Chapter 7. Reflection

Formulae

1. Rule to find the reflection of a point in the x-axis:
   (i) Retain the abscissa i.e. x-coordinate.
   (ii) Change the sign of ordinate i.e. y-coordinate.
2. Rule to find the reflection of a point in the y-axis:
   (i) Change the sign of abscissa i.e., x-coordinate.
   (ii) Retain the ordinate i.e., y-coordinate.
3. Reflection of a point in a line parallel to x-axis. The reflection of the point P(x, y) in the line y = a is the point P(x, -y+2a).
4. Reflection of a point in a line parallel to y-axis. The reflection of the point P(x, y) in the line x = a is the point P' (-x+2a, y).
5. Reflection of a point in the origin:
   (i) Change the sign of abscissa i.e., x-coordinate.
   (ii) Change the sign of ordinate i.e., y-coordinate.
6. A point is called an Invariant point with respect to a given line if and only if lies on the line.

Determine the Following

Question 1. The triangle A(1, 2), B(4, 4) and C(3, 7) is first reflected in the line y = 0 onto triangle A'B'C' and then triangle A'B'C' is reflected in the origin onto triangle A"B"C". Write down the co-ordinates of:

(i) $A', B', C',$ (ii) $A'', B'', C''.$

Solution. (i) $A' \rightarrow (1, -2),$

$B' \rightarrow (4, -4),$

$C' \rightarrow (3, -7).$

(ii) $A'' \rightarrow (-1, 2),$

$B'' \rightarrow (-4, 4),$

$C'' \rightarrow (-3, 7).$

Question 2. The point P (a, b) is first reflected in the origin and then reflected on the Y-axis to p'. If P' has co-ordinates (3, – 4), evaluate a, b.

Solution. $P(a, b)$ after reflection at the origin = $(-a, -b)$ $(-a, -b)$ after reflection on the Y-axis = $p'(a-b)$

According to question,

$\Rightarrow \quad (a, -b) = (3, -4)$

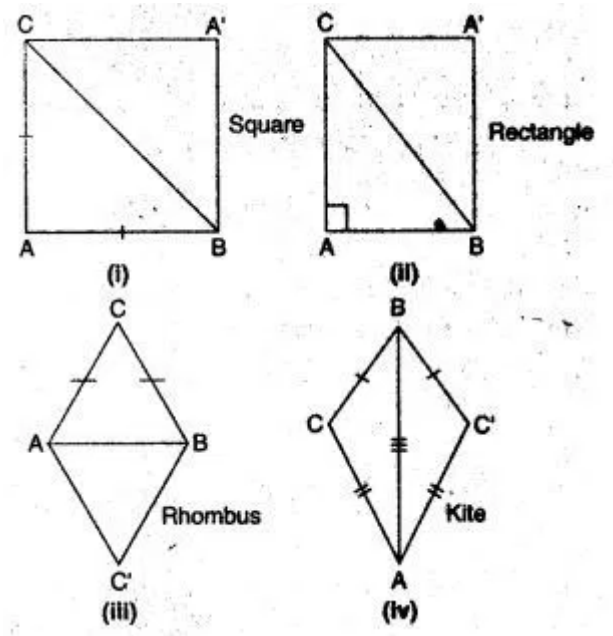
$\Rightarrow \quad a = 3, -b = -4$

$\Rightarrow \quad a = 3, b = 4.$ Ans.

Figure Based Questions

Question 1. Name the figure formed by a triangle and its reflection, when:
(i) An isosceles right-angled triangle is reflected in its hypotenuse.
(ii) A right-angled triangle is reflected in its hypotenuse.
(iii) An isosceles triangle is reflected in its unequal side.
(iv) A scalene triangle is reflected in its greatest side.

Solution:



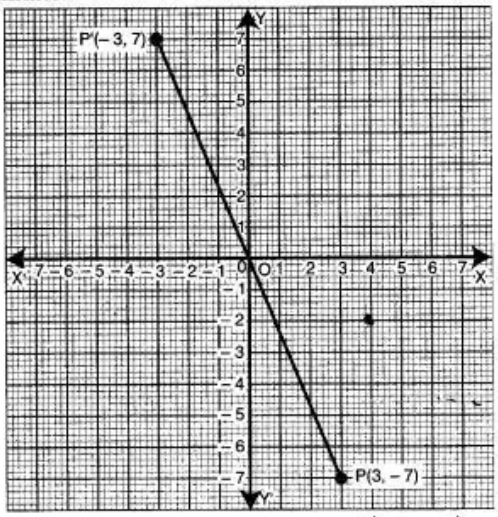
Graphical Depiction

Question 1. Find the co-ordinates of the images of the following under reflection in the origin:

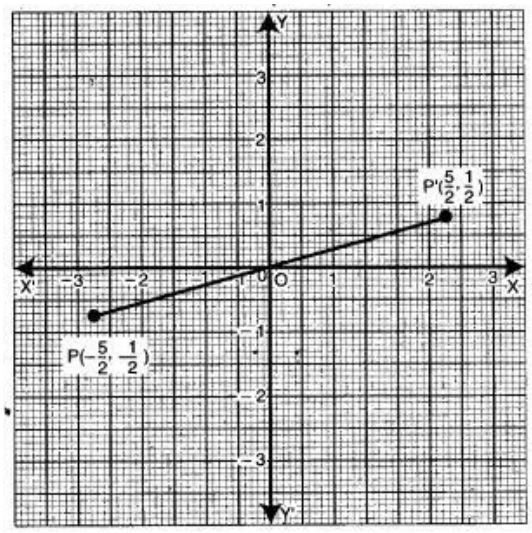
(i) $(3, -7)$ (ii) $\left(\frac{-5}{2}, \frac{-1}{2}\right)$ (iii) $(0, 0)$.

Solution : (i) The reflection (image) of the point $P(3, -7)$ at the origin is the point $P'(-3, 7)$.

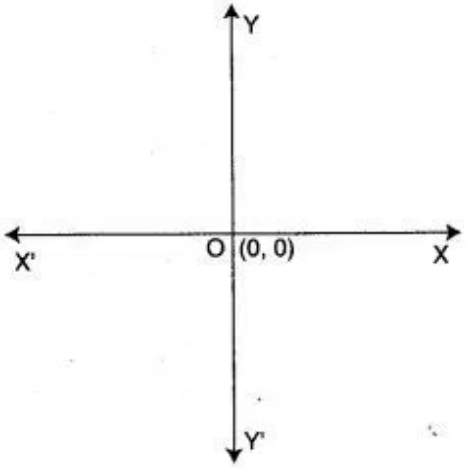
Note : To find the reflection of a point in the origin change : (a) The sign of abscissa; *i.e.*, X co-ordinate. (b) The sign of ordinate; *i.e.*, Y co-ordinate.



