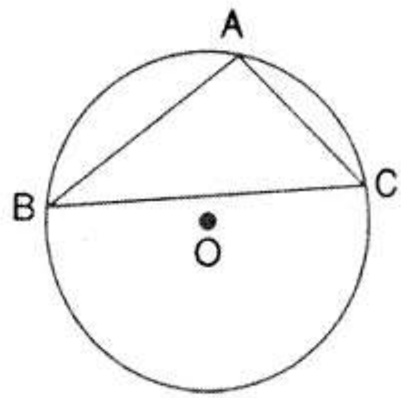
$\Rightarrow \angle ADB + 54° = 180°$

$\Rightarrow \angle ADB = 180° - 54°$

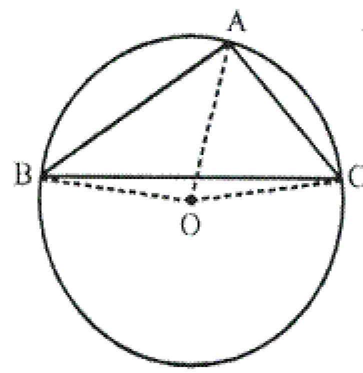
$\Rightarrow \angle ADB = 126°$

Question 9.

The figure shows a circle with centre O. AB is the side of regular pentagon and AC is the side of regular hexagon. Find the angles of triangle ABC.



Solution:



Join OA, OB and OC

Since AB is the side of a regular pentagon,

$\angle AOB = \frac{360°}{5} = 72°$

Again AC is the side of a regular hexagon,

$\angle AOC = \frac{360°}{6} = 60°$

But $\angle AOB + \angle AOC + \angle BOC = 360°$ [*angles at a point*]

$\Rightarrow 72° + 60° + \angle BOC = 360°$

$\Rightarrow 132° + \angle BOC = 360°$

$\Rightarrow \angle BOC = 360° - 132°$

$\Rightarrow \angle BOC = 228°$

Now, Arc BC subtends $\angle BOC$ *at the centre and*

$\angle BAC$ *at the remaining part of the circle.*

$\Rightarrow \angle BAC = \frac{1}{2}\angle BOC$

$\Rightarrow \angle BAC = \frac{1}{2}\angle BOC$

$\Rightarrow \angle BAC = \frac{1}{2} \times 228° = 114°$

Similarly we can prove that

$\Rightarrow \angle ABC = \frac{1}{2}\angle AOC$

$\Rightarrow \angle ABC = \frac{1}{2} \times 60° = 30°$

and

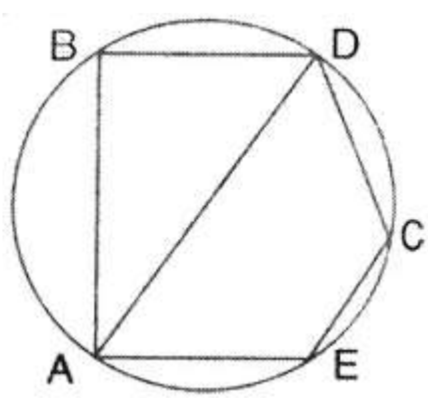
$\Rightarrow \angle ACB = \frac{1}{2}\angle AOB$

$\Rightarrow \angle ACB = \frac{1}{2} \times 72° = 36°$

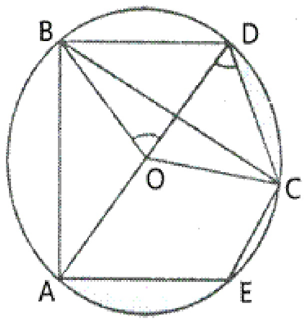
Thus, angles of the triangle are, 114°,30° *and* 36°

Question 10.
In the given figire, BD is a side of a regular hexagon, DC is a side of a regular pentagon and AD is diameter. Calculate:

(i) ∠ ADC
(ii) ∠BAD,
(iii) ∠ABC
(iv) ∠ AEC.



Solution:



Join BC, BO, CO and EO

Since BDis the side of a regular hexagon,

$\angle BOD = \frac{360}{6} = 60°$

Since DC is the side of a regular pentagon,

$\angle COD = \frac{360}{5} = 72°$

In $\Delta BOD . \angle BOD = 60°$ *amd* $ob = od$

$\therefore \angle OBD = \angle ODB = 60°$

(*i*) *In* ΔOCD, $\angle COD = 72°$ *and* $OC = OD$

$$\therefore \angle ODC = \frac{1}{2}(180° - 72°)$$
$$= \frac{1}{2} \times 108°$$
$$= 54°$$

Or, $\angle ADC = 54°$

(*ii*)$\angle BDO = 60°$ *or* $\angle BDA = 60°$

(*iii*)*Arc AC subtends* $\angle AOC$ *at the centre and*

$\angle ABC$ *at the remaining part of the circle.*

$$\therefore \angle ABC = \frac{1}{2}\angle AOC$$
$$= \frac{1}{2}[\angle AOD - \angle COD]$$
$$= \frac{1}{2} \times (180° - 72°)$$
$$= \frac{1}{2} \times 108°$$
$$= 54°$$

(*iv*)*In cyclic quadrilateral AECD*

$\angle AEC + \angle ADC = 180°$ [*sum of opposite angles*]

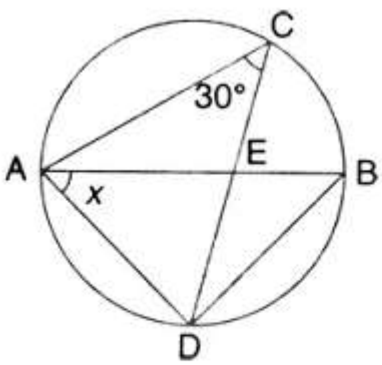
$\Rightarrow \angle AEC + 54° = 180°$

$\Rightarrow \angle AEC = 180° - 54°$

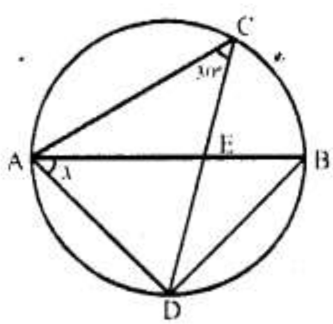
$\Rightarrow \angle AEC = 126°$

Exercise 17 C

Question 1.
In the given circle with diameter AB, find the value of x.

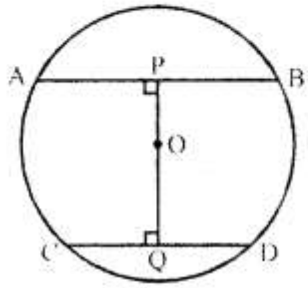


Solution:

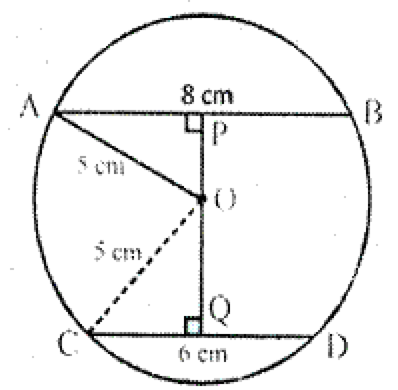


∠ABD = ∠ACD = 30° (Angle in the same segment)
Now in Δ ADB,
∠BAD + ∠ADB + ∠DBA = 180° (Angles of a A)
But ∠ADB = 90° (Angle in a semi-circle)
∴ x + 90° + 30° = 180° ⇒ x + 120° = 180°
∴ x= 180° – 120° = 60° Ans.

Question 1.
In the given figure, O is the centre of the circle with radius 5 cm, OP and OQ are perpendiculars to AB and CD respectively. AB = 8cm and CD = 6cm. Determine the length of PQ.



Solution:



Radius of the circle whose centre is $0 = 5cm$

op $\perp AB$ *and* $OQ \perp CD$, $AB = 8cm$ *and* $CD = 6cm$.

Join OA and OC, then $OA = OC + 5\ cm$

Since $OP \perp AB$, *P is the midpoint of AB.*

Similarly Q is the midpoint of CD.

