SAMPLE QUESTION PAPER (BASIC) - 05

Class 10 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

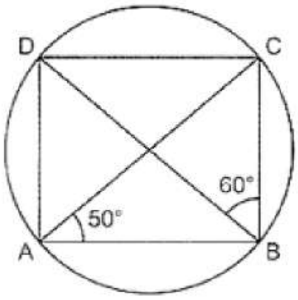
Section A

1. The distance between the points (a cosθ + b sinθ, 0) and (0, a sinθ - b cosθ) is **[1]**

a) $\sqrt{a^2 + b^2}$ b) a + b

c) $a^2 + b^2$ d) $a^2 - b^2$

2. In Fig. ABCD is a cyclic quadrilateral. If $\angle BAC = 50^\circ$ and $\angle DBC = 60^\circ$ then find $\angle BCD$. **[1]**



a) 50° b) 60°

c) 70° d) 55°

3. If the probability of an event is 'p', the probability of its complementary event will be **[1]**

a) p b) p – 1

c) 1 – p d) $1 - \frac{1}{p}$

4. If the point P(2, 1) lies on the line segment joining points A(4, 2) and B(8, 4), then **[1]**

a) $AP = \frac{1}{4}AB$

b) $AP = \frac{1}{2}AB$

c) $AP = \frac{1}{3}AB$

d) AP = PB

5. The graphs of the equations 5x - 15y = 8 and 3x - 9y = $\frac{24}{5}$ are two lines which are **[1]**

a) intersecting exactly at one point

b) coincident

c) perpendicular to each other

d) parallel

6. The abscissa of any point on the y-axis is **[1]**

a) 0

b) 1

c) y

d) -1

7. From the letters of the word MOBILE, a letter is selected. The probability that the letter is a vowel, is **[1]**

a) $\frac{3}{7}$

b) $\frac{1}{6}$

c) $\frac{1}{2}$

d) $\frac{1}{3}$

8. A solid is hemispherical at the bottom and conical (of same radius) above it. If the surface areas of the two parts are equal then the ratio of its radius and the slant height of the conical part is **[1]**

a) 4 : 1

b) 1 : 4

c) 1 : 2

d) 2 : 1

9. A number x is chosen at random from the numbers -3, -2, -1, 0, 1, 2, 3 the probability that |x| < 2 is **[1]**

a) $\frac{1}{7}$

b) $\frac{2}{7}$

c) $\frac{3}{7}$

d) $\frac{5}{7}$

10. If the roots of the equation $ax^2 + bx + c = 0$ are equal then c = ? **[1]**

a) $\frac{b}{2a}$

b) $\frac{b^2}{4a}$

c) $\frac{-b^2}{4a}$

d) $\frac{-b}{2a}$

11. Which of the following is not a quadratic equation? **[1]**

a) $x = x^2 + 3 + 4x^2$

b) $2(x-1)^2 = 4x^2 - 2x + 1$

c) $\left(\sqrt{2}x + \sqrt{3}\right)^2 + x^2 = 3x^2 - 5x$

d) $2x - x^2 = x^2 + 5$

12. $\sec^4 A - \sec^2 A$ is equal to **[1]**

a) $\tan^2 A - \tan^4 A$

b) $\tan^4 A - \tan^2 A$

c) $\tan^2 A + \tan^3 A$

d) $\tan^4 A + \tan^2 A$

13. Which of the following is an irrational number? **[1]**

i. $\frac{22}{7}$

ii. 3.1416

iii. $3.\overline{1416}$

iv. 3.141141114...

a) Option (iv)

b) Option (iii)

c) Option (i)

d) Option (ii)

14. In the given figure P(5, - 3) and Q(3, y) are the points of trisection of the line segment joining A(7, -2) and B(1, **[1]**

-5).

Then, y equals

a) $\frac{-5}{2}$

b) 2

c) -4

d) 4

15. The string of a kite is 100 m long and it makes an angle of 60° with the horizontal. If there is no slack in the string, the height of the kite from the ground is **[1]**

a) $100\sqrt{3}$ m

b) $50\sqrt{2}$ m

c) 100 m

d) $50\sqrt{3}$ m

16. In the formula $\overline{x} = a + h\left(\frac{\sum f_i u_i}{\sum f_i}\right)$, a stands for **[1]**

a) class mark

b) mean

c) class size

d) assumed mean

17. (2+ $\sqrt{5}$) is **[1]**

a) an irrational number

b) not real number

c) a rational number

d) an integer

18. If 2x + 3y = 12 and 3x - 2y = 5 then **[1]**

a) x = 3, y = 2

b) x = 2, y = -3

c) x = 2, y = 3

d) x = 3, y = -2

19. **Assertion (A):** For any two positive integers a and b, HCF (a, b) × LCM (a, b) = a × b **[1]**

Reason (R): The HCF of the two numbers is 8 and their product is 280. Then their LCM is 40.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** In a triangle PQR, X and Y are points on sides PQ and PR respectively, such that XY || QR, then $\frac{PX}{XQ} = \frac{PY}{YR}$. **[1]**

Reason (R): Basic proportionality theorem.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. A bag contains a red ball, a blue ball and a yellow ball. All the balls are being of the same size. Kritika takes out a ball from the bag without looking into it. What is the probability that she takes out the **[2]**

i. yellow ball?

ii. red ball?

iii. blue ball?

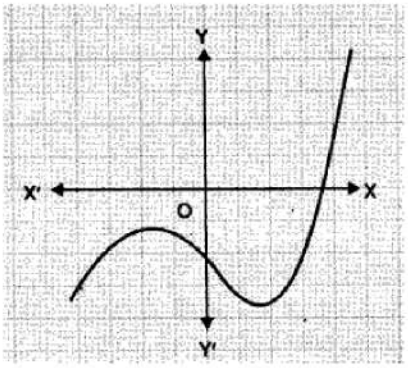
22. The taxi charges in a city comprise of a fixed charge together with the charge for the distance covered. For a **[2]**

journey of 10 km the charge paid is Rs.75 and for a journey of 15 km the charge paid is Rs.110. What will a person have to pay for travelling a distance of 25 km?

