(Imp.)

Inventory / Inventory Control

{Imp For Interview}
Inventory
Quality Control

Inventory - Inventory can be termed as stocks on hand at a given point of time which may be held for the purpose of later use or sale. It as an economic value and it may include raw material, working process inventory, work in process inventory, semifinished inventory and sub assembly and final product [In inventory control our aim is to manage inventory in such a manner that day to day working run smoothly without any delay but at the minimum of the cost.

Inventory Cost :-

- Inventory Cost
  - Purchase Cost
  - ordering or setup Cost
  - Holding or Carrying Cost
  - Shortage or stockout Cost

Purchase Cost: it is the cost of ~~purchasing~~ purchasing inventory item and it depends upon the quantity for the bulk purchase.

$$\boxed{\text{Puchase Cost (P.C.)} = \text{No. of Unit} \times \text{Cost per Unit}}$$

2) Ordering/setup Cost:

Ordering Cost: when inventory is purchase from outside the cost associated with bringing inventory within the production system is termed as ordering cost. It include Cost of tender, cost associated with paper-work processing cost, cumminication cost, inspection cost, transportation cost etc.

Setup Cost: when inventory is produced internaly the cost associated with bringing shutdown producation system again into starting position is termed as setup cost. it include maintenance cost, schdule chart preparation cost, cost of bringing raw material, arrangment of worker, tools, equipment etc

$$\boxed{\text{Ordering Cost (O.C.)} = \text{No. of order} \times \text{Cost/order}}$$

$$\boxed{\text{setup Cost (S.C.)} = \text{No. of setup} \times \text{Cost/setup}}$$

3> Holding or Carrying Cost:- it is the cost associated with storing, keeping and maintaining inventory within production system. it include storage cost, handling cost, damage and depreciation cost, insurance cost, interest etc. This cost depands upon the quantity and period for which inventory is stored.

$$\boxed{\text{Holding Cost (H.C.)} = \text{Average Inventory for a period} \times \text{holding Cost/Unit/time}}$$

4> Shortage or Stockout Cost: shortage simply means absence of inventory and loos associated with not serving the customer is termed as shortage or stockout cost. it include potential profit loos goodwill loss, fast transportation cost disscount cost.

$$\boxed{\text{S.C.} = \text{No. of units short} \times \text{Shortage Cost/unit}}$$

Inventory Classification :

1). Transient or pipe line inventory :- inventory cannot provide service while in tranportection and such inventory called transint or pipeline inventory

2) Buffer or Safety Stock :→

i) if $d' = 15$ Units/day $> d = 10$ Units/day (Avg.) when avg. demand increase

ii) $LT' = 10$ day $> LT = 6$ day when avg. lead time increase

ROL = 60 Unit we use buffer stock

it is reserve stock kept through the year and it is held for protacting aganist the fluctuation in the demand rate and lead time. it is not use under normal working condition and used only adverse condition to prevent stockout.

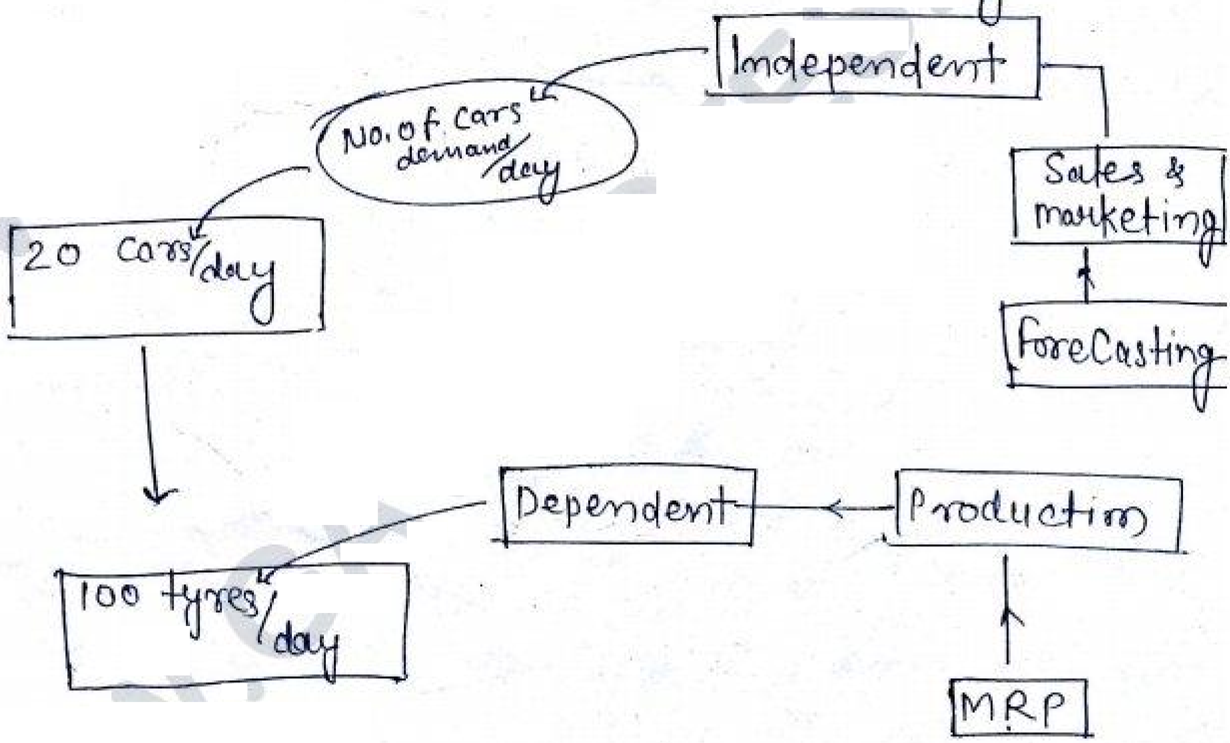
lead time (LT) :- it is the time gap between placing and order and inventory on hand so that it can be consumed or used.

