"In practice, most of the interdependent relationships among characteristics can not be termed as function."

– Unknown

8

Function

Contents :

8.1 Definition

While studying set theory we know that, there may exist a relationship between the elements of two different sets. Some such relations are known as functions. To understand this, let us consider the following figures :

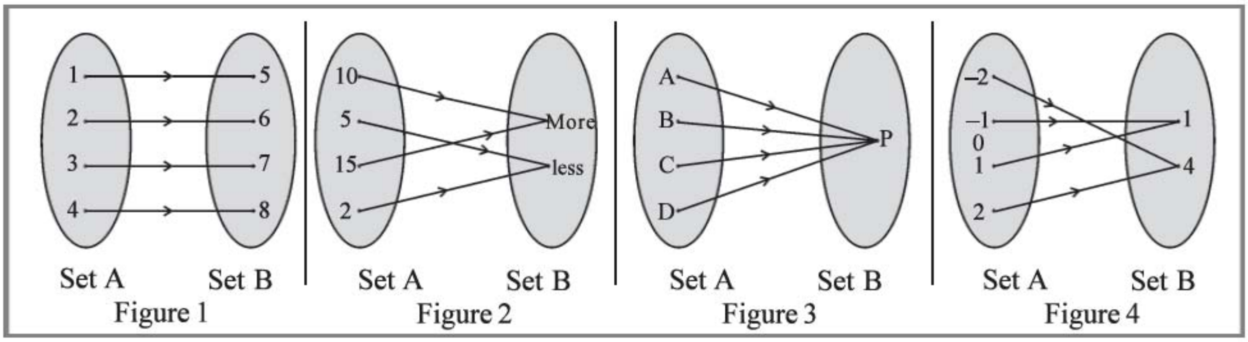


Figure 1 Figure 2 Figure 3 Figure 4

In figure 1, the elements of set A are {1, 2, 3, 4} and the elements of set B are {5, 6, 7, 8}. The rule between them is to add 4 in the elements of set A and hence we get the elements of set B.

In figure 2, elements of set A are {10, 5, 15, 2} which shows the daily demand of a commodity and set B indicates its type {more, less}. Now, if it is decided to bifurcate the demand as "more" if it is 10 or more otherwise it is called "less" demand, then from figure 2 it is clear that there is a relation between the elements of set A and set B.

In figure 3 , set A indicates the names of students of a class and set B indicates name of class teacher, then there exists an association between the elements of these sets.

In figure 4 , the elements of set A are {-2, -1, 0, 1, 2} and that of set B are {1, 4}. Some of the elements of set A are related with the elements of set B. But for the element "0" of set A, there is no corresponding element in set B.

From the above illustrations, it is clear that there exists various types of relations between the elements of two different sets. Can we say that all these relations are functions ? To understand this, let us first understand the definition of a function.

Definition : If A and B are any two non-empty sets and each element of set A is related with one and only one element of set B by some rule, relation or correspondence then it is called a function from set A to set B and is denoted by f, g, h, k, etc.

In the above figure 1, the rule 'add 4 to elements of set A' is called function and is denoted by $f(x) = x + 4,\ x \in A$.

In the figure 2, the relation 'bifurcate the demand as "more" if it is 10 or more otherwise demand is "less"', is also called function.

In figure 3, the correspondence between name of a student and the name of the class teacher is also called function.

Observing the figure 4, we notice that set A is {–2, –1, 0, 1, 2} and set B is {1, 4}. The elements –2, –1, 1, 2 have corresponding images 1, 4 in set B but the element (0) of A does not have an image in B. Hence, it is not a function.

Illustration 1 : Verify whether relations between the elements of the sets given below are functions or not.

(1) A = {1, 2, 3, 4}, B = {3, 5, 7, 9}, and relation is $f(x) = 2x + 1,\ x \in A$

(2) P = $\{-\frac{1}{2}, 0, 1\}$, S = {10}, and rule is $k(x) = 10,\ x \in P$

(3) A = {2, 5, 6}, B = $\{1, \frac{3}{2}, \frac{9}{5}, \frac{11}{7}, \frac{13}{6}\}$, and rule is $y = \frac{2x-1}{x+1},\ x \in A$

(4) B = {–1, 0, 1, 3}, C = {–5, –3, –1, 1, 3}, and rule is $h(x) = 2x + 3,\ x \in B$

(1) We shall determine the image for all $x \in$ A.

