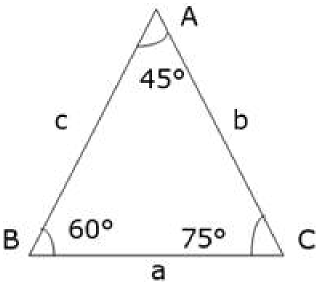
10. Sine and Cosine Formulae and their Applications

Exercise 10.1

1. Question

If in a ΔABC, ∠A = 45^o, ∠B = 60^o, and ∠C = 75^o; find the ratio of its sides.

Answer



Let a, b, c be the sides of the given triangle. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Now substituting the given values we get

$$\Rightarrow \frac{a}{\sin 45°} = \frac{b}{\sin 60°} = \frac{c}{\sin 75°}$$

$$\Rightarrow \frac{a}{\sin 45°} = \frac{b}{\sin 60°} = \frac{c}{\sin(30° + 45°)}$$

$$\Rightarrow \frac{a}{\sin 45°} = \frac{b}{\sin 60°} = \frac{c}{\sin 30° \cos 45° + \sin 45° \cos 30°}$$

(∵ sin (a + b) = sin a cos b + sin b cos a)

Now substituting the corresponding values, we get,

$$\Rightarrow \frac{a}{\frac{1}{\sqrt{2}}} = \frac{b}{\frac{\sqrt{3}}{2}} = \frac{c}{\frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}}$$

$$\Rightarrow \frac{a}{\frac{1}{\sqrt{2}}} = \frac{b}{\frac{\sqrt{3}}{2}} = \frac{c}{\frac{1 + \sqrt{3}}{2\sqrt{2}}}$$

$$\Rightarrow a:b:c = \frac{1}{\sqrt{2}} : \frac{\sqrt{3}}{2} : \frac{1 + \sqrt{3}}{2\sqrt{2}}$$

Multiplying 2√2, we get

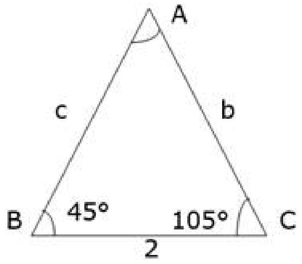
$$a:b:c = 2:\sqrt{6}:(1 + \sqrt{3})$$

Hence the ratio of the sides of the given triangle is $2:\sqrt{6}:(1 + \sqrt{3})$

2. Question

If in any ΔABC, ∠C = 105^o, ∠B = 45^o, a = 2, then find b.

Answer



We know in a triangle,

∠A + ∠B + ∠C = 180°

⇒ ∠A = 180° - ∠B - ∠C

Substituting the given values, we get

∠A = 180° - 45° - 105°

⇒ ∠A = 30°

Let a, b, c be the sides of the given triangle. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{a}{\sin A} = \frac{b}{\sin B}$$

Now substituting the corresponding values we get

$$\Rightarrow \frac{2}{\sin 30^\circ} = \frac{b}{\sin 45^\circ}$$

Substitute the equivalent values of the sine, we get

$$\Rightarrow \frac{2}{\frac{1}{2}} = \frac{b}{\frac{1}{\sqrt{2}}}$$

$$\Rightarrow 4 = b\sqrt{2}$$

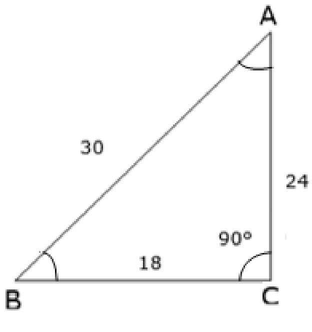
$$\Rightarrow b = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

Hence the value of b is 2√2 units.

3. Question

In ΔABC, if a = 18, b = 24 and c = 30 and ∠C = 90^0, find sin A, sin B and sin C.

Answer



Let a, b, c be the sides of the given triangle. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{a}{\sin A} = \frac{c}{\sin C}$$

Now substituting the given values we get

$$\Rightarrow \frac{18}{\sin A} = \frac{30}{\sin 90°}$$

$$\Rightarrow \sin A = \frac{18 \times \sin 90°}{30}$$

$$\Rightarrow \sin A = \frac{18 \times 1}{30}$$

$$\Rightarrow \sin A = \frac{3}{5}$$

Similarly,

$$\Rightarrow \frac{b}{\sin B} = \frac{c}{\sin C}$$

Now substituting the given values we get

$$\Rightarrow \frac{24}{\sin B} = \frac{30}{\sin 90°}$$

$$\Rightarrow \sin B = \frac{24 \times \sin 90°}{30}$$

$$\Rightarrow \sin B = \frac{24 \times 1}{30}$$

$$\Rightarrow \sin B = \frac{4}{5}$$

And given ∠C = 90°, so sin C = sin 90° = 1.

Hence the values of sin A, sin B, sin C are $\frac{3}{5}, \frac{4}{5}, 1$ respectively

4. Question

In any triangle ABC, prove the following:

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)}$$

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\frac{a}{\sin A} = k$$

⇒a = k sin A

Similarly, b = k sin B

So, a - b = k(sin A - sin B)

And a + b = k(sin A + sin B)

So, the given LHS becomes,

$$\text{LHS} = \frac{a - b}{a + b}$$

$$\Rightarrow = \frac{k(\sin A - \sin B)}{k(\sin A + \sin B)}$$

$$\Rightarrow = \frac{(\sin A - \sin B)}{(\sin A + \sin B)} \ldots (i)$$

But,

$$\sin A - \sin B = 2\sin\left(\frac{A - B}{2}\right)\cos\left(\frac{A + B}{2}\right),$$

$$\sin A + \sin B = 2\sin\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right)$$

Substituting the above values in equation (i), we get

$$\frac{a - b}{a + b} = \frac{\left(2\sin\left(\frac{A - B}{2}\right)\cos\left(\frac{A + B}{2}\right)\right)}{\left(2\sin\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right)\right)}$$

Rearranging the above equation we get,

$$\Rightarrow = \frac{\left(\sin\left(\frac{A - B}{2}\right)\right)}{\left(\cos\left(\frac{A - B}{2}\right)\right)} \times \frac{\cos\left(\frac{A + B}{2}\right)}{\sin\left(\frac{A + B}{2}\right)}$$

$$\Rightarrow = \frac{\left(\tan\left(\frac{A - B}{2}\right)\right)}{1} \times \frac{1}{\tan\left(\frac{A + B}{2}\right)}$$

$$\Rightarrow = \frac{\left(\tan\left(\frac{A - B}{2}\right)\right)}{\left(\tan\left(\frac{A + B}{2}\right)\right)} = \text{RHS}$$

Hence proved

5. Question

In any triangle ABC, prove the following:

$$(a - b)\cos\frac{C}{2} = c\sin\left(\frac{A - B}{2}\right)$$

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\frac{a}{\sin A} = k$$

⇒a = k sin A

Similarly, b = k sin B

So, a - b = k(sin A - sin B)..(i)

So the given LHS becomes,

$$\text{LHS} = (a-b)\cos\frac{C}{2}$$

Substituting equation (i) in above equation, we get

$$\Rightarrow = (k(\sin A - \sin B))\cos\frac{C}{2} \ldots \text{(ii)}$$

But,

$$\sin A - \sin B = 2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right),$$

Substituting the above values in equation (ii), we get

$$(a-b)\cos\frac{C}{2} = \left(k\left(2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)\right)\right)\cos\frac{C}{2}$$

$$\Rightarrow = \left(k\left(2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)\right)\right)\cos\frac{(\pi-(A+B))}{2} \ (\because A+B+C=\pi)$$

$$\Rightarrow = \left(2k\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)\right)\sin\left(\frac{A+B}{2}\right) \ \left(\because \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta\right)$$

Rearranging the above equation we get

$$\Rightarrow = k\sin\left(\frac{A-B}{2}\right)\left(2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A+B}{2}\right)\right)$$

But $\sin(\theta) = 2\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)$

So the above equation becomes,

$$= k\sin\left(\frac{A-B}{2}\right)(\sin(A+B))$$

$$= k\sin\left(\frac{A-B}{2}\right)(\sin(\pi-C)) \ (\because \pi = A+B+C \Rightarrow A+B = \pi - C)$$

$$\Rightarrow = k\sin(C)\sin\left(\frac{A-B}{2}\right) \ (\because \sin(\pi-\theta) = \sin\theta)$$

But from sine rule,

$$\frac{c}{\sin C} = k \Rightarrow c = k\sin C$$

So the above equation becomes,

$$\Rightarrow = c\sin\left(\frac{A-B}{2}\right) = \text{RHS}$$

Hence proved

6. Question

In any triangle ABC, prove the following:

$$\frac{c}{a-b} = \frac{\tan\left(\frac{A}{2}\right)+\tan\left(\frac{B}{2}\right)}{\tan\left(\frac{A}{2}\right)-\tan\left(\frac{B}{2}\right)}$$

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\frac{a}{\sin A} = k$$

⇒a = k sin A

Similarly, b = k sin B

And c = k sin C.....(i)

So, a - b = k(sin A - sin B)..(ii)

So the given LHS becomes,

$$\text{LHS} = \frac{c}{a-b}$$

Substituting equation (i) and (ii) in the above equation, we get

$$\Rightarrow = \frac{k\sin C}{k(\sin A - \sin B)}$$

$$\Rightarrow = \frac{\sin C}{(\sin A - \sin B)} \ldots (ii)$$

Applying half angle rule,

$$\sin C = 2\sin\frac{C}{2}\cos\frac{C}{2} \ldots\ldots (iii)$$

And

$$\sin A - \sin B = 2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right) \ldots . (iv)$$

Substituting equation (iii) and (iv) in equation (ii), we get

$$\frac{c}{(a-b)} = \frac{2\sin\frac{C}{2}\cos\frac{C}{2}}{2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)}$$

$$\Rightarrow = \frac{\sin\left(\frac{\pi-(A+B)}{2}\right)\cos\left(\frac{C}{2}\right)}{\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)} \left(\because A+B+C = \pi \Rightarrow C = \pi-(A+B)\right)$$

$$\Rightarrow = \frac{\cancel{\cos\left(\frac{(A+B)}{2}\right)}\cos\left(\frac{C}{2}\right)}{\sin\left(\frac{A-B}{2}\right)\cancel{\cos\left(\frac{A+B}{2}\right)}} \left(\because \sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta\right)$$

$$\Rightarrow = \frac{\cos\left(\frac{\pi-(A+B)}{2}\right)}{\sin\left(\frac{A-B}{2}\right)} \left(\because A+B+C = \pi \Rightarrow C = \pi-(A+B)\right)$$

$$\Rightarrow = \frac{\sin\left(\frac{(A+B)}{2}\right)}{\sin\left(\frac{A-B}{2}\right)} \ldots (v) \left(\because \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta\right)$$

But $\sin\left(\frac{A+B}{2}\right) = \sin\left(\frac{A}{2}+\frac{B}{2}\right) = \sin\frac{A}{2}\cos\frac{B}{2} + \cos\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)$

And $\sin\left(\frac{A-B}{2}\right) = \sin\left(\frac{A}{2} - \frac{B}{2}\right) = \sin\frac{A}{2}\cos\frac{B}{2} - \cos\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)$

So the above equations in equation (v), we get

$$\Rightarrow = \frac{\sin\frac{A}{2}\cos\frac{B}{2} + \cos\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)}{\sin\frac{A}{2}\cos\frac{B}{2} - \cos\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)}$$

Dividing numerator and denominator by $\cos\frac{A}{2}\cos\frac{B}{2}$, we get

$$\Rightarrow = \frac{\dfrac{\sin\frac{A}{2}\cos\frac{B}{2} + \cos\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)}{\cos\frac{A}{2}\cos\frac{B}{2}}}{\dfrac{\sin\frac{A}{2}\cos\frac{B}{2} - \cos\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)}{\cos\frac{A}{2}\cos\frac{B}{2}}}$$

$$\Rightarrow = \frac{\dfrac{\sin\frac{A}{2}\cos\frac{B}{2}}{\cos\frac{A}{2}\cos\frac{B}{2}} + \dfrac{\cos\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)}{\cos\frac{A}{2}\cos\frac{B}{2}}}{\dfrac{\sin\frac{A}{2}\cos\frac{B}{2}}{\cos\frac{A}{2}\cos\frac{B}{2}} - \dfrac{\cos\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)}{\cos\frac{A}{2}\cos\frac{B}{2}}}$$

By canceling the like terms we get,

$$\Rightarrow = \frac{\dfrac{\sin\frac{A}{2}}{\cos\frac{A}{2}} + \dfrac{\sin\left(\frac{B}{2}\right)}{\cos\frac{B}{2}}}{\dfrac{\sin\frac{A}{2}}{\cos\frac{A}{2}} - \dfrac{\sin\left(\frac{B}{2}\right)}{\cos\frac{B}{2}}}$$

$$\Rightarrow = \frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{\tan\frac{A}{2} - \tan\frac{B}{2}} = \text{RHS}$$

Hence proved

7. Question

In any triangle ABC, prove the following:

$$\frac{c}{a+b} = \frac{1-\tan\left(\dfrac{A}{2}\right)\tan\left(\dfrac{B}{2}\right)}{1+\tan\left(\dfrac{A}{2}\right)\tan\left(\dfrac{B}{2}\right)}$$

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\frac{a}{\sin A} = k$$

$\Rightarrow a = k \sin A$

Similarly, b = k sin B

And c = k sin C.....(i)

So, a + b = k(sin A + sin B)..(ii)

So the given LHS becomes,

$$\text{LHS} = \frac{c}{a+b}$$

Substituting equation (i) and (ii) in the above equation, we get

$$\Rightarrow = \frac{k\sin C}{k(\sin A + \sin B)}$$

$$\Rightarrow = \frac{\sin C}{(\sin A + \sin B)} \ldots (ii)$$

Applying half angle rule,

$$\sin C = 2\sin\frac{C}{2}\cos\frac{C}{2} \ldots\ldots (iii)$$

And

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \ldots . (iv)$$

Substituting equation (iii) and (iv) in equation (ii), we get

$$\frac{c}{(a+b)} = \frac{2\sin\frac{C}{2}\cos\frac{C}{2}}{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}$$

$$\Rightarrow = \frac{\sin\left(\frac{\pi-(A+B)}{2}\right)\cos\left(\frac{\pi-(A+B)}{2}\right)}{\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)} \ (\because A + B + C = \pi)$$

$$\Rightarrow = \frac{\cos\left(\frac{(A+B)}{2}\right)\cancel{\sin\left(\frac{A+B}{2}\right)}}{\cancel{\sin\left(\frac{A+B}{2}\right)}\cos\left(\frac{A-B}{2}\right)} \left(\because \sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta, \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta\right)$$

$$\Rightarrow = \frac{\cos\left(\frac{(A+B)}{2}\right)}{\cos\left(\frac{A-B}{2}\right)} \ldots (v)$$

But $\cos\left(\frac{A+B}{2}\right) = \cos\left(\frac{A}{2}+\frac{B}{2}\right) = \cos\frac{A}{2}\cos\frac{B}{2} + \sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)$

And $\cos\left(\frac{A-B}{2}\right) = \cos\left(\frac{A}{2}-\frac{B}{2}\right) = \cos\frac{A}{2}\cos\frac{B}{2} - \sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)$

So the above equations in equation (v), we get

$$\Rightarrow = \frac{\cos\frac{A}{2}\cos\frac{B}{2} + \sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)}{\cos\frac{A}{2}\cos\frac{B}{2} - \sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)}$$

Dividing numerator and denominator by $\cos\frac{A}{2}\cos\frac{B}{2}$, we get

$$\Rightarrow = \frac{\frac{\cos\frac{A}{2}\cos\frac{B}{2} + \sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)}{\cos\frac{A}{2}\cos\frac{B}{2}}}{\frac{\cos\frac{A}{2}\cos\frac{B}{2} - \sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)}{\cos\frac{A}{2}\cos\frac{B}{2}}}$$

$$\Rightarrow = \frac{\frac{\cos\frac{A}{2}\cos\frac{B}{2}}{\cos\frac{A}{2}\cos\frac{B}{2}} + \frac{\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)}{\cos\frac{A}{2}\cos\frac{B}{2}}}{\frac{\cos\frac{A}{2}\cos\frac{B}{2}}{\cos\frac{A}{2}\cos\frac{B}{2}} - \frac{\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)}{\cos\frac{A}{2}\cos\frac{B}{2}}}$$

By canceling the like terms we get

$$\Rightarrow = \frac{1 + \frac{\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)}{\cos\frac{A}{2}\cos\frac{B}{2}}}{1 - \frac{\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)}{\cos\frac{A}{2}\cos\frac{B}{2}}}$$

$$\Rightarrow = \frac{1 + \tan\frac{A}{2}\tan\frac{B}{2}}{1 - \tan\frac{A}{2}\tan\frac{B}{2}} = \text{RHS}$$

Hence proved

8. Question

In any triangle ABC, prove the following:

$$\frac{a+b}{c} = \frac{\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}}$$

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\frac{a}{\sin A} = k$$

⇒a = k sin A

Similarly, b = k sin B

And c = k sin C.....(i)

So, a + b = k(sin A + sin B)..(ii)

So the given LHS becomes,

$$\text{LHS} = \frac{a + b}{c}$$

Substituting equation (i) and (ii) in above equation, we get

$$\Rightarrow = \frac{k(\sin A + \sin B)}{k(\sin C)}$$

$$\Rightarrow = \frac{(\sin A + \sin B)}{(\sin C)} \ldots (ii)$$

Applying half angle rule,

$$\sin C = 2\sin\frac{C}{2}\cos\frac{C}{2} \ldots\ldots (iii)$$

And

$$\sin A + \sin B = 2\sin\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right) \ldots . (iv)$$

Substituting equation (iii) and (iv) in equation (ii), we get

$$\frac{a + b}{(c)} = \frac{2\sin\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right)}{2\sin\left(\frac{C}{2}\right)\cos\left(\frac{C}{2}\right)}$$

$$\Rightarrow = \frac{\sin\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right)}{\sin\left(\frac{C}{2}\right)\cos\left(\frac{\pi - (A + B)}{2}\right)} (\because A + B + C = \pi)$$

$$\Rightarrow = \frac{\cancel{\sin\left(\frac{A + B}{2}\right)}\cos\left(\frac{A - B}{2}\right)}{\sin\left(\frac{C}{2}\right)\cancel{\sin\left(\frac{A + B}{2}\right)}} \left(\because \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta, \cos(\frac{\pi}{2} - \theta) = \sin\theta\right)$$

$$\Rightarrow = \frac{\cos\left(\frac{(A - B)}{2}\right)}{\sin\left(\frac{C}{2}\right)} = RHS$$

Hence proved

9. Question

In any triangle ABC, prove the following:

$$\sin\left(\frac{B - C}{2}\right) = \frac{b - c}{a}\cos\frac{A}{2}$$

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\frac{c}{\sin C} = k$$

⇒c = k sin C

Similarly, b = k sin B

So, b - c = k(sin B - sin C)..(i)

Here we will consider RHS, so we get

$$RHS = \frac{b - c}{a}\cos\frac{A}{2}$$

Substituting equation (i) in the above equation, we get

$$\Rightarrow = \frac{(k(\sin B - \sin C))}{k \sin A} \cos\frac{A}{2} \ldots \text{(ii)}$$

But,

$$\sin B - \sin C = 2\sin\left(\frac{B-C}{2}\right)\cos\left(\frac{B+C}{2}\right),$$

Substituting the above values in equation (ii), we get

$$\frac{b-c}{a}\cos\frac{A}{2} = \frac{2\sin\left(\frac{B-C}{2}\right)\cos\left(\frac{B+C}{2}\right)}{\sin A}\cos\left(\frac{\pi-(B+C)}{2}\right) (\because A + B + C = \pi)$$

$$\Rightarrow = \frac{2\sin\left(\frac{B-C}{2}\right)\cos\left(\frac{B+C}{2}\right)}{\sin A}\sin\left(\frac{(B+C)}{2}\right) \left(\because \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta\right)$$

Rearranging the above equation we get

$$\Rightarrow = \frac{\sin\left(\frac{B-C}{2}\right)\left(2\sin\left(\frac{(B+C)}{2}\right)\cos\left(\frac{B+C}{2}\right)\right)}{\sin A}$$