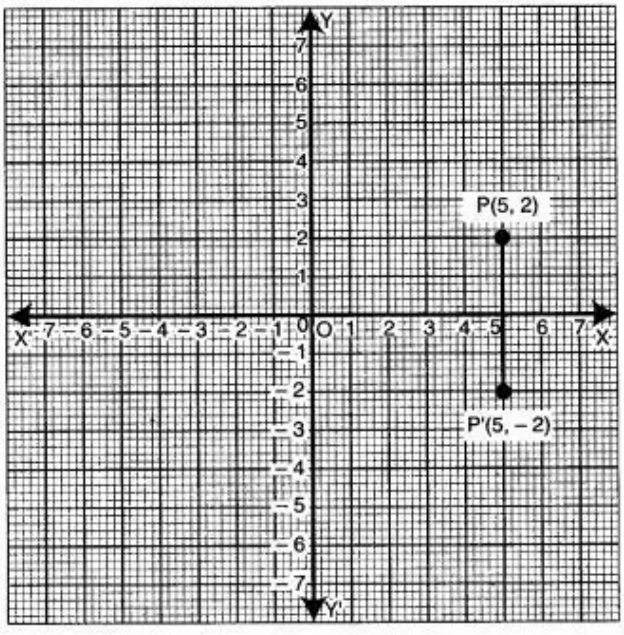
(ii) The reflection of the point $P\left(\frac{-5}{2}, \frac{-1}{2}\right)$ at the origin is the point $P'\left(\frac{5}{2}, \frac{1}{2}\right)$.



(iii) The reflection (image) of the point (0, 0) at the origin is the point (0, 0) itself.



Question 2. The image of a point P under reflection on the X-axis is (5, – 2). Write down the co-ordinates of P.

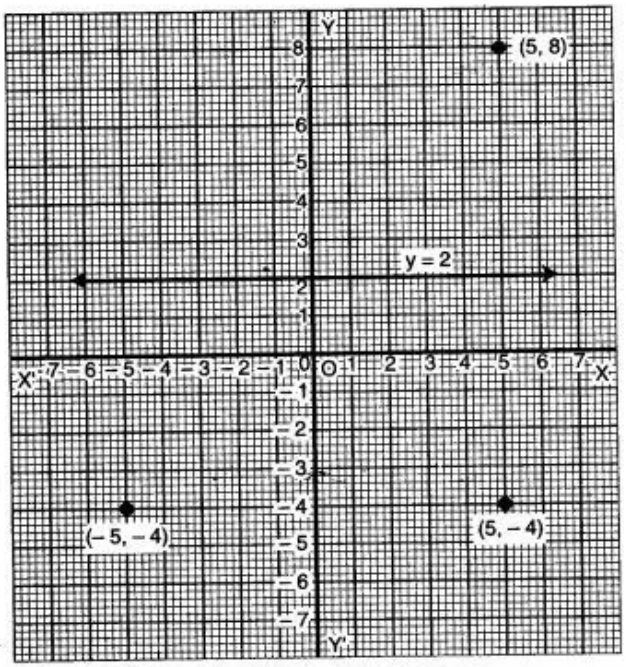


Solution : The image of a point P under reflection on the X-axis is P′(5, – 2).

So, co-ordinates of P = (5, 2).

Question 3. Write down the co-ordinates of the image of (5, – 4).
(i) Reflection in x = 0;
(ii) Reflection in y = 2.

Solution :



(i) Reflection in $x = 0$ is $(-5, -4)$. Ans.

(ii) Reflection in $y = 2$ is $(5, 8)$. Ans.

Question 4. Use a graph paper for this question.
(i) The point P (2, – 4) is reflected about the line x = 0 to get the image Q. Find the coordinates of Q.
(ii) Point Q is reflected about the line y = 0 to get the image R. Find the co-ordinates of R.
(iii) Name the figure PQR.
(iv) Find the area of figure PQR.

Solution :

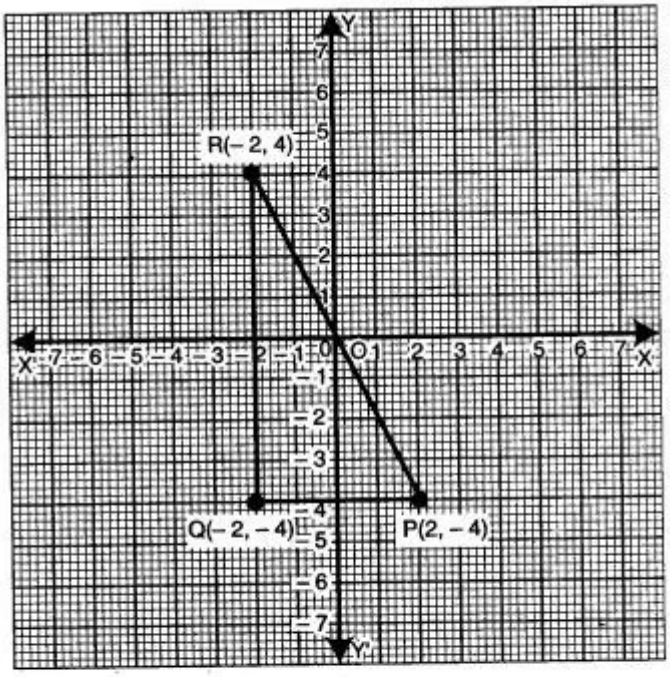
(i) P $(2, -4)$ is reflected in $(x = 0)$ y-axis and Q-image.

$$P(2, -4) \xrightarrow{My} Q(-2, -4) \quad \text{Ans.}$$

(ii) Q $(-2, -4)$ is reflected in $(y = 0)$ x-axis to get R.

$$Q(-2, -4) \xrightarrow{Mx} R(-2, 4) \quad \text{Ans.}$$

(iii) The figure PQR is right angle triangle as shown ahead :



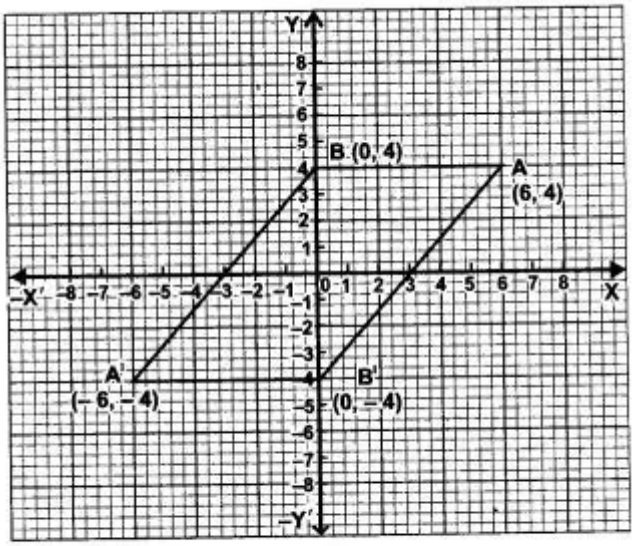
Ans.