The relation is $f(x) = 2x + 1$

For $x = 1, f(1) = 2(1) + 1 = 3$

For $x = 2, f(2) = 2(2) + 1 = 5$

For $x = 3, f(3) = 2(3) + 1 = 7$

For $x = 4, f(4) = 2(4) + 1 = 9$

Thus, for each element of set A, there exists one and only one element in set B i.e. each element of set A is related with one and only one element of set B by the relation $f(x) = 2x + 1$. Hence, $f(x) = 2x + 1$ is a function.

(2) We shall determine the image for all $x \in P$ for $k(x) = 10$

For $x = -\frac{1}{2}$, $k\left(-\frac{1}{2}\right) = 10$

For $x = 0$, $k(0) = 10$

For $x = 1$, $k(1) = 10$

So, for every $x \in P$, there exists an element "10" in set S.

Hence the relation $k(x) = 10$ is a function.

(3) We shall determine the image for all $x \in$ A, for $y = \frac{2x-1}{x+1}$

For $x = 2$, $y = \frac{2(2)-1}{2+1} = \frac{4-1}{3} = \frac{3}{3} = 1$

For $x = 5$, $y = \frac{2(5)-1}{5+1} = \frac{10-1}{6} = \frac{9}{6} = \frac{3}{2}$

For $x = 6$, $y = \frac{2(6)-1}{6+1} = \frac{12-1}{7} = \frac{11}{7}$

So for every $x \in$ A, there exists a unique coresponding element in set B.

Hence, the relation $y = \frac{2x-1}{x+1}$ is a function.

(4) We shall determine the image for all $x \in$ B. The relation is $h(x) = 2x + 3$

For $x = -1$, $h(-1) = 2(-1) + 3 = -2 + 3 = 1$

For $x = 0$, $h(2) = 2(0) + 3 = 0 + 3 = 3$

For $x = 1$, $h(1) = 2(1) + 3 = 2 + 3 = 5$, which not an element of set C

So, there is no corresponding element for $x = 1$ in set C.

Hence, the relation $h(x) = 2x + 3$ is not a function.

8.2 Domain, Co–domain and Range

We know that, function is a unique relationship between the elements of two different non-empty sets. A unique image for elements of set A is obtained in set B. The set A is called domain and set B is called co-domain. A set of images or functional values of all the elements of set A is called range of the function. Domain of the function is denoted by D_f and range of the function is denoted by R_f. In notations, $R_f = f(A) = \{f(x) \mid x \in A\}$. Range of a function is a subset of co-domain or co-domain itself. For the illustration 1 above, in (1) set A is domain, set B is co-damain and it can be easily seen that the range is also co-domain B itself. Whereas in (2), domain is P and co-domain and range is S. In (3), domain is A, co-domain is B and range $R_f = \left\{1, \frac{3}{2}, \frac{11}{7}\right\}$ which is subset of co-domain B.

8.3 Notations of Functions

If f is a function of set A to set B then in notation it is shown as $f : A \rightarrow B$ and in words it is said that f is a function from set A to set B. Set A is called the domain and set B is called the co-domain.

From illustration 1, the relationship can be expressed as under :

(1) $f : A \rightarrow B$, A = {1, 2, 3, 4}, B = {3, 5, 7, 9} and $f(x) = 2x + 1$, $x \in A$

(2) $k : P \rightarrow S$, $P = \{-\frac{1}{2}, 0, 1\}$, S = {10} and $k(x) = 10$, $x \in P$

(3) $f : A \rightarrow B$, A = {2, 5, 6}, $B = \{1, \frac{3}{2}, \frac{9}{5}, \frac{11}{7}, \frac{13}{6}\}$, $y = f(x) = \frac{2x-1}{x+1}$, $x \in A$

(4) The relationship given here is not a function.