OR

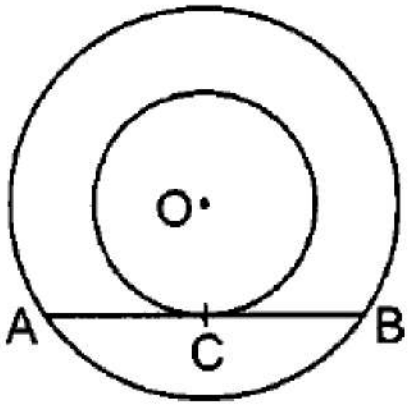
Do the equation $\frac{x}{2}+y+\frac{2}{5}=0$ and $4x+8y+\frac{5}{16}=0$ represent a pair of coincident lines? Justify your answer.

23. Find the number of zeroes of p(x). The graph of y = p(x) is given in figure below, for some polynomial p(x): **[2]**



24. Find a point on the y-axis which is equidistant from the points A(6, 5) and B (- 4, 3). **[2]**

25. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. Using the above, do the following: In figure, O is the centre of the two concentric circles. AB is a chord of the larger circle touching the smaller circle at C. Prove that AC = BC. **[2]**



OR

Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre.

Section C

26. Prove that $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\frac{1}{\sec\theta-\tan\theta}$, using identity $sec^2\theta=1+tan^2\theta$. **[3]**

27. Check whether the pair of equations x + 3y = 6 and 2x – 3y = 12 is consistent. If so, solve them graphically. **[3]**

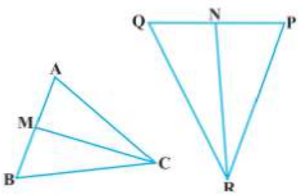
28. Show that $\sqrt{6}+\sqrt{2}$ is irrational. **[3]**

OR

Find the LCM of the following polynomials:a^8-b^8 and $\left(a^4-b^4\right)(a+b)$

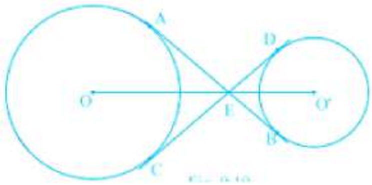
29. In Fig. CM and RN are respectively the medians of $\triangle$ABC and $\triangle$PQR. If $\triangle$ABC $\sim$ $\triangle$PQR, prove that: **[3]**

    i. $\triangle$AMC $\sim$ $\triangle$PNR
    ii. $\frac{CM}{RN}=\frac{AB}{PQ}$
    iii. $\triangle$CMB $\sim$ $\triangle$RNQ



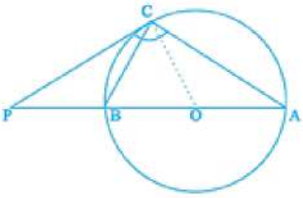
30. The common tangents AB and CD to two circles with centres O and O' intersect at E between their centres. **[3]**

Prove that the points O, E and O' are collinear.



OR

The tangent at a point C of a circle and a diameter AB when extended intersect at P. If ∠PCA = 110°, find ∠CBA. [Hint: Join C with centre O].



31. If the angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary, find the height of the tower. **[3]**

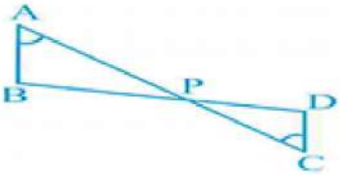
Section D

32. The area of right angled triangle is 480 cm^2 . If the base of triangle is 8 cm more than twice the height (altitude) of the triangle, then find the sides of the triangle. **[5]**

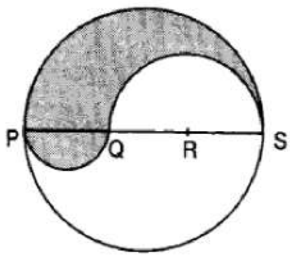
OR

A motorboat whose speed is 18 km/h in still water takes 1 hr 30 minutes more to go 36 km upstream than to return downstream to the same spot. Find the speed of the stream.

33. In the given figure, if ∠A = ∠C, AB = 6cm, BP = 15 cm, AP = 12cm and CP = 4, then find the lengths of PD and CD. **[5]**



34. PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters as shown in Fig. Find the perimeter and area of the shaded region **[5]**



OR

Find the area of the segment of a circle of radius 12 cm whose corresponding sector central angle 60°. (Use π = 3.14).

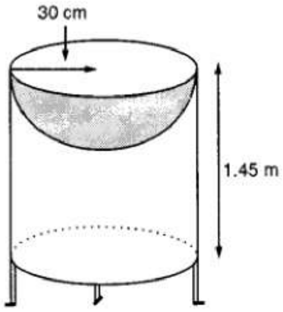
35. The table below gives the percentage distribution of female teachers in the primary schools of rural areas of various states and union territories of India. Find the mean percentage of female teachers by assumed mean method. **[5]**

Percentage of female teachers	15 - 25	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75	75 - 85
Number of states/ U.T.	6	11	7	4	4	2	1

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Mayank a student of class 7^{th} loves watching and playing with birds of different kinds. One day he had an idea in his mind to make a bird-bath on his garden. His brother who is studying in class 10^{th} helped him to choose the material and shape of the birdbath. They made it in the shape of a cylinder with a hemispherical depression at one end as shown in the Figure below. They opted for the height of the hollow cylinder as 1.45 m and its radius is 30 cm. The cost of material used for making bird bath is ₹40 per square meter.



(i) Find the curved surface area of the hemisphere.

(ii) Find the total surface area of the bird-bath. (Take $\pi = \frac{22}{7}$)

(iii) What is total cost for making the bird bath?

OR

Mayank and his brother thought of increasing the radius of hemisphere to 35 cm with same material so that birds get more space, then what is the new height of cylinder?

37. **Read the text carefully and answer the questions:** **[4]**

Kamla and her husband were working in a factory in Seelampur, New Delhi. During the pandemic, they were asked to leave the job. As they have very limited resources to survive in a metro city, they decided to go back to their hometown in Himachal Pradesh. After a few months of struggle, they thought to grow roses in their fields and sell them to local vendors as roses have been always in demand. Their business started growing up and they hired many workers to manage their garden and do packaging of the flowers.

In their garden bed, there are 23 rose plants in the first row, 21 are in the 2^{nd}, 19 in 3^{rd} row and so on. There are 5 plants in the last row.

(i) How many rows are there of rose plants?

(ii) Also, find the total number of rose plants in the garden.

(iii) How many plants are there in 6th row.

