CBSE Test Paper 05

CH-07 Permutations & Combinations

1. The number of 7 digit numbers which can be formed using the digits 1, 2, 3, 2, 3, 3, 4 is
   a. 840
   b. 252
   c. 504
   d. 420
2. If ${}^{n}P_5 = 60{}^{n-1}P_3$, then n is
   a. none of these
   b. 12
   c. 6
   d. 10
3. The remainder obtained when $1! + 2! + 3! + \ .....+200!$ is divided by 14 is
   a. 5
   b. 6
   c. 3
   d. 4
4. The number of all numbers that can be formed by using some or all of the digits 1, 3, 5, 7, 9 (without repetitions) is
   a. none of these
   b. 120
   c. 32
   d. 325

5. The number of even numbers that can be formed by using all the digits 1, 2, 3, 4, and 5 (without repetitions) is
    a. 162
    b. none of these
    c. 1250
    d. 48
6. Fill in the blanks:

    The value of 4! - 3! is ________.
7. Fill in the blanks:

    The number of all permutations of n distinct objects taken all at a time is ________.
8. Prove that n (n - 1) (n - 2)... (n - r + 1) = $\frac{n!}{(n-r)!}$
9. In how many ways, can 5 sportsmen be selected from a group of 10?
10. In how many ways, can 6 different rings be worn on the four fingers of hand?
11. If P_m stands for ${}^mP_m$, then prove that $1 + 1 . P_1 + 2 . P_2 + 3 . P_3 + ... + {}^nP_n = (n + 1)!$.
12. Prove that $\frac{(2n)!}{n!}$ = $\{1.3.5... (2n - 1)\}2^n$.
13. Find the number of 5-digit telephone numbers having at least one of their digits repeated.
14. It is required, to seat 5 men, and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?
15. If ${}^{n+5}P_{n+1} = \frac{11(n-1)}{2}\ {}^{n+3}P_n$, find n.

Solution

1. (d) 420

   Explanation:

   There are 7 digits 1,2,3,2,3,3,4, in which the number 2 is repeating twice and the the number 3 is repeating thrice.

   Hence the number of 7 digit numbers which can be formed =
   $\frac{7!}{2!3!} = \frac{1\times2\times3\times4\times5\times6\times7}{1\times2\times1\times2\times3} = 420$

2. (d) 10

   Explanation:

   $^nP_5 = 60.^{n-1}P_3$

   $\Rightarrow \frac{n!}{(n-5)!} = 60 \cdot \frac{(n-1)!}{(n-4)!}$

   $\Rightarrow n(n-1)(n-2)(n-3)(n-4) = 60(n-1)(n-2)(n-3)$

   $\Rightarrow n(n-4) = 60$

   $\Rightarrow n^2 - 4n - 60 = 0$

   $\Rightarrow (n-10)(n+6) = 0$

   $\Rightarrow n = 10, -6$

   Since n cannot be negative we have n=10

3. (a) 5

   Explanation:

   Consider 1! + 2! + 3! + 4! + 5! + ... + 200!

   We know that 7! = 7 × 6 × 5 × 4 × 3 × 2 × 1, which is divisible by 14

   Now as 8! = 8 × 7!, it is also divisible by 14

   Hence 9!, 10!, 11! ...14! are divisible by 14.

   Thus we can conclude that the remainder obtained when we divide (1! + 2! + 3! + ... + 200!) by 14 will be same as the remainder obtained when we divide $(1! + 2! + 3! + 4! + 5! + 6!)$ by 14.

   Consider, $1! + 2! + 3! + 4! + 5! + 6! = 1 + 2 + 6 + 24 + 120 + 720 = 873.$ Now we have when 873 is divided by 14, the remainder is 5.

Thus, the remainder obtained when 1! + 2! + 3! + ... + 200! is divided by 14 is 5.

4. (d) 325

   Explanation:

   Number of ways of forming 5 digit numbers= $5 \times 4 \times 3 \times 2 \times 1 = 120$

   Number of ways of forming 4 digit numbers= $5 \times 4 \times 3 \times 2 = 120$

   Number of ways of forming 3 digit numbers= $5 \times 4 \times 3 = 60$

   Number of ways of forming 2 digit numbers=$5 \times 4 = 20$

   Number of ways of forming 1 digit numbers= 5

   Hence the total number of ways = 120+ 120 + 60+ 20+ 5 = 325

5. (d) 48

   Explanation:

t th	th	h	t	o
1	2	3	4	2

   Since we need an even number , ones place can be occupied by only two numbers 2 and 4 in two ways.
   Since repetition is not allowed tens place is occupied by remaining 4 numbers in 4 ways and the hundred's place by 3 ways and thousands place in 2 ways.
   Hence total number of even numbers can be formed in 1x2x3x4x2 = 48.