Illustration 2 : Obtain domain, co-domain and range for the following functions :

(1) $f : A \rightarrow B$, A = {–1, 0, 1}, B = {1, 2, 3, 4, 5, 6, 7}, $f(x) = 2x + 5$, $x \in A$

(2) $g : A \rightarrow N$, A = {–1, 2, 3, 4}, $g(x) = 3x + 5$, $x \in A$

(3) $h : P \rightarrow S$, P = {–2, –1, 0, 1}, S = {–4, –3, –2, –1}, $h(x) = x - 2$, $x \in P$

(4) $k : A \rightarrow Z$, $A = \left\{-\frac{1}{2}, 0, \frac{1}{2}\right\}$, $k(x) = 4x^2 + 3$, $x \in A$

(1) Here, Domain A = D_f = {–1, 0, 1}

Co-domain B = {1, 2, 3, 4, 5, 6, 7}

Now, for every $x \in A$, we find $f(x) = 2x + 5$

For $x = -1$, $f(-1) = 2(-1) + 5 = -2 + 5 = 3$

For $x = 0$, $f(0) = 2(0) + 5 = 0 + 5 = 5$

For $x = 1$, $f(1) = 2(1) + 5 = 2 + 5 = 7$

∴ Range of the function $R_f = \{f(-1), f(0), f(1)\} = \{3, 5, 7\}$

(2) Here, Domain A = D_f = {–1, 2, 3, 4}

Co-domain B = N

Now, for every $x \in A$, we find $g(x) = 3x + 5$.

For $x = -1$, $g(-1) = 3(-1) + 5 = 2$

For $x = 2$, $g(2) = 3(2) + 5 = 11$

For $x = 3$, $g(3) = 3(3) + 5 = 14$

For $x = 4$, $g(4) = 3(4) + 5 = 17$

∴ Range of the function R_f = {2, 11, 14, 17} which is subset of co-domain.

(3) Here, Domain A = P = D_f = {–2, –1, 0, 1}

Co-domain B = S = {–4, –3, –2, –1}

Now, for every $x \in P$, we find $h(x) = x - 2$.

For $x = -2$, $h(-2) = -2 - 2 = -4$

For $x = -1$, $h(-1) = -1 - 2 = -3$

For $x = 0$, $h(0) = 0 - 2 = -2$

For $x = 1$, $h(1) = 1 - 2 = -1$

∴ Range of the function R_f = {–4, –3, –2, –1}

Here, the range and co-domain are same.

(4) Here, Domain $A = D_f = \left\{-\frac{1}{2}, 0, \frac{1}{2}\right\}$

Co-domain $B = Z$

Now, for every $x \in A$, we find $k(x) = 4x^2 + 3$.

For $x = -\frac{1}{2}$, $k\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right)^2 + 3 = 1 + 3 = 4$

For $x = 0$, $k(0) = 4(0)^2 + 3 = 0 + 3 = 3$

For $x = \frac{1}{2}$, $k\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^2 + 3 = 1 + 3 = 4$

$\therefore$ Range of the function $R_f = \{3, 4\}$. Here, the range is a subset of co-domain.

Illustration 3 : Find domain, co-domain and range for the following functions :

(1) $f : A \rightarrow N$, $f(x) = x^2 + 1$, $A = \{x \mid -2 \le x < 1, x \in Z\}$