3) Seasonal Inventory:- The demand for these inventory item changes with seasonal variation.

4) Anticipation Inventory:- These inventory items are buildup to meet anticipated demand in future like big selling forecast, govt. policy change, price hike, strike shut down etc.

Characteristics of Inventory Model:-

(1) Dependent and Independent demand inventory:-



Dependent: the demand for these item directed related or link to any other item, useally of a higher level of which it become a part.

Independent: The demand for these item is not directly related or link to any other item it is difficult to compute and it projected with the help of forecasting.

2) Inventory Review System:-

Review system

Q- System
or
Fixed order system
or
Reorder system
or
Two bin system

P- System
or
Fixed Period system
or
Periodic Review system

$d = 10\,unit/day$

$LT = 5\,day$

$\boxed{RoL = 50\,Unit}$

$Q = 500\,unit/order$

450	50

SS-system (optimum Replenishment)
(Combination of both)
Now a day people use SS-systems

SS-System

Fast com (Q-system)

$\boxed{ROL = 50 \text{ Units}}$ — Q $d'' = 15$ (Fixed Quantity)

Slow con P-system

$\boxed{ROP = 45 \text{ days}}$ — P $d'' = 9$

Reorder period

Fixed order System :- in this system as inventory
Q-system decreases to Reorder level of fresh order for fixed quantity is placed at that point. In this system size of order is fixed but the time of order is variable.

Fixed Period System :- In this system inventory
P-system is reviewed after a fix period of time and a fresh order for variable quantity is placed at that point. in this system size of order is variable but the time of order is fixed.

3) Deterministic & Probabilistic Models:-

Deterministic :- In these model demand rate and lead time remain fixed and constant and therefore we need not to carry safety stock.

Probabilistic :- These model represent the real world condition where there is variation and fluctuation in the demand rate & lead time In these model we need to carry safety stocks to prevent stock out during adverse condition.

Notations

D - Annual or yearly demand of inventory (Units/year)

Q - Quantity to be ordered at each order point (units/order)

N - No. of order placed in year (order/year)

$$\boxed{N = \frac{D}{Q}} \text{ order/year.}$$

T - time length of one inventory cycle or time gap between two succesive order (year/order)

$$\boxed{T = \frac{1}{N}} \text{ or } \boxed{T \cdot N = 1}$$

$$\boxed{N = 4 \text{ order/yr}}$$

$T = 3$ months/order

$T = \frac{3 \times 4}{4}$ month/order

$$\boxed{T = \frac{1}{4} \text{ yr/order}}$$

C - Cost of purchasing one unit of inventory (Rs/unit)

C_o - Cost of placing one order (Rs/order)

C_h - Cost of holding one unit in inventory for one complete year (Rs/unit/year)