But $\sin(\theta) = 2\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)$

So the above equation becomes,

$$= \frac{\sin\left(\frac{B-C}{2}\right)(\sin(B+C))}{\sin A}$$

$$= \frac{\sin\left(\frac{B-C}{2}\right)(\sin(\pi-A))}{\sin A} (\because \pi = A + B + C \Rightarrow A + B = \pi - C)$$

$$\Rightarrow = \frac{\sin\left(\frac{B-C}{2}\right)(\cancel{\sin(A)})}{\cancel{\sin A}} (\because \sin(\pi-\theta) = \sin\theta)$$

$$\Rightarrow = \sin\left(\frac{B-C}{2}\right) = \text{LHS}$$

Hence proved

10. Question

In any triangle ABC, prove the following:

$$\frac{a^2-c^2}{b^2} = \frac{\sin(A-C)}{\sin(A+C)}$$

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\frac{c}{\sin C} = k$$

$\Rightarrow c = k \sin C$

Similarly, b = k sin B

And a = k sin A

Here we will consider LHS, so we get

$$\text{LHS} = \frac{a^2 - c^2}{b^2}$$

Substituting corresponding values in the above equation, we get

$$\Rightarrow = \frac{(k\sin A)^2 - (k\sin C)^2}{(k\sin B)^2}$$

$$\Rightarrow = \frac{k^2(\sin^2 A - \sin^2 C)}{k^2\sin^2 B} \ldots (ii)$$

But,

$$\sin^2 A - \sin^2 C = \sin(A + C)\sin(A - C),$$

Substituting the above values in equation (ii), we get

$$\frac{a^2 - c^2}{b^2} = \frac{\sin(A + C)\sin(A - C)}{\sin^2(\pi - (A + C))} \; (\because A + B + C = \pi)$$

$$\Rightarrow = \frac{\sin(A + C)\sin(A - C)}{\sin^2((A + C))} \; (\because \sin(\pi - \theta) = \sin\theta)$$

$$\Rightarrow = \frac{\sin(A - C)}{\sin(A + C)} = \text{RHS}$$

Hence proved

11. Question

In any triangle ABC, prove the following:

b sin B – c sin C = a sin (B – C)

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\frac{c}{\sin C} = k$$

⇒c = k sin C

Similarly, b = k sin B

And a = k sin A

Here we will consider LHS, so we get

LHS = b sin B – c sin C

Substituting corresponding values in the above equation, we get

⇒ = k sin B sin B –k sin C sin C

⇒ = k ($\sin^2$ B – $\sin^2$ C)..........(ii)

But,

$$\sin^2 B - \sin^2 C = \sin(B + C)\sin(B - C),$$

Substituting the above values in equation (ii), we get

⇒ = k(sin(B + C) sin(B - C))

But A + B + C = π⇒ B + C = π –A, so the above equation becomes,

⇒ = k(sin(π –A) sin(B - C))

But sin (π - θ) = sin θ

⇒ = k(sin(A) sin(B - C))

From sine rule, a = k sin A, so the above equation becomes,

⇒ = a sin(B - C) = RHS

Hence proved

12. Question

In any triangle ABC, prove the following:

a^2 sin (B – C) = (b^2 – c^2) sin A

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\frac{c}{\sin C} = k$$

⇒c = k sin C

Similarly, b = k sin B

And a = k sin A

Here we will consider RHS, so we get

RHS = (b^2 – c^2) sin A

Substituting corresponding values in the above equation, we get

⇒ = [(k sin B)^2 - (k sin C)^2] sin A

⇒ = k^2($\sin^2$ B – $\sin^2$ C)sin A..........(ii)

But,

$$\sin^2 B - \sin^2 C = \sin(B + C)\sin(B - C),$$

Substituting the above values in equation (ii), we get

⇒ = k^2(sin(B + C) sin(B - C)) sin A

But A + B + C = π ⇒ B + C = π –A, so the above equation becomes,

⇒ = k^2(sin(π –A) sin(B - C))sin A

But sin (π - θ) = sin θ

⇒ = k^2(sin(A) sin(B - C))sin A

Rearranging the above equation we get

⇒ = (k sin(A))(sin(B - C))(k sin A)

From sine rule, a = k sin A, so the above equation becomes,

⇒ = a^2 sin(B - C) = RHS

Hence proved

13. Question

In any triangle ABC, prove the following:

$$\frac{\sqrt{\sin A}-\sqrt{\sin B}}{\sqrt{\sin A}+\sqrt{\sin B}}=\frac{a+b-2\sqrt{ab}}{a-b}$$

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\Rightarrow \sin A = \frac{a}{k}, \sin B = \frac{b}{k}$$

Here we will consider LHS, so we get

$$\text{LHS} = \frac{\sqrt{\sin A}-\sqrt{\sin B}}{\sqrt{\sin A}+\sqrt{\sin B}}$$

Multiply and divide by $\frac{\sqrt{\sin A}-\sqrt{\sin B}}{\sqrt{\sin A}-\sqrt{\sin B}}$, we get

$$= \frac{\sqrt{\sin A}-\sqrt{\sin B}}{\sqrt{\sin A}+\sqrt{\sin B}} \times = \frac{\sqrt{\sin A}-\sqrt{\sin B}}{\sqrt{\sin A}-\sqrt{\sin B}}$$

$$\Rightarrow = \frac{\left(\sqrt{\sin A}-\sqrt{\sin B}\right)^2}{\left(\sqrt{\sin A}\right)^2-\left(\sqrt{\sin B}\right)^2}$$

$$\Rightarrow = \frac{\left(\sqrt{\sin A}\right)^2+\left(\sqrt{\sin B}\right)^2-\left(2\sqrt{\sin A}\times\sqrt{\sin B}\right)}{\sin A-\sin B}$$

$$\Rightarrow = \frac{\sin A+\sin B-\left(2\sqrt{\sin A\times\sin B}\right)}{\sin A-\sin B}$$

Substituting corresponding values from sine rule in the above equation, we get

$$\Rightarrow = \frac{\frac{a}{k}+\frac{b}{k}-\left(2\sqrt{\frac{a}{k}\times\frac{b}{k}}\right)}{\frac{a}{k}-\frac{b}{k}}$$

$$\Rightarrow = \frac{\frac{1}{k}\left(a+b-2\sqrt{ab}\right)}{\frac{1}{k}(a-b)}$$

$$\Rightarrow = \frac{a+b-2\sqrt{ab}}{a-b} = \text{RHS}$$

Hence proved

14. Question

In any triangle ABC, prove the following:

a(sin B – sin C) + b(sin C – sin A) + c(sin A – sin B) = 0

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

⇒ a = k sin A, b = k sin B, c = k sin C

Here we will consider LHS, so we get

LHS = a(sin B – sin C) + b(sin C – sin A) + c(sin A – sin B)

Substituting corresponding values from sine rule in above equation, we get

⇒ = k sin A(sin B – sin C) + k sin B(sin C – sin A) + k sin C(sin A – sin B)

⇒ = k sin A sin B – k sin A sin C + k sin B sin C – k sin B sin A + k sin C sin A – k sin C sin B

Cancelling the like terms, we get

LHS = 0 = RHS

Hence proved

15. Question

In any triangle ABC, prove the following:

$$\frac{a^2 \sin(B-C)}{\sin A} + \frac{b^2 \sin(C-A)}{\sin B} + \frac{c^2 \sin(A-B)}{\sin C} = 0$$

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

⇒ a = k sin A, b = k sin B, c = k sin C

Here we will consider LHS, so we get

$$\text{LHS} = \frac{a^2 \sin(B-C)}{\sin A} + \frac{b^2 \sin(C-A)}{\sin B} + \frac{c^2 \sin(A-B)}{\sin C}$$

Substituting corresponding values from sine rule in the above equation, we get

$$\Rightarrow = \frac{(k \sin A)^2 \sin(B-C)}{\sin A} + \frac{(k \sin B)^2 \sin(C-A)}{\sin B} + \frac{(k \sin C)^2 \sin(A-B)}{\sin C}$$

$$\Rightarrow = \frac{k^2 \sin^2 A \sin(B-C)}{\sin A} + \frac{k^2 \sin^2 B \sin(C-A)}{\sin B} + \frac{k^2 \sin^2 C \sin(A-B)}{\sin C}$$

Canceling the like terms, we get

$$\Rightarrow = k^2[\sin A \sin(B-C) + \sin B \sin(C-A) + \sin C \sin(A-B)]$$

But sin(A - B) = sin A cos B – cos A sin B, so the above equation becomes

$$\Rightarrow = k^2[\sin A(\sin B \cos C - \cos B \sin C) + \sin B(\sin C \cos A - \cos C \sin A) + \sin C(\sin A \cos B - \cos A \sin B)]$$

$$\Rightarrow = k^2[\sin A \sin B \cos C - \sin A \cos B \sin C + \sin B \sin C \cos A - \sin B \cos C \sin A + \sin C \sin A \cos B - \sin C \cos A \sin B]$$

Cancelling the like terms, we get,

LHS = 0 = RHS

Hence proved

16. Question

In any triangle ABC, prove the following:

$a^2 (\cos^2 B - \cos^2 C) + b^2 (\cos^2 C - \cos^2 A) + c^2 (\cos^2 A - \cos^2 B) = 0$

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

⇒ a = k sin A, b = k sin B, c = k sin C

Here we will consider LHS, so we get

$LHS = a^2 (\cos^2 B - \cos^2 C) + b^2 (\cos^2 C - \cos^2 A) + c^2 (\cos^2 A - \cos^2 B)$

Substituting corresponding values from sine rule in above equation, we get

$= (k \sin A)^2 (\cos^2 B - \cos^2 C) + (k \sin B)^2 (\cos^2 C - \cos^2 A) + (k \sin C)^2 (\cos^2 A - \cos^2 B)$

$= k^2(\sin^2 A \cos^2 B - \sin^2 A \cos^2 C + \sin^2 B \cos^2 C - \sin^2 B \cos^2 A + \sin^2 A \cos^2 A - \sin^2 A \cos^2 B)$

Cancelling the like terms, we get

LHS = 0 = RHS

Hence proved

17. Question

In any triangle ABC, prove the following:

b cos B + c cos C = a cos (B – C)

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

⇒ a = k sin A, b = k sin B, c = k sin C

Here we will consider LHS, so we get

LHS = b cos B + c cos C

Substituting corresponding values from sine rule in above equation, we get

⇒ = k sin B cos B + k sin C cos C

$$\Rightarrow = k\left(\frac{\sin 2B}{2} + \frac{\sin 2C}{2}\right) (\because \sin 2A = 2\sin A\cos A)$$

$$\Rightarrow = \frac{k}{2}(\sin 2B + \sin 2C) \ldots. (i)$$

Now we will consider RHS, so we get

RHS = a cos (B – C)

Substituting corresponding values from sine rule in the above equation, we get

⇒ = k sin A cos (B – C)

But sin(A + B) + sin (A - B) = 2 sin A cos B, so the above equation becomes,

$$\Rightarrow = k\left[\frac{\sin(A + (B - C)) + \sin(A - (B - C))}{2}\right]$$

$$\Rightarrow = \frac{k}{2}[\sin(A + B - C) + \sin(A - B + C)]$$

We know in a triangle, π = A + B + C, hence the above equation becomes,

$$\Rightarrow = \frac{k}{2}[\sin(\pi - C - C) + \sin(\pi - B - B)]$$

$$\Rightarrow = \frac{k}{2}[\sin(\pi - 2C) + \sin(\pi - 2B)]$$

$$\Rightarrow = \frac{k}{2}[\sin(2C) + \sin(2B)] \ldots \ldots (ii)\ (\because \sin(\pi - \theta) = \sin\theta)$$

Comparing equation (i) and (ii),

LHS = RHS

Hence proved

18. Question

In any triangle ABC, prove the following:

$$\frac{(\cos 2A)}{a^2} - \frac{\cos 2B}{b^2} = \frac{1}{a^2} - \frac{1}{b^2}$$

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

⇒ a = k sin A, b = k sin B, c = k sin C

Consider the LHS of the given equation, we get

$$LHS = \frac{(\cos 2A)}{a^2} - \frac{\cos 2B}{b^2}$$

$$\Rightarrow = \frac{1 - 2\sin^2 A}{a^2} - \frac{1 - \sin^2 B}{b^2}\ (\because \cos 2A = 1 - \sin^2 A)$$

Substituting the values from sine rule into the above equation, we get

$$\Rightarrow = \frac{1 - 2\left(\frac{a}{k}\right)^2}{a^2} - \frac{1 - \left(\frac{b}{k}\right)^2}{b^2}$$

$$\Rightarrow = \frac{\frac{k^2 - 2a^2}{k^2}}{a^2} - \frac{\frac{k^2 - 2b^2}{k^2}}{b^2}$$

$$\Rightarrow = \frac{k^2 - 2a^2}{k^2a^2} - \frac{k^2 - 2b^2}{k^2b^2}$$

$$\Rightarrow = \frac{b^2(k^2 - 2a^2) - a^2(k^2 - 2b^2)}{k^2a^2b^2}$$

$$\Rightarrow = \frac{b^2k^2 - 2a^2b^2 - a^2k^2 + 2a^2b^2}{k^2a^2b^2}$$

$$\Rightarrow = \frac{b^2k^2 - a^2k^2}{k^2a^2b^2}$$

$$\Rightarrow = \frac{b^2 - a^2}{a^2b^2}$$

$$\Rightarrow = \frac{b^2}{a^2b^2} - \frac{a^2}{a^2b^2}$$

$$\Rightarrow = \frac{1}{a^2} - \frac{1}{b^2} = \text{RHS}$$

Hence proved

19. Question

In any triangle ABC, prove the following:

$$\frac{\cos^2 B - \cos^2 C}{b+c} + \frac{\cos^2 C - \cos^2 A}{c-a} + \frac{\cos^2 A - \cos^2 B}{a+b} = 0$$

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

⇒ a = k sin A, b = k sin B, c = k sin C

Now,

$$\frac{\cos^2 B - \cos^2 C}{b + c}$$

Substituting the values from sine rule into the above equation, we get

$$\Rightarrow = \frac{(\cos B + \cos C)(\cos B - \cos C)}{k(\sin B + \sin C)}$$

$$(\because \cos^2 B - \cos^2 C = (\cos B + \cos C)(\cos B - \cos C))$$

But $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

And $\cos B - \cos C = 2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)$

And $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

Substituting these we get

$$\Rightarrow = \frac{\left(2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)\right)\left(-2\sin\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right)\right)}{k\left(2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)\right)}$$

By canceling the like terms, we get

$$\Rightarrow = \frac{\left(-2\sin\left(\frac{B-C}{2}\right)\cos\left(\frac{B+C}{2}\right)\right)}{k}$$

$$\Rightarrow = \frac{-(\sin B - \sin C)}{k} \left(\because \sin A - \sin B = 2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)\right)$$

$$\Rightarrow = \frac{\sin C - \sin B}{k} \dots \dots (i)$$

Similarly,

$$\frac{\cos^2 C - \cos^2 A}{c + a}$$

Substituting the values from sine rule into the above equation, we get

$$\Rightarrow = \frac{(\cos C + \cos A)(\cos C - \cos A)}{k(\sin C + \sin A)}$$

$(\because \cos^2 B - \cos^2 C = (\cos B + \cos C)(\cos B - \cos C))$

But $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

And $\cos B - \cos C = 2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)$

And $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

Substituting these we get

$$\Rightarrow = \frac{\left(2\cos\left(\frac{C + A}{2}\right)\cos\left(\frac{C - A}{2}\right)\right)\left(-2\sin\left(\frac{C + A}{2}\right)\sin\left(\frac{C - A}{2}\right)\right)}{k\left(2\sin\left(\frac{C + A}{2}\right)\cos\left(\frac{C - A}{2}\right)\right)}$$

By canceling the like terms, we get

$$\Rightarrow = \frac{\left(-2\sin\left(\frac{C - A}{2}\right)\cos\left(\frac{C + A}{2}\right)\right)}{k}$$

$$\Rightarrow = \frac{-(\sin C - \sin A)}{k}\left(\because \sin A - \sin B = 2\sin\left(\frac{A - B}{2}\right)\cos\left(\frac{A + B}{2}\right)\right)$$

$$\Rightarrow = \frac{\sin A - \sin C}{k} \dots \dots (ii)$$

Similarly,

$$\frac{\cos^2 A - \cos^2 B}{a + b}$$

Substituting the values from sine rule into the above equation, we get

$$\Rightarrow = \frac{(\cos A + \cos B)(\cos A - \cos B)}{k(\sin A + \sin B)}$$

$(\because \cos^2 B - \cos^2 C = (\cos B + \cos C)(\cos B - \cos C))$

But $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

And $\cos B - \cos C = 2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)$

And $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

Substituting these we get

$$\Rightarrow = \frac{\left(2\cos\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right)\right)\left(-2\sin\left(\frac{A + B}{2}\right)\sin\left(\frac{A - B}{2}\right)\right)}{k\left(2\sin\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right)\right)}$$

By canceling the like terms, we get

$$\Rightarrow = \frac{\left(-2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)\right)}{k}$$

$$\Rightarrow = \frac{-(\sin A - \sin B)}{k}\left(\because \sin A - \sin B = 2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)\right)$$

$$\Rightarrow = \frac{\sin B - \sin A}{k} \ldots\ldots(iii)$$

So the LHS of the given equation, we get

$$\frac{\cos^2 B - \cos^2 C}{b+c} + \frac{\cos^2 C - \cos^2 A}{c-a} + \frac{\cos^2 A - \cos^2 B}{a+b}$$

From equation (i), (ii) and (iii), we get

$$\Rightarrow = \frac{\sin C - \sin B}{k} + \frac{\sin A - \sin C}{k} + \frac{\sin B - \sin A}{k}$$

$$\Rightarrow = \frac{\sin C - \sin B + \sin A - \sin C + \sin B - \sin A}{k}$$

$$\Rightarrow = 0 = \text{RHS}$$

Hence proved

20. Question

In any triangle ABC, prove the following:

$$a\sin\frac{A}{2}\sin\left(\frac{B-C}{2}\right)+b\sin\frac{B}{2}\sin\left(\frac{C-A}{2}\right)+c\sin\frac{C}{2}\sin\left(\frac{A-B}{2}\right)=0$$

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

⇒ a = k sin A, b = k sin B, c = k sin C

So the LHS of the given equation, we get

$$\text{LHS} = a\sin\frac{A}{2}\sin\left(\frac{B-C}{2}\right) + b\sin\frac{B}{2}\sin\left(\frac{C-A}{2}\right) + c\sin\frac{C}{2}\sin\left(\frac{A-B}{2}\right)$$

Substituting values from sine rule, we get

$$\Rightarrow = k\sin A\sin\frac{A}{2}\sin\left(\frac{B-C}{2}\right) + k\sin B\sin\frac{B}{2}\sin\left(\frac{C-A}{2}\right) + k\sin C\sin\frac{C}{2}\sin\left(\frac{A-B}{2}\right)\ldots\ldots(i)$$

As A + B + C = π

Hence, $\sin\frac{A}{2} = \sin\left(\frac{\pi}{2}-\frac{B+C}{2}\right) = \cos\left(\frac{B+C}{2}\right)\ldots\ldots(ii),\left(\text{as}\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta\right),$

Similarly, $\sin\frac{B}{2} = \sin\left(\frac{\pi}{2}-\frac{C+A}{2}\right) = \cos\left(\frac{C+A}{2}\right)\ldots\ldots(iii)$

And, $\sin\frac{C}{2} = \sin\left(\frac{\pi}{2}-\frac{A+B}{2}\right) = \cos\left(\frac{A+B}{2}\right)\ldots\ldots(iv)$

$$\Rightarrow = k\left[\sin(A)\cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right) + \sin B\cos\left(\frac{C+A}{2}\right)\sin\left(\frac{C-A}{2}\right) + \sin C\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

Now $\sin A - \sin B = 2\sin\frac{A-B}{2}\cos\frac{A+B}{2}$, so the above equation becomes,

$$\Rightarrow = \frac{k}{2}[\sin(A)(\sin B - \sin C) + \sin B(\sin C - \sin A) + \sin C(\sin A - \sin B)]$$

$$\Rightarrow = \frac{k}{2}[\sin A\sin B - \sin A\sin C + \sin B\sin C - \sin B\sin A + \sin C\sin A - \sin C\sin B]$$

Canceling the like terms we get

$$\Rightarrow = \frac{k}{2}(0) = 0 = \text{RHS}$$

Hence proved

21. Question

In any triangle ABC, prove the following:

$$\frac{b\sec B + c\sec C}{\tan B + \tan C} = \frac{c\sec C + a\sec A}{\tan C + \tan A} = \frac{a\sec A + b\sec B}{\tan A + \tan B}$$

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

⇒ a = k sin A, b = k sin B, c = k sin C

So the LHS of the given equation, we get

$$\text{LHS} = \frac{b\sec B + c\sec C}{\tan B + \tan C}$$

$$\Rightarrow = \frac{\frac{b}{\cos B} + \frac{c}{\cos C}}{\frac{\sin B}{\cos B} + \frac{\sin C}{\cos C}}\left(\because \sec A = \frac{1}{\cos A}, \tan A = \frac{\sin A}{\cos A}\right)$$

$$\Rightarrow = \frac{\frac{b\cos C + c\cos B}{\cos B\cos C}}{\frac{\sin B\cos C + \sin C\cos B}{\cos B\cos C}}$$

$$\Rightarrow = \frac{b\cos C + c\cos B}{\sin B\cos C + \sin C\cos B}$$

Substituting values from sine law, we get

$$\Rightarrow = \frac{k\sin B\cos C + k\sin C\cos B}{\sin B\cos C + \sin C\cos B}$$

$$\Rightarrow = \frac{k(\sin B\cos C + \sin C\cos B)}{\sin B\cos C + \sin C\cos B} = k \ldots..(i)$$