(2) $f : Z \rightarrow N$, $f(x) = x^2 + 2$, $x \in Z$

(3) $f : N \rightarrow N$, $f(x) = 4x$, $x \in N$

(1) Here, Domain $D_f = A = \{-2, -1, 0\}$

Co-domain $B = N$

function $f(x) = x^2 + 1$

For $x = -2$, $f(-2) = 4 + 1 = 5$

For $x = -1$, $f(-1) = 1 + 1 = 2$

For $x = 0$, $f(0) = 0 + 1 = 1$

$\therefore$ Range of the function $R_f = \{5, 2, 1\}$

(2) Domain $D_f = A = Z$

Co-domain $B = N$

For the range of the function $f(x) = x^2 + 2$,

$R_f = \{....., f(-2), f(-1), f(0), f(1), f(2), ...\}$

$= \{....., 2, 3, 6, 11, ...\}$

(3) Domain $D_f = A = N$

Co-domain $B = N$,

The range of function $f(x) = 4x$

$R_f = \{f(1), f(2), f(3), f(4), ...\}$

$= \{4, 8, 12, 16, ...\}$

Activity

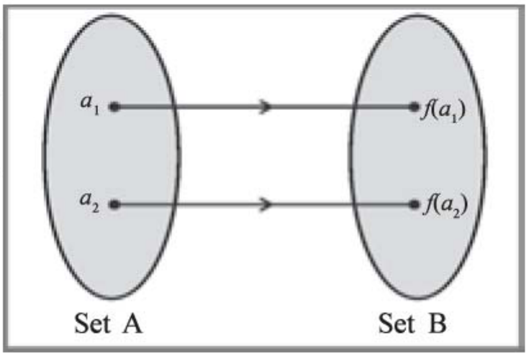
For the last cricket match that you have seen, write a set indicating names of bolwers. Also write a set indicating the possible wickets that they may get. From this information can you say that the relation between the set of bowlers' names and the set of possible wickets is a function ? If yes, then find the domain, co-domain and range of the function.

8.4 Types of Function

Functions are of many types. The main three types among them are as under :

(1) One-One function (2) Many-One function (3) Constant function.

8.4.1 One – One function



Suppose f : A→B. It for any two different elements of domain A, their images or functional values are different then the function f if called one – one function.

i.e. for function f : A→B, $a_1 \neq a_2$, $a_1, a_2 \in$ A and $f(a_1) \neq f(a_2)$ then function f is called one-one function.

e.g., g : X → Y where X = {–2, –1, 0}, Y = {0, 1, 2, 3} and $g(x) = x + 2$

Now, for $x = -2$, $g(-2) = 0$

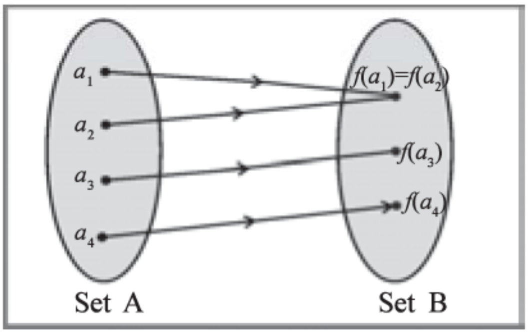
for $x = -1$, $g(-1) = 1$

for $x = 0$, $g(0) = 2$

Thus, for $a_1 \neq a_2$, $a_1, a_2 \in$ A, $g(a_1) \neq g(a_2)$ i.e. for two different values of the domain, there are two different images in co-domain. Hence, the given function g is one–one function.

8.4.2 Many–One function

Suppose f : A→B. If for any two different elements of domain A, their images or functional values are same, then function f is called many–one function.



i.e. for function f : A→B, $a_1 \neq a_2$, $a_1, a_2 \in$ A and $f(a_1) = f(a_2)$ then function f is called many–one function.

e.g., f : A→B, A = {–2, –1, 1, 2} and B = {2, 5}, $f(x) = x^2 + 1$. Then the functional value for each value of the domain is obtained as under :

for $x = -2$, $f(-2) = (-2)^2 + 1 = 5$

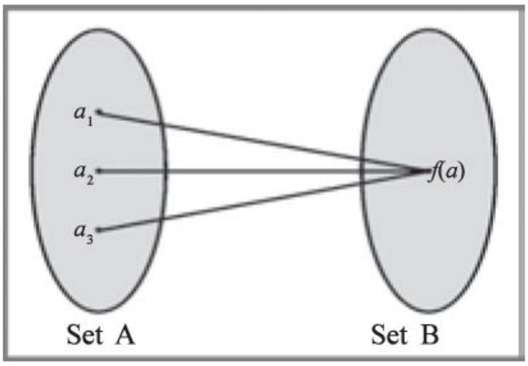
for $x = -1$, $f(-1) = (-1)^2 + 1 = 2$

for $x = 1$, $f(1) = (1)^2 + 1 = 2$

for $x = 2$, $f(2) = (2)^2 + 1 = 5$

Thus, for $a_1 \neq a_2$, $a_1, a_2 \in$ A, $f(a_1) = f(a_2)$ i.e. for two different values of the domain, there are two same images in co-domain. Hence, the given function f is many–one function.

