CBSE Test Paper 04

Chapter 3 Matrices

1. If A and B are two matrices such that A + B and AB are both defined, then
    a. number of columns of A = number of rows of B.
    b. A, B are square matrices not necessarily of same order
    c. A and B can be any matrices
    d. A and B are square matrices of same order
2. The equations 2x + 3y = 7, 14x + 21y = 49 has
    a. infinitely many solutions
    b. finitely many solutions
    c. a unique solution
    d. no solution
3. If A is square matrix such that $A^2 = I$, then A^{-1} is equal to
    a. O
    b. A + I
    c. I
    d. A
4. If $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix}$ then A is a
    a. skew-symmetric matrix
    b. symmetric matrix
    c. none of these
    d. diagonal matrix
5. Let a, b, c, d, u, v be integers. If the system of equations, ax + by = u, cx + dy = v, has a unique solution in integers, then
    a. ad - bc need not be equal to ± 1.
    b. ad - bc = -1
    c. ad - bc = 1
    d. ad – bc = ± 1
6. A matrix which is not a square matrix is called a ________ matrix.
7. The product of any matrix by the scalar ________ is the null matrix.

8. If A is symmetric matrix, then B'AB is ________ matrix.
9. From the following matrix equation, find the value of x.
$$\begin{bmatrix} x+y & 4 \\ -5 & 3y \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -5 & 6 \end{bmatrix}$$
10. If A is a square matrix such that A^2 = A. then write the value of 7A - $(I + A)^3$. where I is an identity matrix. **(1)**
11. Use elementary column operation $C_2 \to C_2 - 2C_1$ in the matrix equation
$$\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}..$$
12. If A = $\begin{bmatrix} -1 & 5 \\ 3 & 2 \end{bmatrix}$, then show that (3A)' =3A'.
13. If A, B are square matrices of same order and B is a skew - symmetric matrix, show that A'BA is skew symmetric.
14. If $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$, then show that A is a skew symmetric matrix.
15. The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are `80, `60 and `40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.
16. i. Show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix.

    ii. Show that the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew symmetric matrix.
17. If $A = \begin{bmatrix} 3 & -4 \\ 1 & 1 \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$; where n is any positive integer.
18. Find x, y, z if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies A' = A^{-1}.

Solution

1. d. A and B are square matrices of same order

   Explanation: If A and B are square matrices of same order, both operations A + B and AB are well defined.

2. a. infinitely many solutions

   Explanation: For infinitely many solutions, $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$., for given system of equations we have : $\frac{2}{14}=\frac{3}{21}=\frac{7}{49}$.

3. d. A

   Explanation: If A amd B are two square matrices of same order and the product AB= I, the matrix B is called inverse of matrix A. Therefore, if A^2= I, then matrix A is the inverse of itself.

4. a. skew-symmetric matrix

   Explanation: The diagonal elements of a skew – symmetric matrix is always zero and the elements $a_{ij} = - a_{ji}$.

5. a. ad - bc need not be equal to ± 1.

   Explanation: ax + by = u , cx +dy = v , $\Delta=\begin{vmatrix}a & b\\ c & d\end{vmatrix}=ad-bc;$

   $\Delta_1=\begin{vmatrix}u & b\\ v & d\end{vmatrix}=ud-bv;\Delta_2=\begin{vmatrix}a & u\\ c & v\end{vmatrix}=av-cu;$

   $\Rightarrow x=\frac{\Delta_1}{\Delta}=\frac{ud-bv}{ad-bc},y=\frac{\Delta_2}{\Delta}=\frac{av-cu}{ad-bc}$

   since the solution is unique in integers. $\Delta=\pm 1$, $ad-bc=\pm 1$

6. rectangular
7. 0
8. symmetric

9. According to the question,

   $\begin{bmatrix}x+y & 4\\ -5 & 3y\end{bmatrix}=\begin{bmatrix}3 & 4\\ -5 & 6\end{bmatrix}$

   Equating the corresponding elements,

$x + y = 3$ ...(i)

and $3y = 6$ ...(ii)

From Eq. (ii), we get

$\Rightarrow y = 2$

On substituting y = 2 in Eq. (i), we get

x + 2 = 3

$\Rightarrow$ x = 1

10. According to the question, $A^2 = A$

Now, $7A - (I + A)^3 = 7A - [I^3 + A^3 + 3I(I + A)]$ [$\because (x + y)^3 = x^3 + y^3 + 3xy(x + y)$]

$= 7A - [I + A^2A + 3A(I + A)]$ [$\because I^3 = I$]

$= 7A - (I + A \cdot A + 3AI + 3A^2)$ [$\because A^2 = A$]

$= 7A - (I + A + 3A + 3A)$ [$\because AI = A$ and $A^2 = A$]

$= 7A - (I + 7A) = -I$

11. According to the question,

$$\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

Applying $C_2 \to C_2 - 2C_1$,

$$\Rightarrow \begin{bmatrix} 4 & 2-8 \\ 3 & 3-6 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0-4 \\ 1 & 1-2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & -6 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & -4 \\ 1 & -1 \end{bmatrix}$$

which is the required answer.