(iv) Area of $\Delta PQR = \frac{1}{2} \times PQ \times QR$

$= \frac{1}{2} \times 4 \times 8 = 16$ sq. unit. Ans.

Question 5. Using a graph paper, plot the points A (6,4) and B (0,4).
(i) Reflect A and B in the origin to get the images A’ and B’.
(ii) Write the co-ordinates of A’ and B’.
(iii) State the geometrical name for. the figure ABA’B’.
(iv) Find its perimeter.

Solution : (i)



(ii) A' (– 6, – 4) and B' (0, – 4) Ans.

(iii) ABA'B' is a parallelogram. Ans.

(iv) From the figure AB = 6, BB' = 8, A'B' = 6

In $\Delta ABB'$, $(AB')^2 = AB)^2 + (BB')^2$

$6^2 + 8^2 = 100$

$\therefore \quad AB' = 10 = A'B$

{ABA'B' is a parallelogram}

$\therefore$ Perimeter of ABA'B'

$= AB + BA' + A'B' + B'A$

$= 6 + 10 + 6 + 10 = 32$ **units**

Question 6. (i) Find the reflection of the point (3, 5) on X-axis.
(ii) Find the reflection of the point (- 3, 5) on X-axis.
(iii) Find the reflection of the point (- 3, – 5) on X-axis.

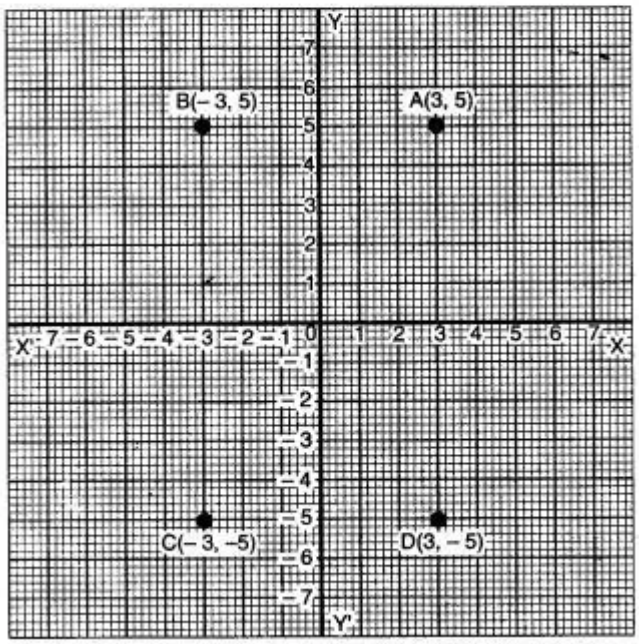
(iv) Find the reflection of the point (3, – 5) on X-axis.

Solution : (i) $A(3, 5) \xrightarrow{Mx} D(3, -5)$

(ii) $B(-3, 5) \xrightarrow{Mx} C(-3, -5)$

(iii) $C(-3, -5) \xrightarrow{Mx} B(-3, 5)$

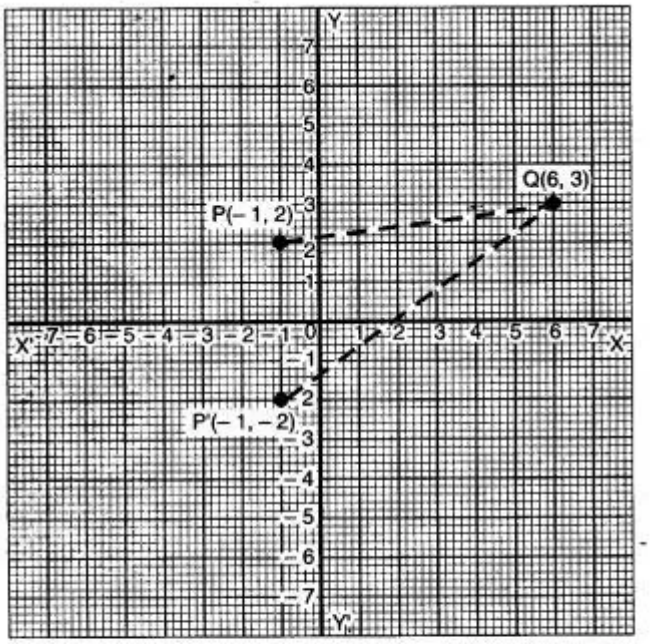
(iv) $D(3, -5) \xrightarrow{Mx} A(3, 5)$



Question 7. P, Q have co-ordinates (-1, 2) and (6, 3) respectively. Reflect P on the X-axis to P'. Find:
(i) The co-ordinate of P'
(ii) Length of P'Q.
(iii) Length of PQ.

(iv) Is P'Q = PQ ?

Solution :



(i) $P' \rightarrow (-1-2)$. Ans.

(ii) $P'Q = \sqrt{(6+1)^2+(3+2)^2}$

$= \sqrt{49+25} = \sqrt{74}$. Ans.

(iii) $PQ = \sqrt{(6+1)^2+(3-2)^2}$

$= \sqrt{49+1} = \sqrt{50}$. Ans.

(iv) No. ($P'Q \neq PQ$) Ans.

Question 8. A point P(4, – 1) is reflected to P' in the line y = 2 followed by the reflection to P" in the line x = -1. Find :

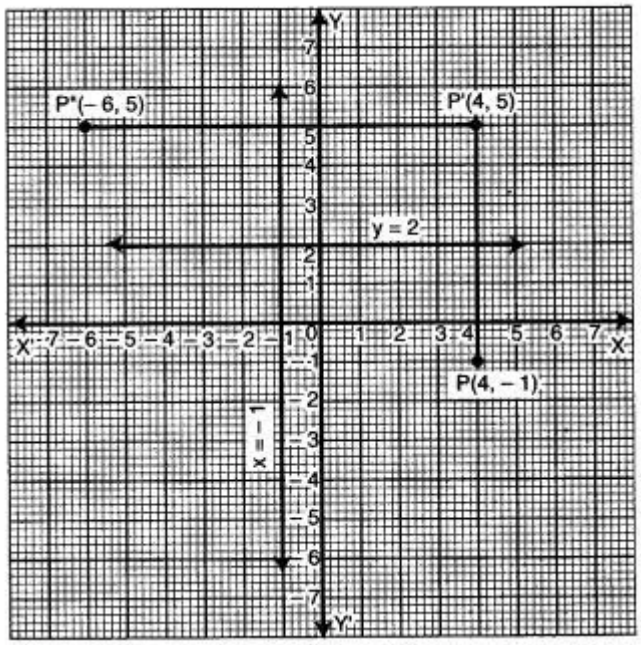
(i) The co-ordinates of P'.