8.4.3 Constant function



Suppose f : A→B. If for each element of domain A, the image or functional value is same then function f is called constant function.

e.g., f = {1, 2, 3} → {4, 5, 6} and $f(x) = 5$. Then, for values 1, 2 and 3 of the variable x, we get f(x) = 5. Thus, f is a constant function.

Illustration 4 : State the type of function f : N→N, $f(x) = x^2$

Domain D_f = N = {1, 2, 3, ...}

Co-domain B = N = {1, 2, 3, ...}

$f(x) = x^2$. By substituting the values of x = 1, 2, 3, ... the images are 1, 4, 9,...

Thus, for two different values of domain their images in co-domain are different. Hence, the given function is one – one function.

Illustration 5 : f : Z→N ∪ {0}, $f(x) = x^2$ then state the type of function f.

Domain D_f = Z = {... –3, –2, –1, 0, 1, 2, 3, ...}

Co-domain B = N ∪ {0} = {0, 1, 2, 3, ...}

By substituting the values of x = –2, –1, 0, 1, 2 the images are 4, 1, 0, 1, 4

Thus, for two different values {–2, 2} and {–1, 1} of domain their images in co-domain are same. Hence, the given function is many – one function.

Illustration 6 :

(i) f : Z→ R and $f(x)$ = 100 then state the type of function f.

Domain A = D_f = Z = {....., –2, –1, 0, 1, 2, ...} and co-domain B = R. For every value of the domain, the image in B is same, which is "100".

Hence, the given function is constant function.

(ii) State the type of the function between dates of a month and days of the week.

In a month, there are four weeks. So, the four different days are repeated four times with different dates of a month.

Hence, it is a many–one function.

Activity

Decide the type of function formed between the elements of sets indicating names of your five friends and the number of their family members.

8.5 Equal Functions

If two different functions f and g satisfy the following conditions then they are said to be equal functions. It is denoted by $f = g$.

(1) Both the functions must have same domain i.e. both the functions must be defined on same domain.

(2) For each element x of the domain, $f(x) = g(x)$ i.e. for each value of the domain their images must be same.

In notation, the above definition can be written as under :

If f : A → B and g : A → C and for each, $x \in$ A, $f(x) = g(x)$ then we can say that $f = g$.

Illustration 7 : If f = N → N, $f(x) = x^2$ and g = Z – {0} → N, $g(x) = x^2$, can we say that f and g are equal functions ?

Domain of function f is D_f = A = N

Domain of function g is D_f = A = Z – {0}

Thus, both the functions are defined on different domains. So, they are not equal functions.

Illustration 8 : f : A→B, A = {1, 3}, B = {1, 4, 9, 16}, $f(x) = x^2$ and g : A→B, A = {1, 3}, B = {1, 4, 7, 9, 11} $g(x) = 4x - 3$. Check the equality of the functions f and g.

Both the functions given here are defined on the same domain A.

For $x = 1, f(1) = 1^2,\quad g(1) = 4(1) - 3$

$= 1 \quad\quad = 1$

$x = 3, f(3) = 3^2,\quad g(3) = 4(3) - 3$

$= 9 \quad\quad = 9$

Thus, for each $x \in A$, $f(x) = g(x)$. Therefore, f and g are equal functions.

Illustration 9 : $f : A \rightarrow B$, $f(x) = 1 - 4x$ and $g : A \rightarrow C$, $g(x) = 6x + 1$ and $A = \{0, 1, 2\}$. Are f and g equal functions ?

Both the functions given here are defined on the same domain.

For $x = 0$, $f(0) = 1 - 4(0) = 1$ and $g(0) = 6(0) + 1 = 1$

For $x = 1$, $f(1) = 1 - 4(1) = -3$ and $g(1) = 6(1) + 1 = 7$

Thus, $f(1) \neq g(1)$. Therefore, f and g are not equal functions.

8.6 Real Functions

If $f : A \rightarrow B$ where $A \subset R$ then f is called function of real variable and if the range of f is also defined on real set R then f is called real function. e.g., for $f : Z \rightarrow R$, domain is Z which is the set of integers and the co-domain is the real set R itself. Therefore, function f is called real function. In short, a function for which domain and co-domain are defined on real set R or on any subset of it, is called real function.