In right ΔOAP,

$OA^2 = OP^2 + AP^2$ [*Pythagoras Theorem*]

$\Rightarrow (5)^2 = OP^2 + (4)^2$ $[\because AP = PB = \frac{1}{2} \times 8 = 4cm]$

$\Rightarrow 25 = OP^2 + 16$

$\Rightarrow OP^2 = 25 - 16$

$\Rightarrow OP^2 = 9$

$\Rightarrow OP = 3cm$

Similarly, in right ΔOCQ,

$OC^2 = OQ^2 + CQ^2$ [*Pythagoras Theorem*]

$\Rightarrow (5)^2 = OQ^2 + (3)^2$

$\Rightarrow 25 = OQ^2 + 9$

$\Rightarrow OQ^2 = 25 - 9$

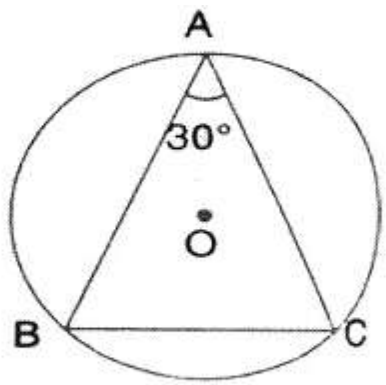
$\Rightarrow OQ^2 = 16$

$\Rightarrow OQ = 4cm$

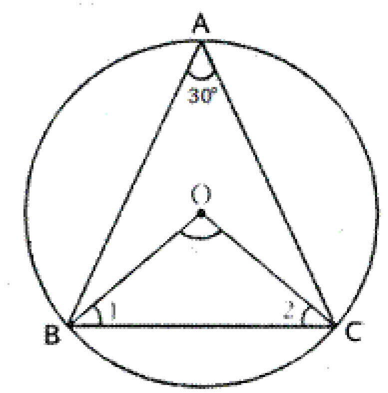
Hence, $PQ = OP + OQ = 3 + 4 = 7cm$

Question 2.

In the given figure, ABC is a triangle in which ∠ BAC = 30° Show that BC is equal to the radius of the circum-circle of the triangle ABC, whose centre is O.



Solution:



Given – In the figure ABC is a triangle in which $\angle A = 30^\circ$.

To prove – BC is the radius of circumcircle of $\triangle ABC$ *whose centre is O.*

Construction – Join OB and OC.

Proof:

$\angle BOC = 2\angle BAC = 2 \times 30^\circ = 60^\circ$

Now in $\triangle OBC$,

$OB = OC$ [*Radii of the same circle*]

$\angle OBC = \angle OCB$

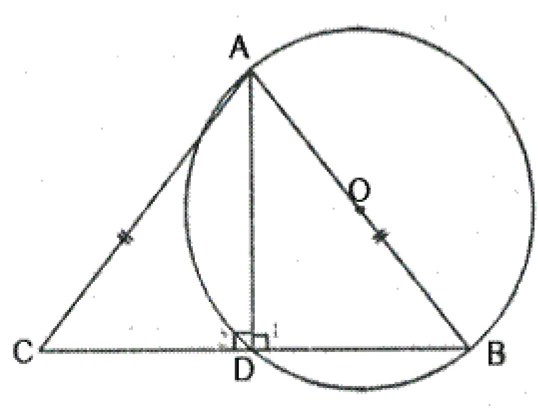
But, in $\triangle BOC$,

$\angle OBC + \angle OCB + \angle BOC = 180^\circ$ [*Angles of a triangle*]

$\Rightarrow \angle OBC + \angle OBC + 60^\circ = 180^\circ$
$\Rightarrow 2\angle OBC + 60^\circ = 180^\circ$
$\Rightarrow 2\angle OBC = 180^\circ - 60^\circ$
$\Rightarrow 2\angle OBC = 120^\circ$
$\Rightarrow \angle OBC = \frac{120^\circ}{2} = 60^\circ$
$\Rightarrow \angle OBC = \angle OCB = \angle BOC = 60^\circ$
$\Rightarrow \Delta BOC$ is an equilateral triangle.
$\Rightarrow BC = OB = OC$
But, OB and OC are the radii of the circum – circle.
$\therefore BC$ is also the radius of the circum – circle.

Question 3.
Prove that the circle drawn on any one a the equal side of an isosceles triangle as diameter bisects the base.

Solution:



Given – In $\Delta ABC, AB = AC$ and a circle with AB as diameter is drawn which intersects the side BC and D.
To prove – D is the mid point of BC.
Construction – Join AD.
Proof – $\angle 1 = 90^\circ$ [Angle in a semi circle]
But $\angle 1 + \angle 2 = 180^\circ$ [Linear pair]
$\therefore \angle 2 = 90^\circ$
Now in right ΔABD and ΔACD,

$Hyp.\ AB = Hyp.AC$ [Given]

$Side\ AD = AD$ [Common]

$\therefore$ By the Right Angle – Hypotenuse – Side criterion of congruence, we have

$\Delta ABD \cong \Delta ACD$ [RHS criterion of congruence]

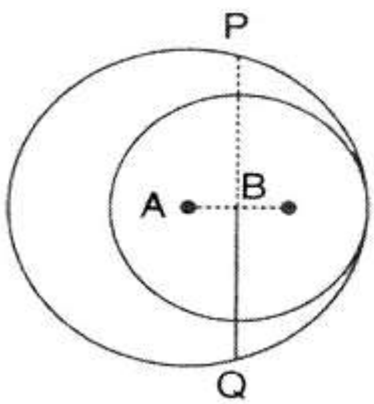
The corresponding parts of the congruent triangles are congruent.

$\therefore BD = DC$ [c.p.c.t]

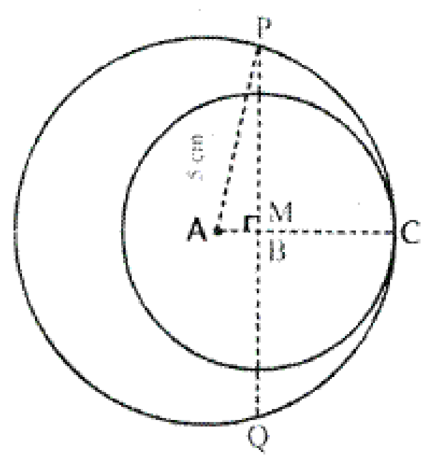
Hence D is the mid point of BC.

Question 3 (old).

The given figure show two circles with centres A and B; and radii 5 cm and 3cm respectively, touching each other internally. If the perpendicular bisector of AB meets the bigger circle in P and Q, find the length of PQ.



Solution:



Join AP and produce AB to meet the bigger circle at C.

$AB = AC - BC = 5cm - 3cm = 2cm.$

But, M is the mid – point of AB.

$\therefore AM = \frac{2}{2} = 1cm$

Now in right ΔAPM,

$AP^2 = MP^2 + AM^2$ [Pythagoras Theorem]

$\Rightarrow (5)^2 = MP^2 + 1^2$

$\Rightarrow 25 = MP^2 1$

$\Rightarrow MP^2 = 25 - 1$

$\Rightarrow MP^2 = 24$

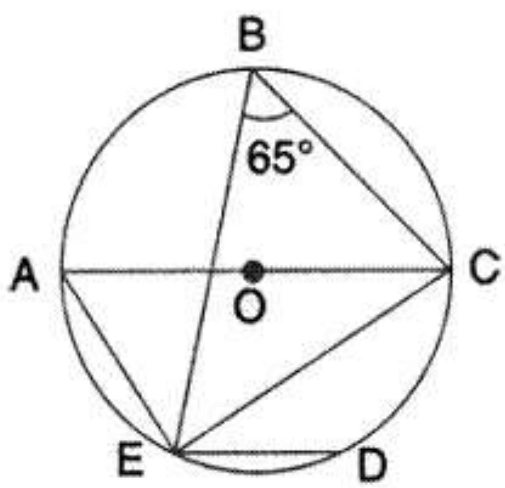
$\Rightarrow MP = \sqrt{24} = \sqrt{4 \times 6} = 2\sqrt{6} cm$

$\therefore PQ = 2MP = 2 \times 2\sqrt{6} = 4\sqrt{6} cm$

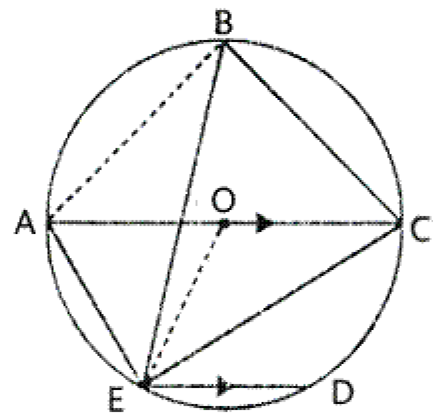
$\Rightarrow PQ = 4 \times 2.45 = 9.8cm$

Question 4.

In the given figure, chord ED is parallel to diameter AC of the circle. Given ∠ CBE = 65°, calculate ∠DEC.



Solution:



Join OE.