Deterministic Models:-

MODEL #1

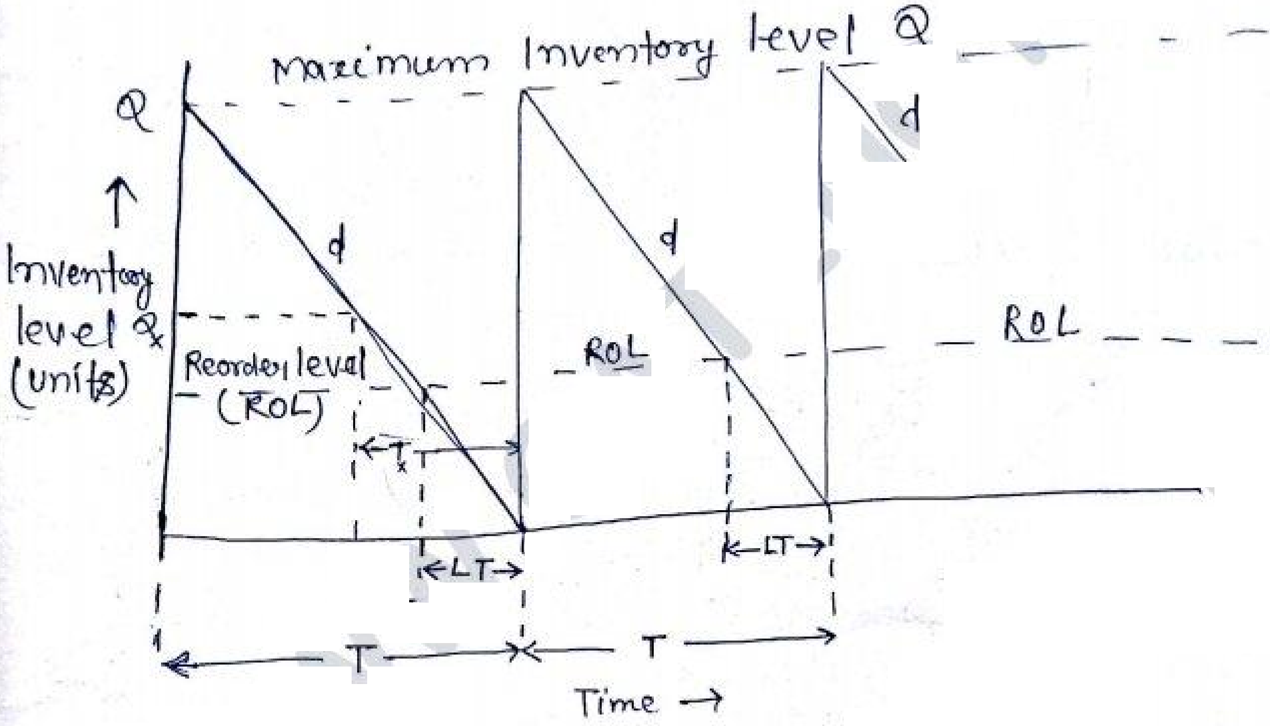
Economic order Quantity (EOQ) model:

or

Harris - Wilson Model

or

Infinite Rate of Replenishment



$$\boxed{Q_x = T_x \cdot d} \qquad \boxed{Q = T \cdot d} \qquad \boxed{ROL = LT \cdot d}$$

$$\boxed{d = \frac{Q}{T} = \frac{Q_x}{T_x} = \frac{ROL}{LT}}$$

Total Annual Cost (TAC) = Puchase Cost (P.C.) + ordering Cost (O.C) + Holding Cost (H.C)

$\boxed{PC = D.C}$

$\boxed{OC = N \cdot C_o = \frac{D}{Q} \cdot C_o}$

HC for period T $= \frac{Q}{2} \cdot C_h \cdot T$

H.C. for period T $= \frac{Q}{2} C_h \cdot T$

eg. 5days
20,15,10, 5, 0

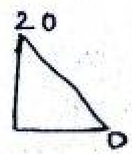
$C_h = 2$ Rs/unit/day

H.C = 40+30+20+10

H.C. = 100 Rs

$Q_{avg} = \frac{50}{5} = 10$ Units

H.C = 10×5×2
= 150 Rs



$Q_{avg} = \frac{20}{2} = \underline{\underline{10}}$

Annual Holding Cost $= \frac{Q}{2} \cdot C_h \cdot T \cdot N$

$$\boxed{T = 1/N}$$

$$\boxed{\text{Annual H.C.} = \frac{Q}{2} \cdot C_h}$$

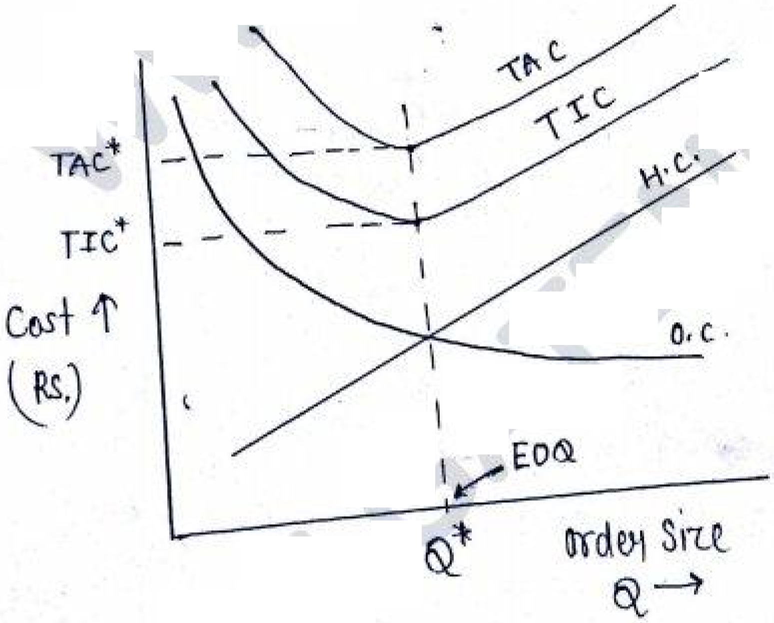
$$\boxed{TAC = D.C + \frac{D}{Q} \cdot C_o + \frac{Q}{2} \cdot C_h}$$