Now consider the second part of the equation, we get

$$\frac{c\sec C + a\sec A}{\tan C + \tan A}$$

$$\Rightarrow = \frac{\frac{c}{\cos C} + \frac{a}{\cos A}}{\frac{\sin C}{\cos C} + \frac{\sin A}{\cos A}} \left(\because \sec A = \frac{1}{\cos A}, \tan A = \frac{\sin A}{\cos A}\right)$$

$$\Rightarrow = \frac{\frac{c\cos A + a\cos C}{\cos A\cos C}}{\frac{\sin C\cos A + \sin A\cos C}{\cos A\cos C}}$$

$$\Rightarrow = \frac{c\cos A + a\cos C}{\sin C\cos A + \sin A\cos C}$$

Substituting values from sine law, we get

$$\Rightarrow = \frac{k\sin C\cos A + k\sin A\cos C}{\sin C\cos A + \sin A\cos C}$$

$$\Rightarrow = \frac{k(\sin C\cos A + \sin A\cos C)}{\sin C\cos A + \sin A\cos C} = k \ldots..(ii)$$

Now consider the third part of the equation, we get

$$\frac{a\sec A + b\sec B}{\tan A + \tan B}$$

$$\Rightarrow = \frac{\frac{a}{\cos A} + \frac{b}{\cos B}}{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}} \left(\because \sec A = \frac{1}{\cos A}, \tan A = \frac{\sin A}{\cos A}\right)$$

$$\Rightarrow = \frac{\frac{a\cos B + b\cos A}{\cos A\cos B}}{\frac{\sin A\cos B + \sin B\cos A}{\cos A\cos B}}$$

$$\Rightarrow = \frac{a\cos B + b\cos A}{\sin A\cos B + \sin B\cos A}$$

Substituting values from sine law, we get

$$\Rightarrow = \frac{k\sin A\cos B + k\sin B\cos A}{\sin A\cos B + \sin B\cos A}$$

$$\Rightarrow = \frac{k(\sin A\cos B + \sin B\cos A)}{\sin A\cos B + \sin B\cos A} = k \ldots..(iii)$$

From equation (i), (ii), and (iii), we get

$$\frac{b\sec B + c\sec C}{\tan B + \tan C} = \frac{c\sec C + a\sec A}{\tan C + \tan A} = \frac{a\sec A + b\sec B}{\tan A + \tan B}$$

Hence proved

22. Question

In any triangle ABC, prove the following:

a cos A + b cos B + c cos C = 2b sin A sin C = 2c sin A sin B

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

⇒ a = k sin A, b = k sin B, c = k sin C

So the LHS of the given equation, we get

LHS = a cos A + b cos B + c cos C

Substituting values from sine law, we get

= k sin A cos A + k sin B cos B + k sin C cos C

$$\Rightarrow = k\left[\frac{\sin 2A}{2} + \frac{\sin 2B}{2} + \sin C\cos C\right] (\because \sin 2A = 2\sin A\cos A)$$

$$\Rightarrow = \frac{k}{2}\left[2\sin\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right) + 2\sin C\cos C\right]$$

$$\left(\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\frac{(A-B)}{2}\right)$$

$$\Rightarrow = \frac{k}{2}[2\sin(A+B)\cos(A-B) + 2\sin C\cos C]$$

$$\Rightarrow = \frac{k}{2}\left[2\sin(\pi - C)\cos(A-B) + 2\sin C\cos\left(\pi - (A+B)\right)\right]$$

(∵ π = A + B + C)

$$\Rightarrow = \frac{k}{2}[2\sin(C)\cos(A-B) - 2\sin C\cos(A+B)]$$

$$(\because \sin(\pi-\theta) = \sin\theta, \cos(\pi-\theta) = -\cos\theta)$$

$$\Rightarrow = k\sin C\,[\cos(A-B) - \cos(A+B)]$$

$$\Rightarrow = k\sin C\,[2\sin A\sin B] \ldots (i)\ (\because \cos(A-B) - \cos(A+B) = 2\sin A\sin B)$$

Now, from sine rule,

k sin C = c

Putting this value in equation (i), we get

LHS = 2c sin A sin B

And also k sin B = b (from sine rule)

Putting this in equation (i), we get

LHS = 2b sin A sin C

Hence LHS = RHS

i.e., a cos A + b cos B + c cos C = 2b sin A sin C = 2c sin A sin B

Hence proved

23. Question

In any triangle ABC, prove the following:

a(cos B cos C + cos A) = b(cos C cos A + cos B) = c(cos A cos B + cos C)

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

⇒ a = k sin A, b = k sin B, c = k sin C

So by considering the LHS of the given equation, we get

a(cos B cos C + cos A)

Now substituting the values from sine rule, we get

⇒ = k sin A(cos B cos C + cos A)

⇒ = k (sin A cos B cos C + sin A cos A)

$$\Rightarrow = k\left\{\left(\frac{\sin(A + B) + \sin(A - B)}{2}\right)\cos C + \sin A\cos A\right\}$$

$(\because \sin(A + B) + \sin(A - B) = 2\sin A\cos B)$

$$\Rightarrow = \frac{k}{2}\{\sin(A + B)\cos C + \sin(A - B)\cos C + 2\sin A\cos A\}$$

$$\Rightarrow = \frac{k}{2}\left\{\frac{\sin(A + B + C) + \sin(A + B - C)}{2} + \frac{\sin(A - B + C) + \sin(A - B - C)}{2} + 2\sin A\cos A\right\}(\because \sin(A + B) + \sin(A - B) = 2\sin A\cos B)$$

$$\Rightarrow = \frac{k}{4}\{\sin(\pi) + \sin((\pi - C) - C) + \sin(\pi - B - B) + \sin(-(B + C - A)) + 2\sin 2A\}(\because \pi = A + B + C, \sin 2A = 2\sin A\cos A)$$

$$\Rightarrow = \frac{k}{4}\{\sin(\pi) + \sin(\pi - 2C) + \sin(\pi - 2B) - \sin((\pi - A - A) + 2\sin 2A\}$$

$$\Rightarrow = \frac{k}{4}\{\sin(2C) + \sin(2B) - \sin(2A) + 2\sin 2A\}(\because \sin(\pi - \theta) = \sin\theta)$$

$$\Rightarrow = \frac{k}{4}\{\sin(2C) + \sin(2B) + \sin 2A\} \ldots.(i)$$

Now consider the second part from the given equation, we get

b(cos A cos C + cos B)

Now substituting the values from sine rule, we get

⇒ = k sin B(cos A cos C + cos B)

⇒ = k (sin B cos A cos C + sin B cos B)

$$\Rightarrow = k\left\{\left(\frac{\sin(B + C) + \sin(B - C)}{2}\right)\cos A + \sin B\cos B\right\}$$

$(\because \sin(A + B) + \sin(A - B) = 2\sin A\cos B)$

$$\Rightarrow = \frac{k}{2}\{\sin(B + C)\cos A + \sin(B - C)\cos A + 2\sin B\cos B\}$$

$$\Rightarrow = \frac{k}{2}\left\{\frac{\sin(A + B + C) + \sin(B + C - A)}{2} + \frac{\sin(B - C + A) + \sin(B - C - A)}{2} + 2\sin B\cos B\right\}(\because \sin(A + B) + \sin(A - B) = 2\sin A\cos B)$$

$$\Rightarrow = \frac{k}{4}\{\sin(\pi) + \sin((\pi - A) - A) + \sin(\pi - C - C) + \sin(-(C + A - B)) + 2\sin 2B\}(\because \pi = A + B + C, \sin 2A = 2\sin A\cos A)$$

$$\Rightarrow = \frac{k}{4}\{\sin(\pi) + \sin(\pi - 2A) + \sin(\pi - 2C) - \sin((\pi - B - B) + 2\sin 2B\}$$

$$\Rightarrow = \frac{k}{4}\{\sin(2A) + \sin(2C) - \sin(2B) + 2\sin 2B\} \ (\because \sin(\pi - \theta) = \sin\theta)$$

$$\Rightarrow = \frac{k}{4}\{\sin(2C) + \sin(2B) + \sin 2A\} \ldots .(ii)$$

Now consider the third part from the given equation, we get

c(cos A cos B + cos C)

Now substituting the values from sine rule, we get

⇒ = k sin C(cos A cos B + cos C)

⇒ = k (sin C cos A cos B + sin C cos C)

$$\Rightarrow = k\left\{\left(\frac{\sin(C + A) + \sin(C - A)}{2}\right)\cos B + \sin C\cos C\right\}$$

$(\because \sin(A + B) + \sin(A - B) = 2\sin A\cos B)$

$$\Rightarrow = \frac{k}{2}\{\sin(A + C)\cos B + \sin(C - A)\cos B + 2\sin C\cos C\}$$

$$\Rightarrow = \frac{k}{2}\left\{\frac{\sin(A + B + C) + \sin(A + C - B)}{2} + \frac{\sin(C - A + B) + \sin(C - A - B)}{2} + 2\sin C\cos C\right\} (\because \sin(A + B) + \sin(A - B) = 2\sin A\cos B)$$

$$\Rightarrow = \frac{k}{4}\{\sin(\pi) + \sin((\pi - B) - B) + \sin(\pi - A - A) + \sin(-(A + B - C)) + 2\sin 2C\} \ (\because \pi = A + B + C, \sin 2A = 2\sin A\cos A)$$

$$\Rightarrow = \frac{k}{4}\{\sin(\pi) + \sin(\pi - 2B) + \sin(\pi - 2A) - \sin((\pi - C - C) + 2\sin 2C\}$$

$$\Rightarrow = \frac{k}{4}\{\sin(2B) + \sin(2A) - \sin(2C) + 2\sin 2C\} \ (\because \sin(\pi - \theta) = \sin\theta)$$

$$\Rightarrow = \frac{k}{4}\{\sin(2C) + \sin(2B) + \sin 2A\} \ldots .(iii)$$

From equation (i), (ii), and (iii), we get

a(cos B cos C + cos A) = b(cos C cos A + cos B) = c(cos A cos B + cos C)

Hence proved

24. Question

In any triangle ABC, prove the following:

$$a(\cos C - \cos B) = 2(b - c)\cos^2\frac{A}{2}$$

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$\Rightarrow$ a = k sin A, b = k sin B, c = k sin C

So by considering the LHS of the given equation, we get

$$\text{LHS} = a(\cos C - \cos B)$$

Substituting the corresponding values from sine rule, we get

$$\Rightarrow = k\sin A\,(\cos C - \cos B)$$

$$\Rightarrow = k(\sin A\cos C - \sin A\cos B)$$

$$\Rightarrow = k\left(\frac{\sin(A + C) + \sin(A - C)}{2} - \frac{\sin(A + B) + \sin(A - B)}{2}\right)$$

$(\because \sin(A + C) + \sin(A - C) = 2\sin A\cos C)$

$$\Rightarrow = \frac{k}{2}(\sin(\pi - B) + \sin(A - C) - (\sin(\pi - C) + \sin(A - B)))(\because \pi = A + B + C)$$

$$\Rightarrow = \frac{k}{2}(\sin(B) + \sin(A - C) - \sin(C) - \sin(A - B))\ (\because \sin(\pi - \theta) = \sin\theta)$$

Rearranging we get

$$\Rightarrow = \frac{k}{2}(\sin(B) - \sin(C) + \sin(A - C) - \sin(A - B))$$

$$\Rightarrow = \frac{k}{2}\left(2\sin\left(\frac{B - C}{2}\right)\cos\left(\frac{B + C}{2}\right) + 2\sin\left(\frac{A - C - (A - B)}{2}\right)\cos\left(\frac{A - C + (A - B)}{2}\right)\right)$$

$$\left(\because \sin A - \sin B = 2\cos\left[\frac{(A + B)}{2}\right]\sin\left[\frac{(A - B)}{2}\right]\right)$$

$$\Rightarrow = \frac{k}{2}\left(2\sin\left(\frac{B - C}{2}\right)\cos\left(\frac{B + C}{2}\right) + 2\sin\left(\frac{B - C}{2}\right)\cos\left(\frac{2A - (C + B)}{2}\right)\right)$$

$$\Rightarrow = k\sin\left(\frac{B - C}{2}\right)\left(\cos\left(\frac{B + C}{2}\right) + \cos\left(\frac{2A - (\pi - A)}{2}\right)\right)\ (\because \pi = A + B + C)$$

$$\Rightarrow = k\sin\left(\frac{B - C}{2}\right)\left(\cos\left(\frac{\pi - A}{2}\right) + \cos\left(\frac{-(\pi - 3A)}{2}\right)\right)\ (\because \pi = A + B + C)$$

$$\Rightarrow = k\sin\left(\frac{B - C}{2}\right)\left(\cos\left(\frac{\pi - A}{2}\right) + \cos\left(\frac{\pi - 3A}{2}\right)\right)\ (\because \cos(-\theta) = \cos\theta)$$

$$\Rightarrow = k\sin\left(\frac{B - C}{2}\right)\left(\sin\frac{A}{2} + \sin\frac{3A}{2}\right)\ \left(\because \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta\right)$$

$$\Rightarrow = k\sin\left(\frac{B - C}{2}\right)\left(2\sin\left(\frac{\frac{A}{2} + \frac{3A}{2}}{2}\right)\cos\left(\frac{\frac{3A}{2} - \frac{A}{2}}{2}\right)\right)$$

$$\left(\because \sin A + \sin B = 2\cos\left[\frac{(A - B)}{2}\right]\sin\left[\frac{(A + B)}{2}\right]\right)$$

$$\Rightarrow = k\sin\left(\frac{B - C}{2}\right)\left(2\sin\left(\frac{2A}{2}\right)\cos\left(\frac{A}{2}\right)\right)$$

$$\Rightarrow = 2k\sin\left(\frac{B-C}{2}\right)\left(\left(2\sin\frac{A}{2}\cos\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)\right) \ (\because \sin 2A = 2\sin A\cos A)$$

$$\Rightarrow = 4k\sin\left(\frac{B-C}{2}\right)\left(\sin\frac{A}{2}\cos^2\left(\frac{A}{2}\right)\right)$$

$$\Rightarrow = 4k\sin\left(\frac{B-C}{2}\right)\cos\left(\frac{\pi}{2}-\frac{A}{2}\right)\cos^2\left(\frac{A}{2}\right) \ \left(\because \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta\right)$$

$$\Rightarrow = 4k\sin\left(\frac{B-C}{2}\right)\cos\left(\frac{B+C}{2}\right)\cos^2\left(\frac{A}{2}\right) \ (\because \pi = A+B+C)$$

Regrouping this we get

$$\Rightarrow = 2k\left(2\sin\left(\frac{B-C}{2}\right)\cos\left(\frac{B+C}{2}\right)\right)\cos^2\left(\frac{A}{2}\right)$$

But $\left(\because \sin A - \sin B = 2\cos\left[\frac{(A+B)}{2}\right]\sin\left[\frac{(A-B)}{2}\right]\right)$

Hence the above equation becomes,

$$\Rightarrow = 2k(\sin B - \sin C)\cos^2\left(\frac{A}{2}\right)$$

$$\Rightarrow = 2(k\sin B - k\sin C)\cos^2\left(\frac{A}{2}\right)$$

(by applying sine rule)

$$\Rightarrow = 2(b-c)\cos^2\left(\frac{A}{2}\right) = \text{RHS}$$

Hence proved

25. Question

In ΔABC prove that, if θ be any angle, then b cos θ = c cos (A - θ) + a cos (C + θ)

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

⇒ a = k sin A, b = k sin B, c = k sin C

So by considering the RHS of the given equation, we get

RHS = c cos(A - θ) + a cos(C + θ)

Substituting the corresponding values from sine rule, we get

⇒ = k sin C cos(A - θ) + k sin A cos(C + θ)

$$\Rightarrow = k\left\{\frac{\sin(C+A-\theta) + \sin\left(C-(A-\theta)\right)}{2} + \frac{\sin(C+A+\theta) + \sin(A-(C+\theta))}{2}\right\}$$

$(\because \sin(A+B) + \sin(A-B) = 2\sin A\cos B)$

$$\Rightarrow = \frac{k}{2}\left\{\sin\left((\pi-B)-\theta\right) + \sin(C+\theta-A)) + \sin\left((\pi-B)+\theta\right) + \sin\left(-(C+\theta-A)\right)\right\} \ (\because \pi = A+B+C)$$

$$\Rightarrow = \frac{k}{2}\{\sin(\pi - (B + \theta)) + \sin(C + \theta - A)) + \sin(\pi - (B - \theta)) - \sin((C + \theta - A))\} (\because \sin(-\theta) = -\sin\theta)$$

$$\Rightarrow = \frac{k}{2}\{\sin(B + \theta) + \sin(C + \theta - A)) + \sin(B - \theta) - \sin(C + \theta - A)\} (\because \sin(\pi - \theta) = \sin\theta)$$

By cancelling like terms we get

$$\Rightarrow = \frac{k}{2}\{\sin(B + \theta) + \sin(B - \theta)\}$$

$$\Rightarrow = \frac{k}{2}\{2\sin B\cos\theta\} (\because 2\sin A\cos B = \sin(A + B) + \sin(A - B))$$

⇒ = k sin B cos θ

⇒ = b cos θ (from sine rule b = k sin B)

= LHS

Hence proved

26. Question

In a ΔABC, if $\sin^2 A + \sin^2 B = \sin^2 C$, show that the triangle is right angled.

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\Rightarrow \sin A = \frac{a}{k}, \sin B = \frac{b}{k}, \sin C = \frac{c}{k} \ldots\ldots (i)$$

So by considering the given condition, we get

$\sin^2 A + \sin^2 B = \sin^2 C$

Substituting the values from equation (i), we get

$$\Rightarrow \left(\frac{a}{k}\right)^2 + \left(\frac{b}{k}\right)^2 = \left(\frac{c}{k}\right)^2$$

$$\Rightarrow \frac{1}{k^2}(a^2 + b^2) = \frac{c^2}{k^2}$$

$$\Rightarrow a^2 + b^2 = c^2$$

This is Pythagoras theorem; hence the given triangle ABC is right - angles triangle

Hence proved

27. Question

In any ΔABC, if a^2, b^2, c^2 are in A.P., prove that cot A, cot B, and cot C are also in A.P

Answer

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k \text{ or } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$$

$$\Rightarrow \sin A = ak, \sin B = bk, \sin C = ck \ldots\ldots (i)$$

So by considering the given condition, we get

a^2, b^2, c^2 are in A.P

Then

$b^2 - a^2 = c^2 - b^2$ (this is the condition for A.P)

Substituting the values from equation (i), we get

$\Rightarrow (k \sin B)^2 - (k \sin A)^2 = (k \sin C)^2 - (k \sin B)^2$

$\Rightarrow k^2 (\sin^2 B - \sin^2 A) = k^2 (\sin^2 C - \sin^2 B)$

$\Rightarrow \sin (B + A) \sin (B - A) = \sin (C + B) \sin (C - B)$

$(\because \sin^2 A - \sin^2 B = \sin (A + B) \sin (A - B))$

$\Rightarrow \sin (\pi - C) \sin (B - A) = \sin (\pi - A) \sin (C - B)$ $(\because \pi = A + B + C)$

$\Rightarrow \sin (C) \sin (B - A) = \sin (A) \sin (C - B)$ $(\because \sin (\pi - \theta) = \sin \theta)$

Shuffling this, we get

$$\Rightarrow \frac{\sin(B-A)}{\sin A} = \frac{\sin(C-B)}{\sin C}$$

$$\Rightarrow \frac{1}{\sin B} \times \frac{\sin(B-A)}{\sin A} = \frac{\sin(C-B)}{\sin C} \times \frac{1}{\sin B}$$

$$\Rightarrow \frac{\sin B \cos A - \cos B \sin A}{\sin A \sin B} = \frac{\sin C \cos B - \cos C \sin B}{\sin C \sin B}$$

$(\because \sin(A-B) = \sin A \cos B - \cos A \sin B)$

$$\Rightarrow \frac{\sin B \cos A}{\sin A \sin B} - \frac{\cos B \sin A}{\sin A \sin B} = \frac{\sin C \cos B}{\sin C \sin B} - \frac{\sin B \cos C}{\sin A \sin B}$$

Canceling the like terms we get

$$\Rightarrow \frac{\cos A}{\sin A} - \frac{\cos B}{\sin B} = \frac{\cos B}{\sin B} - \frac{\cos C}{\sin A}$$

But $\frac{\cos A}{\sin A} = \cot A$, so the above equation becomes,

$\Rightarrow \cot A - \cot B = \cot B - \cot C$

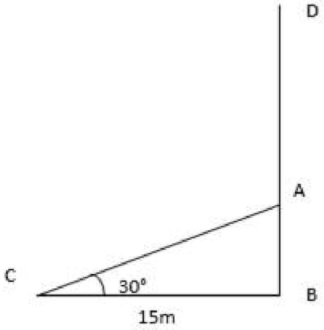
Hence cot A, cot B, cot C are in AP

Hence proved

28. Question

The upper part of a tree broken over by the wind makes an angle of 30^{o} with the ground, and the distance from the root to the point where the top of the tree touches the ground is 15 m. Using sine rule, find the height of the tree.

Answer



Let BD be the tree, let A be the point where the tree is broken by the wind.