ArcEC subtends $\angle EOC$ at the centre and $\angle EBC$ at the remaining part of the circle.

$\angle EOC = 2\angle EBC = 2 \times 65° = 130°.$

Now in $\Delta OEC, OE = OC$ [Radii of the same circle]

$\therefore \angle OEC = \angle OCE$

But, in ΔEOC,

$\angle OEC + \angle OCE + \angle EOC = 180°$ [Angles of a triangle]

$\Rightarrow \angle OCE + \angle OCE + \angle EOC = 180°$

$\Rightarrow 2\angle OCE + 130° = 180°$

$\Rightarrow 2\angle OCE = 180° - 130°$

$\Rightarrow 2\angle OCE = 50°$

$\Rightarrow \angle OCE = \frac{50°}{2} = 25°$

$\therefore AC \parallel ED$ [given]

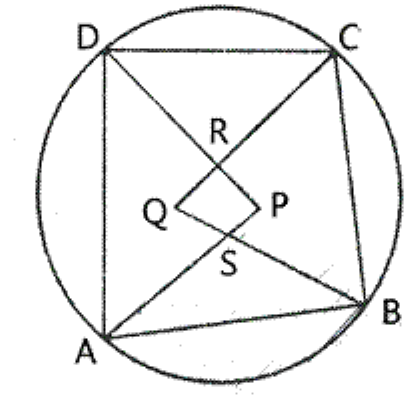
$\therefore \angle DEC = \angle OCE$ [Alternate angles]

$\Rightarrow \angle DEC = 25°$

Question 5.

The quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic. Prove it.

Solution:



Given – ABCD is a cyclic quadrilateral and PQRS is a quadrilateral formed by the angle bisectors of angle $\angle A, \angle B, \angle C$ and $\angle D$.

To prove – PQRS is a cyclic quadrilateral.

Proof – In ΔAPD,

$\angle PAD + \angle ADP + \angle APD = 180°$ ...(1)

Similarly, IN ΔBQC,

$\angle QBC + \angle BCQ + \angle BQC = 180°$ ...(2)

Adding (1) and (2), we get

$\angle PAD + \angle ADP + \angle APD + \angle QBC + \angle BCQ + \angle BQC = 180° + 180°$

$\Rightarrow \angle PAD + \angle ADP + \angle QBC + \angle BCQ + \angle APD + \angle BQC = 360°$ ....(3)

But $\angle PAD + \angle ADP + \angle QBC + \angle BCQ = \frac{1}{2}[\angle A + \angle B + \angle C + \angle D]$

$$= \frac{1}{2} \times 360° = 180°$$

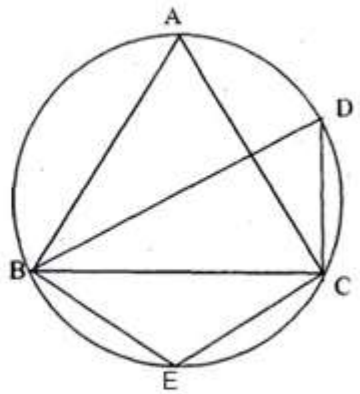
$\therefore \angle APD + \angle BQC = 360° - 180° = 180°$ [*from* (3)]

But these are the sum of opposite angles of quadrilateral PRQS.

$\therefore$ Quad. PRQS is a cyclic quadrilateral.

Question 6.
In the figure, ∠DBC = 58°. BD is a diameter of the circle. Calculate:

(i) ∠BDC
(ii) ∠BEC
(iii) ∠BAC



Solution:

(i) Given that BD is a diameter of the cirde.

The angle in a semicirde is a right angle.

$\therefore \angle BCD = 90°$

Also given that $\angle DBC = 58°$

In ΔBDC,

$\angle DBC + \angle BCD + \angle BDC = 180°$

$\Rightarrow 58° + 90° + \angle BDC = 180°$

$\Rightarrow 148° + \angle BDC = 180°$

$\Rightarrow \angle BDC = 180° - 148°$

$\Rightarrow \angle BDC = 32°$

(ii) We know that the opposite angles of a cyclic quadrilateral are supplementary.

Thus, in cyclic quadrilateral BECD,

$\angle BEC + \angle BDC = 180°$

$\Rightarrow \angle BEC + 32° = 180°$

$\Rightarrow \angle BEC = 180° - 32°$

$\Rightarrow \angle BEC = 148°$

(iii) In cydic quadrilateral ABEC,

$\angle BAC + \angle BEC = 180°$

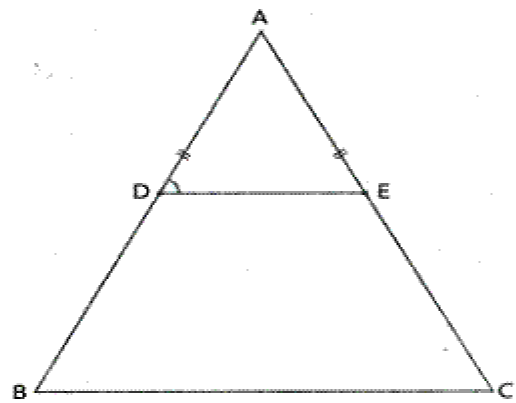
$\Rightarrow \angle BAC + 148° = 180°$

$\Rightarrow \angle BAC = 180° - 148°$

$\Rightarrow \angle BAC = 32°$

Question 7.
D and E are points on equal sides AB and AC of an isosceles triangle ABC such that AD = AE. Proved that the points B, C, E and D are concyclic.

Solution:



Given – In $\triangle ABC$, $AB = AC$ and D and E are points on AB and AC such that $AD = AE$. DE is joined.

To prove B, C, E, D are concyclic.

Proof – In $\triangle ABC$, $AB = AC$

$\therefore \angle B = \angle C$ [Angles opposite to equeal sides]

Similiarly, In $\triangle ADE$, $AD = AE$ [given]

$\therefore \angle ADE = \angle AED$ [Angles opposite to equal sides]

In $\triangle ABC$,

$\because \frac{AP}{AB} = \frac{AE}{AC}$

$\therefore DE \parallel BC$

$\therefore \angle ADE = \angle B$ [corresponoding angles]

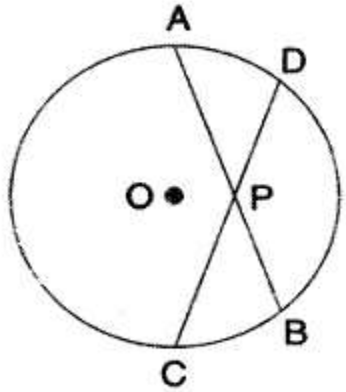
But $\angle B = \angle C$ [proved]

$\therefore$ Ext. $\angle ADE$ = its interior opposite $\angle C$

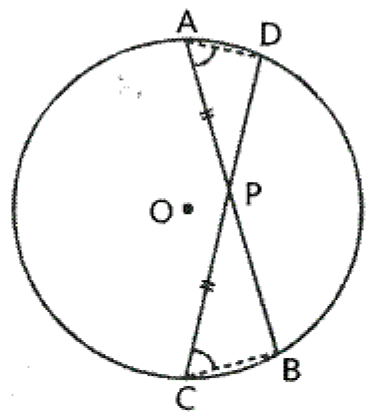
$\therefore$ BCED is a cyclic quadrilateral.

Hence B, C, E and D are concyclic.

Question 7 (old).
Chords AB and CD of a circle intersect each other at point P such that AP = CP.
Show that: AB = CD.



Solution:



Given – Two chords AB and CD intersect each other at P inside the circle with centre O and AP = CP

TO prove – AB = CD

Prood – Two chords AB and CD intersect each other inside the circle at P.

$\therefore AP \times PB = CP \times PD$

$\Rightarrow \frac{AP}{CP} = \frac{PD}{PB}$

But $AP = CP$ ...(1) [*given*]

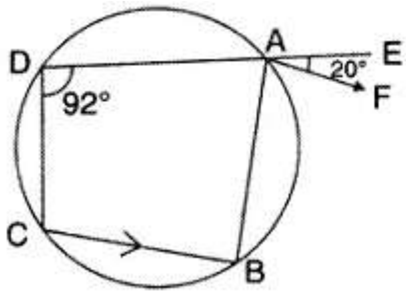
$\therefore PD = PB$ *or* $PB = PD$ ...(2)

Adding (1) *and* (2)

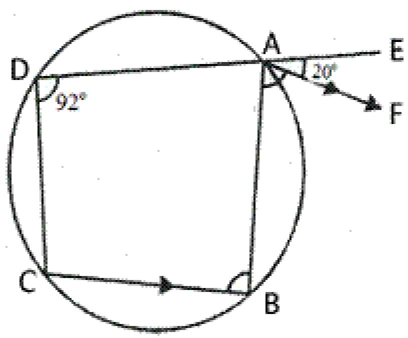
$AP + PB = CP + PD$

$\Rightarrow AB = CD$

Question 8.
In the given figure, ABCD is a cyclic quadrilateral. AF is drawn parallel to CB and DA is produced to point E. If ∠ ADC = 92°, ∠ FAE = 20°; determine ∠ BCD. Given reason in support of your answer.