Constant

Total variable Cost/total inventory Cost $= O.C. + H.C$

$$\boxed{TIC = \frac{D}{Q} \cdot C_o + \frac{Q}{2} \cdot C_h}$$

$$\boxed{TAC = TIC + D.C}$$



At EOQ $\boxed{O.C. = H.C}$

The ordering quantity Q^* at which holding cost equal to ordering cost and total inventory cost is minimum is known as economic order quantity

At EOQ

O.C. = H.C.

$$\frac{D}{Q^*} \cdot C_o = \frac{Q^*}{2} \cdot C_h$$

$$\boxed{Q^* = \sqrt{\frac{2DC_o}{C_h}}}$$

$$TIC^* = \frac{D}{Q^*} C_o + \frac{Q^*}{2} \cdot C_h$$

but as $\frac{D}{Q^*} C_o = \frac{Q^*}{2} \cdot C_h$

$$TIC^* = 2\,\frac{Q^*}{2} \cdot C_h$$

only for EOQ

$$\boxed{TIC^* = \sqrt{2DC_oC_h}}$$

$$\boxed{TIC(Q) = \frac{D}{Q} C_o + \frac{Q}{2} C_h}$$

for EOQ or Non EOQ

for TIC to be minimum

$$\frac{dTIC}{dQ} = 0$$

$$\frac{C_h}{2} - \frac{D}{Q^{*2}} C_o = 0$$

2nd derivative

$$0 - \frac{(-2)DC_o}{Q^{*3}} = \frac{2DC_o}{Q^{*3}} \quad \text{+ve}$$

$$\boxed{Q^* = \sqrt{\frac{2DC_o}{C_h}}}$$

Note:- when holding Cost is given in terms of intrest or percentage it alway corresponds to unit price of inventory and the interest rate should be always yearly.

$$\boxed{C_h = i\% \text{ OF } C}$$

yearly

e.g:-

$C = \text{Rs } 50/\text{unit}$

$i\% = 1.5\%/\text{month}$

$\downarrow \times 12$

$i\% = 18\%/\text{yr}$

$C_h = 0.18 \times 50 = \text{Rs } 9/\text{unit}/\text{year}$

Problem 4: Total inventory cost at the order size of 400 unit and 1200 units are equal then determine EOQ (Q^*).

Solⁿ

$$TIC(Q) = \frac{D}{Q} \cdot C_0 + \frac{Q}{2} C_h$$

$$TIC(400) = TIC(1200)$$

$$\frac{D}{400} C_0 + \frac{400}{2} C_h = \frac{D}{1200} C_0 + \frac{400}{2} C_h$$

$$D \cdot C_0 \left[\frac{1}{400} - \frac{1}{1200}\right] = C_h[600 - 200]$$

$$\frac{D C_0}{C_h} = 240000 \qquad Q^* = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{2 \times 240000}$$

$$Q^* = 692.8$$

Note:

If $TIC(Q_1) = TIC(Q_2)$

then $\boxed{Q^* = \sqrt{Q_1 \cdot Q_2}}$

Problem 5:- Determine EOQ value when annual demand is worth Rs 50,000, ordering cost is 2% of order value and holding cost is 10% of average inventory value.

Sol^n

$D \cdot C_3 = Rs\ 50{,}000$

$C_o = 2\% (Q^* \cdot C) \qquad C_o = 0.02\, Q^* \cdot C$

$C_h = 10\% \text{ of } C \qquad C_h = 0.1 C$

$$Q^* = \sqrt{\frac{2 D C_o}{C_h}} = \sqrt{\frac{2 \times \frac{50000}{C} \times 0.02\, Q^* \cdot C}{0.1\, C}}$$

$$Q^* = \sqrt{\frac{20000\, Q^*}{C}}$$

$$Q^{*2} \cdot C = 20000\, Q^*$$

$$Q^* \cdot C = Rs\ 20{,}000$$

Problem 6:- The demand of soap at a retailor is 40 kg/day the retailor purchase soap from a Company in the bulk at the rate of Rs 50/kg. The retailor spend Rs 200 for each order and holding cost is Rs 0.1/kg/day the lead time of 13 day the retailor current ordering policy to order 200 kg every 5 day determine

i) Economic order Quantity (EOQ)

ii) Amount of saving with EOQ compare the current ordering policy in 30 days

(iii) Reorder level corresponding to EOQ

Sol^n

$d = 40$ kg/day

$LT = 13$ days

$C_h = $ Rs 0.1/kg/day

$C_0 = $ Rs 200/order

$Q = 200$ kg/order

$C = $ Rs 50/kg

(i) $Q^* = \sqrt{\dfrac{2DC_0}{C_h}} = \sqrt{\dfrac{2 \times 40 \times 200}{0.1}}$