And according to the given condition,

AD = AC

Now let AB = x and AC = y = AD

So the total height of the tree is AB + AD = x + y

Now in ΔABC,

∠C = 30°, ∠B = 90°,

So, ∠A = 180° - (∠B + ∠C) (∵ π = A + B + C)

Hence ∠A = 180° - (90° + 30°) = 60°

Now applying sine rule, we get

$$\frac{BC}{\sin A} = \frac{AC}{\sin B} = \frac{AB}{\sin C}$$

Now substituting the values obtained, we get

$$\frac{15}{\sin 60°} = \frac{y}{\sin 90°} = \frac{x}{\sin 30°}$$

Substituting the corresponding values, we get

$$\Rightarrow \frac{15}{\frac{\sqrt{3}}{2}} = \frac{y}{1} = \frac{x}{\frac{1}{2}}$$

$$\Rightarrow \frac{15 \times 2}{\sqrt{3}} = y = 2x \ldots (i)$$

So,

$$\Rightarrow \frac{15 \times 2}{\sqrt{3}} = 2x$$

$$\Rightarrow \frac{15 \times 2}{2\sqrt{3}} = x$$

$$\Rightarrow x = \frac{15 \times 2}{2\sqrt{3}} = 5\sqrt{3}m$$

And also from (i),

$$\Rightarrow \frac{15 \times 2}{\sqrt{3}} = y$$

⇒y = 10√3 m

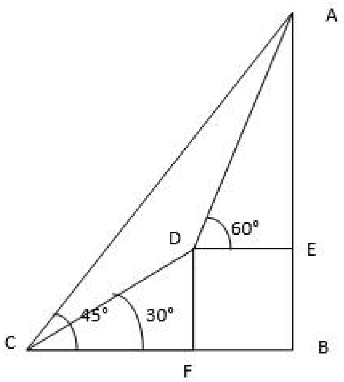
So the height of the tree is

$x + y = 5\sqrt{3} + 10\sqrt{3} = 15\sqrt{3}$ m

29. Question

At the foot of a mountain, the elevation of its summit is 45^o, after ascending 1000 m towards the mountain up a slope of 30^o inclination, the elevation is found to be 60^o. Find the height of the mountain.

Answer



Let AB be the mountain, so the at the foot of a mountain the elevation of its summit is 45^o

So, ∠ACB = 45°

Now when moving on the slope of 30° by a distance of 1000m,

i.e., the CD is the distance moved on the slope of 30° towards the mountain,

Hence CD = 1000m...........(i)

And ∠DCF = 30°

Let EB = FD = x........(ii)

DE = FB = z.......(iii)

CF = y and AE = t.........(iv)

So after moving 1000m, the elevation becomes 60°,

So ∠ADE = 60°

In ΔDFC,

$$\sin 30° = \frac{DF}{CD} \left(\because \sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}\right)$$

$$\Rightarrow \frac{1}{2} = \frac{x}{1000} \text{ (from (ii))}$$

$$\Rightarrow x = 500\text{m} \ldots\ldots \text{(iv)}$$

And

$$\tan 30° = \frac{DF}{CF} \left(\because \tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}\right)$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{y} \text{ (from (ii) and (iv))}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{500}{y} \text{ (from (iv))}$$

Hence $y = 500\sqrt{3}\text{ m} \ldots.. \text{(v)}$

In ΔADE,

$\tan 60° = \frac{AE}{DE}\left(\because \tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}\right)$

$\Rightarrow \sqrt{3} = \frac{t}{z}$

⇒t = z√3.........(vi)

In ΔABC,

$\tan 45° = \frac{AB}{CB}\left(\because \tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}\right)$

$\Rightarrow 1 = \frac{AE + EB}{CF + FB}$

$\Rightarrow 1 = \frac{t + x}{y + z}$

⇒t + x = y + z

⇒z√3 + 500 = 500√3 + z (from (iv), (v), (vi))

⇒z√3-z = 500√3-500

⇒z(√3-1) = 500(√3-1)

⇒z = 500m......(vii)

Hence the equation (vi) becomes,

t = z√3 = (500) √3 m

Hence the height of the mountain is

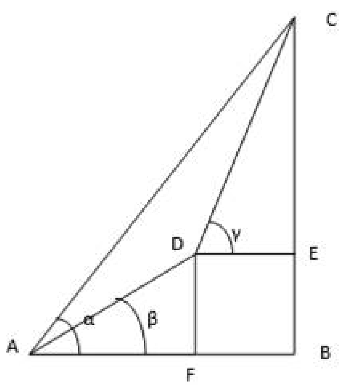
AB = AE + EB = t + x = (500√3 + 500)m

So the height of the mountain is 500(√3 + 1)m.

30. Question

A person observes the angle of elevation of the peak of a hill from a station to be α. He walks c meters along a slope inclined at the angle β and finds the angle of elevation of the peak of the hill to be γ. Show that the height of the peal above the ground is $\frac{c\sin\alpha\sin(\gamma-\beta)}{\sin(\gamma-\alpha)}$

Answer



Let AB be the peak of a hill, so the at the station A the elevation of its summit is α°

So, ∠CAB = α°

Now when moving on the slope of β° by a distance of ‘c’m,

i.e., AD is the distance moved on the slope of β° towards the hill, Hence AD = ’c’m...........(i)

And ∠DAF = β°

Let EB = FD = x........(ii)

DE = FB = z.......(iii)

AF = y and CE = t.........(iv)

So after moving ‘c’m, the elevation becomes γ°,

So ∠CDE = γ°

In ΔDFA,

$$\sin\beta^\circ = \frac{DF}{AD}\left(\because \sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}\right)$$

$$\Rightarrow \sin\beta = \frac{x}{c}\,(\text{from (ii)})$$

$$\Rightarrow x = (c\sin\beta)m \ldots\ldots (iv)$$

And

$$\tan\beta^\circ = \frac{DF}{AF}\left(\because \tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}\right)$$

$$\Rightarrow \tan\beta = \frac{x}{y}\,(\text{from (ii) and (iv)})$$

$$\Rightarrow \frac{\sin\beta}{\cos\beta} = \frac{c\sin\beta}{y}\,(\text{from (iv)})$$

$$\Rightarrow y = (c\cos\beta)\ m \ldots\ldots (v)$$

In ΔCDE,

$$\tan\gamma^\circ = \frac{CE}{DE}\left(\because \tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}\right)$$

$$\Rightarrow \tan\gamma = \frac{t}{z}$$

⇒z = t cot γ.........(vi)

In ΔCBA,

$$\tan\alpha^\circ = \frac{CB}{AB}\left(\because \tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}\right)$$

$$\Rightarrow \tan\alpha = \frac{CE + EB}{AF + FB}$$

$$\Rightarrow \tan\alpha = \frac{t + x}{y + z}$$

$$\Rightarrow \tan\alpha = \frac{t + c\sin\beta}{c\cos\beta + t\cot\gamma}\,(\text{from (iv), (v), (vi)})$$

⇒tan α (c cos β + t cot γ) = t + c sin β

⇒(c tan α cos β + t tan α cot γ) = t + c sin β

⇒t-t tan α cot γ = c tan α cos β-c sin β

⇒t(1 - tan α cot γ) = c(tan α cos β-sin β)

$$\Rightarrow t = c\left(\frac{\tan\alpha\cos\beta - \sin\beta}{1 - \tan\alpha\cot\gamma}\right)$$

$$\Rightarrow t = c\left(\frac{\frac{\sin\alpha}{\cos\alpha}\times\cos\beta - \sin\beta}{1 - \frac{\sin\alpha}{\cos\alpha}\times\frac{\cos\gamma}{\sin\gamma}}\right)$$

$$\Rightarrow t = c\left(\frac{\frac{\sin\alpha\cos\beta - \sin\beta\cos\alpha}{\cos\alpha}}{\frac{\cos\alpha\sin\gamma - \sin\alpha\cos\gamma}{\cos\alpha\sin\gamma}}\right)$$

$$\Rightarrow t = c\left(\frac{\frac{\sin(\alpha-\beta)}{1}}{\frac{\sin(\gamma-\alpha)}{\sin\gamma}}\right)\ (\because\ \sin(A - B) = \sin A\cos B - \cos A\sin B)$$

$$\Rightarrow t = c\left(\frac{\sin\gamma\sin(\alpha-\beta)}{\sin(\gamma-\alpha)}\right) \dots\dots (vii)$$

Now,

AB = AE + EB = t + x

$$AB = c\left(\frac{\sin\gamma\sin(\alpha-\beta)}{\sin(\gamma-\alpha)}\right) + c\sin\beta\ \left(\text{from (iv)and (vii)}\right)$$

$$AB = c\left(\frac{\sin\gamma\sin(\alpha-\beta)}{\sin(\gamma-\alpha)} + \sin\beta\right)$$

$$AB = c\left(\frac{\sin\gamma\sin(\alpha-\beta) + \sin\beta\sin(\gamma-\alpha)}{\sin(\gamma-\alpha)}\right)$$

$$AB = c\left(\frac{\sin\gamma(\sin\alpha\cos\beta - \cos\alpha\sin\beta) + \sin\beta(\sin\gamma\cos\alpha - \cos\gamma\sin\alpha)}{\sin(\gamma-\alpha)}\right)$$

$(\because\ \sin(A - B) = \sin A\cos B - \cos A\sin B)$

$$AB = c\left(\frac{(\sin\alpha\cos\beta\sin\gamma - \cos\alpha\sin\beta\sin\gamma) + (\sin\gamma\cos\alpha\sin\beta - \cos\gamma\sin\alpha\sin\beta)}{\sin(\gamma-\alpha)}\right)$$

$$AB = c\left(\frac{(\sin\alpha\cos\beta\sin\gamma) - \cos\gamma\sin\alpha\sin\beta}{\sin(\gamma-\alpha)}\right)$$

$$AB = c\left(\frac{\sin\alpha(\cos\beta\sin\gamma - \cos\gamma\sin\beta)}{\sin(\gamma-\alpha)}\right)$$

$$AB = c\left(\frac{\sin\alpha(\sin(\gamma-\beta))}{\sin(\gamma-\alpha)}\right)$$

So the height of the hill is $c\left(\frac{\sin\alpha(\sin(\gamma-\beta))}{\sin(\gamma-\alpha)}\right)$

Hence proved

31. Question

If the sides a, b, c of a ΔABC is in H.P., prove that $\sin^2\frac{A}{2}, \sin^2\frac{B}{2}, \sin^2\frac{C}{2}$ are in H.P.

Answer

As a, b, c is in HP (given)

So, $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP

Hence

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b} \ldots.(i)$$

Let a, b, c be the sides of any triangle ABC. Then by applying the sine rule, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

So, a = k sin A, b = k sin B, c = k sin C...(ii)

Substituting equation (ii) in equation (i), we get

$$\frac{1}{k \sin B} - \frac{1}{k \sin A} = \frac{1}{k \sin C} - \frac{1}{k \sin B}$$

$$\Rightarrow \frac{k \sin A - k \sin B}{k^2 \sin B \sin A} = \frac{k \sin B - k \sin C}{k^2 \sin C \sin B}$$

$$\Rightarrow \frac{\sin A - \sin B}{\cancel{\sin B} \sin A} = \frac{\sin B - \sin C}{\sin C \cancel{\sin B}}$$

$$\Rightarrow \frac{\cancel{2} \cos\left(\frac{A + B}{2}\right) \sin\left(\frac{A - B}{2}\right)}{\sin A} = \frac{\cancel{2} \cos\left(\frac{B + C}{2}\right) \sin\left(\frac{B - C}{2}\right)}{\sin C}$$

$$\left(\because \sin A - \sin B = 2 \cos\left(\frac{A + B}{2}\right) \sin\left(\frac{A - B}{2}\right)\right)$$

$$\Rightarrow \frac{\cos\left(\frac{\pi - C}{2}\right) \sin\left(\frac{A - B}{2}\right)}{\cancel{2} \sin\frac{A}{2} \cos\frac{A}{2}} = \frac{\cos\left(\frac{\pi - A}{2}\right) \sin\left(\frac{B - C}{2}\right)}{\cancel{2} \sin\frac{C}{2} \cos\frac{C}{2}}$$

$$(\text{as } \sin 2A = 2 \sin A \cos A \text{ and } \pi = A + B + C)$$

$$\Rightarrow \frac{\sin\frac{C}{2} \sin\left(\frac{A - B}{2}\right)}{\sin\frac{A}{2} \cos\frac{A}{2}} = \frac{\sin\frac{A}{2} \sin\left(\frac{B - C}{2}\right)}{\sin\frac{C}{2} \cos\frac{C}{2}} \left(\because \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta\right)$$

By cross multiplying we get

$$\Rightarrow \sin\frac{C}{2} \sin\left(\frac{A - B}{2}\right) \sin\frac{C}{2} \cos\frac{C}{2} = \sin\frac{A}{2} \sin\left(\frac{B - C}{2}\right) \sin\frac{A}{2} \cos\frac{A}{2}$$

$$\Rightarrow \sin^2\frac{C}{2} \sin\left(\frac{A - B}{2}\right) \cos\left(\frac{(\pi - (A + B))}{2}\right)$$
$$= \sin^2\frac{A}{2} \sin\left(\frac{B - C}{2}\right) \cos\left(\frac{\pi - (B + C)}{2}\right) (\because \pi = A + B + C)$$

Now, $\left(\because \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta\right)$, so above equation becomes,

$$\Rightarrow \sin^2\frac{C}{2}\left(\sin\left(\frac{A - B}{2}\right) \sin\left(\frac{((A + B))}{2}\right)\right) = \sin^2\frac{A}{2}\left(\sin\left(\frac{B - C}{2}\right) \sin\left(\frac{(B + C)}{2}\right)\right)$$

$\left(\text{But } \sin\left(\frac{A + B}{2}\right) \sin\left(\frac{A - B}{2}\right) = \sin^2\frac{A}{2} - \sin^2\frac{B}{2}\right)$, so the above equation becomes,

$$\Rightarrow \sin^2\frac{C}{2}\left(\sin^2\frac{A}{2} - \sin^2\frac{B}{2}\right) = \sin^2\frac{A}{2}\left(\sin^2\frac{B}{2} - \sin^2\frac{C}{2}\right)$$

$$\Rightarrow \sin^2\frac{C}{2}\sin^2\frac{A}{2} - \sin^2\frac{C}{2}\sin^2\frac{B}{2} = \sin^2\frac{A}{2}\sin^2\frac{B}{2} - \sin^2\frac{A}{2}\sin^2\frac{C}{2}$$

Divide both sides by $\sin^2\frac{C}{2}\sin^2\frac{A}{2}\sin^2\frac{B}{2}$, we get

$$\Rightarrow \frac{\sin^2\frac{C}{2}\sin^2\frac{A}{2} - \sin^2\frac{C}{2}\sin^2\frac{B}{2}}{\sin^2\frac{C}{2}\sin^2\frac{A}{2}\sin^2\frac{B}{2}} = \frac{\left(\sin^2\frac{A}{2}\sin^2\frac{B}{2} - \sin^2\frac{A}{2}\sin^2\frac{C}{2}\right)}{\sin^2\frac{C}{2}\sin^2\frac{A}{2}\sin^2\frac{B}{2}}$$

$$\Rightarrow \frac{\sin^2\frac{C}{2}\sin^2\frac{A}{2}}{\sin^2\frac{C}{2}\sin^2\frac{A}{2}\sin^2\frac{B}{2}} - \frac{\sin^2\frac{C}{2}\sin^2\frac{B}{2}}{\sin^2\frac{C}{2}\sin^2\frac{A}{2}\sin^2\frac{B}{2}} = \frac{\sin^2\frac{A}{2}\sin^2\frac{B}{2}}{\sin^2\frac{C}{2}\sin^2\frac{A}{2}\sin^2\frac{B}{2}} - \frac{\sin^2\frac{A}{2}\sin^2\frac{C}{2}}{\sin^2\frac{C}{2}\sin^2\frac{A}{2}\sin^2\frac{B}{2}}$$

Now canceling the like terms we get

$$\Rightarrow \frac{1}{\sin^2\frac{B}{2}} - \frac{1}{\sin^2\frac{A}{2}} = \frac{1}{\sin^2\frac{C}{2}} - \frac{1}{\sin^2\frac{B}{2}}$$

Hence $\frac{1}{\sin^2\frac{A}{2}}, \frac{1}{\sin^2\frac{B}{2}}, \frac{1}{\sin^2\frac{C}{2}}$ are in AP

Therefore,

$\sin^2\frac{A}{2}, \sin^2\frac{B}{2}, \sin^2\frac{C}{2}$ are in HP

Hence proved

Exercise 10.2

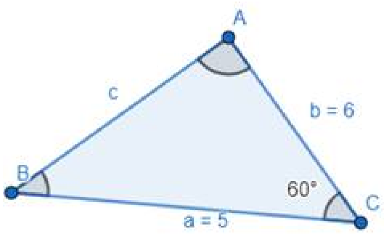
1. Question

In a ΔABC, if a = 5, b = 6 and C = 60°, show that its area is $\frac{15\sqrt{3}}{2}$ sq. units.

Answer

Note: In any ΔABC we define ‘a’ as length of side opposite to ∠A , ‘b’ as length of side opposite to ∠B and ‘c’ as length of side opposite to ∠C .

Key point to solve the problem:



Area of ΔABC = $\frac{1}{2}ab\sin\theta$,where θ is the angle between sides BC and AC , a is the length of BC and b is length of AC

We have,

a = 5 , b = 6 and ∠C = 60°

∴ Area of ΔABC = $\frac{1}{2} \times 5 \times 6 \times \sin 60°$

$= \frac{30}{2} \times \frac{\sqrt{3}}{2} = \frac{15\sqrt{3}}{2}\ sq\ units$

1. Question

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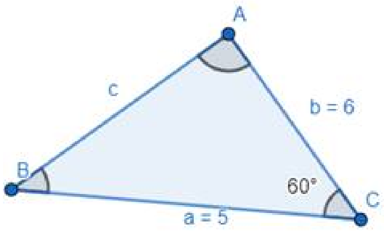


Area of ΔABC = $\frac{1}{2}ab\sin\theta$,where θ is the angle between sides BC and AC , a is the length of BC and b is length of AC

We have,

a = 5 , b = 6 and ∠C = 60°

∴ Area of ΔABC = $\frac{1}{2} \times 5 \times 6 \times \sin 60°$

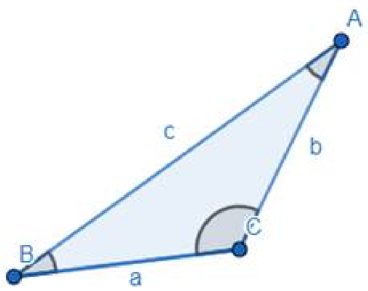
$= \frac{30}{2} \times \frac{\sqrt{3}}{2} = \frac{15\sqrt{3}}{2}\ sq\ units$

2. Question

In a ΔABC, if $a = \sqrt{2}, b = \sqrt{3} \text{ and } c = \sqrt{5},$ show that its area is $\frac{1}{2}\sqrt{6}$ sq.units.

Answer

Note: In any ΔABC we define 'a' as length of side opposite to ∠A , 'b' as length of side opposite to ∠B and 'c' as length of side opposite to ∠C .



Key point to solve the problem:

• Area of ΔABC = 0.5×(product of any two sides)×(sine of angle between them)

• Idea of cosine formula: Cos A = $\frac{b^2+c^2-a^2}{2bc}$

Given $a = \sqrt{2}, b = \sqrt{3}\ and\ c = \sqrt{5}$

∴ Cos A = $\frac{(\sqrt{3})^2+(\sqrt{5})^2-(\sqrt{2})^2}{2\sqrt{3}\sqrt{5}} = \frac{3+5-2}{2\sqrt{15}} = \frac{3}{\sqrt{15}}$

∵ Area of ΔABC = $\frac{1}{2}\, bc \sin A$

We need to find sin A

As we know that - $\sin^2 A = 1 - \cos^2 A$ {using trigonometric identity}

∴ sin A = $\sqrt{1-\left(\frac{3}{\sqrt{15}}\right)^2} = \sqrt{1-\frac{9}{15}} = \sqrt{\frac{6}{15}}$

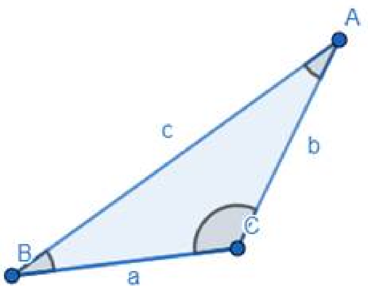
∴ ar(ΔABC) = $\frac{1}{2} \times \sqrt{3} \times \sqrt{5} \times \sqrt{\frac{6}{15}} = \frac{1}{2}\sqrt{6}$ sq units. **...ans**

2. Question

In a ΔABC, if $a = \sqrt{2}, b = \sqrt{3} \text{ and } c = \sqrt{5},$ show that its area is $\frac{1}{2}\sqrt{6}$ sq.units.

Answer

Note: In any ΔABC we define 'a' as length of side opposite to ∠A , 'b' as length of side opposite to ∠B and 'c' as length of side opposite to ∠C .