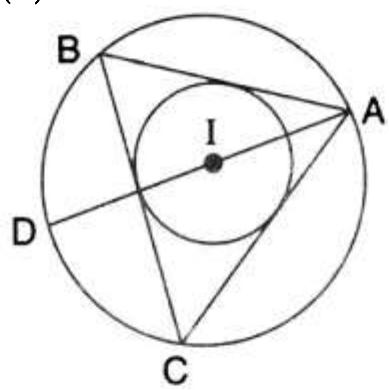


Solution:

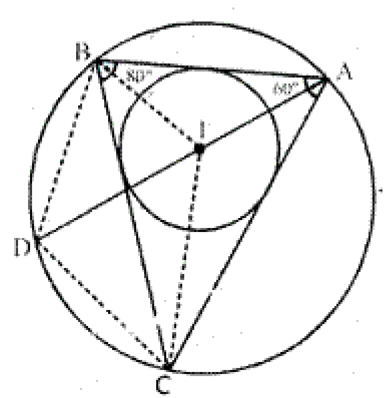


In cyclic quad. ABCD,
$AF \parallel CB$ and DA is produced to E such that $\angle ADC = 92°$ and $\angle FAE = 20°$
Now we need to find the measure of $\angle BCD$
In cyclic quad. ABCD,
$\angle B + \angle D = 180°$
$\Rightarrow \angle B + 92° = 180°$
$\Rightarrow \angle B = 180° - 92°$
$\Rightarrow \angle B = 88°$
Since $AF \parallel CB$, $\angle FAB = \angle B = 88°$
But, $\angle FAE = 20°$ [given]
Ext. $\angle BAE = \angle BAF + \angle FAE$
$= 88° + 22° = 108°$
But, Ext. $\angle BAE = \angle BCD$
$\therefore \angle BCD = 108°$

Question 9.
If I is the in centre of triangle ABC and AI when produced meets the circumcircle of triangle ABC in points D. if ∠ BAC = 66° and ∠ = 80o.calculate:
(i) ∠ DBC
(ii) ∠ IBC
(iii) ∠ BIC.



Solution:



Join DB and DC, IB and IC,
$\angle BAC = 66°, \angle ABC = 80°$, *I is the incentre of the* ΔABC,
(i) Since $\angle DBC$ *and* $\angle DAC$ *are in the same segment,*
$\angle DBC = \angle DAC$
$But, \angle DAC = \frac{1}{2}\angle BAC = \frac{1}{2} \times 66° = 33°$
$\therefore \angle DBC = 33°$
(ii) Since I is the incentre of ΔABC, *IB bisects* $\angle ABC$
$\therefore \angle IBC = \frac{1}{2}\angle ABC = \frac{1}{2} \times 80° = 40°$

(iii) $\therefore \angle BAC = 66^\circ$ and $\angle ABC = 80^\circ$

In $\triangle ABC, \angle ACB = 180^\circ - (\angle ABC + \angle BAC)$

$\Rightarrow \angle ACB = 180^\circ - (80^\circ + 66^\circ)$

$\Rightarrow \angle ACB = 180^\circ - (156^\circ)$

$\Rightarrow \angle ACB = 34^\circ$

Since IC bisects the $\angle C$,

$\therefore \angle ICB = \frac{1}{2}\angle C = \frac{1}{2} \times 34^\circ = 17^\circ$

Now in $\triangle IBC$,

$\angle IBC + \angle ICB + \angle BIC = 180^\circ$

$\Rightarrow 40^\circ + 17^\circ + \angle BIC = 180^\circ$

$\Rightarrow 57^\circ + \angle BIC = 180^\circ$

$\Rightarrow \angle BIC = 180^\circ - 57^\circ$

$\Rightarrow \angle BIC = 123^\circ$

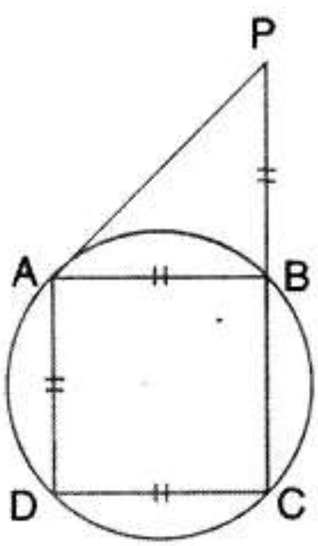
Question 10.

In the given figure, AB = AD = DC = PB and ∠ DBC = x°. Determine, in terms of x:

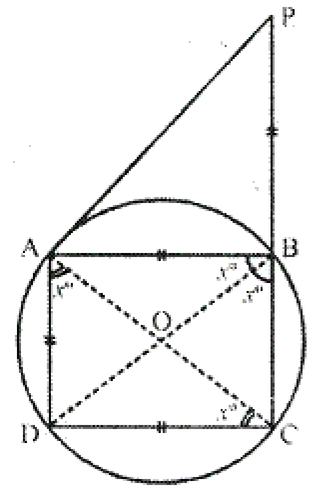
(i) ∠ ABD,

(ii) ∠ APB.

Hence or otherwise, prove that AP is parallel to DB.



Solution:



Given – In the figure, $AB = AD = DC = PB$ *and* $DBC = X^\circ$

Join AC and BD.

To find : the measure of $\angle ABD$ *and* $\angle APB$.

Proof : $\angle DAC = \angle DBC = X$ [*angles in the same segment*]

But $\angle DCA = \angle DAC = X$ [$\because AD = DC$]

Also, we have, $\angle ABD = \angle DAC$ [*angles in the same segment*]

In ΔABP, *ext.*$\angle ABC = \angle BAP + \angle APB$

But, $\angle BAP = \angle APB$ [$\because AB = BP$]

$2 \times X = \angle APB + \angle APB = 2\angle APB$

$\therefore 2\angle APB = 2X$

$\Rightarrow \angle APB = X$

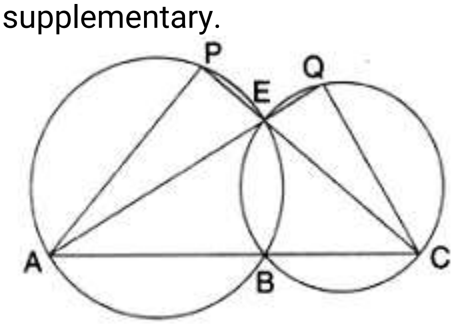
$\therefore \angle APB = \angle DBC = X,$

But these are corresponding angles

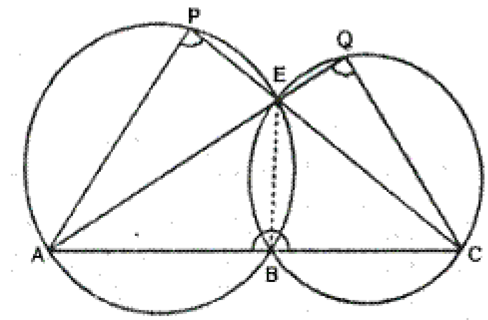
$\therefore AP \parallel DB$

Question 11.

In the given figure; ABC, AEQ and CEP are straight lines. Show that ∠APE and ∠ CQE are supplementary.



