Chapter 7

Quadratic Equations

Exercise 7.1

Solve the following (1 to 12) equations:

Question 1.

(i) $x^2 – 11x + 30 = 0$

(ii) $4x^2 – 25 = 0$

Solution:

(i) $x^2 – 11x + 30 = 0$

Let us simplify the given equation,

By factorizing, we get

$X^2 – 5x – 6x + 30 = 0$

$X(x – 5) – 6 (x – 5) = 0$

$(x – 5) (x – 6) = 0$

So,

$(x – 5) = 0$ or $(x – 6) = 0$

$X = 5$ or $x = 6$

Value of $x = 5,6$

(ii) $4x^2 – 25 = 0$

Let us simplify the given equation,

$4x^2 = 25$

$X^2 = \frac{25}{4}$

$X = \pm \sqrt{\frac{25}{4}}$

$= \pm \frac{5}{2}$

Value of x = $+\frac{5}{2} - \frac{5}{2}$

Question 2.

(i) $2x^2 – 5x = 0$

(ii) $x^2 – 2x = 48$

Solution:

(i) $2x^2 – 5x = 0$

Let us simplify the given equation,

X(2x – 5) = 0

So,

X = 0 or 2x – 5 = 0

X = 0 or 2x = 5

X = 0 or x = $\frac{5}{2}$

Value of x = 0 , $\frac{5}{2}$

(ii) $x^2 – 2x = 48$

Let us simplify the given equation,

By factorizing , we get

$X^2 – 2x – 48 = 0$

$X^2 – 8x + 6x – 48 = 0$

$X (x – 8) +6 (x – 8) = 0$

$(x – 8) (x + 6) = 0$

So,

$(x – 8) = 0$ or $(x + 6) = 0$

$X = 8$ or $x = -6$

Value of $x = 8$ or $x = -6$

Question 3.

(i) $6 + x = x^2$

(ii) $2x^2 + 3x + 1$

Solution:

(i) $6 + x = x^2$

Let us simplify the given equation,

$6 + x – x^2 = 0$

$X^2 – x – 6 = 0$

By factorizing, we get

$X^2 – 3x + 2x – 6 = 0$

$X (x -3) + 2(x -3) = 0$

$(x – 3) (x + 2) = 0$

So ,

(x – 3) = 0 or (x + 2) = 0

X = 3 or x =-2

Value of x = 3 , -2

(ii) $2x^2 + 3x + 1 = 0$

Let us simplify the given equation,

By factorizing, we get

$2x^2 – 2x – x + 1 = 0$

2x (x – 1) -1 (x – 1) = 0

(x – 1) (2x – 1) = 0

So,

(x – 1) = 0 or (2x – 1) = 0

X = 1 or 2x = 1

X = 1 or x = $\frac{1}{2}$

Value of x = 1 , $\frac{1}{2}$

Question 4.

(i) $3x^2 = 2x + 8$

(ii) $4x^2 + 15 = 16x$

Solution;

(i) $3x^2 = 2x + 8$

Let us simplify the given equation,

$3x2 = 2x + 8 = 0$

By factorizing , we get

$3x^2 – 6x + 4x – 8 = 0$

$3x (x – 2) +4(x – 2) = 0$

$(x – 2) (3x + 4) = 0$

So,

$(x – 2) = 0$ or $(3x + 4) = 0$

$X = 2$ or$3x = -4$

$X = 2$ or $x = -\frac{4}{3}$

(ii) $4x^2 + 15 = 16x$

Let us simplify the given equation,

$4x^2 – 16x + 15 = 0$

By factorizing , we get

$4x^2 – 6x – 10x + 15 = 0$

$2x (2x – 3) -5 (2x – 3) = 0$

$(2x – 3) (2x – 5) = 0$

So,

$(2x – 3) = 0$ or $(2x – 5) = 0$

$2x = 3$ or $2x = 5$

$X = \frac{3}{2}$ or $x = \frac{5}{2}$

Question 5 .