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∴ ar(ΔABC) = $\frac{1}{2} \times \sqrt{3} \times \sqrt{5} \times \sqrt{\frac{6}{15}} = \frac{1}{2}\sqrt{6}$ sq units. **...ans**

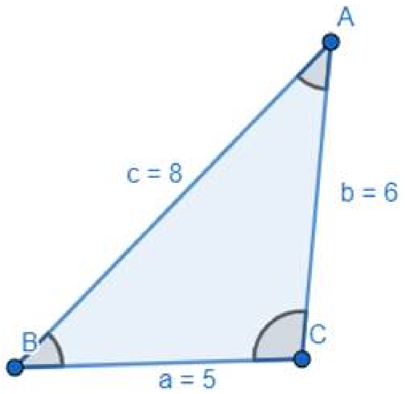
3. Question

The sides of a triangle are a = 5, b = 6 and c = 8,

show that: 8 cos A + 16 cos B + 4 cos C = 17

Answer

Note: In any ΔABC we define 'a' as length of side opposite to ∠A , 'b' as length of side opposite to ∠B and 'c' as length of side opposite to ∠C .



Key point to solve the problem:

Idea of cosine formula in ΔABC

• Cos A = $\frac{b^2+c^2-a^2}{2bc}$

• Cos B = $\frac{a^2+c^2-b^2}{2ac}$

• Cos C = $\frac{b^2+a^2-c^2}{2ac}$

As we have a = 5, b = 6 and c = 8

∴ Cos A = $\frac{6^2+8^2-5^2}{2\times8\times6} = \frac{75}{96}$

Cos B = $\frac{5^2+8^2-6^2}{2\times5\times8} = \frac{53}{80}$

Cos C = $\frac{6^2+5^2-8^2}{2\times6\times5} = \frac{-3}{60} = -\frac{1}{20}$

We have to prove:

8 cos A + 16 cos B + 4 cos C = 17

LHS = 8 cos A + 16 cos B + 4 cos C

Putting the values of cos A, cos B and cos C in LHS

LHS = $8 \times \frac{75}{96} + 16 \times \frac{53}{80} - 4 \times \frac{1}{20} = \frac{75}{12} + \frac{53}{5} - \frac{1}{5} = \frac{999}{60} = 16.65$

LHS ≠ RHS

From cosine expressions we have:

96 cos A = 75 , 80 cos B = 53 and 20 cos C = -1

Adding all we have,

96 cos A + 80 cos B +20 cos C = 75+53-1 = 127

∴ 96 cos A + 80 cos B +20 cos C = 127

Please check it....

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The sides of a triangle are a = 5, b = 6 and c = 8,

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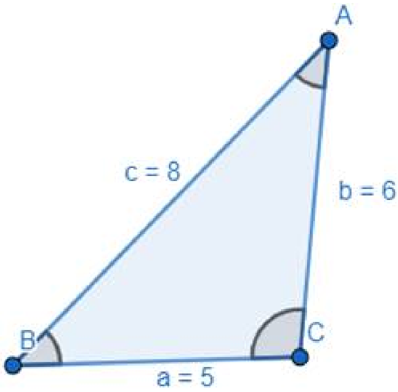

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∴ 96 cos A + 80 cos B +20 cos C = 127

Please check it….

4. Question

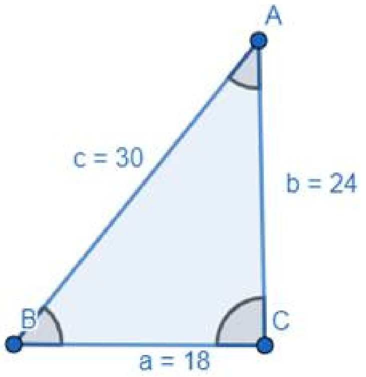
In a ΔABC, if a = 18, b = 24, c = 30, find cos A, cos B and cos C.

Answer

Note: In any ΔABC we define 'a' as length of side opposite to ∠A , 'b' as length of side opposite to ∠B and 'c' as length of side opposite to ∠C .

Key point to solve the problem:

Idea of cosine formula in ΔABC

- 

Cos A = $\frac{b^2+c^2-a^2}{2bc}$

- Cos B = $\frac{a^2+c^2-b^2}{2ac}$
- Cos C = $\frac{b^2+a^2-c^2}{2ac}$

As we have a = 18, b = 24 and c = 30

∴ Cos A = $\frac{24^2+30^2-18^2}{2\times24\times30} = \frac{24}{30} = \frac{4}{5}$

Cos B = $\frac{18^2+30^2-24^2}{2\times18\times30} = \frac{18}{30} = \frac{3}{5}$

Cos C = $\frac{18^2+24^2-30^2}{2\times18\times24} = \frac{0}{864} = 0$

4. Question

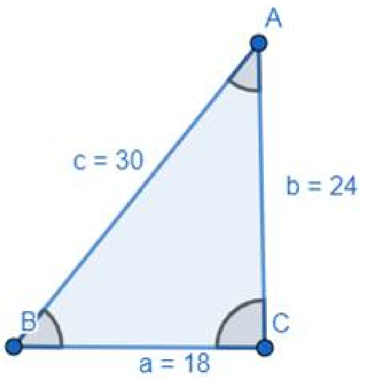
In a ΔABC, if a = 18, b = 24, c = 30, find cos A, cos B and cos C.

Answer

Note: In any ΔABC we define 'a' as length of side opposite to ∠A , 'b' as length of side opposite to ∠B and 'c' as length of side opposite to ∠C .

Key point to solve the problem:

Idea of cosine formula in ΔABC

- 

Cos A = $\frac{b^2+c^2-a^2}{2bc}$

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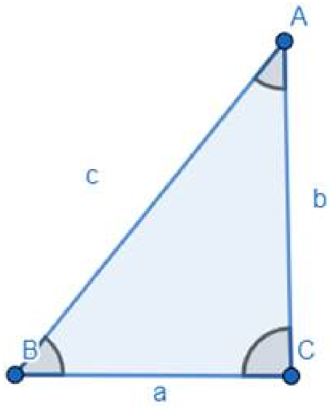
Cos C = $\frac{18^2+24^2-30^2}{2\times18\times24} = \frac{0}{864} = 0$

5. Question

For any ΔABC, show that - b (c cos A – a cos C) = c^2 – a^2

Answer

Note: In any ΔABC we define 'a' as length of side opposite to ∠A , 'b' as length of side opposite to ∠B and 'c' as length of side opposite to ∠C .



Key point to solve the problem:

Idea of cosine formula in ΔABC

- Cos A = $\frac{b^2+c^2-a^2}{2bc}$
- Cos B = $\frac{a^2+c^2-b^2}{2ac}$
- Cos C = $\frac{b^2+a^2-c^2}{2ab}$

As we have to prove:

b (c cos A – a cos C) = c^2 – a^2

As LHS contain bc cos A and ab cos C which can be obtained from cosine formulae.

∴ From cosine formula we have:

Cos A = $\frac{b^2+c^2-a^2}{2bc}$

⇒ bc cos A = $\frac{b^2+c^2-a^2}{2}$ .....eqn 1

And Cos C = $\frac{b^2+a^2-c^2}{2ab}$

⇒ ab cos C = $\frac{b^2+a^2-c^2}{2}$ ......eqn 2

Subtracting eqn 2 from eqn 1:

bc cos A - ab cos C = $\frac{b^2+c^2-a^2}{2}$ - $\frac{b^2+a^2-c^2}{2}$

⇒ bc cos A - ab cos C = c^2 - a^2

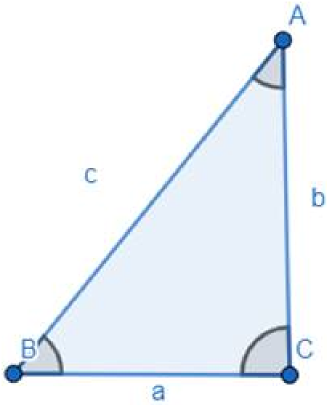
∴ b(c cos A - a cos C) = c^2 - a^2 ...**proved**

5. Question

For any ΔABC, show that - b (c cos A – a cos C) = c^2 – a^2

Answer

Note: In any ΔABC we define 'a' as length of side opposite to ∠A , 'b' as length of side opposite to ∠B and 'c' as length of side opposite to ∠C .



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⇒ bc cos A - ab cos C = c^2 - a^2

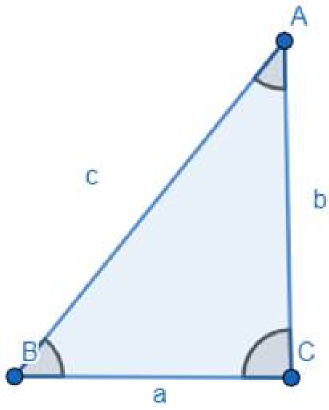
∴ b(c cos A - a cos C) = c^2 - a^2 ...**proved**

6. Question

For any Δ ABC show that - c (a cos B – b cos A) = a^2 – b^2

Answer

Note: In any ΔABC we define ‘a’ as length of side opposite to ∠A , ‘b’ as length of side opposite to ∠B and ‘c’ as length of side opposite to ∠C .



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Idea of cosine formula in ΔABC

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- Cos B = $\frac{a^2+c^2-b^2}{2ac}$
- Cos C = $\frac{b^2+a^2-c^2}{2ab}$

As we have to prove:

c (a cos B – b cos A) = a^2 – b^2

As LHS contain ca cos B and cb cos A which can be obtained from cosine formulae.

∴ From cosine formula we have:

Cos A = $\frac{b^2+c^2-a^2}{2bc}$

⇒ bc cos A = $\frac{b^2+c^2-a^2}{2}$ .....eqn 1

And Cos B = $\frac{c^2+a^2-b^2}{2ac}$

⇒ ac cos B = $\frac{c^2+a^2-b^2}{2}$ ......eqn 2

Subtracting eqn 1 from eqn 2:

ac cos B - bc cos A = $\frac{c^2+a^2-b^2}{2} - \frac{b^2+c^2-a^2}{2}$

⇒ ac cos B - bc cos A = $a^2 - b^2$

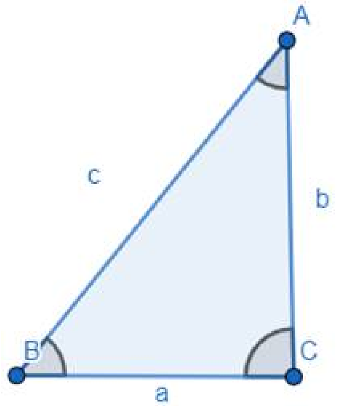
∴ c (a cos B – b cos A) = a^2 – b^2 ...proved

6. Question

For any Δ ABC show that - c (a cos B – b cos A) = a^2 – b^2

Answer

Note: In any ΔABC we define ‘a’ as length of side opposite to ∠A , ‘b’ as length of side opposite to ∠B and ‘c’ as length of side opposite to ∠C .



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• Cos C = $\frac{b^2+a^2-c^2}{2ab}$

As we have to prove:

c (a cos B – b cos A) = $a^2 - b^2$

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Cos A = $\frac{b^2+c^2-a^2}{2bc}$

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⇒ ac cos B = $\frac{c^2+a^2-b^2}{2}$ ......eqn 2

Subtracting eqn 1 from eqn 2:

ac cos B - bc cos A = $\frac{c^2+a^2-b^2}{2} - \frac{b^2+c^2-a^2}{2}$

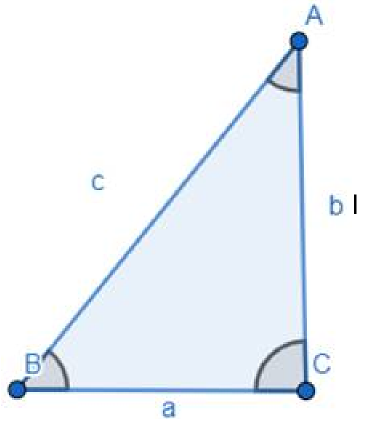
⇒ ac cos B - bc cos A = $a^2 - b^2$

∴ c (a cos B – b cos A) = $a^2 - b^2$ …proved

7. Question

For any Δ ABC show that-

2 (bc cos A + ca cos B + ab cos C) = $a^2 + b^2 + c^2$



Answer

Note: In any ΔABC we define 'a' as length of side opposite to ∠A , 'b' as length of side opposite to ∠B and 'c' as length of side opposite to ∠C .

Key point to solve the problem:

Idea of cosine formula in ΔABC

• Cos A = $\frac{b^2+c^2-a^2}{2bc}$

• Cos B = $\frac{a^2+c^2-b^2}{2ac}$

• Cos C = $\frac{b^2+a^2-c^2}{2ab}$

As we have to prove:

2 (bc cos A + ca cos B + ab cos C) = $a^2 + b^2 + c^2$

As LHS contain 2ca cos B, 2ab cos C and 2cb cos A ,which can be obtained from cosine formulae.

∴ From cosine formula we have:

Cos A = $\frac{b^2+c^2-a^2}{2bc}$

⇒ 2bc cos A =$b^2 + c^2 - a^2$ …..eqn 1

Cos C = $\frac{b^2+a^2-c^2}{2ab}$

⇒ 2ab cos C = $b^2 + a^2 - c^2$ …eqn 2

And, Cos B = $\frac{c^2+a^2-b^2}{2ac}$

⇒ 2ac cos B = $c^2 + a^2 - b^2$ ……eqn 3

Adding eqn 1,2 and 3:-

2bc cos A + 2ab cos C + 2ac cos B = $c^2 + a^2 - b^2 + b^2 + a^2 - c^2 + b^2 + c^2 - a^2$

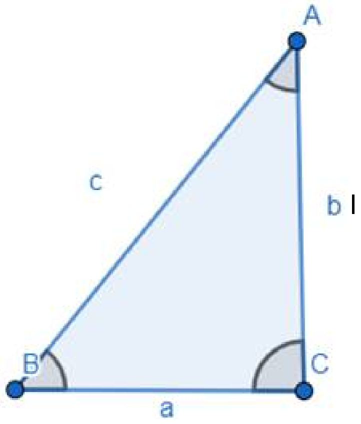
⇒ 2bc cos A + 2ab cos C + 2ac cos B = $c^2 + a^2 + b^2$

⇒ 2(bc cos A + ab cos C + ac cos B) = $a^2 + b^2 + c^2$ …**proved**

7. Question

For any Δ ABC show that-

2 (bc cos A + ca cos B + ab cos C) = $a^2 + b^2 + c^2$



Answer

Note: In any ΔABC we define 'a' as length of side opposite to ∠A , 'b' as length of side opposite to ∠B and 'c' as length of side opposite to ∠C .

Key point to solve the problem:

Idea of cosine formula in ΔABC

• Cos A = $\frac{b^2+c^2-a^2}{2bc}$

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Cos A = $\frac{b^2+c^2-a^2}{2bc}$

⇒ 2bc cos A =$b^2 + c^2 - a^2$ .....eqn 1

Cos C = $\frac{b^2+a^2-c^2}{2ab}$

⇒ 2ab cos C = $b^2 + a^2 - c^2$ ...eqn 2

And, Cos B = $\frac{c^2+a^2-b^2}{2ac}$

⇒ 2ac cos B = $c^2 + a^2 - b^2$ ......eqn 3

Adding eqn 1,2 and 3:-

2bc cos A + 2ab cos C + 2ac cos B = $c^2 + a^2 - b^2 + b^2 + a^2 - c^2 + b^2 + c^2 - a^2$

⇒ 2bc cos A + 2ab cos C + 2ac cos B = $c^2 + a^2 + b^2$

⇒ 2(bc cos A + ab cos C + ac cos B) = $a^2 + b^2 + c^2$ ...**proved**

8. Question

For any Δ ABC show that-

$(c^2 - a^2 + b^2)$ tan A = $(a^2 - b^2 + c^2)$ tan B = $(b^2 - c^2 + a^2)$ tan C

Answer

Note: In any ΔABC we define 'a' as length of side opposite to ∠A , 'b' as length of side opposite to ∠B and 'c' as length of side opposite to ∠C .

The key point to solve the problem:

The idea of cosine formula in ΔABC

• Cos A = $\frac{b^2+c^2-a^2}{2bc}$

• Cos B = $\frac{a^2+c^2-b^2}{2ac}$

• Cos C = $\frac{b^2+a^2-c^2}{2ab}$

The idea of sine formula in ΔABC

• $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

As we have to prove:

$(c^2 - a^2 + b^2) \tan A = (a^2 - b^2 + c^2) \tan B = (b^2 - c^2 + a^2) \tan C$

As LHS contain $(c^2 - a^2 + b^2)$, $(a^2 - b^2 + c^2)$and $(b^2 - c^2 + a^2)$,which shows resemblance with cosine formulae.

∴ From cosine formula we have:

Cos A = $\frac{b^2+c^2-a^2}{2bc}$

⇒ 2bc cos A =$b^2 + c^2 - a^2$

Multiplying with tan A both sides to get the form desired in proof

2bc cos A tan A = $(b^2 + c^2 - a^2)\tan A$

2bc sin A = $(b^2 + c^2 - a^2)\tan A$ .....eqn 1

Cos C = $\frac{b^2+a^2-c^2}{2ab}$

⇒ 2ab cos C = $b^2 + a^2 - c^2$

Multiplying with tan C both sides to get the form desired in proof

2ab cos C tan C = $(b^2 + a^2 - c^2)$tan C

2ab sin C = $(b^2 + a^2 - c^2)$tan C .....eqn 2

And, Cos B = $\frac{c^2+a^2-b^2}{2ac}$

⇒ 2ac cos B = $c^2 + a^2 - b^2$

Multiplying with tan B both sides to get the form desired in proof

2ac cos B tan B = $(c^2 + a^2 - b^2)\tan B$

2ac sin B = $(c^2 + a^2 - b^2)\tan B$ ......eqn 3

As we are observing that sin terms are being involved so let's try to use sine formula.

From sine formula we have,

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Multiplying abc to each fraction:-

$\frac{\text{abc} \sin A}{a} = \frac{\text{abc} \sin B}{b} = \frac{\text{abc} \sin C}{c}$

⇒ bc sin A = ac sin B = ab sin C

⇒ 2bc sin A = 2ac sin B = 2ab sin C

∴ From eqn 1, 2 and 3 we have:

$(b^2 + c^2 - a^2)\tan A = (c^2 + a^2 - b^2)\tan B = (b^2 + a^2 - c^2)$ tan C

Hence, proved.

8. Question

For any Δ ABC show that-

$(c^2 - a^2 + b^2)$ tan A = $(a^2 - b^2 + c^2)$ tan B = $(b^2 - c^2 + a^2)$ tan C

Answer

Note: In any ΔABC we define 'a' as length of side opposite to ∠A , 'b' as length of side opposite to ∠B and 'c' as length of side opposite to ∠C .

The key point to solve the problem:

The idea of cosine formula in ΔABC

- Cos A = $\frac{b^2+c^2-a^2}{2bc}$
- Cos B = $\frac{a^2+c^2-b^2}{2ac}$
- Cos C = $\frac{b^2+a^2-c^2}{2ab}$

The idea of sine formula in ΔABC

- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

As we have to prove:

$(c^2 - a^2 + b^2)$ tan A = $(a^2 - b^2 + c^2)$ tan B = $(b^2 - c^2 + a^2)$ tan C

As LHS contain $(c^2 - a^2 + b^2)$, $(a^2 - b^2 + c^2)$and $(b^2 - c^2 + a^2)$,which shows resemblance with cosine formulae.

∴ From cosine formula we have:

Cos A = $\frac{b^2+c^2-a^2}{2bc}$

⇒ 2bc cos A =$b^2 + c^2 - a^2$

Multiplying with tan A both sides to get the form desired in proof

2bc cos A tan A = $(b^2 + c^2 - a^2)\tan A$

2bc sin A = $(b^2 + c^2 - a^2)\tan A$ .....eqn 1

Cos C = $\frac{b^2+a^2-c^2}{2ab}$

⇒ 2ab cos C = $b^2 + a^2 - c^2$

Multiplying with tan C both sides to get the form desired in proof

2ab cos C tan C = $(b^2 + a^2 - c^2)$tan C

2ab sin C = $(b^2 + a^2 - c^2)$tan C .....eqn 2

And, Cos B = $\frac{c^2+a^2-b^2}{2ac}$

⇒ 2ac cos B = $c^2 + a^2 - b^2$

Multiplying with tan B both sides to get the form desired in proof

2ac cos B tan B = $(c^2 + a^2 - b^2)\tan B$

2ac sin B = $(c^2 + a^2 - b^2)\tan B$ ……eqn 3

As we are observing that sin terms are being involved so let's try to use sine formula.

From sine formula we have,

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Multiplying abc to each fraction:-

$$\frac{\text{abc}\sin A}{a} = \frac{\text{abc}\sin B}{b} = \frac{\text{abc}\sin C}{c}$$

⇒ bc sin A = ac sin B = ab sin C

⇒ 2bc sin A = 2ac sin B = 2ab sin C

∴ From eqn 1, 2 and 3 we have:

$(b^2 + c^2 - a^2)\tan A = (c^2 + a^2 - b^2)\tan B = (b^2 + a^2 - c^2)\tan C$

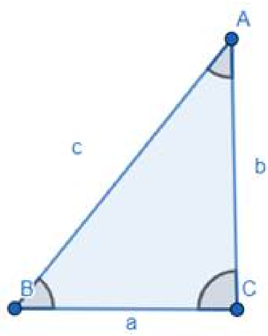
Hence, proved.