Solution:



Given – In the figure, ABC, AEQ and CEP are straight line

To prove – $\angle APE + \angle CQE = 180^\circ$

Construction – Join EB

Proof – In cyclic quad. $ABEP$,

$\angle APE + \angle ABE = 180^\circ$ ....(1)

Similarly, in cyclic quad. $BCQE$,

$\angle CQE + \angle CBE = 180^\circ$ ......(2)

Adding (1) and (2),

$\angle APE + \angle ABE + \angle CQE + \angle CBE = 180^\circ + 180^\circ = 360^\circ$

$\Rightarrow \angle APE + \angle ABE + \angle CBE = 360^\circ$

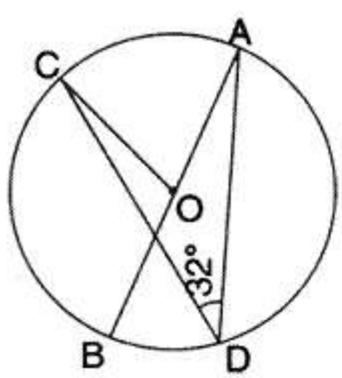
But, $\angle ABE + \angle CBE = 180^\circ$ [Linear pair]

$\therefore \angle APE + \angle CQE + 180^\circ = 360^\circ$

$\Rightarrow \angle APE + \angle CQE = 360^\circ - 180^\circ = 180^\circ$

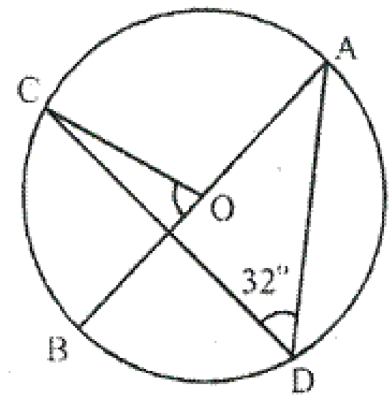
Hence $\angle APE$ AND $\angle CQE$ are supplementary.

Question 12.
In the given, AB is the diameter of the circle with centre O.



If ∠ ADC = 32°, find angle BOC.

Solution:



Arc AC subtends $\angle AOC$ at the centre and $\angle ADC$ at the remaining part of the circle

$\therefore \angle AOC = 2\angle ADC$

$\Rightarrow \angle AOC = 2 \times 32° = 64°$

Since $\angle AOC$ and $\angle BOC$ are linear pair, we have

$\angle AOC + \angle BOC = 180°$

$\Rightarrow 64° + \angle BOC = 180°$

$\Rightarrow \angle BOC = 180°$

$\Rightarrow \angle BOC = 180° - 64°$

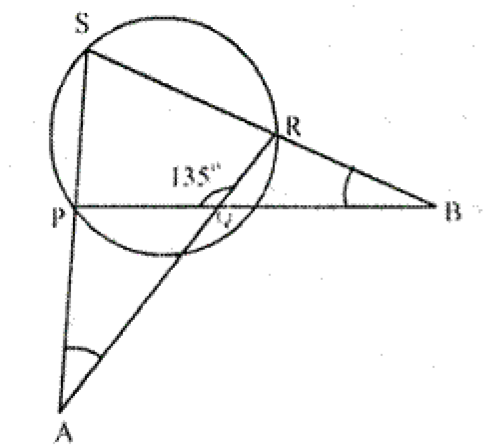
$\Rightarrow \angle BOC = 116°$

Question 13.

In a cyclic-quadrilateral PQRS, angle PQR = 135°. Sides SP and RQ produced meet at point A: whereas sides PQ and SR produced meet at point B.

If ∠A: ∠B = 2 : 1;find angles A and B.

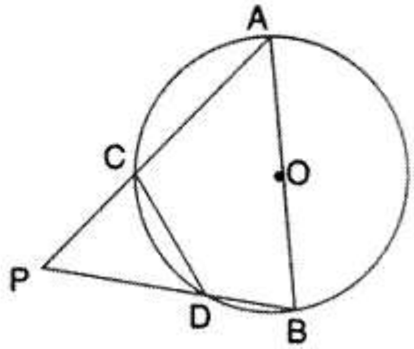
Solution:



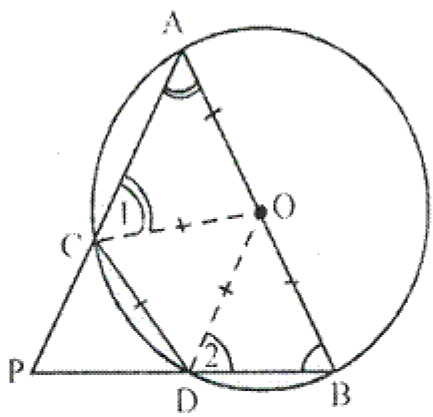
PQRS is a cyclic quadrilateral in which $\angle PQR = 135°$
Sides SP and RQ are produced to meet at A and
Sides PQ and SR are produced to meet at B.
$\angle A = \angle B = 2:1$
Let $\angle A = 2x$, *then* $\angle B - x$
Now, in cyclic quad. PQRS,
Since, $\angle PQR = 135°, \angle S = 180° - 135° = 45°$
[*Since sum of opposite angles of a cyclic quadrilateral are supplementary*]
Since, $\angle PQR$ *and* $\angle PQA$ *are linear pair,*
$\angle PQR + \angle PQA = 180°$
$\Rightarrow 135° + \angle PQA = 180°$
$\Rightarrow \angle PQA = 180° - 135° = 45°$
Now, in ΔPBS,
$\angle P = 180° - (45° + x) = 180° - 45° - x = 135° - x$ ....(1)
Again, in ΔPQA,
Ext. $\angle P = \angle PQA + \angle A = 45° + 2x$ ....(2)
From (1) *and* (2),
$45° + 2x = 135° - x$
$\Rightarrow 2x + x = 135° - 45°$
$\Rightarrow 3x = 90°$
$\Rightarrow x = 30°$
Hence, $\angle A = 2x = 2 \times 30° = 60°$
and $\angle B = x = 30°$

Question 17 (old).
If the following figure, AB is the diameter of a circle with centre O and CD is the chord

with lenght equal radius OA.
If AC produced and BD produced meet at point p; show that ∠APB = 60°



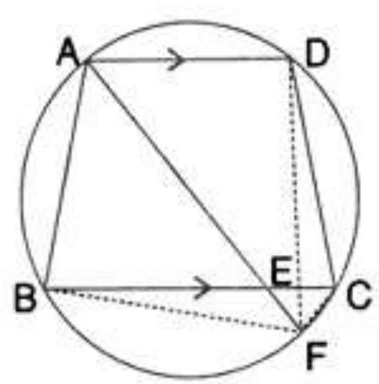
Solution:



Given – In the figure, AB is the diameter of a circle with centre O.
CD is the chord with lengh equal radius OA.
AC and BD produced meet at point P
To prove : $\angle APB = 60°$
Construction – Join OC and OD
Proof – We have $CD = OC = OD$ [given]
Therefore, ΔOCD *is an equilateral triangle*
$\therefore \angle OCD = \angle ODC = \angle COD = 60°$
In $\Delta AOC, OA = OC$ [radii of the same circle]
$\therefore \angle A = \angle 1$
Similarly, in $\Delta BOD, OB = OD$ [radii of the same circle]
$\therefore \angle B = \angle 2$
Now, in cyclic quad. ACDB,

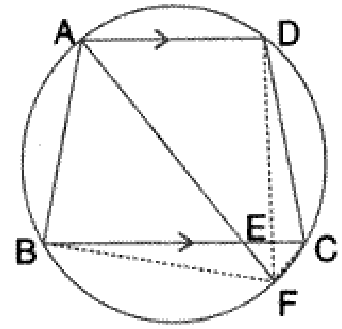
$\text{since}, \angle ACD + \angle B = 180°$
[*Since sum of opposite angles of a cyclic quadrilateral are supplementary*]
$\Rightarrow 60° + \angle 1 + \angle B = 180°$
$\Rightarrow \angle 1 + \angle B = 180° - 60°$
$\Rightarrow \angle 1 + \angle B = 120°$
But, $\angle 1 = \angle A$
$\therefore \angle A + \angle B = 120° \quad ....(1)$
Now, *in* ΔAPB,
$\angle P + \angle A + \angle B = 180°$ [*Sum of angles of a triangles*]
$\Rightarrow \angle P + 120° = 180°$
$\Rightarrow \angle P = 180° - 120°$ [*from* (1)]
$\Rightarrow \angle P = 60°$ *or* $\angle APB = 60°$

Question 14.
In the following figure, ABCD is a cyclic quadrilateral in which AD is parallel to BC.