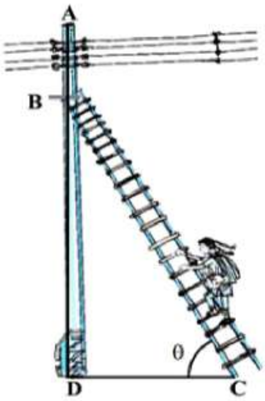
OR

If total number of plants are 80 in the garden, then find number of rows?

38. **Read the text carefully and answer the questions:** **[4]**

In a village, group of people complained about an electric fault in their area. On their complaint, an electrician reached village to repair an electric fault on a pole of height 10 m. She needs to reach a point 1.5 m below the top of the pole to undertake the repair work (see the adjoining figure). She used ladder, inclined at an angle of θ

to the horizontal such that $\cos\theta = \frac{\sqrt{3}}{2}$, to reach the required position.



(i) Find the length BD?

(ii) Find the length of ladder.

(iii) How far from the foot of the pole should she place the foot of the ladder?

OR

If the height of pole and distance BD is doubled, then what will be the length of the ladder?

Solution

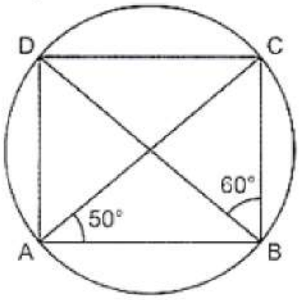
SAMPLE QUESTION PAPER (BASIC) - 05

Class 10 - Mathematics

Section A

1. **(a)** $\sqrt{a^2+b^2}$
   Explanation: Distance between $(a\cos\theta+b\sin\theta,0)$ and $(0,a\sin\theta-b\cos\theta)$
   $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
   $=\sqrt{\sqrt{(0-(a\cos\theta+b\sin\theta))^2+\{(a\sin\theta-b\cos\theta-0)\}^2}}$
   $=\sqrt{\{0-(a\cos\theta+b\sin\theta)\}^2+\{(a\sin\theta-b\cos\theta-0)\}^2}$
   $=\sqrt{\begin{pmatrix}a^2\cos^2\theta+b^2\sin^2\theta+2ab\sin\theta\cos\theta+\\ a^2\sin^2\theta+b^2\cos^2\theta-2ab\sin\theta\cos\theta\end{pmatrix}}$
   $=\sqrt{a^2\times1+b^2\times1}=\sqrt{a^2+b^2}$
   $\{\because\sin^2\theta+\cos^2\theta=1\}$

2. **(c)** 70°
   Explanation: Here $\angle BDC=\angle BAC=50^\circ$ (angles in same segment are equal)



   In ΔBCD, we have
   $\angle BCD=180^\circ-(\angle BDC+\angle DBC)$
   $=180^\circ-(50^\circ+60^\circ)$
   $=70^{\circ}$

3. **(c)** 1 – p
   Explanation: If the probability of an event is p, the probability of its complementary event will be 1 - p. because we know that the sum of probability of an event and its complementary event is always 1.
   Hence, p + 1 - p = 1

4. **(b)** $AP=\frac{1}{2}AB$
   Explanation: AP = $\sqrt{(2-4)^2+(1-2)^2}$
   = $\sqrt{4+1}$ = $\sqrt{5}$ = units
   AB = $\sqrt{(8-4)^2+(4-2)^2}$
   = $\sqrt{16+4}$ = $\sqrt{20}$ = $2\sqrt{5}$ units
   Here AB = 2 $\times$ AP
   $\therefore$ AP = $\frac{1}{2}$AB

5. **(b)** coincident
   Explanation: We have,
   5x - 15y - 8 = 0
   And, 3x - 9y - $\frac{24}{5}$ = 0
   Here, a_1 = 5, b_1 = - 15 and c_1 = - 8
   And, a_2 = 3, b_2 = - 9 and c_2 = $\frac{-24}{5}$
   $\therefore\frac{a_1}{a_2}=\frac{5}{3},\frac{b_1}{b_2}=\frac{-15}{-9}=\frac{5}{3}$ and $\frac{c_1}{c_2}=-8\times\frac{5}{-24}=\frac{5}{3}$
   Clearly, $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$
   Hence, the given system has a unique solution and the lines are coincident.

6. **(a)** 0

Explanation: Since coordinates of any point on y-axis is (0, y)

Therefore, the abscissa is 0.

7. **(c)** $\frac{1}{2}$

Explanation: No. of total letters in the word MOBILE = 6

No, of vowels = {O,I,E}, i, e = 3

Probability of being a vowel = $\frac{3}{6} = \frac{1}{2}$

8. **(c)** 1 : 2

Explanation: $2\pi r^2 = \pi rl \Rightarrow \frac{r}{l} = \frac{1}{2}$

9. **(c)** $\frac{3}{7}$

Explanation: Total possible number of events (n) = 7

Now |x| < 2

x < 2 or -x < 2

$\Rightarrow$ x > -2

$\therefore$ x

$\Rightarrow$ x = 1, 0, -1, -2, -3 or x = -1, 0, 1, 2, 3

$\therefore$ x = -1, 0, 1

$\therefore$ m = 3

$\therefore$ Probability = $\frac{m}{n} = \frac{3}{7}$

10. **(b)** $\frac{b^2}{4a}$

Explanation: Since the roots are equal, we have D = 0

$\therefore b^2 - 4ac = 0 \Rightarrow 4ac = b^2 \Rightarrow c = \frac{b^2}{4a}$.

11. **(c)** $\left(\sqrt{2}x + \sqrt{3}\right)^2 + x^2 = 3x^2 - 5x$

Explanation: In equation $\left(\sqrt{2}x + \sqrt{3}\right)^2 + x^2 = 3x^2 - 5x$

$\Rightarrow 2x^2 + 3 + 2\sqrt{6}x + x^2 = 3x^2 - 5x$

$\Rightarrow 3x^2 - 3x^2 + 5x + 2\sqrt{6}x + 3 = 0$

$\Rightarrow \left(5 + 2\sqrt{6}\right)x + 3 = 0$

It is not the quadratic equation because its degree is not 2.

12. **(d)** $\tan^4$A + $\tan^2$A

Explanation: We have, $\sec^4$A - $\sec^2$A = $\sec^2$A ($\sec^2$A - 1)

= (1 + $\tan^2$A) $\tan^2$A

= $\tan^2$A + $\tan^4$A

= $\tan^4$A + $\tan^2$A

13. **(a)** Option (iv)

Explanation: 3.141141114 is an irrational number because it is a non-repeating and non-terminating decimal.