9. Question

For any Δ ABC show that-

$$\frac{c - b\cos A}{b - c\cos A} = \frac{\cos B}{\cos C}$$

Answer



Note: In any ΔABC we define 'a' as the length of the side opposite to ∠A, 'b' as the length of the side opposite to ∠B and 'c' as the length of the side opposite to ∠C.

Key point to solve the problem:

Idea of projection Formula:

• c = a cos B + b cos A

• b = c cos A + a cos C

• a = c cos B + b cos C

As we have to prove:

$$\frac{c - b\cos A}{b - c\cos A} = \frac{\cos B}{\cos C}$$

We can observe that we can get terms c – b cos A and b – c cos A from projection formula

∴ from projection formula we have-

c = a cos B + b cos A

⇒ c – b cos A = a cos B .....eqn 1

Also,

b = c cos A + a cos C

⇒ b – c cos A = a cos C ......eqn 2

Dividing eqn 1 by eqn 2, we have-

$$\frac{c - b\cos A}{b - c\cos A} = \frac{a\cos B}{a\cos C}$$

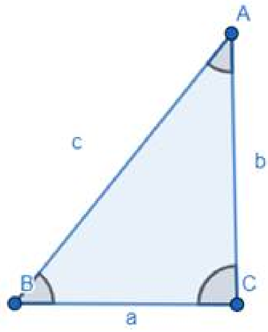
⇒ $\frac{c-b\cos A}{b-c\cos A} = \frac{\cos B}{\cos C}$ Hence proved.

9. Question

For any Δ ABC show that-

$$\frac{c - b\cos A}{b - c\cos A} = \frac{\cos B}{\cos C}$$

Answer



Note: In any ΔABC we define 'a' as the length of the side opposite to ∠A, 'b' as the length of the side opposite to ∠B and 'c' as the length of the side opposite to ∠C.

Key point to solve the problem:

Idea of projection Formula:

• c = a cos B + b cos A

• b = c cos A + a cos C

• a = c cos B + b cos C

As we have to prove:

$$\frac{c - b\cos A}{b - c\cos A} = \frac{\cos B}{\cos C}$$

We can observe that we can get terms c – b cos A and b – c cos A from projection formula

∴ from projection formula we have-

c = a cos B + b cos A

⇒ c – b cos A = a cos B .....eqn 1

Also,

b = c cos A + a cos C

⇒ b – c cos A = a cos C ……eqn 2

Dividing eqn 1 by eqn 2, we have-

$$\frac{c - b\cos A}{b - c\cos A} = \frac{a\cos B}{a\cos C}$$

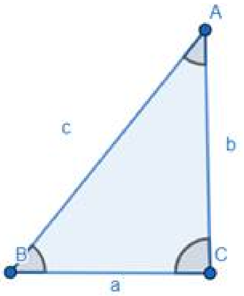
⇒ $\frac{c-b\cos A}{b-c\cos A} = \frac{\cos B}{\cos C}$ Hence proved.

10. Question

For any Δ ABC show that-

a(cos B + cos C – 1) + b(cos C + cos A –1) + c(cos A + cos B – 1) = 0

Answer



Note: In any ΔABC we define ‘a’ as the length of the side opposite to ∠A, ‘b’ as the length of the side opposite to ∠B and ‘c’ as the length of the side opposite to ∠C.

Key point to solve the problem:

Idea of projection Formula:

• c = a cos B + b cos A

• b = c cos A + a cos C

• a = c cos B + b cos C

As we have to prove:

a(cos B + cos C – 1) + b(cos C + cos A –1) + c(cos A + cos B – 1) = 0

We can observe that we all the terms present in equation to be proved are also present in expressions of projection formula ,so we have to apply the formula with slight modification -

∴ from projection formula we have-

c = a cos B + b cos A

⇒ b cos A + a cos B – c = 0 …..eqn 1

Also,

b = c cos A + a cos C

⇒ c cos A + a cos C – b = 0 ……eqn 2

Also,

a = c cos B + b cos C

⇒ c cos B + b cos C – a = 0 …..eqn 3

Adding eqn 1 ,2 and 3 –

We have,

b cos A + a cos B – c + c cos A + a cos C – b + c cos B + b cos C – a = 0 b cos A – b + b cos C + a cos B + a cos C – a + c cos A + c cos B – c = 0

⇒ b(cos A + cos C – 1) + a(cos B + cos C – 1) + c(cos A + cos B -1) = 0

Hence,

a(cos B + cos C – 1) + b(cos C + cos A –1) + c(cos A + cos B – 1) = 0

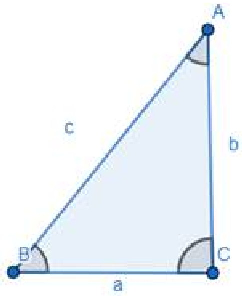
….proved

10. Question

For any Δ ABC show that-

a(cos B + cos C – 1) + b(cos C + cos A –1) + c(cos A + cos B – 1) = 0

Answer



Note: In any ΔABC we define 'a' as the length of the side opposite to ∠A, 'b' as the length of the side opposite to ∠B and 'c' as the length of the side opposite to ∠C.

Key point to solve the problem:

Idea of projection Formula:

• c = a cos B + b cos A

• b = c cos A + a cos C

• a = c cos B + b cos C

As we have to prove:

a(cos B + cos C – 1) + b(cos C + cos A –1) + c(cos A + cos B – 1) = 0

We can observe that we all the terms present in equation to be proved are also present in expressions of projection formula ,so we have to apply the formula with slight modification -

∴ from projection formula we have-

c = a cos B + b cos A

⇒ b cos A + a cos B – c = 0 …..eqn 1

Also,

b = c cos A + a cos C

⇒ c cos A + a cos C – b = 0 ……eqn 2

Also,

a = c cos B + b cos C

⇒ c cos B + b cos C – a = 0 …..eqn 3

Adding eqn 1 ,2 and 3 –

We have,

b cos A + a cos B – c + c cos A + a cos C – b + c cos B + b cos C – a = 0 b cos A – b + b cos C + a cos B + a cos C – a + c cos A + c cos B – c = 0

⇒ b(cos A + cos C – 1) + a(cos B + cos C – 1) + c(cos A + cos B -1) = 0

Hence,

a(cos B + cos C – 1) + b(cos C + cos A –1) + c(cos A + cos B – 1) = 0

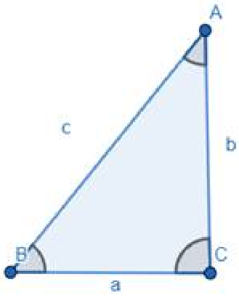
….proved

11. Question

For any Δ ABC show that -

a cos A + b cos B + c cos C = 2b sin A sin C

Answer



Note: In any ΔABC we define 'a' as the length of the side opposite to ∠A, 'b' as the length of the side opposite to ∠B and 'c' as the length of the side opposite to ∠C.

The key point to solve the problem:

The idea of sine Formula:

- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

As we have to prove:

a cos A + b cos B + c cos C = 2b sin A sin C

We can observe that we all the terms present in the equation to be proved are not showing any resemblance with known formula but the term is RHS side has sine terms, so there is a possibility that sine formula can solve our problem

∴ from sine formula we have-

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2k\ (say)$$

∴ a = 2k sin A , b = 2k sin B , c = 2k sin C

As,

LHS = a cos A + b cos B + c cos C

= 2k sin A cos A + 2k sin B cos B + 2k sin C cos C

= k(2sin A cos A + 2sin B cos B + 2sin C cos C

LHS = k(sin 2A + sin 2B + sin 2C) {using 2 sin X cos X = sin 2X }

Using transformation formula – sin X + sin Y = $2\sin\left(\frac{X+Y}{2}\right)\cos\left(\frac{X-Y}{2}\right)$

LHS = k (2sin(A + B) cos (A – B) + sin 2C)

∵ ∠ A + ∠ B + ∠ C = π

∴ A + B = π – C

∴ LHS = k { 2sin (π – C) cos (A – B) + 2 sin C cos C }

[as sin (π – θ) = sin θ]

LHS = k{ 2 sin C cos (A – B) + 2 sin C cos C }

LHS = 2k sin C { cos (A – B) + cos C }

Using transformation formula – cos X + cos Y = $2\cos\left(\frac{X+Y}{2}\right)\cos\left(\frac{X-Y}{2}\right)$

LHS = 2k sin C { $2\cos\left(\frac{A+C-B}{2}\right)\cos\left(\frac{A-B-C}{2}\right)$ }

LHS = 4k sin C $\cos\left(\frac{\pi-2B}{2}\right)\cos\left(\frac{A-(\pi-A)}{2}\right)$ {∵∠ A + ∠ B + ∠ C = π }

LHS = 4k sin C sin B sin A

∵ 2k sin B = b

We have,

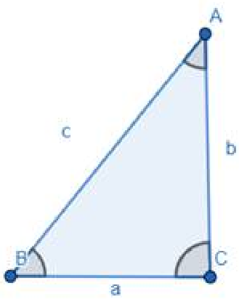
LHS = 2b sin A sin C = RHS **.....Hence proved.**

11. Question

For any Δ ABC show that -

a cos A + b cos B + c cos C = 2b sin A sin C

Answer



Note: In any ΔABC we define 'a' as the length of the side opposite to ∠A, 'b' as the length of the side opposite to ∠B and 'c' as the length of the side opposite to ∠C.

The key point to solve the problem:

The idea of sine Formula:

- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

As we have to prove:

a cos A + b cos B + c cos C = 2b sin A sin C

We can observe that we all the terms present in the equation to be proved are not showing any resemblance with known formula but the term is RHS side has sine terms, so there is a possibility that sine formula can solve our problem

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As,

LHS = a cos A + b cos B + c cos C

= 2k sin A cos A + 2k sin B cos B + 2k sin C cos C

= k(2sin A cos A + 2sin B cos B + 2sin C cos C

LHS = k(sin 2A + sin 2B + sin 2C) {using 2 sin X cos X = sin 2X }

Using transformation formula – sin X + sin Y = $2\sin\left(\frac{X+Y}{2}\right)\cos\left(\frac{X-Y}{2}\right)$

LHS = k (2sin(A + B) cos (A – B) + sin 2C)

∵ ∠ A + ∠ B + ∠ C = π

∴ A + B = π - C

∴ LHS = k { 2sin (π – C) cos (A – B) + 2 sin C cos C }

[as sin (π – θ) = sin θ]

LHS = k{ 2 sin C cos (A – B) + 2 sin C cos C }

LHS = 2k sin C { cos (A – B) + cos C }

Using transformation formula – cos X + cos Y = $2\cos\left(\frac{X+Y}{2}\right)\cos\left(\frac{X-Y}{2}\right)$

LHS = 2k sin C { $2\cos\left(\frac{A+C-B}{2}\right)\cos\left(\frac{A-B-C}{2}\right)$ }

LHS = 4k sin C $\cos\left(\frac{\pi-2B}{2}\right)\cos\left(\frac{A-(\pi-A)}{2}\right)$ {∵∠ A + ∠ B + ∠ C = π }

LHS = 4k sin C sin B sin A

∵ 2k sin B = b

We have,

LHS = 2b sin A sin C = RHS **.....Hence proved.**

12. Question

For any Δ ABC show that - $a^2 = (b+c)^2 - 4bc\cos^2\frac{A}{2}$

Answer

Note: In any ΔABC we define 'a' as the length of the side opposite to ∠A, 'b' as the length of the side opposite to ∠B and 'c' as the length of the side opposite to ∠C.

The key point to solve the problem:

The idea of cosine formula in ΔABC

• Cos A = $\frac{b^2+c^2-a^2}{2bc}$

• Cos B = $\frac{a^2+c^2-b^2}{2ac}$

• Cos C = $\frac{b^2+a^2-c^2}{2ab}$

As we have to prove: $a^2 = (b+c)^2 - 4bc\,cos^2\frac{A}{2}$

The form required to prove contains similar terms as present in cosine formula.

∴ Cosine formula is the perfect tool for solving the problem.

As we see the expression has bc term so we will apply the formula of cosA

As cos A = $\frac{b^2+c^2-a^2}{2bc}$

⇒ 2bc cos A = $b^2 + c^2 - a^2$

We need $(b + c)^2$ in our proof so adding 2bc both sides –

∴ 2bc + 2bc cos A = $b^2 + c^2 + 2bc - a^2$

⇒ 2bc (1 + cos A) = $(b + c)^2 - a^2$

∵ 1 + cos A = $2\cos^2 (A / 2)$ { using multiple angle formulae }

∴ $a^2 = (b+c)^2 - 2bc\ 2cos^2\frac{A}{2}$

⇒ $a^2 = (b+c)^2 - 4bc\,cos^2\frac{A}{2}$ **.....Hence proved.**

12. Question

For any Δ ABC show that - $a^2 = (b+c)^2 - 4bc\cos^2\frac{A}{2}$

Answer

Note: In any ΔABC we define 'a' as the length of the side opposite to ∠A, 'b' as the length of the side opposite to ∠B and 'c' as the length of the side opposite to ∠C.

The key point to solve the problem:

The idea of cosine formula in ΔABC

• Cos A = $\frac{b^2+c^2-a^2}{2bc}$

• Cos B = $\frac{a^2+c^2-b^2}{2ac}$

• Cos C = $\frac{b^2+a^2-c^2}{2ab}$

As we have to prove: $a^2 = (b+c)^2 - 4bc\,cos^2\frac{A}{2}$

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As cos A = $\frac{b^2+c^2-a^2}{2bc}$

⇒ 2bc cos A = $b^2 + c^2 - a^2$

We need $(b + c)^2$ in our proof so adding 2bc both sides –

∴ 2bc + 2bc cos A = $b^2 + c^2 + 2bc - a^2$

⇒ 2bc (1 + cos A) = $(b + c)^2 - a^2$

∵ 1 + cos A = $2\cos^2 (A / 2)$ { using multiple angle formulae }

∴ $a^2 = (b+c)^2 - 2bc\ 2cos^2\frac{A}{2}$

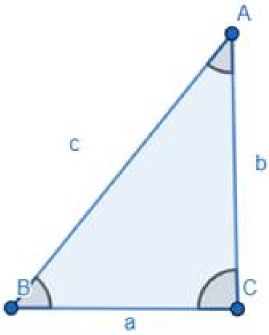
⇒ $a^2 = (b+c)^2 - 4bc\,cos^2\frac{A}{2}$ **.....Hence proved.**

13. Question

For any Δ ABC show that –

$4\left(bc\cos^2\frac{A}{2}+ca\cos^2\frac{B}{2}+ab\cos^2\frac{C}{2}\right)$ = $(a+b+c)^2$

Answer



Note: In any ΔABC we define ‘a’ as the length of the side opposite to ∠A, ‘b’ as the length of the side opposite to ∠B and ‘c’ as the length of the side opposite to ∠C.

The key point to solve the problem:

The idea of cosine formula in ΔABC

- Cos A = $\frac{b^2+c^2-a^2}{2bc}$
- Cos B = $\frac{a^2+c^2-b^2}{2ac}$
- Cos C = $\frac{b^2+a^2-c^2}{2ab}$

As we have to prove:

$4\left(bc\,cos^2\frac{A}{2}+ca\,cos^2\frac{B}{2}+ab\,cos^2\frac{C}{2}\right)$ = $(a+b+c)^2$

The form required to prove contains similar terms as present in cosine formula.

∴ Cosine formula is the perfect tool for solving the problem.

As we see the expression has bc, ac and ab terms so we will apply the formula of cos A, cos B, and cos C all.

As cos A = $\frac{b^2+c^2-a^2}{2bc}$

⇒ 2bc cos A = $b^2 + c^2 - a^2$

We need $(b + c)^2$ in our proof so adding 2bc both sides –

∴ 2bc + 2bc cos A = $b^2 + c^2 + 2bc - a^2$

⇒ 2bc (1 + cos A) = $(b + c)^2 - a^2$

∵ 1 + cos A = $2\cos^2$ (A / 2) { using multiple angle formulae }

∴ $a^2 = (b+c)^2 - 2bc\ 2cos^2\frac{A}{2}$

⇒ $a^2 = (b+c)^2 - 4bc\,cos^2\frac{A}{2}$

⇒ $4bc\,cos^2\frac{A}{2} = (b+c)^2 - a^2$ ....eqn 1

Similarly,

$4ac\,cos^2\frac{B}{2} = (a+c)^2 - b^2$ ....eqn 2

And, $4ab\cos^2\frac{C}{2} = (a+b)^2 - c^2$ ....eqn 3

Adding equation 1, 2 and 3 we have –

$$4bc\cos^2\frac{A}{2} + 4ac\cos^2\frac{B}{2} + 4ab\cos^2\frac{C}{2} = (b+c)^2 - a^2 + (a+c)^2 - b^2 + (a+b)^2 - c^2$$

$$4\left(bc\cos^2\frac{A}{2} + ca\cos^2\frac{B}{2} + ab\cos^2\frac{C}{2}\right) = a^2 + b^2 + c^2 + 2bc + 2ca + 2ab$$

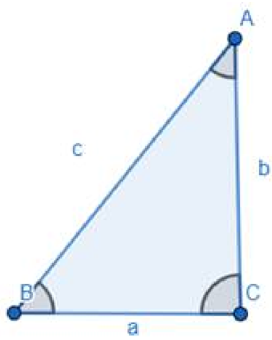
$4\left(bc\cos^2\frac{A}{2} + ca\cos^2\frac{B}{2} + ab\cos^2\frac{C}{2}\right) = (a+b+c)^2$ **...Hence proved**

13. Question

For any Δ ABC show that –

$4\left(bc\cos^2\frac{A}{2} + ca\cos^2\frac{B}{2} + ab\cos^2\frac{C}{2}\right) = (a+b+c)^2$

Answer



Note: In any ΔABC we define 'a' as the length of the side opposite to ∠A, 'b' as the length of the side opposite to ∠B and 'c' as the length of the side opposite to ∠C.

The key point to solve the problem:

The idea of cosine formula in ΔABC

• Cos A = $\frac{b^2+c^2-a^2}{2bc}$

• Cos B = $\frac{a^2+c^2-b^2}{2ac}$

• Cos C = $\frac{b^2+a^2-c^2}{2ab}$

As we have to prove:

$4\left(bc\cos^2\frac{A}{2} + ca\cos^2\frac{B}{2} + ab\cos^2\frac{C}{2}\right) = (a+b+c)^2$

The form required to prove contains similar terms as present in cosine formula.

∴ Cosine formula is the perfect tool for solving the problem.

As we see the expression has bc, ac and ab terms so we will apply the formula of cos A, cos B, and cos C all.