If the bisector of angle A meet BC at point E and the given circle at point F, prove that:
(i) EF = FC
(ii) BF =DF

Solution:

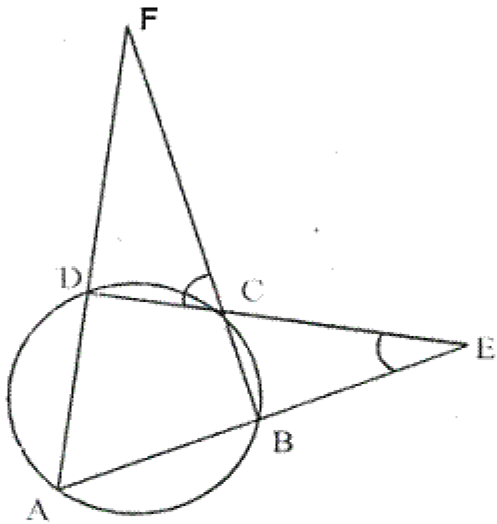


Given $-$ *ABCD is a cyclic quadrilateral in which* $AD \parallel BC$
Bisector of $\angle A$ *meets BC at E and the given circle at F.*
DF and BF are joined.
To prove $-$
(i) $EF = FC$
(ii) $BF = DF$
Proof $-$ *ABCD is a cyclic quadrilateral and* $AD \parallel BC$
$\because AF$ *is the bisector of* $\angle A, \angle BAF = \angle DAF$
Also, $\angle DAE = \angle BAE$
$\angle DAE = \angle AEB$ [*Alternate angles*]
(i)*In* $\Delta\ ABE$, $\angle ABE = 180° - 2\angle AEB$
$\angle CEF = \angle AEB$ [*Vertically Opposite angles*]
$\angle ADC = 180° - \angle ABC = 180° - (180° - 2\angle AEB)$
$\angle ADC = 2\angle AEB$
$\angle AFC = 180° - \angle ADC$
$= 180° - 2\angle AEB$ [*Since ADCF is a cyclic quadrilateral*]
$\angle ECF = 180° - (\angle AFC + \angle CEF)$
$= 180° - (180° - 2\angle AEB + \angle AEB)$
$= \angle AEB$
$\therefore EC = EF$
(ii) $\therefore$ *Arc BF = Arc DF* [*Equal arcs subtends equal angles*]
$\Rightarrow BF = DF$ [*Equal arcs have equal chords*]

Question 15.
ABCD is a cyclic quadrilateral. Sides AB and DC produced meet at point e; whereas sides BC and AD produced meet at point F. I f ∠ DCF : ∠F : ∠E = 3 : 5 : 4, find the angles

of the cyclic quadrilateral ABCD.

Solution:



$Given - In\ a\ circle,\ ABCD\ is\ a\ cyclic\ quadrilateral\ AB\ and\ DC$

$are\ produced\ to\ meet\ at\ E\ and\ BC\ and\ AD\ are\ produced\ to\ meet\ at\ F.$

$\angle DCF : \angle F : \angle E = 3:5:4$

$Let\ \angle DCF = 3X, \angle F = 5x, \angle E = 4x$

$Now,\ we\ have\ to\ find, \angle A, \angle B, \angle C\ AND\ \angle D$

$In\ cyclic\ quad.\ ABCD,\ BC\ is\ produced.$

$\therefore \angle A = \angle DCF = 3x$

$In\ \Delta CDF,$

$Ext.\angle CDA = \angle DCF + \angle F = 3x + 5x = 8x$

$In\ \Delta BCE,$

$Ext.\angle ABC = \angle BCE + \angle E \quad [\angle BCE = \angle DCF, vertically\ opposite\ angles]$

$= \angle DCF + \angle E$

$= 3x + 4x = 7x$

$Now,\ in\ cyclic\ quad.ABCD,$

$since, \angle B + \angle D = 180°$

$[Since\ sum\ of\ opposite\ of\ a\ cyclic\ quadrilateral\ are\ supplementary]$

$\Rightarrow 7x + 8x = 180°$

$\Rightarrow 15x = 180°$

$$\Rightarrow x = \frac{180°}{15} = 12°$$

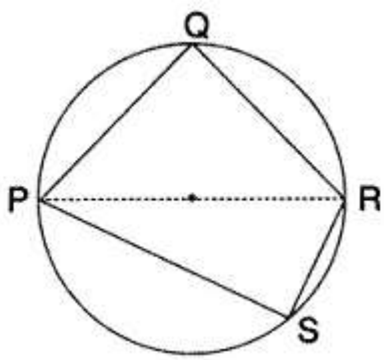
$\therefore \angle A = 3x = 3 \times 12° = 36°$

$\angle B = 7x = 7 \times 12° = 84°$

$\angle C = 180° - \angle A = 180° - 36° = 144°$

$\angle D = 8x = 8 \times 12° = 96°$

Question 16.
The following figure shows a circle with PR as its diameter. If PQ = 7 cm and QR = 3RS = 6 cm, Find the perimeter of the cyclic quadrilateral PORS.



Solution:

In the figure, PQRS is a cyclic quadrilateral in which PR is a diameter

$PQ = 7 cm$

$QR = 3RS = 6cm$

$3RS = 6\ cm \Rightarrow RS = 2\ cm$

Now in ΔPQR,

$\angle Q = 90°$ [*Angles in a semi circle*]

$\therefore PR^2 = PQ^2 + QR^2$ [*Pythagoras Theorem*]

$= 7^2 + 6^2$

$= 49 + 36$

$= 85$

Again in right $\Delta PSQ, PR^2 = PS^2 + RS^2$

$\Rightarrow 85 = PS^2 + 2^2$

$\Rightarrow PS^2 = 85 - 4 = 81 = (9)^2$

$\therefore PS = 9cm$

$Now, perimeter\ of\ quad.PQRS = PQ + QR + RS + SP$

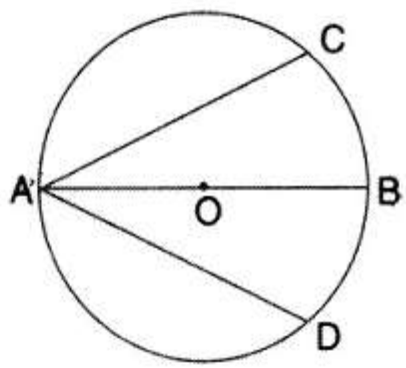
$= (7 + 9 + 2 + 6)cm$

$= 24$

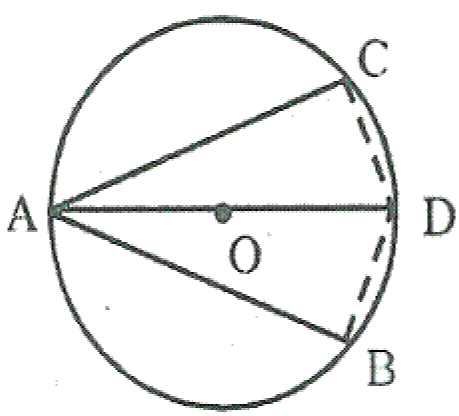
Question 17.
In the following figure, AB is the diameter of a circle with centre O. If chord AC = chord AD. Prove that:

(i) arc BC = arc DB
(ii) AB is bisector of ∠ CAD.

Further if the lenght of arc AC is twice the length of arc BC find :
(a) ∠ BAC
(b) ∠ ABC



Solution:

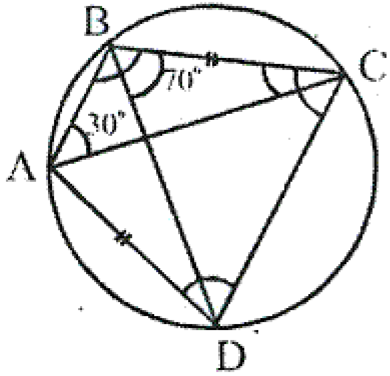


$Given - In\ a\ circle\ with\ centre\ O,\ AB\ is\ the\ diameter\ and\ AC\ and$
$AD\ are\ two\ chords\ such\ that\ AC = AD.$
$To\ prove: (i)\ arc\ BC = arc\ DB$
$(ii)\ AB\ is\ the\ bisector\ of\ \angle CAD$
$(iii)\ If\ arc\ AC = 2arc\ BC,\ then\ find$
$(a) \angle BAC\quad (b) \angle ABC$
$Construction: Join\ BC\ and\ BD$
$Proof: In\ right\ angled\ \Delta ABC\ and\ \Delta ABD$
$Side\ AC = AD \qquad [given]$
$Hyp.\ AB = AB \qquad [common]$
$\therefore By\ Right\ Angle - Hypotenuse - Side\ criterion\ of\ congruence,$
$\Delta ABC \cong \Delta ABD$
$(i)\ The\ corresponding\ parts\ of\ the\ congruent\ triangles\ are\ congruent.$
$\therefore BC = BD \qquad [c.p.c.t]$
$\therefore Arc\ BC = ArcBD \qquad [equal\ chords\ have\ equal\ arcs]$
$(ii)\ \angle BAC = \angle BAD$
$\therefore AB\ is\ the\ bisector\ of\ \angle CAD$
$(iii)\ If\ Arc\ AC = 2\ arc\ BC,$
$then\ \angle ABC = 2\angle BAC$
$But\ \angle ABC + \angle BAC = 90°$
$\Rightarrow 2\angle BAC + \angle BAC = 90°$
$\Rightarrow 3\angle BAC = 90°$
$$\Rightarrow \angle BAC = \frac{90°}{3} = 30°$$
$\angle ABC = 2\angle BAC \Rightarrow \angle ABC = 2 \times 30° = 60°$