Illustration 10 : If $f(x)\ \dfrac{x^3 + 1}{x^2 - 2x + 1}$ where $x \in Z - \{1\}$ then find $f(-2)$, $f(-1)$ and $f(0)$.

$$f(x) = \frac{x^3 + 1}{x^2 - 2x + 1}$$

$$f(-2) = \frac{(-2)^3 + 1}{(-2)^2 - 2(-2) + 1}; \quad f(-1) = \frac{(-1)^3 + 1}{(-1)^2 - 2(-1) + 1}$$

$$= \frac{-8 + 1}{4 + 4 + 1} \quad\quad = \frac{-1 + 1}{1 + 2 + 1}$$

$$= -\frac{7}{9} \quad\quad = \frac{0}{4}$$

$$= 0$$

$$f(0) = \frac{0^3 + 1}{0^2 - 2(0) + 1} = \frac{1}{1} = 1$$

Illustration 11 : If $f(x) = x^3 + 3^x - x^2 - 2^x$ then find $f(3) - 6f(2)$.

$f(x) = x^3 + 3^x - x^2 - 2^x$

$f(3) = (3)^3 + 3^3 - (3)^2 - (2)^3$ and $f(2) = 2^3 + 3^2 - 2^2 - 2^2$

$= 27 + 27 - 9 - 8 \quad\quad = 8 + 9 - 4 - 4$

$= 37 \quad\quad = 9$

$\therefore \quad f(3) - 6f(2) = 37 - 6(9)$

$= 37 - 54$

$= -17$

Illustration 12 : $f(x) = x(2x - 7)$ where $x \in R$. If $f(x) = 15$ then find the value of x.

$f(x) = 15$

$\therefore \quad x(2x - 7) = 15$

$\therefore \quad 2x^2 - 7x - 15 = 0$

$\therefore \quad 2x^2 + 3x - 10x - 15 = 0$

$\therefore \quad x(2x + 3) - 5(2x + 3) = 0$

$\therefore \quad (2x + 3)(x - 5) = 0$

$\therefore \quad (2x + 3) = 0$ or $(x - 5) = 0$

$\therefore \quad x = -\frac{3}{2}$ or $x = 5$

Illustration 13 : If $f(x) = x^2 - 4x + 8$ then for which value of x, is $f(2x) = 2f(x)$?

$f(x) = x^2 - 4x + 8$

$\therefore \quad f(2x) = (2x)^2 - 4(2x) + 8$

$= 4x^2 - 8x + 8$

Now, $f(2x) = 2f(x)$

$\therefore \quad 4x^2 - 8x + 8 = 2x^2 - 8x + 16$

$\therefore \quad 2x^2 - 8 = 0$

$\therefore \quad x^2 - 4 = 0$

$\therefore \quad x^2 = 4$

$\therefore \quad x = \pm 2$

$\therefore$ When $x = \pm 2$, $f(2x) = 2f(x)$.

Summary

- Unique relation between elements of two non-empty sets is called function.
- If $f : A \to B$ is a function then set A is called domain and B is called co-domain.
- A set of all images obtained for each value of the domain is called range of a function.
- Range of a function is subset of the co-domain or is the co-domain itself.
- If for any two different values of the domain of a function, their images in co-domain are also different then it is called one - one function.
- If for any two different values of the domain of a function, their images in co-domain are same then it is called many - one function.
- If for each value of the domain of a function, their images in co-domain are same then it is called constant function.
- For equality of two different functions, they must be defined on same domain and their functional values for each value of the domain must be same.
- If the domain of a function is a subset of real set R then the function f is called a function of real variable.
- If the domain and co-domain of a function are defined on real set R then it is called real function.

EXERCISE 8

Section A

Find the correct option for the following multiple choice questions :

1. Which of the following statement is true ?

(a) f : { 1, 2, 3, 4 } → { 3, 4, 5 }, the rule 'add 2 to the elements of domain' is not a function.

(b) f : A → B, A = {–2, –1, 0, 1, 2 }, B = {0, 1, 2, 3, 4}, $f(x) = x^2$ is not a function.