(h) The co-ordinates of P".

(iii) The length of PP'.

(iv) The length of P'P".

Solution :



(i) $P' \rightarrow (4, 5)$. Ans.

(ii) $P'' \rightarrow (-6, 5)$. Ans.

(iii) $PP' = 6$ units Ans.

(iv) $P'P'' = 10$ units Ans.

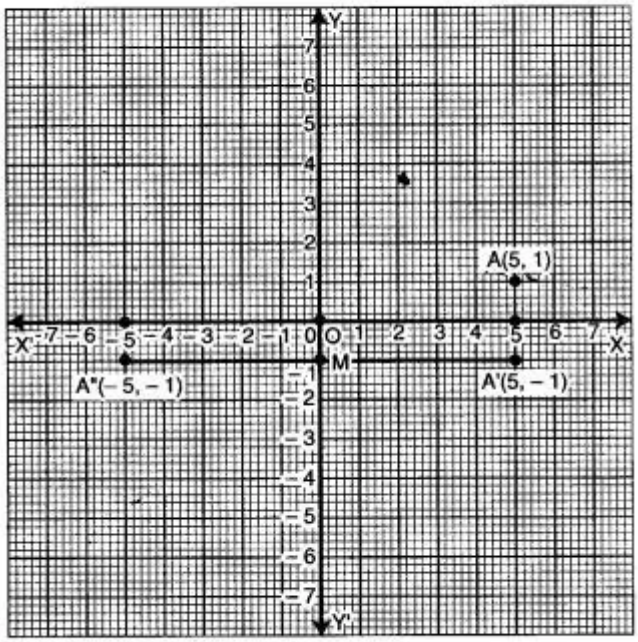
Question 9. Point A (5, 1) on reflection on X- axis is mapped as A'. Also A on reflection on Y-axis is mapped as A".
(i) Write the co-ordinates of A'.
(ii) Write the co-ordinates of A".
(iii) Calculate the distance A' A".

(iv) On which coordinate axis does the middle point M of A"A' lie ?

Solution : See graph,

(i) $A' \rightarrow (5, -1)$. Ans.

(ii) $A'' \rightarrow (-5, -1)$. Ans.



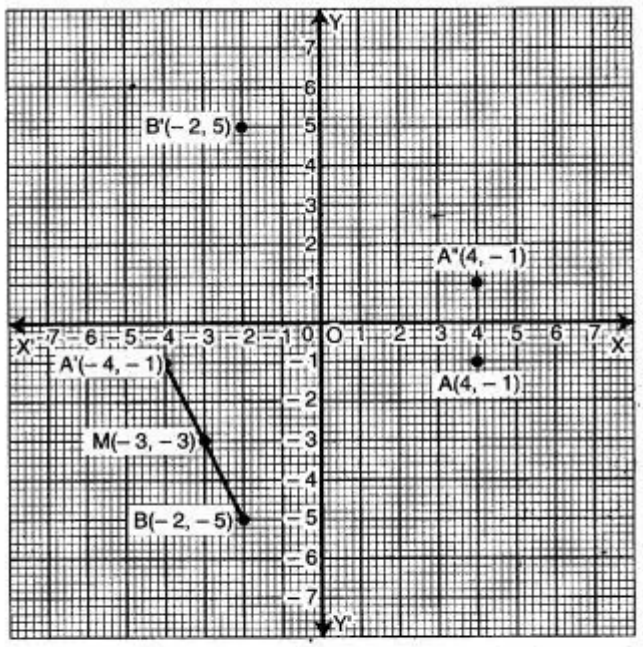
(iii) Distance $A'A'' = 5 + 5 = 10$. Ans.

(iv) M lies on Y-axis. Ans.

Question 10. Point A(4, – 1) is reflected as A' on Y-axis. Point B on refletion on X-axis is mapped as B' (- 2, 5).
(i) Write the co-ordinates of A'.
(ii) Write the co-ordinates of B.
(iii) Write the co-ordinates of the middle point
M of the segment A'B.

(iv) Write the co-ordinates of the point of reflection A" of A on X-axis.

Solution :



(i) $A' \rightarrow (-4, -1)$. Ans.

(ii) $B \rightarrow (-2, -5)$. Ans.

(iii) $M \rightarrow (-3, -3)$. Ans.

(iv) $A'' \rightarrow (4, 1)$. Ans.

Question 11. Point A (2, -4) is reflected in origin as A'. Point B (- 3, 2) is reflected on X-axis as B'.

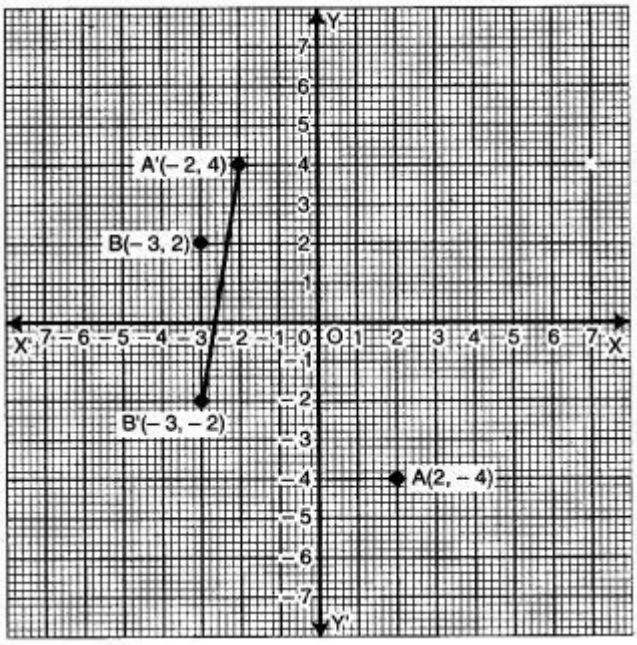
(i) Write the co-ordinates of A'.

(ii) Write the co-ordinates of B'.

(iii) Calculate the distance A'B'.

Give your answer correct to 1 decimal place, (do not consult tables).

Solution :



(i) $A' \rightarrow (-2, 4)$. Ans.