Question 18.
In cyclic quadrilateral ABCD; AD = BC, ∠ = 30° and ∠ = 70°; find;

(i) ∠ BCD
(ii) ∠BCA
(iii) ∠ABC
(iv) ∠ ADC

Solution:



$ABCD$ is a cyclic quadrilateral and $AD = BC$

$\angle BAC = 30°, \angle CBD = 70°$

We have

$\angle DAC = \angle CBD$ [angles in the same segment]

$\Rightarrow \angle DAC = 70°$ $[\because \angle CBD = 70°]$

$\Rightarrow \angle BAD = \angle BAC + \angle DAC = 30° + 70° = 100°$ .....(1)

Since the sum of opposite angles of cyclic quadrilateral is supplementary

$\angle BAD + \angle BCD = 180°$

$\Rightarrow 100° + \angle BCD = 180°$ [from (1)]

$\Rightarrow \angle BCD = 180° - 100° = 80°$

Since $AD = BC, \angle ACD = \angle BDC$ [Equal chords subtends equal angles]

But $\angle ACB = \angle ADB$ [angles in the same segment]

$\therefore \angle ACD + \angle ACB = \angle BDC + \angle ADB$

$\Rightarrow \angle BCD = \angle ADC = 80°$

But in ΔBCD,

$\angle CBD + \angle BCD + \angle BDC = 180°$ [angles oaf a triangle]

$\Rightarrow 70° + 80° + \angle BDC = 180°$

$\Rightarrow 150° + \angle BDC = 180°$

$\therefore \angle BDC = 180° - 150° = 30°$

$\Rightarrow \angle ACD = 30°$ $[\because \angle ACD = \angle BDC]$

$\therefore \angle BCA = \angle BCD - \angle ACD = 80° - 30° = 50°$

Since the sum of opposite angles of cyclic quadrilateral is supplementary,

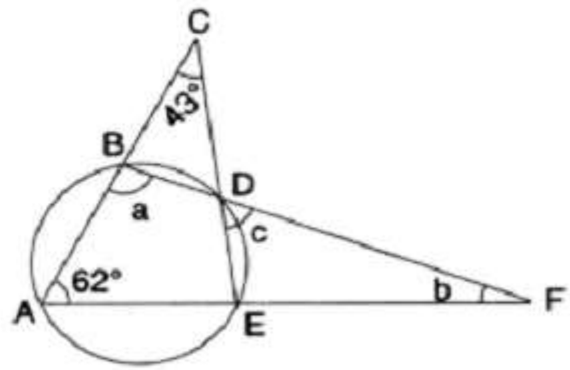
$\angle ADC + \angle ABC = 180°$

$\Rightarrow 80° + \angle ABC = 180°$

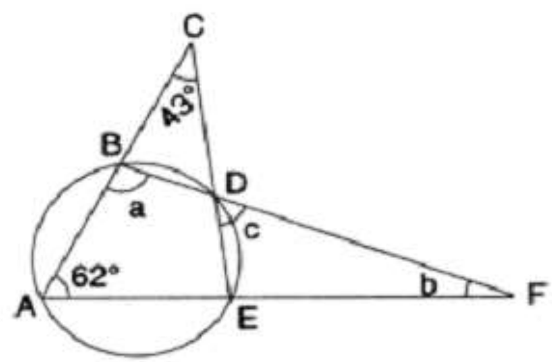
$\Rightarrow \angle ABC = 180° - 80° = 100°$

Question 19.

In the given figure, ∠ACE = 43° and ∠ = 62°; find the values of a, b and c.



Solution:



Now, $\angle ACE = 43°$ and $\angle CAF = 62°$ [given]

In ΔAEC,

$\therefore \angle ACE + \angle CAE + \angle AEC = 180°$

$\Rightarrow 43° + 62° + \angle AEC = 180°$

$\Rightarrow 105° + \angle AEC = 180°$

$\Rightarrow \angle AEC = 180° - 105° = 75°$

Now, $\angle ABD + \angle AED = 180°$

[Opposite angles of a cyclic quad and $\angle AED = \angle AEC$]

$\Rightarrow a + 75° = 180°$

$\Rightarrow a = 180° - 75°$

$\Rightarrow a = 105°$

$\angle EDF = \angle BAE$ [angles in the alternate segments]

$\therefore c = 62°$

In $\Delta BAF, a + 62° + b = 180°$

$\Rightarrow 105° + 62° + b = 180°$

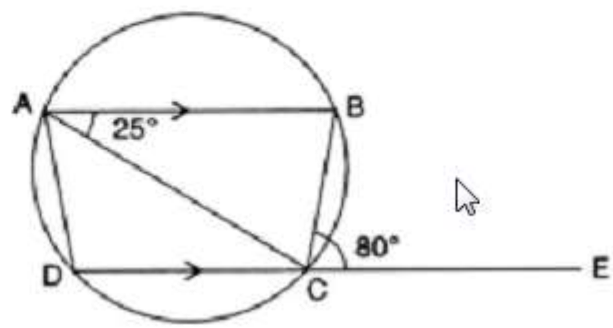
$\Rightarrow 167° + b = 180°$

$\Rightarrow b = 180° - 167° = 13°$

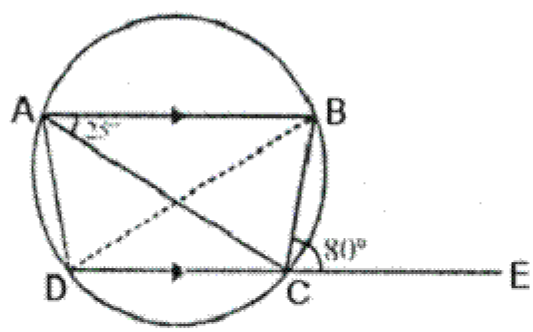
Hence, $a = 105°, b = 13°$ and $c = 62°$

Question 20.
In the given figure, AB is parallel to DC, ∠BCE = 80° and ∠ BAC = 25°

Find
(i) ∠ CAD
(ii) ∠ CBD
(iii) ∠ ADC



Solution:



In the given figure,
ABCD is a cyclic quad in which $AB \parallel DC$
$\therefore$ *ABCD is an isosceles trapezium*
$\therefore AD = BC$
(i)Join BD and we have,

$Ext.\angle BCE = \angle BAD$ $\left[\begin{array}{l}\text{Exterior angle of a cyclic qud is} \\ \text{equal to interior opposite angle}\end{array}\right]$

$\therefore \angle BAD = 80°$ $[\because \angle BCE = 80°]$
But $\angle BAC = 25°$
$\therefore \angle CAD = \angle BAD - \angle BAC$
$= 80° - 25°$
$= 55°$

$(ii) \angle CBD = \angle CAD$ [angle of the same segment]

$= 55°$

$(iii) \angle ADC = \angle BCD$ [angles of the isosceles trapezium]

$= 180° - \angle BCE$

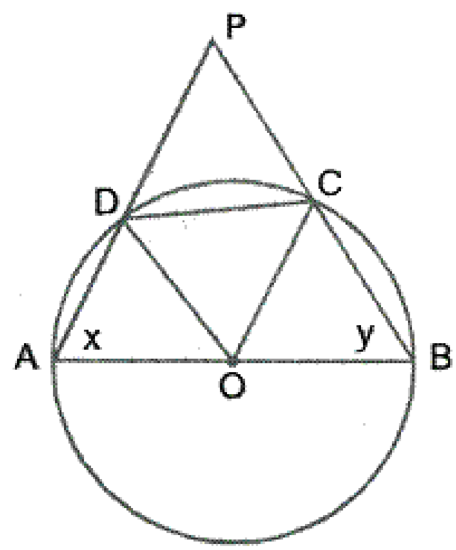
$= 180° - 80°$

$= 100°$

Question 21.