14. **(c)** -4

Explanation: Q (3, y) divides AB in the ratio 2 : 1

so Q is $\left(\frac{2\times1+1\times7}{2+1}, \frac{2\times(-5)+1\times(-2)}{2+1}\right)$, is

hence, = -4

15. **(d)** $50\sqrt{3}$ m

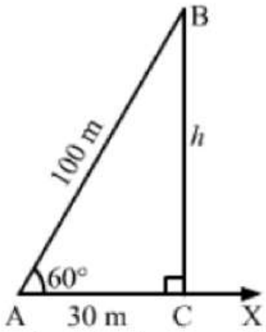
Explanation:

Let BA be the string of the kite and AX be the horizontal line.

In $\triangle$BAC If BC $\perp$ AX, then AB = 100 and $\angle$BAC = 60^o.

Let:

BC = h



In the right $\triangle$ACB, we have:

$\frac{BC}{AB}$ = sin 60^o = $\frac{\sqrt{3}}{2}$

$\Rightarrow \frac{h}{100} = \frac{\sqrt{3}}{2}$

$\Rightarrow h = \frac{100\sqrt{3}}{2} = 50\sqrt{3}$ m

Hence, the height of the kite is $50\sqrt{3}$ m.

16. **(d)** assumed mean

Explanation: In the formula, $\overline{x} = a + h\left(\frac{\sum f_i u_i}{\sum f_i}\right)$ 'a' stands for Assumed Mean i.e., assumed mean value is the mid value (x_i) of class intervals of a set of grouped data.

17. **(a)** an irrational number

Explanation: The sum of a rational and an irrational number is an irrational number hence it is an irrational number.

18. **(a)** x = 3, y = 2

Explanation: We have:

2x + 3y = 12 …(i)

3x – 2y = 5 …(ii)

Now, by multiplying (i) by 2 and (ii) by 3 and then adding them we get:

4x + 9x = 24 + 15

13x = 39

x = $\frac{39}{13}$ = 3

Now putting the value of x in (i), we get

2 × 3 + 3y = 12

$\therefore$ y = $\frac{12-6}{3}$ = 2

19. **(c)** A is true but R is false.

Explanation: HCF (a, b) $\times$ LCM (a, b) = a $\times$ b

$\Rightarrow$ 8 $\times$ LCM = 280

$\Rightarrow$ LCM = $\frac{280}{8}$ = 35

A is true but R is false.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Statement of Basic Proportionality Theorem (Thale's Theorem).

Section B

21. Kritika takes out a ball from the bag without looking into it. So, it is equally likely that she takes out any one of them.

Let Y be the event that 'the ball taken out is yellow', B be the event that 'the ball taken out is blue' and R be the event that 'the ball taken out is red'.

Now, the number of possible outcomes = 3.

i. The number of outcomes favourable to the event Y = 1.

So, $P(Y) = \frac{1}{3}$

Similarly,

ii. $P(R) = \frac{1}{3}$

iii. $P(B) = \frac{1}{3}$

22. Let the fixed charges of taxi be Rs. x and the running charges be Rs.y km/hr.

According to the given condition, we have

For a journey of 10 km the charge paid is Rs.75

$x+10y=75$ ....(i)

and for a journey of 15 km the charge paid is Rs.110.

$x+15y=110$ ....(ii)

Subtracting equation (ii) from equation (i), we get

$-5y=-35$

$\Rightarrow y=7$

Putting y = 7 in equation (i), we get

$x+10(7)=75$

$\Rightarrow x+70=75$

$\Rightarrow x=5$

$\therefore$ Total charges for travelling a distance of 25 km = $x+25y$

= Rs. (5 + 25 $\times$ 7)

= Rs (5 + 175)

= Rs.180.

OR

Condition for coincident lines,

$a_1/a_2 = b_1/b_2 = c_1/c_2$;

No, given pair of linear equations are

$\frac{x}{2}+y+\frac{2}{5}=0$ and $4x+8y+\frac{5}{16}=0$

Comparing with ax + by + c = 0;

Here, $a_1 = 1/2$, $b_1 = 1$, $c_1 = 2/5$;

And $a_2 = 4$, $b_2 = 8$, $c_2 = 5/16$;

$a_1/a_2 = 1/8$

$b_1/b_2 = 1/8$

$c_1/c_2 = 32/25$

Here, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, i.e. parallel lines

Hence, the given pair of linear equations has no solution.

23. The number of zeroes is 1 as the graph intersects the x-axis at one point only.

24. We have to find a point on the y-axis which is equidistant from the points A(6, 5) and B (- 4, 3).

We know that a point on y-axis is of the form (0, y). So, let the required point be P (0, y).

Then,

PA = PB

$\Rightarrow \sqrt{(0-6)^2+(y-5)^2}=\sqrt{(0+4)^2+(y-3)^2}$

$\Rightarrow 36 + (y - 5)^2 = 16 + (y - 3)^2$

$\Rightarrow 36 + y^2 - 10y + 25 = 16 + y^2 - 6y + 9$

$\Rightarrow 4y = 36$

$\Rightarrow y = 9$

So, the required point is (0, 9).

25. Construction : Draw OC

Proof : Line AB is tangent to smaller circle at point C.

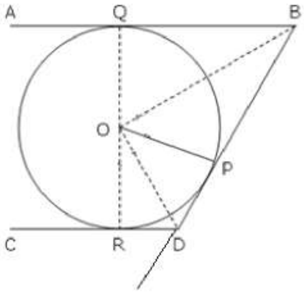
$\therefore$ segment $OC \perp AB$

AB is chord to larger circle and

as perpendicular drawn from centre to chord bisects the chord.

$\therefore AC=CB$

OR



Given: AB and CD are two parallel tangents, another tangent BD intersects them at B and D respectively. The intercept BD subtends ∠BOD at center O.

To prove: $\angle \mathrm{BOD} = 90^\circ$

Proof: In △BOP and △BOQ,

OP = OQ = radius

OB is common

BP = BQ (Tangents from one point B)

So $\triangle \mathrm{BOP} \cong \triangle \mathrm{BOQ}$ (By SSS criteria)

Hence ∠OBP = ∠OBQ

So ∠QBP = 2∠OBP........(1)

Similarly in △DOP and △DOR,

∠ODP = ∠ODR

and ∠RDP = 2∠ODP ......(2)

Now, $AB \parallel CD$ and BD is a transversal line.