(ii) $B' \rightarrow (-3, -2)$. Ans.

(iii) Distance $A'B' = \sqrt{(-3+2)^2 + (-2-4)^2}$

$= \sqrt{1+36}$

$= \sqrt{37}$

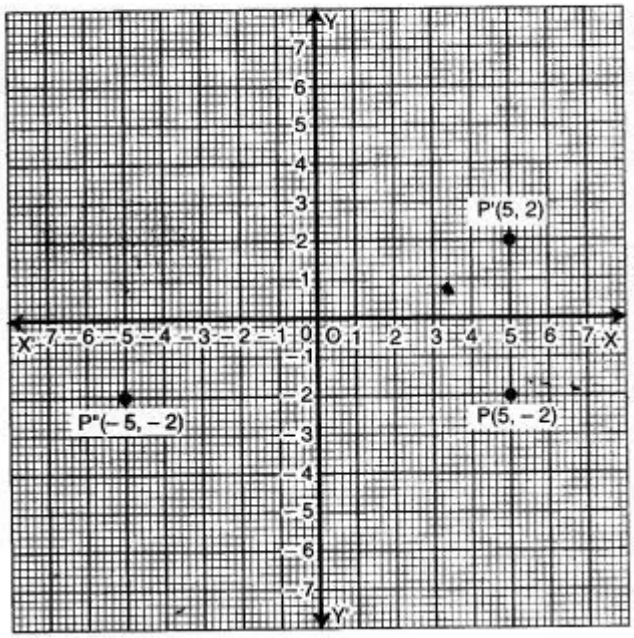
$= 6{\cdot}1$ Ans.

Question 12. (i) Point P(a, b) reflected on the X-axis to P'(5, 2). Write down the value of a and b.
(ii) P" is the image of P when reflected on the Y-axis. Write down the co-ordinates of P".
(iii) Name a single transformation that maps P' to P".

Solution : (i) The value of $a = 5$ and $b = -2$. Ans.

(ii) Co-ordinates of $P'' = (-5, -2)$. Ans.

(iii) $(x, y) \rightarrow (-x, -y)$. Ans.



Question 13. Points (3, 0) and (-1, 0) are invarient points under reflection in the line L_1; point (0, -3) and (0, 1) are invarient points on reflection in line L_2.

(i) Write the equation of the line L_1 and L_2.

(ii) Write down the images of points P(3, 4) and Q(– 5, – 2) on reflection in L_1. Name the images as P' and Q' respectively.

(iii) Write down the images of P and Q on reflecion in L_2. Name the images as P'' and Q'' respectively.

Solution : (i) (3, 0) and (– 1, 0) lies on X-axis, so these are invarient under reflection on the X-axis. Hence, L_1 lies on X-axis. So, equation of line L_1, is $y = 0$.

(0, – 3) and (0, 1) lies on Y-axis, so these are invarient under reflection on the Y-axis. So, equation of line L_2 is $y = 0$. Ans.

(ii) $P' \Rightarrow (3, -4)$

$Q' \Rightarrow (-5, 2)$ Ans.

(iii) $P'' \Rightarrow (-3, 4)$

$Q'' \Rightarrow (5, -2)$ Ans.

Question 14. A point P(a, b) is reflected in the X-axis to P'(2, – 3). Write down the value of a & b. P" is the image of P, when reflected on the Y-axis. Write down the co-ordinates of P" when

P is reflected in the line parallel to the Y-axis, such that x = 4.

Solution : Reflection of P (a, b) on the X-axis

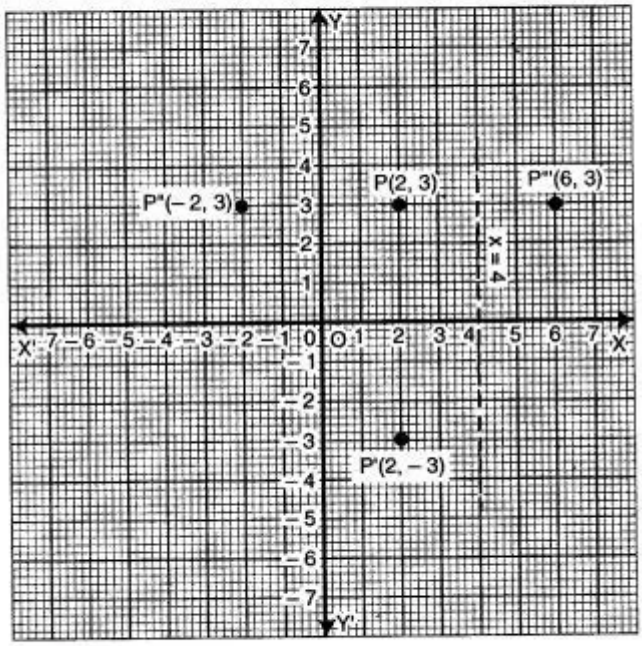
$P'(a, -b)$

(i) $\therefore \quad P'(a, -b) = P'(2, -3)$

$+a = +2, \therefore a = 2$

$-b = -3, \therefore b = 3$

P (2, 3) Ans.



(ii) P′′ is the image of $P(2, 3)$ under reflection

Y-axis = P′′(– 2, 3). Ans.

(iii) P′′′ is the image of P (2, 3) under reflection

$x = 4 \Rightarrow$ P′′′(6, 3). Ans.

Question 15. Use a graph paper to answer the following questions. (Take 1 cm = 1 unit on both axis):

(i) Plot A (4, 4), B (4, – 6) and C (8, 0), the vertices of a triangle ABC.

(ii) Reflect ABC on the y-axis and name it as A'B'C'.

(iii) Write the coordinates of the images A', B' and C'.

(iv) Give a geometrical name for the figure AA' C'B' BC.

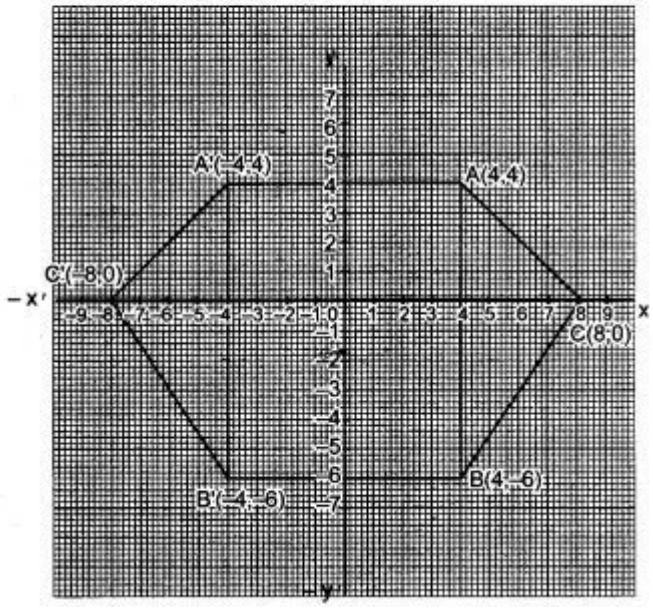
(v) Identify the line of symmetry of AA′ C′ B′ BC.