(i) $x(2x + 5) = 25$

(ii) $(x + 3) (x – 3) = 40$

Solution:

(i) $x(2x + 5) = 25$

Let us simplify the given equation,

$2x^2 + 5x – 25 = 0$

By factorizing, we get

$2x^2 + 10x – 5x – 25 = 0$

$2x (x + 5) – 5 (x + 5) = 0$

$(x + 5) (2x – 5) = 0$

So,

$(x + 5) = 0$ or $(2x – 5) = 0$

$X = -5$ or $2x = 5$

$X = -5$ or $x = \frac{5}{2}$

(ii) $(x + 3) (x – 3) = 40$

Let us simplify the given equation,

$X^2 – 3x + 3x – 9 = 40$

$X^2 – 9 – 40 = 0$

$X^2 – 49 = 0$

$X^2 = 49$

X = $\sqrt{49}$

= $\pm$ 7

Value of x = 7, -7

Question 6.

(i) (2x + 3) (x – 4) = 6

(ii) (3x + 1) (2x + 3) = 3

Solution:

(i) (2x + 3) (x – 4) = 6

Let us simplify the given equation,

$2x^2 – 8x + 3x – 12 – 6 = 0$

$2x^2 – 5x -18 = 0$

By factorizing , we get

$2x^2 – 9x + 4x – 18 = 0$

X (2x – 9) + 2(2x – 9) = 0

(2x – 9) (x +2) = 0

So,

(2x – 9) = 0 or (x + 2) = 0

2x = 9 or x = -2

X = $\frac{9}{2}$ $or\ x = -2$

Value or x = $\frac{9}{2}$, -2

(ii) (3x + 1) (2x + 3) = 3

Let us simplify the given equation,

$6x^2 + 9x + 2x + 3 - 3 = 0$

$6x^2 + 11x = 0$

X(6x + 11) = 0

So,

X = 0 or 6x + 11 = 0

X = 0 or 6x = -11

X = 0 or $x = -\frac{11}{6}$

Value of x = 0 , $-\frac{11}{6}$

Question 7.

(i) $4x^2 + 4x + 1 = 0$

(ii) $(x - 4)^2 + 5^2 = 13^2$

Solution:

(i) $4x^2 + 4x + 1 = 0$

Let us simplify the given equation,

By factorizing , we get

$4x^2 + 2x + 2x + 1 = 0$

2x (2x + 1) +1(2x + 1) = 0

(2x + 1) (2x + 1) = 0

So,

(2x + 1) = 0 or (2x + 1) = 0

2x = -1 or 2x =-1

X =$-\frac{1}{2}$ $or\ x = -\frac{1}{2}$

Value of X =$-\frac{1}{2}, -\frac{1}{2}$

(ii) $(x – 4)^2 + 5^2 = 13^2$

Let us simplify the given equation,

X^2 + 16 -2(x) (4) + 25 – 169 = 0

X^2 – 8x – 128 = 0

By factorizing , we get

X^2 – 16x + 8x – 128 = 0

X(x – 16) + 8(x – 16) = 0

(x – 16) (x + 8) = 0

So,

(x – 16) = 0 or (x + 8) = 0

X = 16 or x = -8

Value of x = 16, -8

Question 8.

(i) $21x^2 = 4(2x + 1)$

(ii) $\frac{2}{3x^2} - \frac{1}{3x} - 1 = 0$

Solution:

(i) $21x^2 = 4(2x + 1)$

Let us simplify the given equation,

$21x^2 = 8x + 4$

$21x^2 - 8x - 4 = 0$

By factorizing, we get

$21x^2 - 14x + 6x - 4 = 0$

$7x(3x - 2) + 2(3x - 2) = 0$

$(3x - 2)(7x + 2) = 0$

So,

$(3x - 2) = 0$ or $(7x + 2) = 0$

$3x = 2$ or $7x = -2$

Value of $x = \frac{2}{3}\ or\ -\frac{2}{7}$

(ii) $\frac{2}{3x^2} - \frac{1}{3x} - 1 = 0$

Let us simplify the given equation,

By taking 3 as LCM and cross multiplying

$2x^2 - x - 3 = 0$

By factorizing , we get

$2x^2 – 3x + 2x - 3 = 0$

X (2x – 3) + 1 (2x – 3) = 0

(2x – 3) (x + 1) = 0

So,

(2x – 3) = 0 or (x + 1) = 0

2x = 3 or x = -1

X = $\frac{3}{2}$ $or\ x = -1$

Value of x = $\frac{3}{2}$, -1

Question 9.

(i) 6x + 29 = $\frac{5}{x}$

(ii) x + $\frac{1}{x} = 2\frac{1}{2}$

Solution:

(i) 6x + 29 = $\frac{5}{x}$

Let us simplify the given equation,

By cross multiplying , we get

$6x^2 + 29x – 5 = 0$

By factorizing , we get

$6x^2 + 30x – x – 5 = 0$

6x (x + 5) – 1 (x + 5) = 0

(x + 5) (6x – 1) = 0

So,

(x + 5) = 0 (6x – 1) = 0

X = -5 or 6x = 1

X = -5 or $x=\frac{1}{6}$

Value of x = -5 , $\frac{1}{6}$

(ii) $x+\frac{1}{x}=2\frac{1}{2}$

$X+\frac{1}{x}=\frac{5}{2}$

Let us simplify the given equation,

By taking LCM

$X^2 + 1 = \frac{5x}{2}$

By cross multiplying,

$2x^2 + 2 – 5x = 0$

$2x^2 – 5x + 2 = 0$

By factorizing, we get

$2x^2 – x – 4x + 2 = 0$

X (2x – 1) -2 (2x – 1) = 0

(2x – 1) (x – 2) = 0

So,

(2x – 1) = 0 or (x – 2) = 0

$2x = 1$ or $x = 2$

$X = \frac{1}{2}$ or $x = 2$

Value of $x = \frac{1}{2}$, 2

Question 10.

(i) $3x - \frac{8}{x} = 2$

(ii) $\frac{x}{3} + \frac{9}{x} = 4$

Solution:

(i) $3x - \frac{8}{x} = 2$

Let us simplify the given equation,

By taking LCM and cross multiplying

$3x^2 - 8 = 2x$

$3x^2 - 2x - 8 = 0$

By factorizing, we get

$3x^2 - 6x + 4x - 8 = 0$

$3x(x - 2) + 4(x - 2) = 0$

$(x - 2) (3x + 4) = 0$

So,

$(x - 2) = 0$ or $(3x + 4) = 0$

$X = 2$ or $3x = -4$

X =2 or = $-\frac{4}{3}$

Value or x = 2 , $-\frac{4}{3}$

(ii) $\frac{x}{3}+\frac{9}{x}=4$

Let us simplify the given equation,

By taking 3x as LCM and cross multiplying

$X^2 + 27 = 12x$

$X^2 – 12+ 27 = 0$

By factorizing, we get

$X^2 – 3x – 9x + 27 = 0$

X (x – 3) – 9 (x – 3) = 0

(x – 3) (x – 9) = 0

So, (x – 3) = 0 or (x – 9) = 0

X = 3 or x = 9

Value or x = 3, 9

Question 11.