So ∠QBP +∠RDP = 180° (The interior angles formed on the same side of the transversal line)

From eqn (1) and (2),

2 ∠OBP + 2 ∠ODP = 180°

So ∠OBP + ∠ODP = 90°

Now in △BOD,

∠BOD + ∠OBP + ∠ODP = 180°

∠BOD + 90 = 180°

Therefore ∠BOD = 90°

Hence proved.

Section C

26. We have to prove that, $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1} = \frac{1}{\sec\theta-\tan\theta}$ using identity $sec^2\theta = 1 + tan^2\theta$

LHS = $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1} = \frac{\tan\theta-1+\sec\theta}{\tan\theta+1-\sec\theta}$ [dividing the numerator and denominator by $\cos\theta$.]

$= \frac{(\tan\theta+\sec\theta)-1}{(\tan\theta-\sec\theta)+1} = \frac{\{(\tan\theta+\sec\theta)-1\}(tan\theta-\sec\theta)}{\{(\tan\theta-\sec\theta)+1\}(\tan\theta-\sec\theta)}$ [Multiplying and dividing by $(\tan\theta - \sec\theta)$]

$= \frac{(\tan^2\theta-\sec^2\theta)-(tan\theta-\sec\theta)}{\{(\tan\theta-\sec\theta)+1\}(\tan\theta-\sec\theta)}$ $[\because (a-b)(a+b) = a^2 - b^2$]

$= \frac{-1-\tan\theta+\sec\theta}{(\tan\theta-\sec\theta+1)(\tan\theta-\sec\theta)}$ $[\because \tan^2\theta - \sec^2\theta = -1$]

$= \frac{-(\tan\theta-\sec\theta+1)}{(\tan\theta-\sec\theta+1)(\tan\theta-\sec\theta)} = \frac{-1}{\tan\theta-\sec\theta}$

$= \frac{1}{\sec\theta-\tan\theta}$ =**RHS**

Hence Proved.

27. The solution of pair of linear equations:

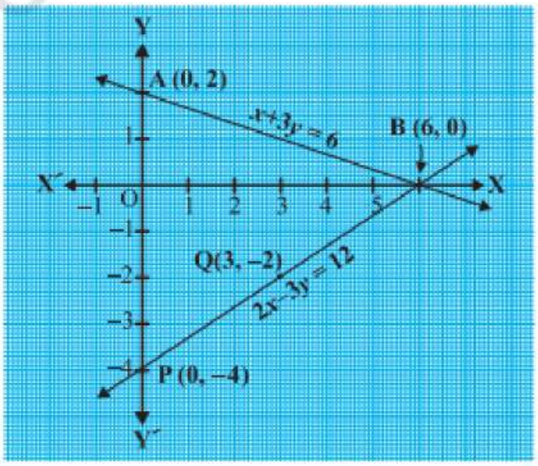
x + 3y = 6 and 2x – 3y = 12

x	0	6
$y = \frac{6-x}{3}$	2	0

and

x	0	3
$y = \frac{2x-12}{3}$	-4	-2

Plot the points A(0, 2), B(6, 0), P(0, – 4) and Q(3, – 2) on graph paper, and join the points to form the lines AB and PQ



We observe that there is a point B (6, 0) common to both the lines AB and PQ. So, the solution of the pair of linear equations is x = 6 and y = 0, i.e., the given pair of equations is consistent.

28. Let $\sqrt{6}+\sqrt{2}$ be rational number

$\sqrt{6}+\sqrt{2} = \frac{p}{q}$

$\sqrt{2} = \frac{p}{q} - \sqrt{6}$

$\sqrt{2} = \frac{p-q\sqrt{6}}{q}$

$2q^2 = p^2 + 6q^2 - 2\sqrt{6}q$

$2q^2 - p^2 - 6q^2 = -2\sqrt{6}q$

$\sqrt{6} = \frac{2q^2-p^2-6q^2}{-2q}$

as $\frac{2q^2-p^2-6q^2}{-2q}$ is in $\frac{p}{q}$ form it is rational number, so $\sqrt{6}$ should be rational number but in general $\sqrt{6}$ is irrational.

So our assumption is wrong.

Therefore given number is irrational.

OR

$P(x) = a^8 - b^8 = \left(a^4+b^4\right)\left(a^4-b^4\right)$

$= \left(a^4+b^4\right)\left(a^2+b^2\right)\left(a^2-b^2\right)$

$= \left(a^4+b^4\right)\left(a^2+b^2\right)(a+b)(a-b)$

$Q(x) = (a+b)\left(a^4-b^4\right)$

$= (a+b)\left(a^2+b^2\right)\left(a^2-b^2\right)$

$= (a+b)\left(a^2+b^2\right)(a+b)(a-b)$ {Using Identity $a^2 - b^2 = (a+b)(a-b)$}

Common factors: $\left(a^2+b^2\right), (a-b), (a+b)$

Uncommon factors: $\left(a^4+b^4\right), (a+b)$

$\therefore LCM$ of $P(x)$ and $Q(x)$

$= \left(a^2+b^2\right)(a-b)(a+b) \times \left(a^4+b^4\right)(a+b)$

$= \left(a^4+b^4\right)\left(a^2+b^2\right)(a+b)^2(a-b)$

29. i. $\triangle$ABC $\sim \triangle$PQR (Given)

So, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ ...(1) (corresponding sides of similar triangles are proportional)

and $\angle$A = $\angle$P, $\angle$B = $\angle$Q and $\angle$C = $\angle$R ...(2)

But AB = 2 AM and PQ = 2 PN (As CM and RN are medians)

So, from (1),

i.e., $\frac{AM}{PN} = \frac{CA}{RP}$ ...(3)

Also, $\angle$ MAC = $\angle$ NPR [From (2)] ...(4)

So, from (3) and (4),

$\triangle$AMC $\sim \triangle$PNR (SAS similarity criterion) ...(5)

ii. From (5), $\frac{\text{CM}}{\text{RN}} = \frac{\text{CA}}{\text{RP}}$ ...(6) (corresponding sides of similar triangles are proportional)

But $\frac{CA}{RP} = \frac{AB}{PQ}$ [From (1)] ...(7)

Therefore, $\frac{CM}{RN} = \frac{AB}{PQ}$ [From (6) and (7)] ...(8)

iii. Again, $\frac{AB}{PQ} = \frac{BC}{QR}$ [From (1)]