6. 18

7. n!

8. LHS = n (n — 1) (n — 2)... (n — r +1)
   $= \frac{n(n-1)(n-2)\ldots(n-(r-1)\}}{1} \times \frac{(n-r)!}{(n-r)!}$
   [multiplying numerator and denominator by (n - r)!]
   $= \frac{n!}{(n-r)!}$ = RHS
   Hence proved.

9. The required number of ways = $^{10}C_5$
   $= \frac{10!}{5!(10-5)!} = \frac{10!}{5!5!} = \frac{10\times9\times8\times7\times6\times5!}{5!\times5\times4\times3\times2\times1}$
   = $3\times2\times42$ = 252 $\left[\because {}^nC_r = \frac{n!}{r!(n-r)!}\right]$

10. Each ring can be worn on any of four fingers and this can be done in 4 ways.

Hence, the required number of ways = $(4)^6$.

11. We have, $P_m = {}^mP_m = m!$

$\therefore 1 + 1 . P_1 + 2 . P_2 + 3 . P_3 + \ldots + n . P_n$

$= 1 + \sum_{r=1}^{n} r . r! = 1 \sum_{r=1}^{n} [(r + 1) - 1]r!$

$= 1 + \sum_{r=1}^{n} [(r + 1)r! - r!]$

$= 1 + \sum_{r=1}^{n} [(r + 1)! - r!]$

$= 1 + [(2! -1!) + (3! - 2!) + (4! - 3!) + \ldots + (n +1)! - n!]$

$= 1 + [(n + !)! - 1!] = (n + 1)!$

Hence proved.

12. $\text{LHS} = \frac{(2n)!}{n!}$

$= \frac{1\cdot2\cdot3\cdot4\cdot5...(2n-2)(2n-1)(2n)}{n!}$

$= \frac{\{1\cdot3\cdot5...(2n-1)\}\{2\cdot4\cdot6...(2n-2)(2n)\}}{n!}$

$= \frac{\{1\cdot3\cdot5...(2n-1)\}2^n\{1\cdot2\cdot3...(n-1)n\}}{n!}$

$= \frac{\{1\cdot3\cdot5...(2n-1)\}\cdot2^n\cdot n!}{n!}$ $[\because 1.2.3 \ldots (n - 1).n = n!]$

$= [1.3.5 \ldots (2n - 1)]2^n = \text{RHS}$

Hence proved.

13. Using the digits 0, 1, 2, ..., 9, the number of 5-digits telephone numbers which can be formed is 10^5 (since repetition is allowed).

The number of 5-digits telephone numbers, which have none of the digits repeated

$= {}^{10}P_5 = \frac{10!}{5!} = 30240$

Hence, the required number of telephone numbers having at least one of their digits repeated

$= 10^5 - {}^{10}P_5$

$= 10^5 - 10 \times 9 \times 8 \times 7 \times 6$

$= 100000 - 30240$

= 69760

14. Here we have 5 men and 4 women. It is given that women occupy the even places.

1	2	3	4	5	6	7	8	9
M	W	M	W	M	W	M	W	M

Here we have four even places. So we can arrange four women's in 4! ways and 5 men in 5! ways in remaining five places.

$\therefore$ Required number of ways $= 4! \times 5!$

$= 24 \times 120 = 2880$

15. We have,

$^{n+5}P_{n+1} = \frac{11(n-1)}{2} \underset{n}{\overset{n+3}{P}}$

$\Rightarrow \frac{(n+5)!}{[n+5-(n+1)]!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{[n+3-n]!}$

$\Rightarrow \frac{(n+5)!}{[n+5-n-1]!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{3!}$

$\Rightarrow \frac{(n+5)!}{4!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{3!}$

$\Rightarrow \frac{(n+5)(n+4)(n+3)!}{4!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{3!}$

$\Rightarrow \frac{(n+5)(n+4)}{4\times3!} = \frac{11(n-1)}{2\times3!}$

$\Rightarrow$ (n + 5) (n + 4) = $\frac{11(n-1)\times4}{2}$

$\Rightarrow$ (n + 5) (n + 4) = 22 (n − 1)

$\Rightarrow$ n^2 + 4n + 5n + 20 = 22n − 22

$\Rightarrow$ n^2 + 9n − 22n + 20 + 22 = 0

$\Rightarrow$ n^2 − 13n + 42 = 0

$\Rightarrow$ n^2 − 6n − 7n + 42 = 0

$\Rightarrow$ n (n − 6) − 7 (n − 6) = 0

$\Rightarrow$ n = 6 or, n = 7

Hence, n = 6 or, 7