$Q^* = 400$ kg/order

(ii) for 30 days

$C_h = $ Rs 3/kg/month

$D = 30 \times 40 = 1200$ kg

(a) current policy

$TIC = \dfrac{D}{Q} C_0 + \dfrac{Q}{2} \cdot C_h = \dfrac{1200}{200} \times 200 + \dfrac{200}{2} \times 3$

$TIC = 1200 + 300$

$TIC = $ Rs 1500

(b) EOQ i.e. $= Q = 400$ kg/order

$$TIC = \frac{1200}{400} \times 200 + \frac{400}{2} \times 3$$

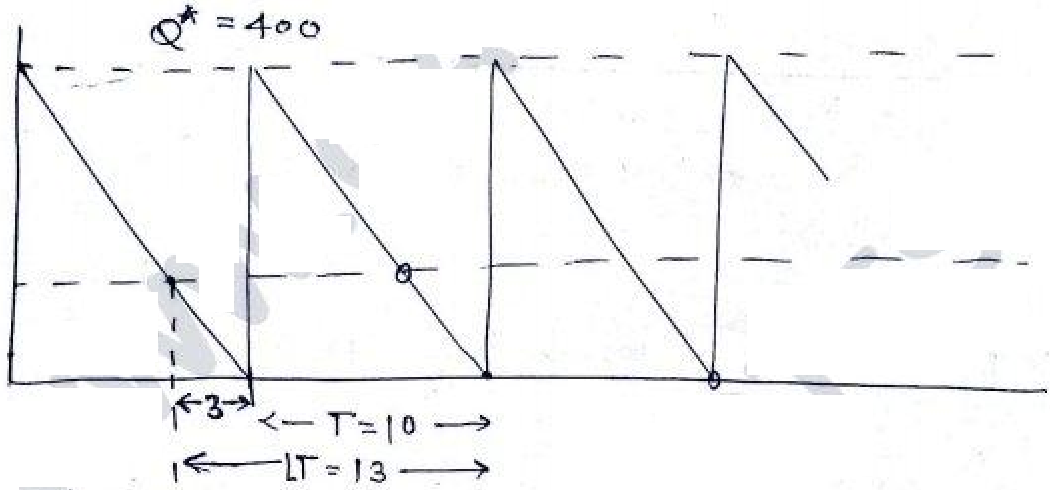
$$TIC = Rs\ 1200$$

Saving $= 1500 - 1200 = Rs\ 300$

(3) Reorder ~~level~~ level (ROL) $= LT \times d$

$= 13 \times 40$

ROL $= 520$ kg



$$T^* = \frac{400}{40} = 10 \text{ days}$$

$\underline{LT > T}$

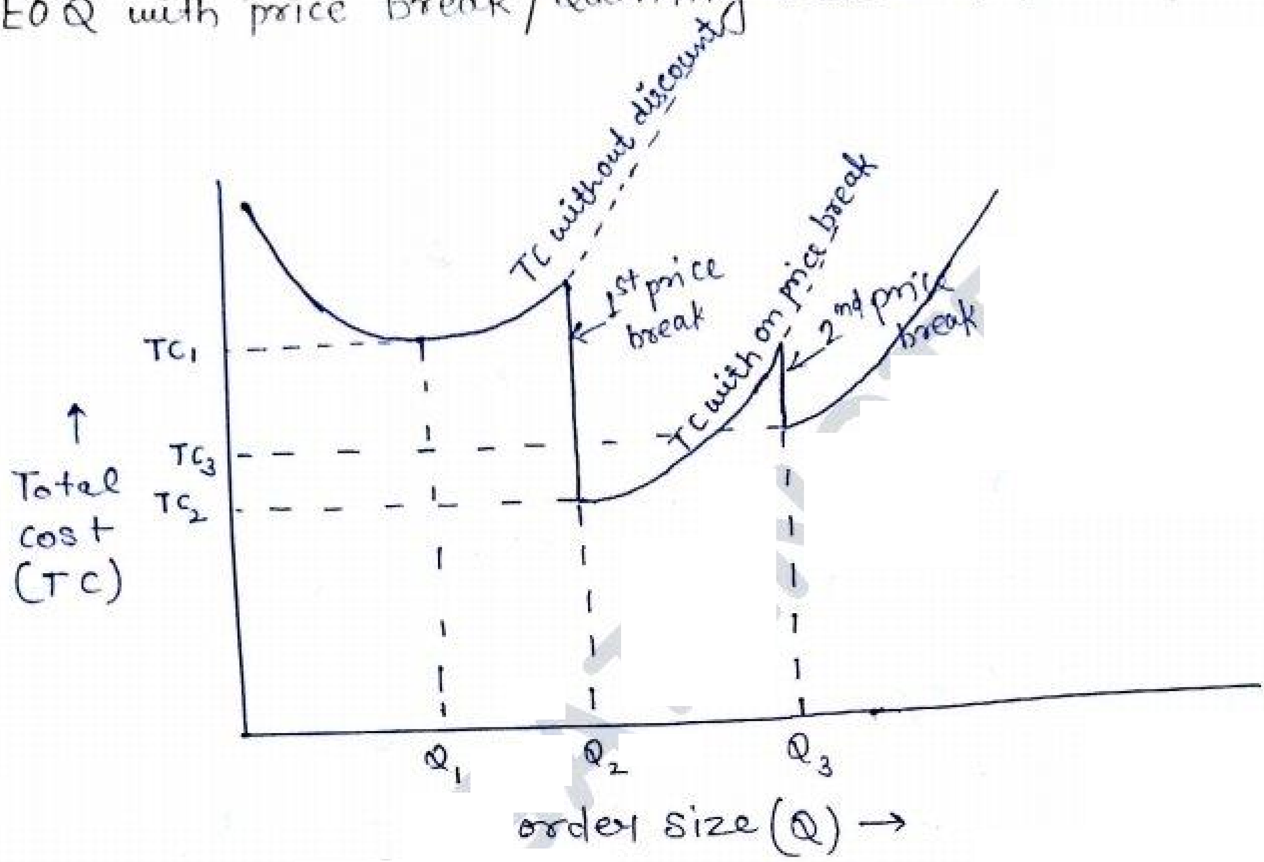
effective lead time

$LT = 13 - 10 = 3$

ROL $= 3 \times 40 = 120$ kg

MODEL #2

EOQ with price break / Quantity discount :-



$$TC = D \cdot C + \frac{D}{Q} \cdot C_o + \frac{Q}{2} \cdot C_h$$

In some condition discount is offered on unit price of inventory for large quantity purchase. These discount take the form of price break. Discount is always offered on unit price of inventory. In order to determine the best order size we need to consider purchasing cost along with ordering and holding cost. Whenever the total cost comes out to be minimum give the best order size.

In most cases discount always Advantage