Therefore $\frac{\mathrm{CM}}{\mathrm{RN}} = \frac{\mathrm{BC}}{\mathrm{QR}}$ [From (8)] ...(9)

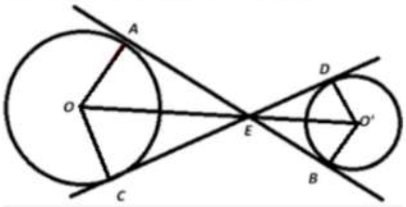
Also, $\frac{CM}{RN} = \frac{AB}{PQ} = \frac{2BM}{2QN}$

i.e., $\frac{\mathrm{CM}}{\mathrm{RN}} = \frac{\mathrm{BM}}{\mathrm{QN}}$ ...(10)

i.e., $\frac{\mathrm{CM}}{\mathrm{RN}} = \frac{\mathrm{BC}}{\mathrm{QR}} = \frac{\mathrm{BM}}{\mathrm{QN}}$ [From (9) and (10)]

Therefore, $\triangle\mathrm{CMB} \sim \triangle\mathrm{RNQ}$ (SSS similarity criterion)

30. Construction : Join OA and OC.



$\angle AEC = \angle DEB$ ....(vertically opposite angles)

In ΔOAE and ΔOCE,

OA = OC ...(Radii of the same circle)

OE = OE ...(Common side)

$\angle OAE = \angle OCE$ ....(each is 90°)

$\Rightarrow \Delta OAE \cong \Delta OCE$ ....(RHS congruence criterion)

$\Rightarrow \angle AEO = \angle CEO$ ....(cpct)

Similarly, for the circle with centre O',

$\angle DEO' = \angle BEO'$

Now, $\angle AEC = \angle DEB$

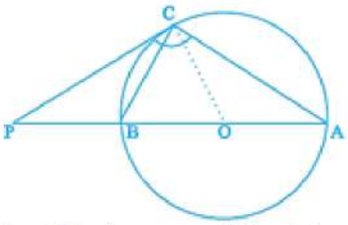
$\Rightarrow \frac{1}{2}\angle AEC = \frac{1}{2}\angle DEB$

$\Rightarrow \angle AEO = \angle CEO = \angle DEO' = \angle BEO'$

Hence, all the fours angles are equal and bisected by OE and O'E.

So, O, E and O' are collinear.

OR



Let D be the centre of the circle.

A, D, B, P all are on the same line and P and C are points on the tangent.

Now, $\angle$BCA is inscribed in a semi-circle, $\angle BCA = 90^\circ$

C is the point on the circle where the tangent touches the circle.

So, $\angle \mathrm{DCP} = 90^\circ$

$\angle\mathrm{PCA} = \angle\mathrm{PCD} + \angle\mathrm{DCA}$

$\Rightarrow 110^o = 90^o + \angle\mathrm{DCA}$

$\Rightarrow \angle DCA = 20^\circ$

In $\triangle$ADC,

AD = DC .... (Radii of the same circle)

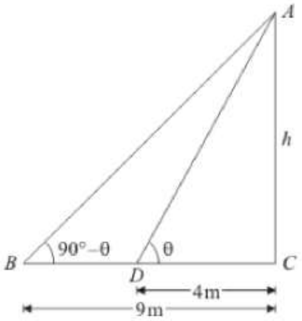
$\Rightarrow \angle DCA = \angle CAD = 20^\circ$

In $\triangle$ABC,

$\angle\mathrm{BCA} = 90^\circ, \angle\mathrm{CAB} = 20^\circ$

So, $\angle CBA = 70^\circ$

31. Let AC be the height of tower is h meters.



Given that: angle of elevation are $\angle B = 90^o - \theta$ and $\angle D = \theta$ and also CD = 4 m and BC = 9 m.

Here we have to find height of tower.

So we use trigonometric ratios.

In a triangle ADC,

$\tan\theta = \frac{h}{4}$

Again in a triangle ABC,

$\Rightarrow \tan(90^\circ - \theta) = \frac{AC}{BC}$

$\Rightarrow \cot\theta = \frac{h}{9}$

$\Rightarrow \frac{1}{\tan\theta} = \frac{h}{9}$

Put $\tan\theta = \frac{h}{4}$

$\Rightarrow \frac{4}{h} = \frac{h}{9}$

$\Rightarrow h^2 = 36$

$\Rightarrow h = 6$

Hence height of tower is 6 meters.

Section D

32. Let the altitude of the triangle = x cm

base = (2x+8) cm

area = 480 sq cm

$\frac{1}{2} \times$ base $\times$ altitude = 480

$\Rightarrow x(2x+8) = 2 \times 480$

$\Rightarrow 2x^2 + 8x - 960 = 0$

$\Rightarrow x^2 + 4x - 480 = 0$

$\Rightarrow x^2 + 24x - 20x - 480 = 0$

$\Rightarrow x(x + 24) - 20(x + 24) = 0$

$\Rightarrow (x + 24)(x - 20) = 0$

$\Rightarrow x + 24 = 0$ or $x - 20 = 0$

$\Rightarrow x = -24$ or $x = 20$

Length never negative so value of x = 20

base = 2x + 8= 2(20) + 8 = 48 cm

By Pythagoras theorem

$\text{hypotenuse}^2 = \text{base}^2 + \text{altitude}^2$

$= (48)^2 + (20)^2$

= 2304 + 400

= 2704

hypotenuse = 52

Three sides of a triangle are 48cm,20cm and 52cm

OR

Given:

Speed of the boat in still water= 18km/hr

Let speed of stream be x km/hr

Speed of boat in upstream = (18 - x)km/hr

Speed of boat in downstream = (18 + x)km/hr

We know that,

$Speed = \frac{\text{Distance}}{\text{Time taken}}$

Time taken to travel 36 km in upstream $= \frac{36}{18-x} hr$

Time taken to travel 36 km in upstream $= \frac{36}{18+x} hr$

According to question,

Time taken for upstream - time taken for downstream = 1hr 30 minutes

$\Rightarrow \frac{36}{18-x} - \frac{36}{18+x} = 1\frac{30}{60}$

$\Rightarrow \frac{36(18+x)-36(18-x)}{(18-x)(18+x)} = \frac{3}{2}$

$\Rightarrow 2[36(18 + x - 18 + x] = 3[(18)^2 - (x)^2]$ ...[$\because (a - b)(a + b) = (a^2 - b^2)$]