As cos A = $\frac{b^2+c^2-a^2}{2bc}$

⇒ 2bc cos A = $b^2 + c^2 - a^2$

We need $(b + c)^2$ in our proof so adding 2bc both sides –

$\therefore 2bc + 2bc \cos A = b^2 + c^2 + 2bc - a^2$

$\Rightarrow 2bc(1 + \cos A) = (b + c)^2 - a^2$

$\because 1 + \cos A = 2\cos^2 (A/2)$ { using multiple angle formulae }

$\therefore a^2 = (b+c)^2 - 2bc\ 2cos^2\frac{A}{2}$

$\Rightarrow a^2 = (b+c)^2 - 4bc\,cos^2\frac{A}{2}$

$\Rightarrow 4bc\,cos^2\frac{A}{2} = (b+c)^2 - a^2$ ....eqn 1

Similarly,

$4ac\,cos^2\frac{B}{2} = (a+c)^2 - b^2$ ....eqn 2

And, $4ab\cos^2\frac{C}{2} = (a+b)^2 - c^2$ ....eqn 3

Adding equation 1, 2 and 3 we have –

$$4bc\cos^2\frac{A}{2} + 4ac\cos^2\frac{B}{2} + 4ab\cos^2\frac{C}{2} = (b+c)^2 - a^2 + (a+c)^2 - b^2 + (a+b)^2 - c^2$$

$$4\left(bc\cos^2\frac{A}{2} + ca\cos^2\frac{B}{2} + ab\cos^2\frac{C}{2}\right) = a^2 + b^2 + c^2 + 2bc + 2ca + 2ab$$

$$4\left(bc\cos^2\frac{A}{2} + ca\cos^2\frac{B}{2} + ab\cos^2\frac{C}{2}\right) = (a+b+c)^2$$ **...Hence proved**

14. Question

In a Δ ABC prove that

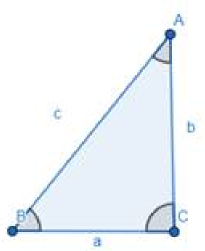
$\sin^3 A \cos(B - C) + \sin^3 B \cos(C - A) + \sin^3 C \cos(A - B) = 3 \sin A \sin B \sin C$

Answer

The key point to solve the problem:

The idea of sine Formula:

- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



Idea of projection Formula:

- c = a cos B + b cos A
- b = c cos A + a cos C
- a = c cos B + b cos C

As we have to prove:-

$\sin^3 A \cos (B - C) + \sin^3 B \cos (C - A) + \sin^3 C \cos (A - B) = 3 \sin A \sin B \sin C$

as there is no resemblance of above expression with any formula so first we need to simplify the expression

LHS = $\sin^3 A \cos (B - C) + \sin^3 B \cos (C - A) + \sin^3 C \cos (A - B)$

LHS = $\sin^2 A \sin A \cos (B - C) + \sin^2 B \sin B \cos (C - A) + \sin^2 C \sin C \cos (A - B)$

LHS = $\sin^2 A \sin\{\pi - (B+C)\}\cos (B - C) + \sin^2 B \sin\{\pi - (A+C)\}\cos (C - A) + \sin^2 C \sin \{\pi - (A + B)\} \cos (A - B)$

LHS = $\sin^2 A \sin (B+C) \cos(B-C) + \sin^2 B \sin(A + C)\cos(C - A) + \sin^2 C \sin(B + C) \cos (A - B)$

Using the relation $\sin (X + Y)\cos(X - Y) = \sin 2X + \sin 2Y$, we have –

LHS = $\sin^2 A (\sin 2B + \sin 2C) + \sin^2 B (\sin 2A + \sin 2C) + \sin^2 C (\sin 2B + \sin 2A)$

Using $\sin 2X = 2\sin X \cos X$, we have –

LHS = $\sin^2 A (2\sin B \cos B + 2\sin C \cos C) + \sin^2 B (2\sin A \cos A + 2\sin C \cos C) + \sin^2 C (2\sin B\cos B + 2\sin A \cos A)$

Using sine formula we have –

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k(\text{say})$$

∴ $\sin A = ka$, $\sin B = kb$ and $\sin C = kc$ ...eqn 1

Putting the values in LHS:-

LHS = $k^2a^2(2kb\cos B + 2kc\cos C) + k^2b^2(2ka\cos A + 2kc\cos C) + k^2c^2(2kb\cos B + 2ka\cos A)$

LHS = $k^3ab(a\cos B + b\cos A) + k^3ac(a\cos C + c\cos A) + k^3bc(b\cos C + c\cos B)$

Using projection formula

- $c = a \cos B + b \cos A$
- $b = c \cos A + a \cos C$
- $a = c \cos B + b \cos C$

We have

LHS = $k^3 abc + k^3 abc + k^3abc$

= $3k^3abc = 3(ka)(kb)(kc)$ {using eqn 1}

= $3\sin A \sin B \sin C$ = RHS **....Hence proved**

14. Question

In a Δ ABC prove that

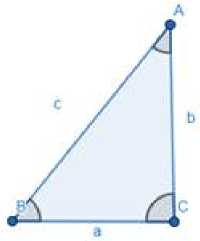
$\sin^3 A \cos (B - C) + \sin^3 B \cos (C - A) + \sin^3 C \cos (A - B) = 3 \sin A \sin B \sin C$

Answer

The key point to solve the problem:

The idea of sine Formula:

- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



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• c = a cos B + b cos A

• b = c cos A + a cos C

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As we have to prove:-

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LHS = $\sin^3 A \cos(B - C) + \sin^3 B \cos(C - A) + \sin^3 C \cos(A - B)$

LHS = $\sin^2 A \sin A \cos(B - C) + \sin^2 B \sin B \cos(C - A) + \sin^2 C \sin C \cos(A - B)$

LHS = $\sin^2 A \sin\{\pi - (B+C)\}\cos(B - C) + \sin^2 B \sin\{\pi - (A+C)\}\cos(C - A) + \sin^2 C \sin\{\pi - (A + B)\}\cos(A - B)$

LHS = $\sin^2 A \sin(B+C)\cos(B-C) + \sin^2 B \sin(A + C)\cos(C - A) + \sin^2 C \sin(B + C)\cos(A - B)$

Using the relation $\sin(X + Y)\cos(X - Y) = \sin 2X + \sin 2Y$, we have –

LHS = $\sin^2 A (\sin 2B + \sin 2C) + \sin^2 B (\sin 2A + \sin 2C) + \sin^2 C (\sin 2B + \sin 2A)$

Using $\sin 2X = 2\sin X \cos X$, we have –

LHS = $\sin^2 A (2\sin B \cos B + 2\sin C \cos C) + \sin^2 B (2\sin A \cos A + 2\sin C \cos C) + \sin^2 C (2\sin B\cos B + 2\sin A \cos A)$

Using sine formula we have –

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k(\text{say})$$

∴ sin A = ka , sin B = kb and sin C = kc …eqn 1

Putting the values in LHS:-

LHS = $k^2a^2(2kb\cos B + 2kc\cos C) + k^2b^2(2ka\cos A + 2kc\cos C) + k^2c^2(2kb\cos B + 2ka\cos A)$

LHS = $k^3ab(a\cos B + b\cos A) + k^3ac(a\cos C + c\cos A) + k^3bc(b\cos C + c\cos B)$

Using projection formula

• c = a cos B + b cos A

• b = c cos A + a cos C

• a = c cos B + b cos C

We have

LHS = $k^3 abc + k^3 abc + k^3 abc$

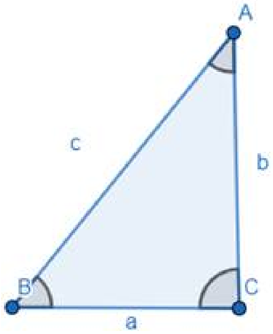
= $3k^3 abc = 3(ka)(kb)(kc)$ {using eqn 1}

= $3\sin A \sin B \sin C$ = RHS **….Hence proved**

15. Question

In any Δ ABC, $\frac{b+c}{12}=\frac{c+a}{13}=\frac{a+b}{15}$, then prove that $\frac{\cos A}{2}=\frac{\cos B}{7}=\frac{\cos C}{11}$.

Answer



Note: In any ΔABC we define 'a' as the length of the side opposite to ∠A, 'b' as the length of the side opposite to ∠B and 'c' as the length of the side opposite to ∠C.

The key point to solve the problem:

The idea of cosine formula in ΔABC

- Cos A = $\frac{b^2+c^2-a^2}{2bc}$
- Cos B = $\frac{a^2+c^2-b^2}{2ac}$
- Cos C = $\frac{b^2+a^2-c^2}{2ab}$

As we have to prove $\frac{\cos A}{2}=\frac{\cos B}{7}=\frac{\cos C}{11}$ under given conditions.

∵ Only cos terms are involved so we will apply cosine formula to find cos A , cos B, and cos C and we will take their ratio.

$\because \frac{b+c}{12}=\frac{c+a}{13}=\frac{a+b}{15}=k(\text{say})$

∴ b + c = 12k ….eqn 1

c + a = 13k ….eqn 2

a + b = 15k ….eqn 3

But only above relation is not sufficient to find cosines as k is unknown, either we need to express k in terms of a , b or c or express a , b , c in terms of k. Later part is easier.

∴ we will find a,b,c in terms of k

Adding eqn 1,2 and 3 we have –

2 (a + b + c) = 40k

∴ a + b+ c = 20k

∴ a = 20k – (b + c) = 20k – 12k = 8k

Similarly, b = 20k – (c + a) = 20k – 13k = 7k

And c = 20k – (a + b) = 20k – 15k = 5k

Hence,

Cos A = $\frac{b^2+c^2-a^2}{2bc} = \frac{49k^2+25k^2-64k^2}{70k^2} = \frac{10k^2}{70k^2} = \frac{1}{7}$

Cos B = $\frac{a^2+c^2-b^2}{2ac} = \frac{64k^2+25k^2-49k^2}{80k^2} = \frac{40k^2}{80k^2} = \frac{1}{2}$

cos C = $\frac{b^2+a^2-c^2}{2ab} = \frac{49k^2+64k^2-25k^2}{112k^2} = \frac{88k^2}{112k^2} = \frac{11}{14}$

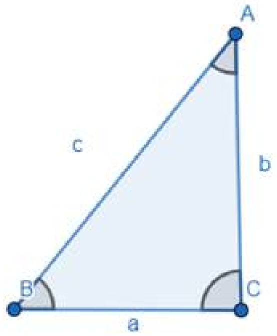
$\therefore \cos A : \cos B : \cos C = \frac{1}{7} : \frac{1}{2} : \frac{11}{14} = \frac{2}{14} : \frac{7}{14} : \frac{11}{14}$

$\therefore \frac{\cos A}{2} = \frac{\cos B}{7} = \frac{\cos C}{11}$ **....Hence proved.**

15. Question

In any Δ ABC, $\frac{b+c}{12} = \frac{c+a}{13} = \frac{a+b}{15}$, then prove that $\frac{\cos A}{2} = \frac{\cos B}{7} = \frac{\cos C}{11}$.

Answer



Note: In any ΔABC we define 'a' as the length of the side opposite to ∠A, 'b' as the length of the side opposite to ∠B and 'c' as the length of the side opposite to ∠C.

The key point to solve the problem:

The idea of cosine formula in ΔABC

- Cos A = $\frac{b^2+c^2-a^2}{2bc}$
- Cos B = $\frac{a^2+c^2-b^2}{2ac}$
- Cos C = $\frac{b^2+a^2-c^2}{2ab}$

As we have to prove $\frac{\cos A}{2} = \frac{\cos B}{7} = \frac{\cos C}{11}$ under given conditions.

∵ Only cos terms are involved so we will apply cosine formula to find cos A , cos B, and cos C and we will take their ratio.

$\because \frac{b+c}{12} = \frac{c+a}{13} = \frac{a+b}{15} = k(\text{say})$

∴ b + c = 12k ....eqn 1

c + a = 13k ....eqn 2

a + b = 15k ....eqn 3

But only above relation is not sufficient to find cosines as k is unknown, either we need to express k in terms of a , b or c or express a , b , c in terms of k. Later part is easier.

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∴ a + b+ c = 20k

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Hence,

Cos A = $\frac{b^2+c^2-a^2}{2bc} = \frac{49k^2+25k^2-64k^2}{70k^2} = \frac{10k^2}{70k^2} = \frac{1}{7}$

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cos C = $\frac{b^2+a^2-c^2}{2ab} = \frac{49k^2+64k^2-25k^2}{112k^2} = \frac{88k^2}{112k^2} = \frac{11}{14}$

$\therefore \cos A : \cos B : \cos C = \frac{1}{7} : \frac{1}{2} : \frac{11}{14} = \frac{2}{14} : \frac{7}{14} : \frac{11}{14}$

$\therefore \frac{\cos A}{2} = \frac{\cos B}{7} = \frac{\cos C}{11}$ **....Hence proved.**

16. Question

In a Δ ABC, if ∠B = 60^o, prove that (a + b + c) (a – b + c) = 3ca

Answer

The key point to solve the problem:

The idea of cosine formula in ΔABC

Cos A = $\frac{b^2+c^2-a^2}{2bc}$ Cos B = $\frac{a^2+c^2-b^2}{2ac}$ Cos C = $\frac{b^2+a^2-c^2}{2ab}$.

As we have to prove : (a + b + c) (a – b + c) = 3ca

LHS = (a + c+ b) (a + c– b) = $(a + c)^2 – b^2$ { using (x + y)(x – y) = $x^2 – y^2$ }

Now the above expression gives us hint that we need to apply cosine formula as terms has resemblance.

∴ cos B = $\frac{a^2+c^2-b^2}{2ac}$

cos 60° = $\frac{a^2+c^2-b^2}{2ac}$

$\Rightarrow \frac{1}{2} = \frac{a^2 + c^2 - b^2}{2ac}$

$\therefore$ ac = $a^2 + c^2 - b^2$

Adding 2bc both sides to get the term present in final term-

$\therefore$ 3ac = $a^2 + c^2 + 2ac - b^2$

$\Rightarrow$ 3ac = $(a + c)^2 - b^2$

using (x + y)(x – y) = $x^2 - y^2$, we have –

3ac = (a + c+ b) (a + c– b)

Or **(a + b + c) (a – b + c) = 3ca** ...**Hence proved**

16. Question

In a Δ ABC, if ∠B = 60°, prove that (a + b + c) (a – b + c) = 3ca

Answer

The key point to solve the problem:

The idea of cosine formula in ΔABC

Cos A = $\frac{b^2 + c^2 - a^2}{2bc}$ Cos B = $\frac{a^2 + c^2 - b^2}{2ac}$ Cos C = $\frac{b^2 + a^2 - c^2}{2ab}$.

As we have to prove : (a + b + c) (a – b + c) = 3ca

LHS = (a + c+ b) (a + c– b) = $(a + c)^2 - b^2$ { using (x + y)(x – y) = $x^2 - y^2$ }

Now the above expression gives us hint that we need to apply cosine formula as terms has resemblance.

$\therefore$ cos B = $\frac{a^2 + c^2 - b^2}{2ac}$

cos 60° = $\frac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow \frac{1}{2} = \frac{a^2 + c^2 - b^2}{2ac}$

$\therefore$ ac = $a^2 + c^2 - b^2$

Adding 2bc both sides to get the term present in final term-

$\therefore$ 3ac = $a^2 + c^2 + 2ac - b^2$

$\Rightarrow$ 3ac = $(a + c)^2 - b^2$

using (x + y)(x – y) = $x^2 - y^2$, we have –

3ac = (a + c+ b) (a + c– b)

Or **(a + b + c) (a – b + c) = 3ca** ...**Hence proved**

17. Question

In a Δ ABC $\cos^2 A + \cos^2 B + \cos^2 C = 1$, prove that the triangle is right angled.

Answer

The key point to solve the problem:

The idea of basic trigonometric formulae, i.e. transformation and T – ratios of multiple angles

Given,

$\cos^2 A + \cos^2 B + \cos^2 C = 1$

Multiplying 2 to both sides so that we can change it in Trigonometric ratios of multiple angles so that we can get the value of angle.

As $2\cos^2 X = 1 + \cos 2X$

$\therefore 2\cos^2 A + 2\cos^2 B + 2\cos^2 C = 2$

$\Rightarrow 1 + \cos 2A + 1 + \cos 2B + 1 + \cos 2C = 2$

$\Rightarrow \cos 2A + \cos 2B + \cos 2C = -1$

Using, $\cos 2X + \cos 2Y = 2\cos(X + Y)\cos(X - Y)$

$\Rightarrow 2\cos(A + B)\cos(A - B) = -1(1 + \cos 2C)$

As $2\cos^2 X = 1 + \cos 2X$ and $A + B + C = \pi$

We have,

$2\cos(\pi - C)\cos(A - B) = -2\cos^2 C$

$-2\cos C\cos(A - B) = -2\cos^2 C$ $\{\because \cos(\pi - \theta) = -\cos\theta\}$

$\therefore 2\cos C(\cos C + \cos(A - B)) = 0$

Either $\cos C = 0 \Rightarrow \angle C = 90°$

Or $\cos C = -\cos(A - B) \Rightarrow C = \pi - (A - B)$ which is not possible as in ΔABC, $A + B + C = \pi$

$\therefore C = 90°$ is the only satisfied solution.

Hence, Δ ABC is a right triangle, right angled at $\angle C$ **…proved**

17. Question

In a Δ ABC $\cos^2 A + \cos^2 B + \cos^2 C = 1$, prove that the triangle is right angled.

Answer

The key point to solve the problem:

The idea of basic trigonometric formulae, i.e. transformation and T – ratios of multiple angles

Given,

$\cos^2 A + \cos^2 B + \cos^2 C = 1$

Multiplying 2 to both sides so that we can change it in Trigonometric ratios of multiple angles so that we can get the value of angle.

As $2\cos^2 X = 1 + \cos 2X$

$\therefore 2\cos^2 A + 2\cos^2 B + 2\cos^2 C = 2$

$\Rightarrow 1 + \cos 2A + 1 + \cos 2B + 1 + \cos 2C = 2$

$\Rightarrow \cos 2A + \cos 2B + \cos 2C = -1$

Using, $\cos 2X + \cos 2Y = 2\cos(X + Y)\cos(X - Y)$

$\Rightarrow 2\cos(A + B)\cos(A - B) = -1(1 + \cos 2C)$

As $2\cos^2 X = 1 + \cos 2X$ and $A + B + C = \pi$

We have,

$2 \cos(\pi - C) \cos(A - B) = -2 \cos^2 C$

$-2 \cos C \cos(A - B) = -2 \cos^2 C$ {$\because \cos(\pi - \theta) = -\cos\theta$ }

$\therefore 2 \cos C (\cos C + \cos(A - B)) = 0$

Either $\cos C = 0 \Rightarrow \angle C = 90°$

Or $\cos C = -\cos(A - B) \Rightarrow C = \pi - (A - B)$ which is not possible as in ΔABC, $A + B + C = \pi$

$\therefore C = 90°$ is the only satisfied solution.

Hence, Δ ABC is a right triangle, right angled at ∠ C **...proved**

18. Question

In a ΔABC, if $\cos C = \frac{\sin A}{2\sin B}$, prove that the triangle is isosceles.

Answer

The key point to solve the problem:

To prove a triangle isosceles our task is to show either any two angles equal or two sides equal.

Idea of cosine formula - Cos C = $\frac{b^2 + a^2 - c^2}{2ab}$

The idea of sine Formula:

- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Given, $\cos C = \frac{\sin A}{2\sin B}$

As it has sin terms involved so that sine formula can work, and cos C is also there so we might need cosine formula too.

Let’s apply sine formula keeping a target to prove any two sides equal.

Using sine formula we have –

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k(\text{say})$$

$\therefore \sin A = ak$ and $\sin B = bk$

$\therefore \cos C = \frac{ak}{2bk} = \frac{a}{2b}$

If we apply cosine formula, we will get an equation in terms of sides only that may give us any two sides equal.

Using, Cos C = $\frac{b^2 + a^2 - c^2}{2ab}$

We have,

$$\frac{b^2 + a^2 - c^2}{2ab} = \frac{a}{2b}$$

$\Rightarrow b^2 + a^2 - c^2 = a^2$

$\Rightarrow b^2 = c^2$

$\Rightarrow b = c$

Hence 2 sides are equal.

∴ Δ ABC is isosceles. **....proved**

18. Question

In a ΔABC, if $\cos C = \frac{\sin A}{2\sin B}$, prove that the triangle is isosceles.

Answer

The key point to solve the problem:

To prove a triangle isosceles our task is to show either any two angles equal or two sides equal.

Idea of cosine formula - Cos C = $\frac{b^2 + a^2 - c^2}{2ab}$

The idea of sine Formula:

- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Given, $\cos C = \frac{\sin A}{2\sin B}$

As it has sin terms involved so that sine formula can work, and cos C is also there so we might need cosine formula too.

Let's apply sine formula keeping a target to prove any two sides equal.

Using sine formula we have –

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k(\text{say})$$

∴ sin A = ak and sin B = bk

∴ cos C = $\frac{ak}{2bk} = \frac{a}{2b}$

If we apply cosine formula, we will get an equation in terms of sides only that may give us any two sides equal.

Using, Cos C = $\frac{b^2 + a^2 - c^2}{2ab}$

We have,

$$\frac{b^2 + a^2 - c^2}{2ab} = \frac{a}{2b}$$

$\Rightarrow b^2 + a^2 - c^2 = a^2$

$\Rightarrow b^2 = c^2$

$\Rightarrow b = c$

Hence 2 sides are equal.

∴ Δ ABC is isosceles. **....proved**

19. Question

Two ships leave a port at the same time. One goes 24 km/hr in the direction N 38^o E and other travels 32 km/hr in the direction S 52^o E. Find the distance between the ships at the end of 3 hrs.

Answer

The key point to solve the problem:

The idea of cosine formula –

Cos C = $\frac{b^2+a^2-c^2}{2ab}$ Cos A = $\frac{b^2+c^2-a^2}{2bc}$ Cos B = $\frac{a^2+c^2-b^2}{2ac}$

According to the question:

One ship goes in north east direction while other in southeast direction.