12. $3A = \begin{bmatrix} -3 & 15 \\ 9 & 6 \end{bmatrix}$

$(3A)' = \begin{bmatrix} -3 & 9 \\ 15 & 6 \end{bmatrix}$ ---(i)

$3A' = 3\begin{bmatrix} -1 & 3 \\ 5 & 2 \end{bmatrix}$

$= \begin{bmatrix} -3 & 9 \\ 15 & 6 \end{bmatrix}$ ----(ii)

From (i) and (ii) (3A)' = 3A'

13. Since, A and B are square matrices of same order and B is a skew-symmetric matrix
$\therefore$ B'=-B............(1)
Now, we have to prove that A’BA is a skew-symmetric matrix.
(A'BA)' = [A'(BA)]' = (BA)'(A')'. [$\because$(AB)'=B'A']
= (A'B')A. [$\because$ (A')'=A]
= A'(-B)A [using (1)]
= -(A'BA)
Hence, A’BA is a skew-symmetric matrix.

14. $A' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$

$A' = -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

Therefore A'= -A

Hence A is a skew symmetric matrix.

15. Let the number of books as a 1 $\times$ 3 matrix=
$B = \begin{bmatrix} 10\ dozen & 8\ dozen & 10\ dozen \\ 10 \times 12 = 120 & 8 \times 12 = 96 & 10 \times 12 \times 120 \end{bmatrix}$
Let the selling prices of each book as a 3 $\times$ 1 matrix $S = \begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$

$\therefore$ Total amount received by selling all books = $BS = \begin{bmatrix} 120 & 96 & 120 \end{bmatrix} \begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$

= [120(80) + 96(60) + 120(40)] = [9600 + 5760 + 4800] = [20160]
Therefore, Total amount received by selling all the books = Rs 20160

16. i. Given: $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ ....(i)

Changing rows of matrix A as the columns of new matrix

$$A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} = A$$

∴ A’ = A

Therefore, by definitions of symmetric matrix, A is a symmetric matrix.

ii. Given: $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ ....(i)

$$\therefore A' = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

Taking (-1) common, $A' = -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -A$ [From eq. (i)]

Therefore, by definition matrix A is a skew-symmetric matrix

17. For n = 1

$$\therefore\ A' = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

Hence, result is true for n =1

Let the result is true for n = k. Then,

$A^K = \begin{bmatrix} 1+2K & -4K \\ K & 1-2K \end{bmatrix}$ ....(i)

Now, we prove that the result is true for n = k + 1.

$A^{k+1} = A. A^k$

$= \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1+2K & -4K \\ K & 1-2K \end{bmatrix}$ (using (i))

$= \begin{bmatrix} 2K+3 & -4K-4 \\ K+1 & -2K-1 \end{bmatrix}$

∴ P (K + 1) is true.

Hence P (n) is true.

18. We have, $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ and $A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$

Also, $A' = A^{-1}$

$\Rightarrow AA' = AA^{-1}$ [$\because AA^{-1} = I$]

$\Rightarrow AA' = I$

$$\Rightarrow \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4y^2+z^2 & 2y^2-z^2 & -2y^2+z^2 \\ 2y^2-z^2 & x^2+y^2+z^2 & x^2-y^2-z^2 \\ -2y^2+z^2 & x^2-y^2-z^2 & x^2+y^2-z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow 2y^2 - z^2 = 0 \Rightarrow 2y^2 = z^2$

$\Rightarrow 4y^2 + z^2 = 1$

$\Rightarrow 2.z^2 + z^2 = 1$

$z = \pm\frac{1}{\sqrt{3}}$

$\therefore y^2 = \frac{z^2}{2} \Rightarrow y = \pm\frac{1}{\sqrt{6}}$

Also, $x^2 + y^2 + z^2 = 1$

$\Rightarrow x^2 = 1 - y^2 - z^2 = 1 - \frac{1}{6} - \frac{1}{3}$

$= 1 - \frac{3}{6} = \frac{1}{2}$

$\Rightarrow x = \pm\frac{1}{\sqrt{2}}$

$\therefore x = \pm\frac{1}{\sqrt{2}},\ y = \pm\frac{1}{\sqrt{6}}$

and $z = \pm\frac{1}{\sqrt{3}}$