(i) $\frac{x-1}{x+1}=\frac{2x-5}{3x-7}$

(ii) $\frac{1}{x+2}+\frac{1}{x}=\frac{3}{4}$

Solution:

(i) $\frac{x-1}{x+1} = \frac{2x-5}{3x-7}$

Let us simplify the given equation,

By cross multiplying,

(x – 1) (3x – 7) = (2x – 5) (x + 1)

$3x^2$ – 7x- 3x + 7 = $2x^2$ + 2x – 5x – 5

$3x^2$ – 10x + 7 – $2x^2$ + 3x + 5 = 0

X^2 – 7x + 12 = 0

By factorizing ,we get

X^2 – 4x – 3x + 12 = 0

X (x – 4) -3(x – 4) = 0

(x – 4) (x – 3) = 0

So,

(x – 4) = 0 or (x – 3) = 0

X = 4 or x = 3

Value or x = 4, 3

(ii) $\frac{1}{x+2} + \frac{1}{x} = \frac{3}{4}$

Let us simplify the given equation,

By taking x(x + 2) as LCM

$$\frac{(x + x + 2)}{x(x + 2)} = \frac{3}{4}$$

By cross multiplying,

$4(2x + 2) = 3x (x + 2)$

$8x + 8 = 3x^2 + 6x$

$3x^2 + 6x – 8x – 8 = 0$

$3x^2 – 2x – 8 = 0$

By factorizing, we get

$3x^2 – 6x + 4x – 8 = 0$

$3x (x – 2) + 4 (x -2) = 0$

$(x – 2) (3x + 4) = 0$

So,

$(x – 2) = 0$ or $(3x + 4) = 0$

$X = 2$ or $3x = -4$

$X = 2$ or $x = -\frac{4}{3}$

Value of $x = 2 , -\frac{4}{3}$

Question 12.

(i) $\frac{8}{x+3} - \frac{3}{2-x} = 2$

(ii) $\frac{x}{x+1} + \frac{x+1}{x} = \frac{21}{6}$

Solution:

(i) $\frac{8}{x+3} - \frac{3}{2-x} = 2$

Let us simplify the given equation,

By taking (x + 3) (2 – x) as LCM

$\frac{[\,8\,(\,2\,-x\,)-\,3\,(\,x+3)]}{(x+3)(\,2-x\,)}=2$

$\frac{[16-8x-3x-9]}{2x-x^2+6-3x}=2$

$7 – 11x = -2x^2 – 2x +12$

$2x^2 + 2x – 11x – 12 + 7$

$2x^2 -9x – 5 = 0$

By factorizing, we get

$2x^2 – 10x + x – 5 = 0$

$2x (x – 5) + 1 (x – 5) = 0$

$(x – 5) (2x + 1) = 0$

So,

$(x – 5) = 0$ or $(2x + 1) = 0$

$X = 5$ or $2x = -1$

$X = 5$ or $x = -\frac{1}{2}$

(ii) $\frac{x}{x+1}+\frac{x+1}{x}=\frac{21}{6}$

Let us simplify the given equation,

By taking x(x + 1) as LCM

$\frac{[x(x)+(x+1)(x+1)}{x(x+1)}=\frac{13}{6}$

$6[x^2 + x^2 + x + x + 1]= 13x (x +1)$

$6[2x^2 + 2x +1] = 13x^2 + 13x$

$12x^2 + 12x + 6 - 13x^2 - 13x = 0$

$-x^2 - x + 6 = 0$

$X^2 + x - 6 = 0$

By factorizing, we get

$X^2 + 3x - 2x - 6 = 0$

$X(x + 3) -2(x + 3) = 0$

$(x + 3) (x - 2) = 0$

So,

$(x + 3) = 0$ or $(x - 2) = 0$

$X = -3$ or $x = 2$

Value of $x = -3, 2$

Chapter Test

Question 1.