$\Rightarrow 2[36(2x)] = 3[324 - x^2]$

$\Rightarrow 144x = 972 - 3x^2$

$\Rightarrow 3x^2 + 144x - 972 = 0$

$\Rightarrow x^2 + 48x - 324 = 0$

Comparing eq. with $ax^2 + bx + c = 0$

Here a = 1, b = 48 and c = -324

We know that,

$D = b^2 - 4ac$

$= (48)^2 - 4(1)(-324)$

= 2304 + 1296

= 3600

So, the roots to the equation are

$x = \frac{-b\pm\sqrt{D}}{2a}$

$x = \frac{-48\pm\sqrt{3600}}{2\times1}$

$x = \frac{-48\pm\sqrt{36\times100}}{2}$

$x = \frac{-48\pm\sqrt{6\times6\times10\times10}}{2}$

$x = \frac{-48\pm60}{2}$

x = -24 ± 30

⇒ x = -24 + 30 or x = -24 - 30

⇒ x = 6 or x = -54

Since, x is the speed, so it can’t be negative

So, x = 6 km /hr

33. Given that in fig, if ∠A = ∠C, AB = 6cm, BP = 15 cm, AP = 12cm and CP = 4,

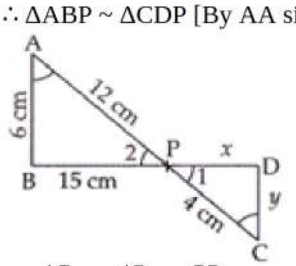
we have to find the lengths of PD and CD.

Now,In ΔABP and ΔCDP, we have,

∠A = ∠C [Given]

∠2 = ∠1 [Vertically opposite angles]

∴ ΔABP ~ ΔCDP [By AA similarity criterion]



$\Rightarrow \frac{AB}{CD} = \frac{AP}{CP} = \frac{BP}{DP}$ (Since corresponding sides of two similar triangles are proportional)

$\Rightarrow \frac{6}{y} = \frac{12}{4} = \frac{15}{x}$

$\Rightarrow \frac{6}{y} = \frac{12}{4}$

$\Rightarrow y = \frac{6\times4}{12} = 2cm$

and $\frac{15}{x} = \frac{12}{4}$

$\Rightarrow \frac{15}{x} = 3$

$\Rightarrow$ x = 5 cm

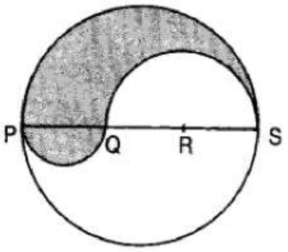
Therefore, PD = 5 cm and CD = 2 cm.

34. PS = Diameter of a circle of radius 6 cm = 12 cm

$\therefore$ PQ = QR = RS = $\frac{12}{3}$ = 4cm, QS = QR + RS = (4 + 4)cm = 8 cm

Let P be the perimeter and A be the area of the shaded region.

P = Arc of semi-circle of radius 6 cm + Arc of semi-circle of radius 4 cm + Arc of semi-circle of radius 2 cm



$\Rightarrow P = (\pi \times 6 + \pi \times 4 + \pi \times 2)\text{cm} = 12\pi\text{cm}$

and, A = Area of semi-circle with PS as diameter + Area of semi-circle with PQ as diameter - Area of semi-circle with QS as diameter.

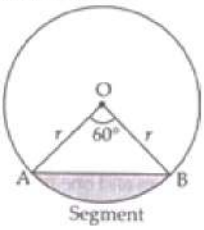
$\Rightarrow A = \frac{1}{2} \times \frac{22}{7} \times (6)^2 + \frac{1}{2} \times \frac{22}{7} \times 2^2 - \frac{1}{2} \times \frac{22}{7} \times (4)^2$

$\Rightarrow A = \frac{1}{2} \times \frac{22}{7}\left(6^2 + 2^2 - 4^2\right) = \frac{1}{2} \times \frac{22}{7} \times 24 = \frac{264}{7}\text{cm}^2$ = 37.71 cm^2

OR

Area of minor segment = Area of sector – Area of $\triangle$OAB

In $\triangle$OAB,



θ = 60°

OA = OB = r = 12 cm

$\angle$B = $\angle$A = x [$\angle$s opp. to equal sides are equal]

$\Rightarrow$ $\angle$A + $\angle$B + $\angle$O = 180°

$\Rightarrow$ x + x + 60° = 180°

$\Rightarrow$ 2x = 180° – 60°

$\Rightarrow$ x = $\frac{120^\circ}{2}$ = 60°

$\therefore$ $\triangle$OAB is equilateral $\triangle$ with each side (a) = 12 cm

Area of the equilateral $\triangle = \frac{\sqrt{3}}{4}a^2$

Area of minor segment = Area of the sector – Area of $\triangle$OAB

$= \frac{\pi r^2 \theta}{360^\circ} - \frac{\sqrt{3}}{4}a^2$

$= \frac{3.14 \times 12 \times 12 \times 60^\circ}{360^\circ} - \frac{\sqrt{3}}{4} \times 12 \times 12$

= 6.28 $\times$ 12 – $36\sqrt{3}$

$\therefore$ Area of minor segment = (75.36 – $36\sqrt{3}$) cm^2.

35. Let, $a = 50$

C.I.	Number of states/ U.T. (f_i)	x_i	$d_i = x_i - 50$	$f_i d_i$
15 - 25	6	20	-30	-180
25 - 35	11	30	-20	-220
35 - 45	7	40	-10	-70
45 - 55	4	50	0	0
55 - 65	4	60	10	40

65 - 75	2	70	20	40
75 - 85	1	80	30	30

From table, $\Sigma f_i d_i = -360, \Sigma f_i = 36$

we know that, mean=$\overline{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$

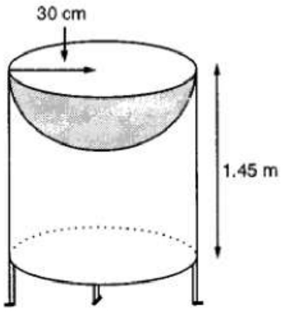
$= 50 + \frac{-360}{35}$

$= 39.71$

Section E

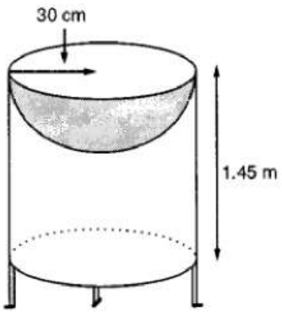
36. **Read the text carefully and answer the questions:**

Mayank a student of class 7th loves watching and playing with birds of different kinds. One day he had an idea in his mind to make a bird-bath on his garden. His brother who is studying in class 10th helped him to choose the material and shape of the birdbath. They made it in the shape of a cylinder with a hemispherical depression at one end as shown in the Figure below. They opted for the height of the hollow cylinder as 1.45 m and its radius is 30 cm. The cost of material used for making bird bath is ₹40 per square meter.