Problem 7: For a production system annual demand is 8000 units. Ordering cost is Rs 1800 per order and holding cost is 10% of unit price. Item can be purchase in lot as given below determine the best order size

ESE 2008

Lot size	Unit price (Rs/unit)
1 – 999	220
1000 – 1499	200
1500 – 1999	190
2000 & above	185

Sol^n $D = 8000$ unit $\quad C_0 = $ Rs 1800/order

we know that

EOQ i.e. $Q^* = \sqrt{\frac{2DC_0}{C_h}}$

where $C_h = 10\%$ of C

starting from the lowest unit price & searching feasible EOQ

$$Q^* = \sqrt{\frac{2 \times 8000 \times 1800}{185 \times 0.1}}$$

$Q^* = 1247.7$ Unit/order

It is not feasible as for $C = $ Rs 185/unit
$Q \geq 2000$

proceeding to next higher unit price of Rs 190/unit

$$Q^* = \sqrt{\frac{2 \times 1800 \times 8000}{190 \times 0.1}}$$

$Q^* = 1231.17$ Unit/order

Again not feasible

$C =$ Rs 200/unit

$$Q^* = \sqrt{\frac{2 \times 8000 \times 1800}{(200 \times 0.1)}}$$

$Q^* = 1200$ unit/order

Now it is feasible as for $C =$ Rs 200/unit

Q must be between 1000 to 1499

Now we compute total cost at feasible EOQ i.e. $Q^* = 1200$ and the next higher price break point of $Q = 1500$ and $Q = 2000$ unit

$$TC(Q) = D \cdot C + \frac{D}{Q} C_o + \frac{Q}{2} \cdot C_h$$

$$T(1200) = 8000 \times 200 + \frac{8000}{1200} \times 1800 + \frac{1200}{2} \times (200 \times 0.1)$$

$T(1200) =$ ~~Rs 1612 6400~~ Rs 1624000

$$T(1500) = 8000 \times 190 + \frac{8000}{1500} \times 1800 + \frac{1500}{2} \times (190 \times 0.1)$$