Solution : (i) and (ii) see the given graph.

(iii) A′ (– 4, 4), B′ (– 4, – 6), C′ (– 8, 0) Ans.

(iv) AA′ C′ B′ BC is a Hexagon. Ans.

(v) y-axis is the line of symmetry. Ans.



Question 16. Use a graph paper for this question. (Take 10 small divisions = 1 unit on both axis). P and Q have co-ordinates (0, 5) (- 2, 4).

(i) P is invariant when reflected in an axis. Name the axis.

(ii) Find the image of Q on reflection in the axis found in (i).

(iii) (0, k) on reflection in the origin is invariant. Write the value of k.

(iv) Write the co-ordinates of the image of Q, obtained by reflecting it in the origin following by reflection in x-axis.

Solution : (i) The axis is y-axis or $x = 0$.

(ii) Image of 'Q'

$$Q' = M_{x=0}(-2, 4)$$
$$= (2, 4)$$

(iii) $\because \quad M_0(a, b) = (-a, -b)$

$\therefore \quad M_0(0, k) = (0, -k)$

$\because \quad k = -k$

$\therefore \quad 2k = 0$

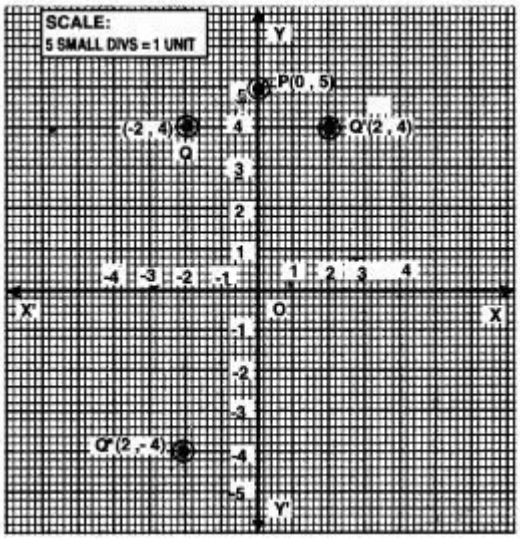
$\therefore \quad k = 0$

(iv) $Q'' = M_x M_0 Q$

$= M_x M_0 (-2, 4)$

$= Mx (2, -4)$

$= (-2, -4)$



Question 17. The image of triangle OXY under reflection in the origin, O is the triangle OX_1Y_1, where $X_1(-3, -4)$ is the image of X and Y_1, $(0, -5)$ is the image of Y.

(i) Draw a diagram to represent this informa-tion and write down the co-ordinates of X and Y.

(ii) What kind of figure is the quadrilateral XYX_1Y_1 ? Give reason for your answer. State, with a reason, whether the figure XYX_1Y_1 has any lines of symmetry.

(iii) Find the co-ordinates of X_2, the image of X under reflection in the origin followed by reflection on the Y-axis.

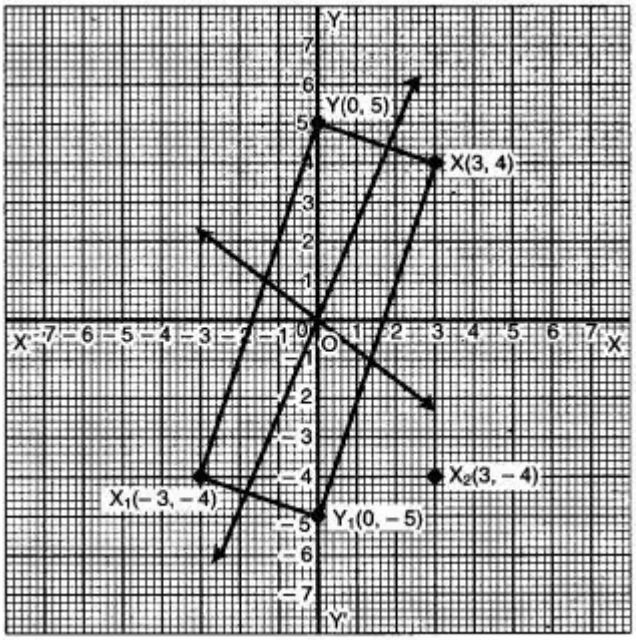
(iv) Find the co-ordinaes of Y_2, the image of Y under reflection on the X-axis followed by reflection in the origin.

Solution : (i) $X_1 \rightarrow (-3, -4)$

$X \rightarrow (3, 4)$

$Y_1 \rightarrow (0, -5)$

$Y \rightarrow (0, 5)$. Ans.



(ii) Rectangle.

Reason : $X_1Y_1 = YX$

and $X_1Y = Y_1X$

Also each angle of the quadrilateral XYX_1Y_1 is 90°.

It has two lines of symmetry. These are the perpendicular bisectors of each pair of opposite sides. Ans.

(iii) $X_2 = (3, -4)$. Ans.

(iv) $Y_2 = (0, 5)$. Ans.

Question 18. The point P(3, 4) is reflected to P' in the x-axis and O' is the image of O (the Origin) in the line PP' Find :

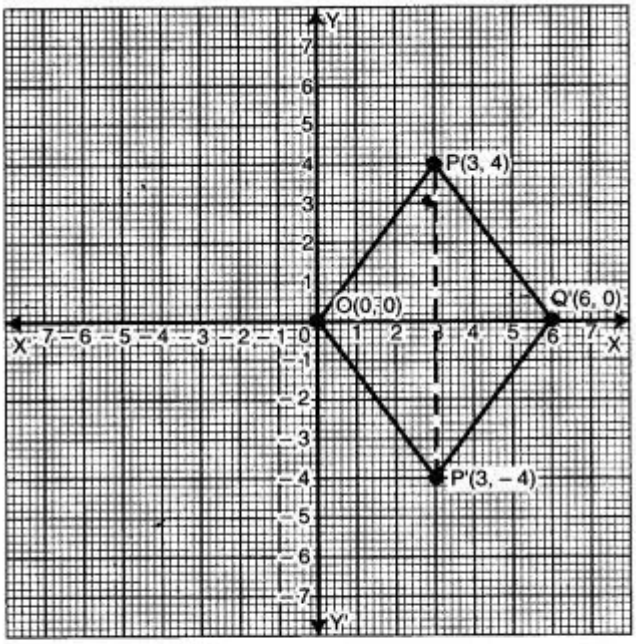
(i) The coordinates of P' and O'.