After 3 hours ship going in north east will be at a distance

Speed of ship A = 24km/hr

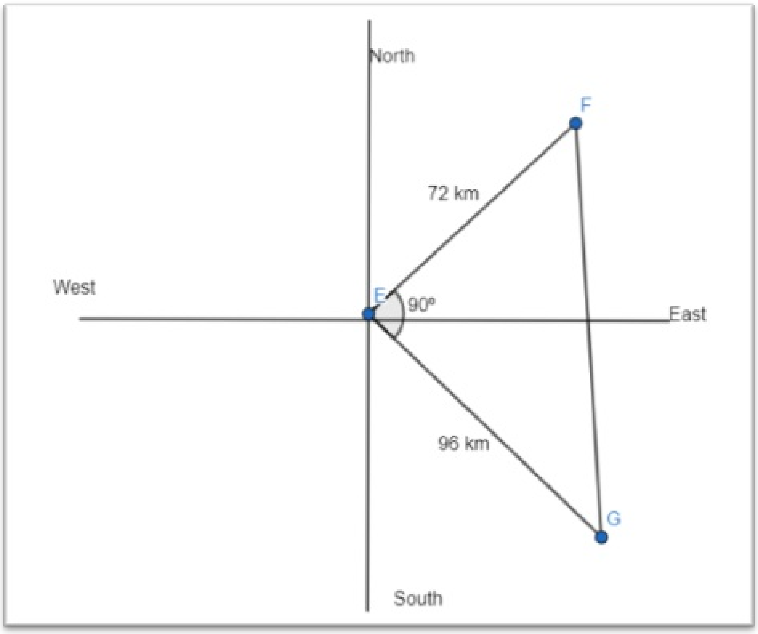
Speed of ship B = 32km/hr

Distance travelled by ship A after 3 hours = 24 × 3 = 72 km

Distance travelled by ship B after 3 hours = 32 × 3 = 96 km

We have to find the distance between the ships :

See the figure :



Now in Δ EFG,

EF is the distance traveled by ship A

And EG is the distance traveled by ship B

we have to find FG,

Applying cosine formula, we have-

Cos E = $\frac{EF^2+EG^2-FG^2}{2\times EF\times FG}$

Cos E = Cos 90° = 0

∴ $FG^2 = EF^2 + EG^2$

⇒ FG = $\sqrt{72^2 + 96^2}$

= $\sqrt{14400} = 120 \text{km}$

∴ distance between ships after 3 hours = 120 KM ….**ans**

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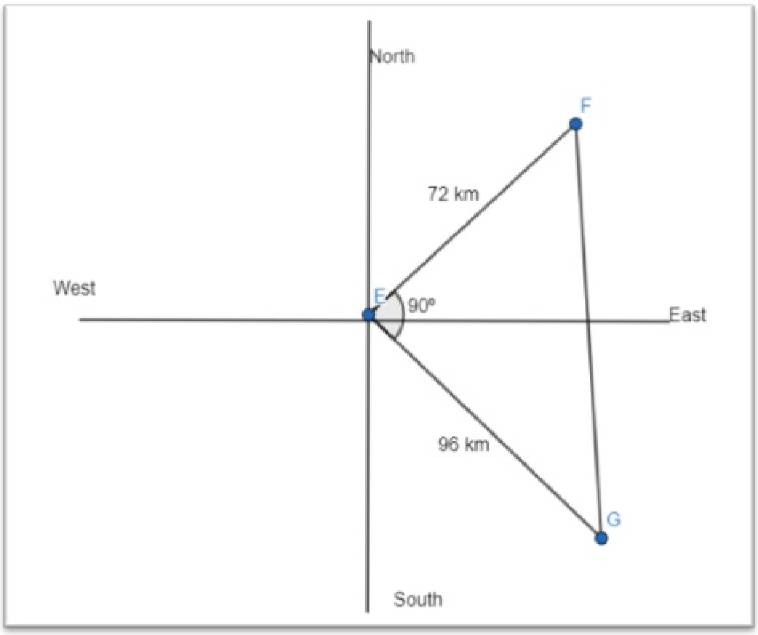

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⇒ FG = $\sqrt{72^2 + 96^2}$

= $\sqrt{14400} = 120\,km$

∴ distance between ships after 3 hours = 120 KM ….**ans**

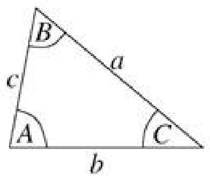
Very Short Answer

1. Question

Find the area of the triangle ΔABC in which a = 1, b = 2 and ∠c = 60^o.

Answer

Given,



a=1, b=2 and ∠C=60°

By Cosine law,

$\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$

$\frac{1}{2} = \frac{1^2 + 2^2 - c^2}{2 \times 1 \times 2}$

$2 = 1 + 4 - c^2$

$c^2 = 5 - 2$

$c^2 = 3$

$c = \sqrt{3}$

$S = \frac{a + b + c}{2}$

$= \frac{1 + 2 + \sqrt{3}}{2}$

$= \frac{3 + \sqrt{3}}{2}$

By Heron’s Law,

Area of Triangle,

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$$A = \sqrt{\frac{3+\sqrt{3}}{2} \times \frac{1+\sqrt{3}}{2} \times \frac{\sqrt{3}-1}{2} \times \frac{3-\sqrt{3}}{2}}$$

$$A = \frac{1}{4}\sqrt{12}$$

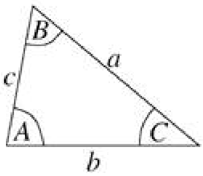
$A = \frac{\sqrt{3}}{2}$ sq. units

2. Question

In a ΔABC, if b = √3, c = 1 and ∠A = 30°, find a.

Answer

Given,



c=1, b=√3 and ∠A=30°

By Cosine law,

$$\cos(A) = \frac{c^2 + b^2 - a^2}{2cb}$$

$$\frac{\sqrt{3}}{2} = \frac{1^2 + (\sqrt{3})^2 - a^2}{2 \times \sqrt{3} \times 1}$$

$3=1+3-a^2$

$a^2=1$

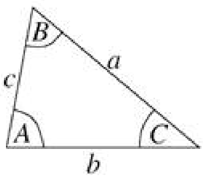
a=1

3. Question

In a ΔABC, if $\cos A = \frac{\sin B}{2 \sin C}$, then show that c = a.

Answer

Given,



By Sine Law,

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

$$b = \frac{c \times \sin(B)}{\sin(C)}$$

By Cosine law,

$$\cos(A) = \frac{c^2 + b^2 - a^2}{2cb}$$

$$\cos(A) = \frac{c^2 + \left(\frac{c \times \sin(B)}{\sin(C)}\right)^2 - a^2}{2 \times c \times \left(\frac{c \sin(B)}{\sin(C)}\right)}$$

$$\cos(A) = \frac{\frac{c^2 \times \sin(C)^2 + c^2 \times \sin(B)^2 - a^2 \sin(C)^2}{\sin(C)^2}}{2 \times \left(\frac{\sin(B)}{\sin(C)}\right)}$$

$$\cos(A) = \frac{c^2 \times \sin(B)^2 + (c^2 - a^2)(\sin(C))^2}{2 \times \sin(B) \times \sin(C)}$$

As it given

$$\cos(A) = \frac{\sin(B)}{2\sin(C)}$$

So,

$$\cos(A) = \frac{\sin(B)}{2\sin(C)} = \frac{c^2 \times \sin(B)^2 + (c^2 - a^2)(\sin(C))^2}{2 \times \sin(B) \times \sin(C)}$$

For the above Equation to be true,

$(c^2 - a^2)(\sin(C))^2 = 0$

So

$(c^2 - a^2) = 0$

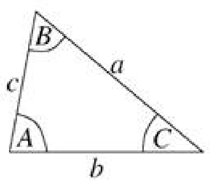
c=a

Hence Proved.

4. Question

In a ΔABC, if b = 20, c = 21 and $\sin A = \frac{3}{5}$, find a.

Answer



We know,

$$\sin\theta^2 + \cos\theta^2 = 1$$

Given,

$$\sin(A) = \frac{3}{5}$$

So

$$\cos(A)^2 = 1 - \left(\frac{3}{5}\right)^2$$

$\cos(A) = \frac{4}{5}$

Now by putting cosine law

By Cosine law,

$\cos(A) = \frac{c^2 + b^2 - a^2}{2cb}$

$\frac{4}{5} = \frac{21^2 + (20)^2 - a^2}{2 \times 21 \times 20}$

a^2 =400+441-672

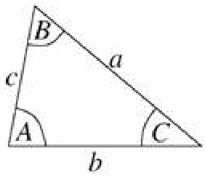
=169

a=13

5. Question

In a ΔABC, if sin A and sin B are the roots of the equation c^2x^2 – c (a + b) x + ab = 0, then find∠C.

Answer



According to the product of the roots

$\sin A \times \sin B = \frac{ab}{c^2}$

But by sine law we know,

$\sin A = \frac{a \times \sin C}{c}$

and

$\sin B = \frac{b \times \sin C}{c}$

So,

$\frac{a \times \sin C}{c} \times \frac{b \times \sin C}{c} = \frac{ab}{c}$

$\frac{ab \times (\sin c)^2}{c^2} = \frac{ab}{c}$

Therefore,

$(\sin C)^2 = 1$

So,

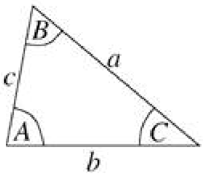
sin C=1

C=90°

6. Question

In a ΔABC, if a = 8, b = 10, c = 12 and C = λA, find the value of λ.

Answer



Given a=8, b=10,c=12

Now by putting cosine law

By Cosine law,

$$\cos(A) = \frac{c^2 + b^2 - a^2}{2cb}$$

$$\cos(A) = \frac{12^2 + 10^2 - 8^2}{2 \times 10 \times 12} = \frac{3}{4}$$

$$A = \cos^{-1}\left(\frac{3}{4}\right) = 41.40962211$$

$$\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos(C) = \frac{8^2 + 10^2 - 12^2}{2 \times 10 \times 8}$$

$$= \frac{1}{8}$$

$$C = \cos^{-1}\left(\frac{1}{8}\right)$$

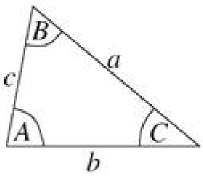
=82.81924422

Therefore, C=2×A and λ=2

7. Question

If the sides of a triangle are proportional to 2, √6 and √3 – 1, find the measure of its greatest angle.

Answer



Lets say

a = 2x, b=√6x and c=(√3-1)x

where x is any constant

Now by putting cosine law

By Cosine law,

$$\cos(A) = \frac{c^2 + b^2 - a^2}{2cb}$$

$$\cos(A) = \frac{((\sqrt{3} - 1)x)^2 + (\sqrt{6}x)^2 - 2x^2}{2 \times (\sqrt{3} - 1)x \times \sqrt{6}x}$$

$$\cos(A) = \frac{3+1-2\sqrt{3}+6-4}{2\times(\sqrt{3}-1)\times\sqrt{6}}$$

$$\cos A = \frac{2(3-\sqrt{3})}{2\times(\sqrt{3}-1)\times\sqrt{6}}$$

$$\cos A = \frac{\sqrt{3}(\sqrt{3}-1)}{(\sqrt{3}-1)\times\sqrt{6}}$$

$$\cos A = \frac{1}{\sqrt{2}} = 45°$$

Putting Cosine law for B,

$$\cos(B) = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos(B) = \frac{(2x)^2+((\sqrt{3}-1)x)^2-(\sqrt{6}x)^2}{2\times(\sqrt{3}-1)x\times 2x}$$

$$\cos(B) = \frac{4+3+1-2\sqrt{3}-6}{2\times(\sqrt{3}-1)\times 2}$$

$$\cos B = \frac{2(1-\sqrt{3})}{2\times(\sqrt{3}-1)\times 2}$$

$$\cos B = \frac{-1}{2}$$

$$B = 120°$$

Putting Cosine law for C,

$$\cos(C) = \frac{a^2+b^2-c^2}{2ab}$$

$$\cos(C) = \frac{(2x)^2+(\sqrt{6}x)^2-((\sqrt{3}-1)x)^2}{2\times\sqrt{6}x\times 2x}$$

$$\cos(C) = \frac{4+6-3-1+2\sqrt{3}}{2\times\sqrt{6}\times 2}$$

$$\cos C = \frac{2\sqrt{3}(\sqrt{3}+1)}{2\times 2\times\sqrt{6}}$$

$$\cos C = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

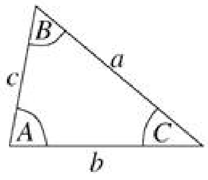
$$C = 15°$$

So, The largest Angle is 120° i.e. ∠B

8. Question

If in a ΔABC, $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$, then find the measures of angles A, B, C.

Answer



According to sine law,

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} = K$$

So,

$a = K \times \sin(A)$

$b = K \times \sin(B)$

$c = K \times \sin(C)$

Substituting in the given equation,

$$\frac{\cos A}{k \times \sin A} = \frac{\cos B}{k \times \sin B} = \frac{\cos C}{k \times \sin C}$$

$$\frac{\cos A}{\sin A} = \frac{\cos B}{\sin B} = \frac{\cos C}{\sin C}$$

cot A= cot B=cot C

Or

tan A=tan B=tan C

Which implies

A=B=C

And we know

A+B+C=180°

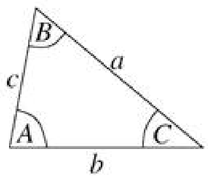
So,

A=B=C=60°

9. Question

In any triangle ABC, find the value of a sin (B – C) + b sin (C – A) + c sin (A – B).

Answer



Lets consider a equilateral as it is given any triangle,

So Here,

A=B=C=60°

Which implies

sin (B – C)=sin (60-60)

sin 0=0

sin (C-A)=sin(60-60)

sin 0=0

sin (A-B)=sin (60-60)

sin 0=0

Therefore,

The Equation will be,

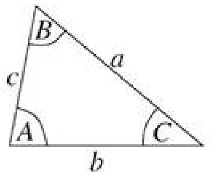
a sin (B – C)+ b sin (C – A)+ c sin (A – B)

a sin 0+ b sin 0+ c sin 0=0

10. Question

In any ΔABC, find the value of ∑ a (sin B – sin C)

Answer



$a(\sin B - \sin C) + b(\sin C - \sin A) + c(\sin A - \sin B)$

a(sinB - sinC) + b(sinC - sinA) + c(sinA - sinB)

We know,

By Sine law,

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} = K$$

a(Kb-Kc)+b(Kc-Ka)+c(Ka-Kb)

K(ab-ac+bc-ab+ac-bc)=0

MCQ

1. Question

Mark the Correct alternative in the following:

In any ΔABC, $\sum a^2(\sin B - \sin C) =$

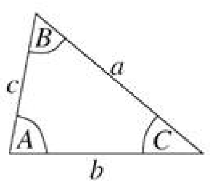
A. $a^2 + b^2 + c^2$

B. a^2

C. b^2

D. 0

Answer



$a^2(\sin B - \sin C) + b^2(\sin C - \sin A) + c^2(\sin A - \sin B)$

$a^2\sin B - a^2\sin C + b^2\sin C - b^2\sin A + c^2\sin A - c^2\sin B$

$a^2\sin B - c^2\sin B + b^2\sin C - a^2\sin C + c^2\sin A - b^2\sin A$

$(a^2 - c^2)\sin B + (b^2 - a^2)\sin C + (c^2 - b^2)\sin A$

By Sine law,

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} = \frac{1}{k}$$

$(a^2 - c^2)bk + (b^2 - a^2)ck + (c^2 - b^2)ak$

$a^2bk - c^2bk + b^2ck - a^2ck + c^2ak - b^2ak$

Considering it as equilateral,

$a=b=c$

$a^2bk - c^2bk + b^2ck - a^2ck + c^2ak - b^2ak=0$

Option D

2. Question

Mark the Correct alternative in the following:

In a ΔABC, if a = 2, ∠B = 60° and ∠C = 75°, then b =

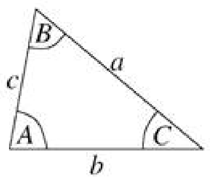
A. $\sqrt{3}$

B. $\sqrt{6}$

C. $\sqrt{9}$

D. $1+\sqrt{2}$

Answer



∠A = 180-(60+75) = 45°

By Sine law,

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} = K$$

$$\frac{2}{\sin(45)} = \frac{b}{\sin(60)} = \frac{c}{\sin(C)} = K$$

$$b = \frac{2 \times \sqrt{3} \times \sqrt{2}}{2} = \sqrt{6}$$

Option B

3. Question

Mark the Correct alternative in the following:

In the sides of triangle are in the ratio $1:\sqrt{3}:2,$ then the measure of its greatest angle is

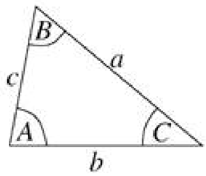
A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\frac{2\pi}{3}$

Answer



By Cosine law,

$$\cos(A) = \frac{c^2 + b^2 - a^2}{2cb}$$

$$\cos(A) = \frac{(2x)^2 + (\sqrt{3}x)^2 - 1x^2}{2 \times (2)x \times \sqrt{3}x}$$

$$\cos(A) = \frac{4 + 3 - 1}{2 \times 2 \times \sqrt{3}}$$

$$\cos A = \frac{\sqrt{3}}{2}$$

A=30°

Putting Cosine law for B,

$$\cos(B) = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos(B) = \frac{(1x)^2 + (2x)^2 - (\sqrt{3}x)^2}{2 \times (1)x \times 2x}$$

$$\cos(B) = \frac{2}{2 \times 1 \times 2}$$

$$\cos B = \frac{1}{2}$$

B=60°

Putting Cosine law for C,

$$\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos(C) = \frac{(1x)^2 + (\sqrt{3}x)^2 - (2x)^2}{2 \times \sqrt{3}x \times 1x}$$

$\cos(C) = \frac{1+3-4}{2 \times \sqrt{3} \times 1}$

$\cos C = \frac{0}{2\sqrt{3}}$

C=90°

So, The largest Angle is ∠C i.e. 90° or $\frac{\pi}{2}$.

Option C

4. Question

Mark the Correct alternative in the following:

In any ΔABC, (bc cos A + ca cos B + ab cos C) =

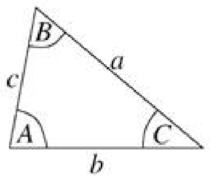
A. abc

B. a + b + c

C. $\frac{a^2+b^2-c^2}{2}$

D. $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$

Answer



By Cosine law,

$\cos(A) = \frac{c^2+b^2-a^2}{2cb}$ and $\cos(B) = \frac{a^2+c^2-b^2}{2ac}$ and $\cos(C) = \frac{a^2+b^2-c^2}{2ab}$

So,ATQ

$$bc\cos A + ca\cos B + ab\cos C = \left(bc \times \frac{c^2+b^2-a^2}{2cb}\right) + \left(ca \times \frac{a^2+c^2-b^2}{2ac}\right) + \left(ab \times \frac{a^2+b^2-c^2}{2ab}\right)$$

$$= \frac{c^2+b^2-a^2+a^2+c^2-b^2+a^2+b^2-c^2}{2}$$

$$= \frac{a^2+b^2-c^2}{2}$$

Option C

5. Question

Mark the Correct alternative in the following:

In a triangle ABC, a = 4, b = 3, ∠A = 60^o then c is a roof of the equation

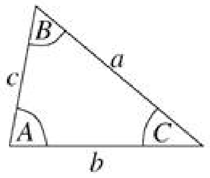
A. $c^2 - 3c - 7 = 0$

B. $c^2 + 3c + 7 = 0$

C. $c^2 - 3c + 7 = 0$

D. $c^2 + 3c - 7 = 0$

Answer



We know by cosine law,

$$\cos(A) = \frac{c^2 + b^2 - a^2}{2cb}$$

$$\frac{1}{2} = \frac{c^2 + 3^2 - 4^2}{2 \times c \times 3}$$

$$1 = \frac{c^2 + 9 - 16}{3 \times c}$$

$3c = c^2 - 7$

$c^2 - 3c - 7 = 0$

Option C

6. Question

Mark the Correct alternative in the following:

In a ΔABC, if (c + a + b) (a + b – c) = ab, then the measure of angle C is

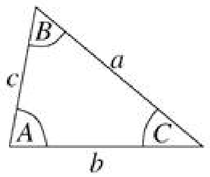
A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\frac{2\pi}{3}$

D. $\frac{\pi}{2}$

Answer



$ac + bc - c^2 + a^2 + ab - ac + ab + b^2 - bc - ab = 0$

$- c^2 + a^2 + ab + b^2 = 0$

$a^2 + b^2 - c^2 = - ab$

By Cosine law,

$$\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos(C) = \frac{-ab}{2ab}$$

$$\cos C = \frac{-1}{2}$$

$$C = 120° = \frac{2\pi}{3}$$

Option C

7. Question

Mark the Correct alternative in the following:

In any ΔABC, the value of 2ac $\sin\left(\frac{A - B + C}{2}\right)$ is

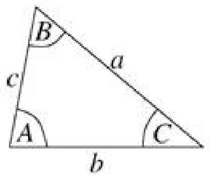
A. $a^2 + b^2 - c^2$

B. $c^2 + a^2 - b^2$

C. $b^2 - c^2 - a^2$

D. $c^2 - a^2 - b^2$

Answer



A+B+C=180°

$$2ac \times \sin\left(\frac{A - B + C}{2}\right)$$

$$2ac \times \sin\left(\frac{\pi - 2B}{2}\right)$$

$$2ac \times \sin\left(\frac{\pi}{2} - B\right)$$

$$2ac \times \cos B$$

But We Know,

$$\cos(B) = \frac{a^2 + c^2 - b^2}{2ac}$$

So the equation will be,

$$2ac \times \frac{a^2 + c^2 - b^2}{2ac}$$

$a^2+c^2-b^2 = c^2+a^2-b^2$

Option B

8. Question

Mark the Correct alternative in the following:

In any ΔABC, a (b cos C – c cos B) =

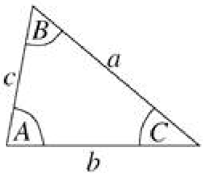
A. a^2

B. $b^2 - c^2$

C. 0

D. $b^2 + c^2$

Answer



$a\,(b\cos C - c\cos B) = ab\cos C - ac\cos B$

we know by Cosine Law,

$\cos(B) = \frac{a^2+c^2-b^2}{2ac}$ and $\cos(C) = \frac{a^2+b^2-c^2}{2ab}$

$$ab\cos C - ac\cos B = \left(ab \times \frac{a^2+b^2-c^2}{2ab}\right) - \left(ac \times \frac{a^2+c^2-b^2}{2ac}\right)$$

$$\frac{a^2+b^2-c^2-a^2-c^2+b^2}{2} = b^2 - c^2$$

Option B