$T(1500) =$ Rs 1543850

$$TC\ (2000) = 8000 \times 185 + \frac{8000}{2000} \times 1800 + \frac{2000}{2} \times (185 \times 0.1)$$

$$TC(2000) = Rs\ 1505700$$

So Best order size $Q = 2000$ Unit/order

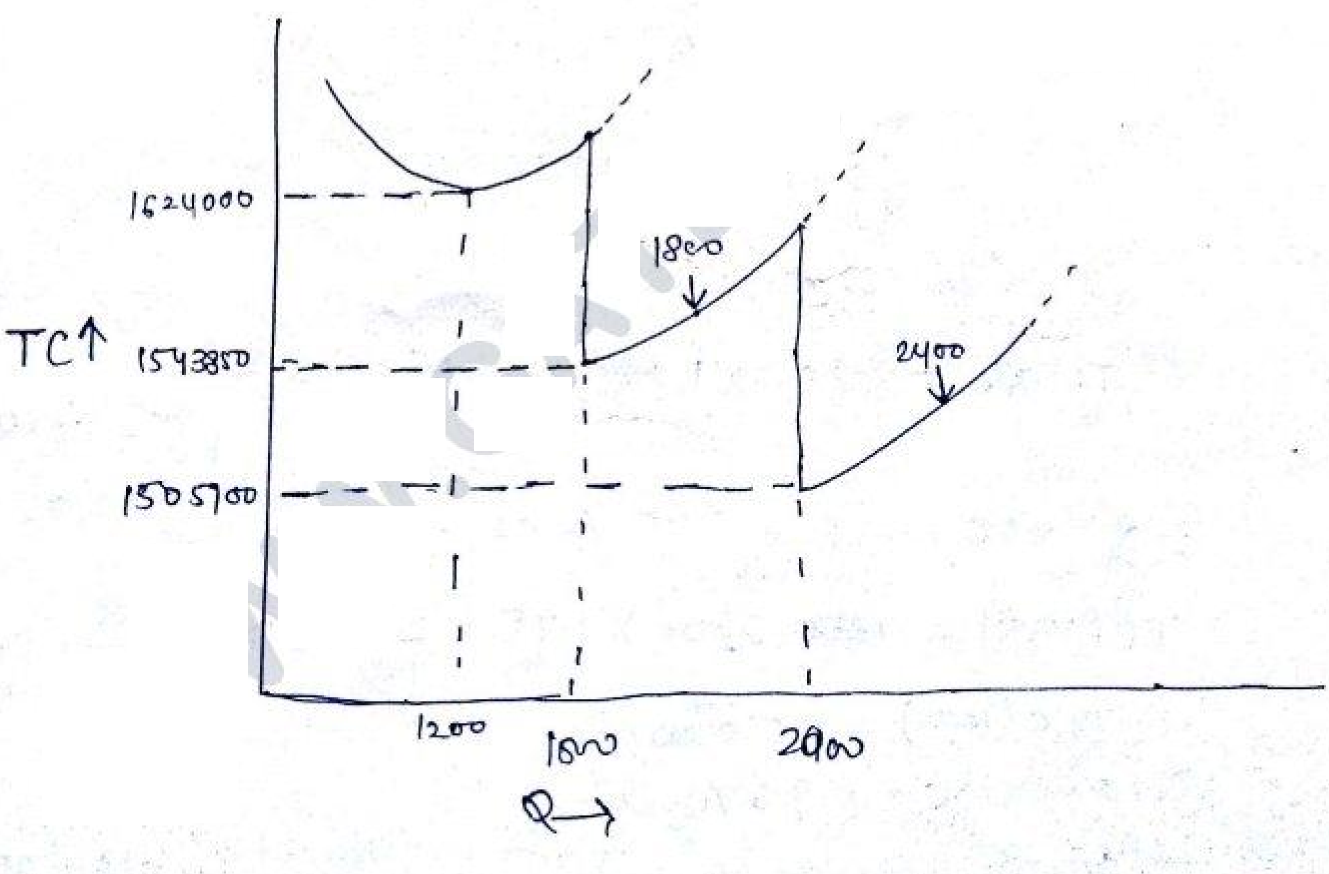
where total Cost is minimum.

$$TC(1800) = 8000 \times 190 + \frac{8000}{1800} \times 1800 + \frac{1800}{2} \times (190 \times 0.1)$$

$$TC(1800) = Rs\ 1545100$$

$$TC(2400) = 8000 \times 185 + \frac{8000}{2400} \times 1800 + \frac{2400}{2} (185 \times 0.1)$$

$$T(2400) = Rs\ 1508200$$



Problem 8:- $D = 2000$ unit/yr $\qquad C_0 = $ Rs 10/order

$C = $ Rs 1/unit $\qquad C_h = $ Rs 0.16/unit/yr.