(i) Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.

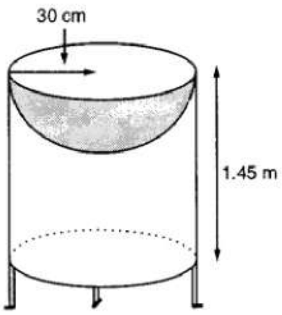


Curved surface area of the hemisphere = $2\pi r^2$

$= 2 \times 3.14 \times 30^2 = 0.56\ m^2$

(ii) Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.



Let S be the total surface area of the birdbath.

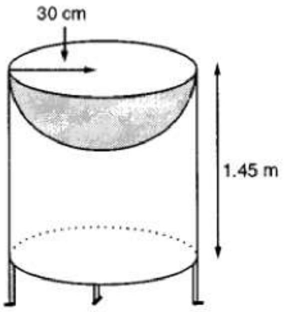
S = Curved surface area of the cylinder + Curved surface area of the hemisphere

$\Rightarrow S = 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

$\Rightarrow S = 2 \times \frac{22}{7} \times 30(145 + 30)$ = 33000 cm^2 = 3.3 m^2

(iii) Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.



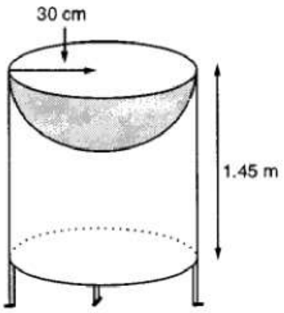
Total Cost of material = Total surface area $\times$ cost per sq m^2

= 3.3 $\times$ 40 = ₹132

OR

Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.



r = 35 cm = $\frac{35}{100}$m

We know that S.A = 3.3 m^2

S = $2\pi r(r + h)$

$\Rightarrow 3.3 = 2 \times \frac{22}{7} \times \frac{35}{100}\left(\frac{35}{100} + h\right)$

$\Rightarrow 3.3 = \frac{22}{10}\left(\frac{35}{100} + h\right)$

$\Rightarrow \frac{33}{22} = \frac{35}{100} + h$

$\Rightarrow h = \frac{3}{2} - \frac{7}{20} = \frac{23}{20}$ = 1.15 m

37. **Read the text carefully and answer the questions:**

Kamla and her husband were working in a factory in Seelampur, New Delhi. During the pandemic, they were asked to leave the job. As they have very limited resources to survive in a metro city, they decided to go back to their hometown in Himachal Pradesh. After a few months of struggle, they thought to grow roses in their fields and sell them to local vendors as roses have been always in demand. Their business started growing up and they hired many workers to manage their garden and do packaging of the flowers.

In their garden bed, there are 23 rose plants in the first row, 21 are in the 2^{nd}, 19 in 3^{rd} row and so on. There are 5 plants in the last row.

(i) The number of rose plants in the 1^{st}, 2^{nd}, ... are 23, 21, 19, ... 5

a = 23, d = 21 - 23 = - 2, a_n = 5

$\therefore a_n = a + (n - 1)d$

or, 5 = 23 + (n - 1)(-2)

or, 5 = 23 - 2n + 2

or, 5 = 25 - 2n

or, 2n = 20

or, n = 10

(ii) Total number of rose plants in the flower bed,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2(23) + (10-1)(-2)]$

S_{10} = 5[46 - 20 + 2]

S_{10} = 5(46 - 18)

S_{10} = 5(28)

S_{10} = 140

(iii) a_n = a + (n - 1)d

$\Rightarrow a_6 = 23 + 5 \times (-2)$

$\Rightarrow a_6 = 13$

OR

$S_n = 80$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 80 = \frac{n}{2}[2 \times 23 + (n-1) \times -2]$

$\Rightarrow 80 = 23n - n^2 + n$

$\Rightarrow n^2 - 24n + 80 = 0$

$\Rightarrow (n - 4)(n - 20) = 0$

$\Rightarrow$ n = 4 or n = 20

n = 20 not possible

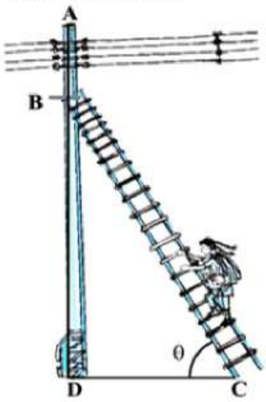
$a_{20} = 23 + 19 \times (-2) = -15$

Number of plants cannot be negative.

n = 4

38. **Read the text carefully and answer the questions:**

In a village, group of people complained about an electric fault in their area. On their complaint, an electrician reached village to repair an electric fault on a pole of height 10 m. She needs to reach a point 1.5 m below the top of the pole to undertake the repair work (see the adjoining figure). She used ladder, inclined at an angle of θ to the horizontal such that $\cos\theta = \frac{\sqrt{3}}{2}$, to reach the required position.



(i) Length BD = AD - AB = 10 - 2.5 = 8.5

(ii) The length of ladder BC

In $\triangle$BDC

$\cos\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 30^o$

$\sin 30^o = \frac{BD}{BC}$

$\Rightarrow \frac{1}{2} = \frac{8.5}{BC}$

$\Rightarrow BC = 2 \times 8.5$ = 17 m

(iii) Distance between foot of ladder and foot of wall CD

In $\triangle$BDC

$\cos 30^o = \frac{CD}{BC}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{CD}{17}$
$\Rightarrow CD = 8.5\sqrt{3}$ m

OR

If the height of pole and distance BD is doubled, then the length of the ladder is

sin 30^o = $\frac{BD}{BC}$
$\Rightarrow \frac{1}{2} = \frac{17}{BC}$
$\Rightarrow BC = 2 \times 17$ = 34 m