(c) g : P → Q, P = {–1, 0, 1}, Q = {$-\frac{1}{3}$ –1, 3}, $g(x) = \frac{x+2}{x-2}$, then g is called function.

(d) g : {2, 3, 4, 5} → {–1, 0, 1} and $g(x) = 4x - 3$ is a function.

2. Which of the following statements is true for the range of function f: A → B ?

(a) $f(A) = \{f(x) \mid x \in A\}$

(b) It is not a co-domain or subset of co-domain.

(c) Domain itself is the range

(d) $f(A) = \{f(x) \mid x \in B\}$

3. Which of the following statements is true for the relation g : X → Y, X = {–1, 0}, Y = {2, 4,}, $g(x) = 4 - 2x$?

(a) g is called a function.

(b) g is not a function.

(c) X is called function.

(d) Y is called function.

4. What is the type of function f: A → B, wherein, for two different values of domain their functional values are same ?

(a) One – one function

(b) Many – one function

(c) One – many function

(d) Many – many function

5. What is the type of function f: A → B, where each value of domain A has the same image in set B ?

(a) Not a function (b) One – one function (c) Constant function (d) Many – one function

6. Which of the following statements is true for a one–one function ?

(a) Only for two values of the domain, their images should be different.

(b) For any two values of the domain, their images are same.

(c) For any two different values of the domain, their images are different.

(d) For each value of the domain, their images are same.

7. What is the type of function f : Z – {0} → N and $f(x) = x^2$, $x \in$ Z – {0} ?

(a) One–one function

(b) Many–one function

(c) Constant function

(d) None of the above

8. Which of the following is a sufficient condition for two different functions to be equal ?

(a) Domains of both the functions must be same.

(b) Ranges of both the functions must be same.

(c) (a) and (b)

(d) (a) or (b)

Section B

Answer the following questions in one sentence :

1. Give the necessary condition for defining a function.
2. $f : A \to B$, $A = \{-3\ -1,\ 1,\ 3\}$, $B = \{1,\ 0,\ 9\}$, $f(x) = x^2$. Is f a function ?
3. $g : N \to N$, 'subtract 2 from the elements of the domain'. Can this rule be called a function ?
4. Define one–one function in notations.
5. Define many–one function in notations.
6. Define constant function in notations.
7. $f : \{1, 2, 3\} \to N$, $g : \{2, 3, 4\} \to N$, $f(x) = 2x + 1$ and $g(x) = x - 1$. Can these two functions f and g be equal functions? Why ?
8. $f : Z \to N$, $f(t) = t^2 + 1$ $t \in Z$. Determine the type of function f.
9. $f : N \to N$, $f(t) = t^2 + 1$ $t \in N$. Determine the type of function f.
10. Define a function of real variable.

Section C

Answer the following questions :

1. Give definition of a function.
2. Define domain and co-domain of a function.
3. Define range of a function.
4. $g : A \to N$, $A = \{x \mid x \in N,\ 1 < x \le 4\}$, $g(x) = x + 1$. Find range of function g.
5. $k : X \to Y$, $X = \{t \mid t \in Z;\ -3 \le t \le 3\}$, $Y = \{\ a \mid a \in N,\ 1 \le a \le 20\}$, $k(t) = t^2 + 2$. State the type of function k.
6. $h : A \to B$, $A = \{1, 2, 3\}$, $B = \{3, 4, 5, 6, 7, 8\}$, $h(x) = x + 5$. State the type of function h.
7. If $P : A \to B$, $P(x) = 2x - 3$ and $R_f = \{-2, -1, 0\}$ then find domain of the function.
8. If $f(x) = 1 - \frac{1}{1-x^2}$, $x \in R - \{-1, 1\}$ then find $f(2) - f(-2)$.
9. If domain of $f(x) = \frac{x-3}{x+4}$ is $\{0, 3, 6\}$ then find its range.
10. If $f(x) = \frac{x^2(x+1)^2}{4}$ is a real function then find the value of $f(3) - f(2)$.
11. If $f : R \to R$ and $f(x) = x^2 + 2x - 1$ then state the type of function f.
12. If $f(x) = \frac{2x-4}{x+7}$ is a real function then for which value of x the image is zero ?
13. $f : Z - \{2\} \to Z$, $f(x) = \frac{x^2+x-6}{x-2}$. State the type of the function.
14. For a real function, $f(x) = 6x^3 - 5^x + 15$, find the value of $f(0)$.
15. If $f(x) = x^3 - 2x + \frac{1}{x}$ is a real function then find the value of $f(3) + f(-3)$.