(ii) The length of segment PP' and OO'.

(iii) The perimeter of the quadrilateral POP'O'

(iv) What is the special name of the quadrilateral POP'O'.

Solution : (i) P' (3, – 4), O'(6, 0) Ans.



(ii) $PP' = \sqrt{(3-3)^2 + (-4-4)^2} = \sqrt{64} = 8$ units

$OO' = \sqrt{(0-6)^2 + (0-0)^2}$

$= \sqrt{36} = 6$ units. Ans.

(iii) Perimeter $= 5 + 5 + 5 + 5 = 20$ units. Ans.

(iv) Rhombus. Ans.

Question 19. Use graph paper for this question.
The points A (2, 3), B (4, 5) and C (7, 2) are the ,vertices of A ABC.
(i) Write down the coordinates of A', B', C' if Δ A'B'C' is the image of Δ ABC, when reflected in the origin.
(ii) Write down the co-ordinates of A", B", C" if A"B"C" is the image of Δ ABC, when reflected in the x-axis.
(iii) Mention the special name of the quadrilateral BCC"B" and find its area.

The points A (2, 3), B (4, 5) and C (7, 2) are the vertices of Δ ABC.

(i) Write down the coordinates of A', B', C' if ΔA'B'C' is the image of Δ ABC, when reflected in the origin.

(ii) Write down the co-ordinates of A", B", C" if A"B"C" is the image of Δ ABC, when reflected in the x-axis.

(iii) Mention the special name of the quadrilateral BCC"B" and find its area.

Solution : The point A (2, 3), B (4, 5) and C (7, 2).

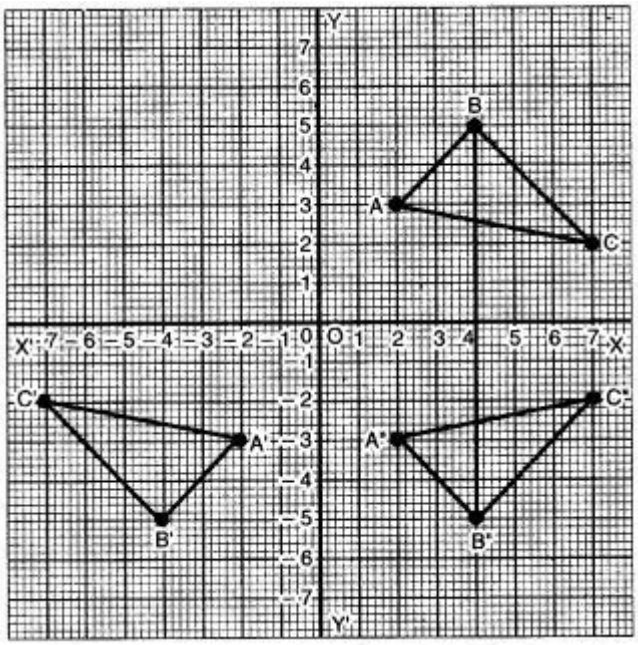
(i) Reflection in origin

$$(x, y) \xrightarrow{M_0} = (-x, -y)$$

$$\therefore \quad A(2, 3) \xrightarrow{M_0} = A'(-2, -3)$$

$$B(4, 5) \xrightarrow{M_0} = B'(-4, -5)$$

$$C(7, 2) \xrightarrow{M_0} = C'(-7, -2)$$



(ii) Now A, B, C is reflected in X axis. Ans.

Reflection in X axis

$$(x, y) \xrightarrow{M_x} = (x, -y)$$

$$\therefore \quad A(2, 3) \xrightarrow{M_x} = A''(2, -3)$$

$$B(4, 5) \xrightarrow{M_x} = B''(4, -5)$$

$$C(7, 2) \xrightarrow{M_x} = C''(7, -2) \quad \text{Ans.}$$

(iii) BCC"B" is an isosceles trapezium.

$$CD = 7 - 4 = 3$$

$$CC'' = 2 + 2 = 4 \text{ and}$$

$$BB'' = 5 + 5 = 10$$

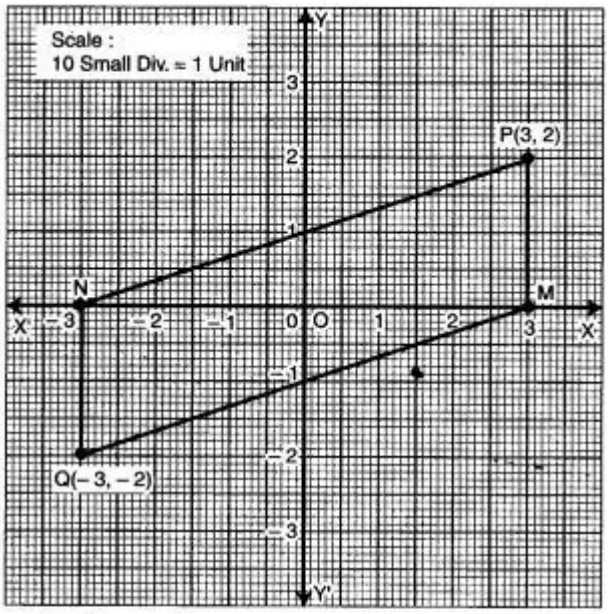
$$\text{Area of Trapezium} = \frac{1}{2}(CC'' + BB'') \times CD$$

$$= \frac{1}{2}(4 + 10) \times 3$$

$$= \frac{1}{2} \times 14 \times 3 = 21 \text{ sq. unit}$$

Question 20. Use a graph paper for this question (take 10 small divisions = 1 unit on both axis).
Plot the points P (3, 2) and Q (-3, -2), from P and Q draw perpendicular PM and QN on the X-axis.
(i) Name the image of P on reflection at the origin.
(ii) Assign, the . special name to the geometrical figure. PMQN and find its area.
(iii) Write the co-ordinates of the point to which M is mapped on reflection in (i) X- axis, (ii) Y-axis, (iii) origin.

Solution :



In the graph paper

(i) Q (−3, −2) Ans.

(ii) Parallelgoram;

$$\text{Area of } \Delta PMN = \frac{1}{2} PM \times MN$$

$$= \frac{1}{2} \times 2 \times 63$$

$$\therefore \text{ Area of PMQN} = 2 \times \Delta PMN$$

$$= 2 \times 6$$

$$= 12 \text{ square unit} \quad \text{Ans.}$$

(iii) Co-ordinates of M (3, 0)

(i) (3, 0), (ii) (−3, 0), (iii) (−3, 0) Ans.