ABCD is a cyclic quadrilateral of a circle with centre o such that AB is a diameter of this circle and the length of the chord CD is equal to the radius of the circle if AD and BC produced meet at P, show that APB =60°

Solution:



In a circle, ABCD is a cyclic quadrilateral in which AB is the diametre and chord CD is equal to the radius of the circle

To prove $- \angle APB = 60°$

Construction − Join OC and OD

Proof − Since chord $CD = CO = DO$ [radii of the circle]

$\therefore \Delta DOC$ is an equilateral triangle

$\therefore \angle DOC = \angle ODC = \angle DCO = 60^\circ$

Let $\angle A = x$ and $\angle B = y$

Since $OA = OB = OC = OD$ [radii of the same circle]

$\therefore \angle ODA = \angle OAD = x$ and

$\angle OCB = \angle OBC = y$

$\therefore \angle AOD = 180^\circ - 2x$ and $\angle BOC = 180^\circ - 2y$

But AOB is a straight line

$\therefore \angle AOD + \angle BOC + \angle COD = 180^\circ$

$\Rightarrow 180^\circ - 2x + 180^\circ - 2y + 60^\circ = 180^\circ$

$\Rightarrow 2x + 2y = 240^\circ$

$\Rightarrow x + y = 120^\circ$

But $\angle A + \angle B + \angle P = 180^\circ$ [Angles of a triangle]

$\Rightarrow 120^\circ + \angle P = 180^\circ$

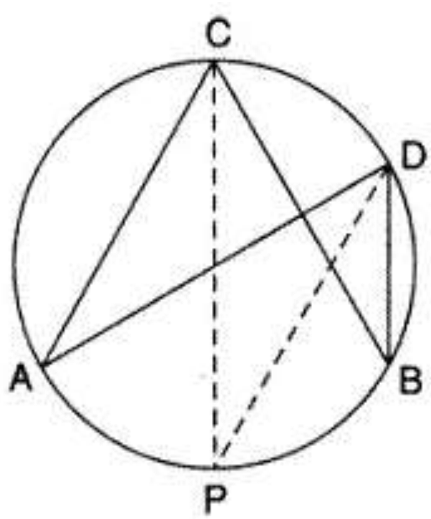
$\Rightarrow \angle P = 180^\circ - 120^\circ$

$\Rightarrow \angle P = 60^\circ$

Hence $\angle APB = 60^\circ$

Question 22.

In the figure, given alongside, CP bisects angle ACB. Show that DP bisects angle ADB.

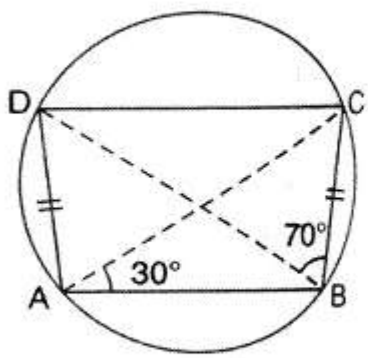


Solution:

Given − *In the figure, CP is the bisector of* $\angle ABC$
To prove − *DP is the bisector of* $\angle ADB$
Proof − *Since CP is the bisector of* $\angle ACB$
$\therefore \angle ACP = \angle BCP$
But $\angle ACP = \angle ADP$ [*Angles in the same segment of the circle*]
and $\angle BCP = \angle BDP$
But $\angle ACP = \angle BCP$
$\therefore \angle ADP = \angle BDP$
$\therefore$ *DP is the bisector of* $\angle ADB$

Question 23.
In the figure, given below, AD = BC, ∠ BAC = 30° and ∠ = 70° find:

(i) ∠ BCD
(ii) ∠ BCA
(iii) ∠ ABC
(iv) ∠ADC



Solution:

In the figure, ABCD is a cyclic quadrilateral
AC and BD are its diagonals.
$\angle BAC = 30°$ and $\angle CBD = 70°$
Now we have to find the measures of
$\angle BCD, \angle BCA, \angle ABC$ and $\angle ADB$
We have $\angle CAD = \angle CBD = 70°$ [Angles in the same segment]
Similarly, $\angle BAC = \angle BDC = 30°$
$\therefore \angle BAD = \angle BAC + \angle CAD$
$= 30° + 70°$
$= 100°$
(i) Now $\angle BCD + \angle BAD = 180°$ [opposite angles of cyclic quadrilateral]
$\Rightarrow \angle BCD + \angle BAD = 180°$
$\Rightarrow \angle BCD + 100° = 180°$
$\Rightarrow \angle BCD = 180° - 100°$
$\Rightarrow \angle BCD = 80°$
(ii) Since $AD = BC$, ABCD is an isosceles trapezium and $AB \parallel DC$
$\angle BAC = \angle DCA$ [alternata angles]
$\Rightarrow \angle DCA = 30°$
$\therefore \angle ABD = \angle DAC = 30°$ [angles in the same segment]
$\therefore \angle BCA = \angle BCD - \angle DAC$
$= 80° - 30°$
$= 50°$
(iii) $\angle ABC = \angle ABD + \angle CBD$
$= 30° + 70°$
$= 100°$
(iv) $\angle ADB = \angle BCA = 50°$ [angles in the same segment]

Question 24.
In the figure given below, AD is a diameter. O is the centre of the circle. AD is parallel to BC and ∠CBD = 32°. Find :

(i) ∠OBD
(ii) ∠AOB
(iii) ∠BED (2016)

Solution:

i. AD is parallel to BC, i.e., OD is parallel to BC and BD is transversal.
$\Rightarrow \angle ODB = \angle CBD = 32°$ ....(Alternate angles)
In $\triangle OBD$,
$OD = OB$ ....(Radii of the same circle)
$\Rightarrow \angle ODB = \angle OBD = 32°$

ii. AD is parallel to BC, i.e., AO is parallel to BC and OB is transversal.

$\Rightarrow \angle AOB = \angle OBC$ ....(Alternate angles)

$\Rightarrow \angle OBC = \angle OBD + \angle DBC$

$\Rightarrow \angle OBC = 32^\circ + 32^\circ$

$\Rightarrow \angle OBC = 64^\circ$

$\Rightarrow \angle AOB = 64^\circ$

iii. In ΔOAB,

$OA = OB$ ....(Radii of the same circle)

$\Rightarrow \angle OAB = \angle OBA = x$ (say)

$\Rightarrow \angle OAB + \angle OBA + \angle AOB = 180^\circ$

$\Rightarrow x + x + 64^\circ = 180^\circ$

$\Rightarrow 2x = 180^\circ - 64^\circ$

$\Rightarrow 2x = 116^\circ$

$\Rightarrow x = 58^\circ$

$\Rightarrow \angle OAB = 58^\circ$

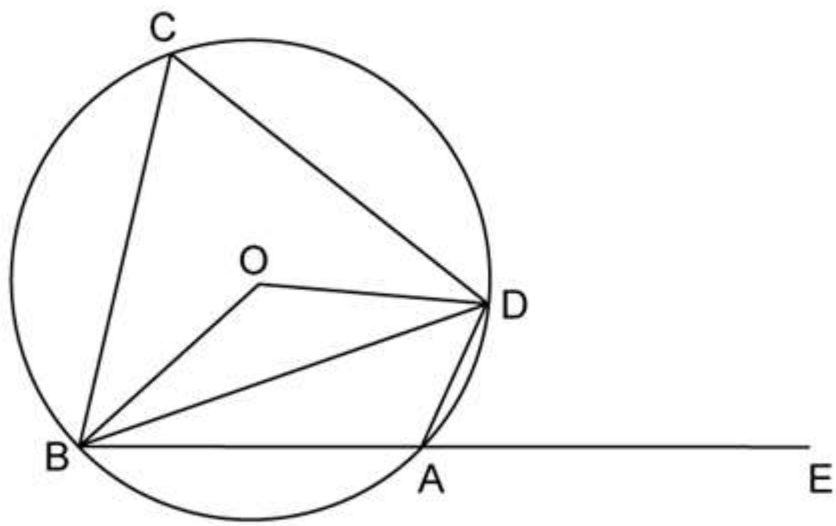
i.e., $\angle DAB = 58^\circ$

$\Rightarrow \angle DAB = \angle BED = 58^\circ$ ....(Angles inscribed in the same arc are equal)

Question 25.

In the figure given, O is the centre of the circle. ∠DAE = 70°. Find giving suitable reasons, the measure of

i. ∠BCD
ii. ∠BOD
iii. ∠OBD



Solution:

∠DAE and ∠DAB are linear pair
So,
∠DAE + ∠DAB = 180°
∴∠DAB = 110°

Also,
∠BCD + ∠DAB = 180°……Opp. Angles of cyclic quadrilateral BADC
∴∠BCD = 70°
∠BCD = $\frac{1}{2}$ ∠BOD…angles subtended by an arc on the centre and on the circle
∴∠BOD = 140°

In ΔBOD,
OB = OD……radii of same circle
So,
∠OBD =∠ODB……isosceles triangle theorem
∠OBD + ∠ODB + ∠BOD = 180°……sum of angles of triangle
2∠OBD = 40°
∠OBD = 20°