Section D

Answer the following questions :

1. For $f : A \rightarrow B$, $A = \{10, 20, 30\}$, $B = \{18, 48, 98, 128, 148\}$, $f(x) = 5x - 2$, obtain domain, co-domain and range.

2. Obtain domain, co-domain and range for $f : P \rightarrow Q$, $P = \left\{-\frac{1}{2}, 1, \frac{1}{2}, \frac{3}{2}\right\}$,

   $Q = \left\{-\frac{1}{5}, 1, \frac{1}{3}, 3\right\}$, $f(x) = \frac{x}{2-x}$.

3. If $f : R - \{0\} \rightarrow R$, $f(x) = \frac{1}{x}\left(1 + \frac{1}{x}\right) - 1$ then find the value of $f(-1)$, $f(-2)$ and $f\left(\frac{1}{2}\right)$.

4. For the function $f : A \rightarrow B$, $f(x) = 4x - 3$, $R_f = \{9, 13, 17, 25\}$ then find D_f.

5. For the real function $f(x) = 2x^2 - 5x + 4$ find the value of x, for which $f(3x) - 3f(x) + 5 = 0$.

6. If $f : A \rightarrow M$, $A = \{x \mid x \in N, 1 \leq x < 5\}$ and $M = \{x \mid x \in N, 1 \leq x \leq 20\}$ and $f(x) = x^2 + 1$ then find the range of f.

7. If $f(x) = \frac{x^2-4}{x-2}$ where $x \in Z\text{-}\{2\}$ then find the value of $f(0) + f(1) - f(-2)$.

8. If the domain of a function $f : A \rightarrow N \cup \{0\}$, $f(x) = \sqrt{x^2-16}$ is $A = \{4, 5\}$ then find its range.

9. If $f(x) = x^2$ and $g(x) = 5x - 6$ where $x \in \{2, 3, 4\}$, check the equality of the functions.

10. If $k : R \rightarrow R$, $k(x) = x^2 + 3x - 12$ then determine the type of the function k.

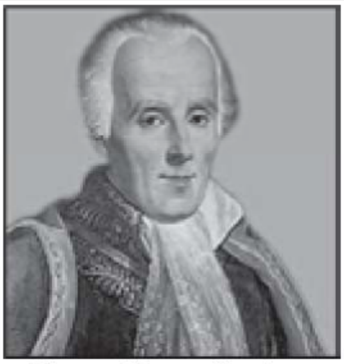
11. If $f(x) = x(3x-2)$, $g(x) = x^3$ and $x \in \{0, 1, 2,\}$ then prove that f and g are equal functions.

12. If $f(x) = \frac{2x+3}{5x+2}$, $x \in R - \left\{\frac{-2}{5}\right\}$ then find the value of $f(2) \cdot f\left(\frac{1}{2}\right)$.

13. If $f(x) = 2x^2 + \frac{1}{x}$, $x \in R - \{0\}$ obtain the value of $f(3) + f(-3)$

14. If $f(x) = 15x^3 - 4x^2 + x + 10$, $x \in R$, obtain the value of $\frac{f(2)}{f(1)}$.

15. If $f : A \rightarrow N \cup \{0\}$, $A = \{500, 1000, 1300, 1400\}$, $f(x) = \sqrt{5600-4x}$, $x \in R$ then find the value of $f(x)$ for $x = 1000$. Also, for which value of x, $f(x) = 20$?



Pierre-Simon Laplace
(1749 - 1827)

Pierre-Simon Laplace : He was an influential French scholar whose work was important to the development in the fields of mathematics, statistics, physics, and astronomy. His work translated the geometric study of classical mechanics to the one based on calculus, opening up a broader range of problems in statistics. He was a pioneer in the development of classical probability theory. The Bayesian interpretation of probability was developed by Laplace.