Question 21. Using graph paper and taking 1 cm = 1 unit along both x-axis and y-axis.
(i) Plot the points A (- 4, 4) and B (2, 2).
(ii) Reflect A and B in the origin to get the images A' and B' respectively.
(iii) Write down the co-ordinates of A' and B'.
(iv) Give the geometrical name for the figure ABA'B'.

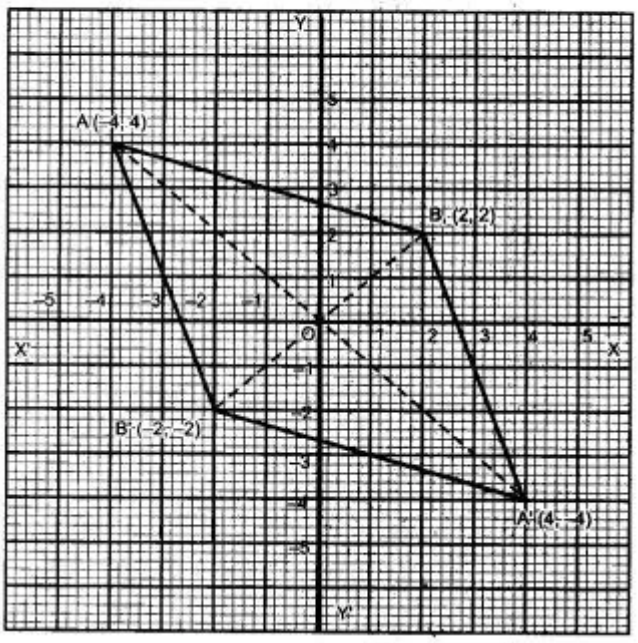
(v) Draw and name its lines of symmetry.

Solution : (i), (ii) (In the graph paper)

(iii) A′ (4, – 4) B′ (– 2, – 2)

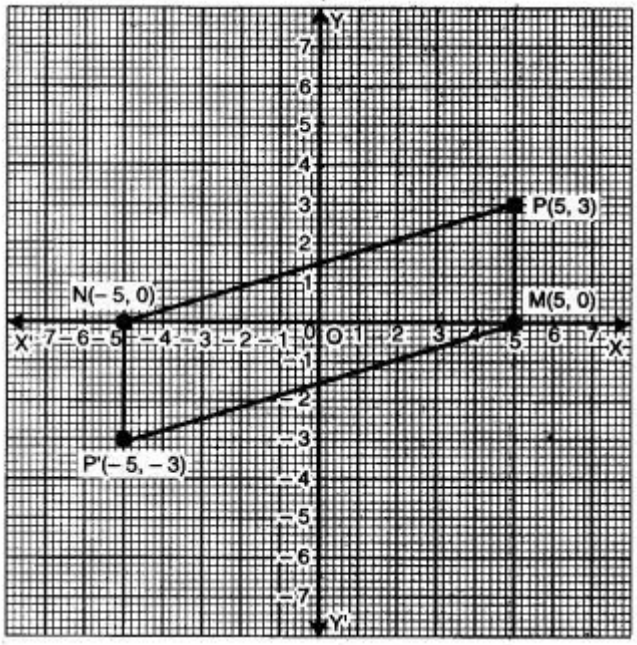
(iv) Rhombus

(v) AA′, and BB′



Question 22. Use graph paper for this question.
The point P (5, 3) was reflected in the origin to get the image P’.
(i) Write down the co-ordinates of P’.
(ii) If M is the foot of the perpendicular from of P to the X-axis, find the co-ordinates of M.
(iii) If N is the foot of the perpendicular from of P’ to the X-axis, find the co-ordinates of N.
(iv) Name the figure PMP’N.
(v) Find the area of die figure PMP’N.

Solution :



(i) P' (– 5, – 3).

(ii) M (5, 0).

(iii) N (– 5, 0).

(iv) The figure PMP'N is a parallelogram.

(v) The area of figure PMP'N

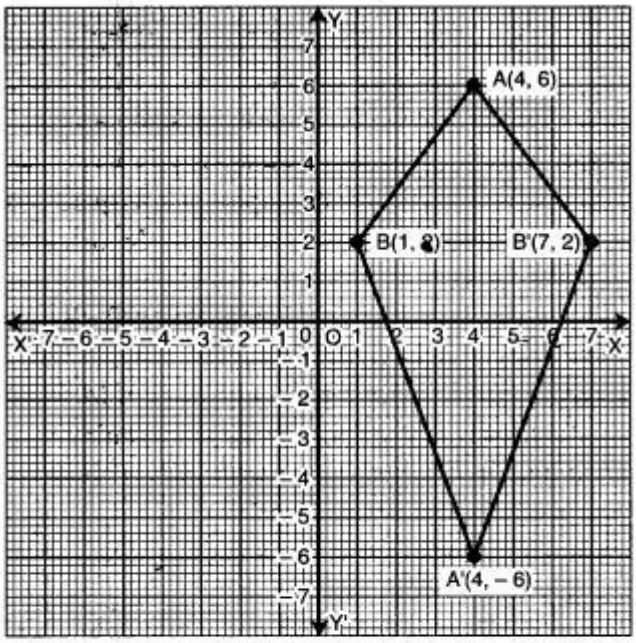
$$= \frac{1}{2} \times 10 \times 3 + \frac{1}{2} \times 10 \times 3$$

$$= 15 + 15$$

$$= 30 \text{ sq. units.}$$

Question 23. Use graph paper to answer this question:
(i) Plot the points A (4,6) and B (1, 2).
(ii) A' is the image of A when reflected in X-axis,
(iii) B' is the image of B when B is reflected in the line AA'.

(iv) Give the geometrical name for the figure ABA'B'.



Solution : (i) In the graph paper.

(ii) A (4, 6) → A' (4, – 6)

(iii) B (1, 2) → B' (7, 2)

(iv) ABA'B' is a kite. Ans.

Question 24. Use graph paper to answer the following questions. (Take 2 cm = 1 unit on both axis).

(i) Plot the points A (- 4, 2) and B (2, 4).

(ii) A' is the image of A when reflected in the y-axis. Plot it on the graph paper and write the coordinates of A'.

(iii) B' is the image of B when reflected in the line AA'. Write the coordinates of B'.

(iv) Write the geometric name of the figure ABA'B'.

(v) Name a line of symmetry of the figure formed.

Solution :

(i) On the graph.

(ii) Co-ordinates of A' = (4, 2).

(iii) Co-ordinates of B' = (2, 0).

(iv) Geometric name of Fig. ABA'B' is Kite

(v) Line of symmetry = AA'.