(i) x (2x + 5) = 3

(ii) $3x^2 – 4x – 4 = 0$

Solution:

(i) x(2x + 5) = 3

We can write it as

$2x^2 + 5x – 3 = 0$

By further calculation

$2x^2 + 6x – x – 3 = 0$

By taking out the common terms

2x (x + 3) -1 (x +3) = 0

So we get

(x + 3) (2x – 1) = 0

Here

X + 3 = 0 then x = -3

2x – 1 = 0 then 2x = 1 where x = $\frac{1}{2}$

Therefore ,

X = -3, $\frac{1}{2}$

(ii) $3x^2 – 4x – 4 = 0$

We can write it as

$3x^2 – 6x + 2x – 4 = 0$

By taking out the common terms

$3x (x – 2) +2(x – 2) = 0$

So we get

$(x – 2) (3x + 2) = 0$

Here

$x – 2 = 0$ than $x = 2$

$3x + 2 = 0$ then $3x = -2$ where $x = -\frac{2}{3}$

Therefore, $x = 2 \ -\frac{2}{3}$

Question 2.

(i) $4x^2 – 2x + \frac{1}{4} = 0$

(ii) $2x^2 + 7x + 6 = 0$

Solution:

(i) $4x^2 – 2x + \frac{1}{4}$

Multiply the equation by 4

$16x^2 – 8x + 1 = 0$

We can write it as

$16x^2 – 4x – 4x + 1 = 0$

Taking out the common terms

$4x\ (\ 4x - 1)\ -1\ (\ 4x - 1) = 0$

So we get

$(\ 4x - 1)\ (\ 4x - 1) = 0$

$(\ 4x - 1)2 = 0$

Here

$4x - 1 = 0$

$4X = 1$

By division

$X = \frac{1}{4}\ , \frac{1}{4}$

(ii) $2x^2 + 7x + 6 = 0$

We can write it as

$2x^2 + 4x + 3x + 6 = 0$

By further calculation

$2x\ (\ x + 2) + 3\ (\ x + 3) = 0$

So we get

$(\ x + 2)\ (\ 2x + 3) = 0$

Here

$X + 2 = 0$ then $x = -2$

$2x + 3 = 0$ then $2x = -3$ where $x = -\frac{3}{2}$

$X = -2\ , -\frac{3}{2}$

Question 3.

(i) $\frac{x-1}{x-2}+\frac{x-3}{x-4}=\frac{31}{3}$

(ii) $\frac{6}{x}-\frac{2}{x-1}=\frac{1}{x-2}$

Solution:

(i) $\frac{x-1}{x-2}+\frac{x-3}{x-4}=\frac{31}{3}$

By taking LCM

$$\frac{[(x-1)(x-4)+(x-2)(x-3)]}{(x-2)(x-4)}=\frac{10}{3}$$

By further calculation

$$\frac{x^2-5x+4+x^2-5x+6}{x^2-6x+8}=\frac{10}{3}$$

By cross multiplication

$10x^2 - 60x + 80 = 6x^2 - 30x + 30$

By further simplification

$10x^2 - 60x + 80 - 6x^2 + 30x - 30 = 0$

So we get

$4x^2 - 30x + 50 = 0$

Dividing by 2

$2x^2 - 15x + 25 = 0$

It can be written as

$2x^2 - 10x - 5x + 25 = 0$

Taking out the common terms

$2x(x - 5) - 5(x - 5) = 0$

(x – 5) (2x – 5) = 0

Here

X – 5 = 0 then x = 5

2x – 5 = 0 then 2x = 5 where x = $\frac{5}{2}$

Therefore, x = 5, $\frac{5}{2}$

(ii). $\frac{6}{x} - \frac{2}{x-1} = \frac{1}{x-2}$

Taking LCM

$\frac{(6x-6-2x)}{x(x-1)} = \frac{1}{x-2}$

By cross multiplication

$4x^2 – 8x – 6x + 12 = x^2 – x$

So we get

$4x^2 – 14x + 12 – x^2 + x = 0$

$3x^2 – 13x + 12 = 0$

$3x^2 – 4x – 9x + 12 = 0$

Taking out the common terms

X(3x – 4) -3 (3x – 4) = 0

(3x – 4) (x – 3) = 0

Here

3x – 4 = 0 then 3x = 4 where x = $\frac{4}{3}$

X – 3 = 0 then x = 3

Therefore, x = 3, $\frac{4}{3}$