1) find Q^* & TIC^*

2) if $Q = 1000$, 5% discount on c

& if $Q = 2000$, 7% discount on c

determine best order size

Sol^n

1) $$Q^* = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2 \times 2000 \times 10}{0.16}}$$

$$Q^* = 500 \text{ unit/order}$$

$$TIC^* = \sqrt{2DC_hC_0}$$

$$= \sqrt{2 \times 2000 \times 10 \times 0.16}$$

$$TIC^* = 80$$

$$TC = 2000 \times 1 + 80 = 2080 \text{ Rs}$$

2) $Q = 1000 \qquad C = 0.95$ /unit $\qquad \boxed{Q = 1000 \text{ best order size}}$

$$TC(1000) = 2000 \times 0.95 + \frac{2000}{1000} \times 10 + \frac{1000}{2} \times 0.16$$

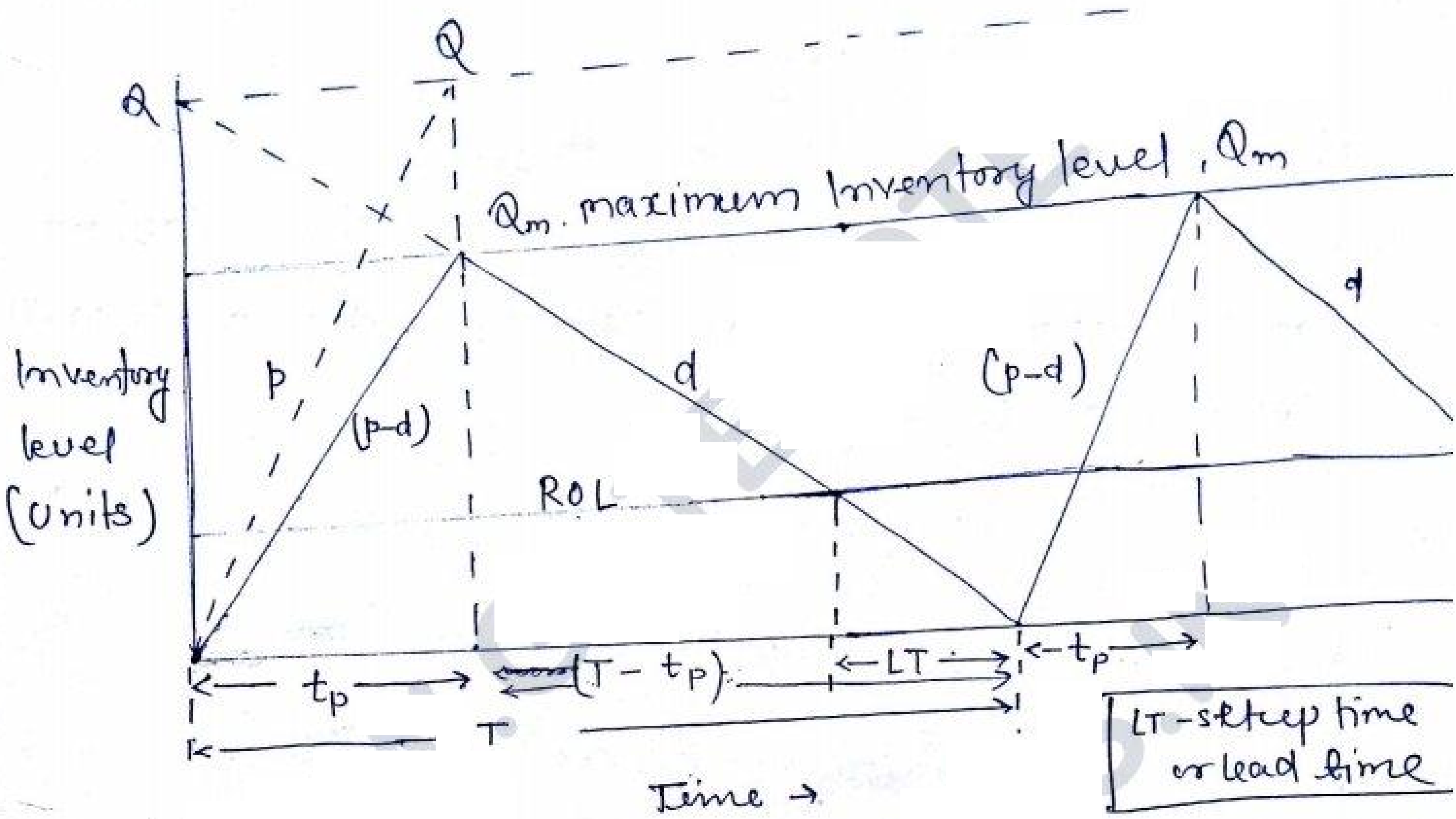
$$TC(1000) = 2000 \text{ Rs}$$

$Q = 2000 \quad C = 0.93$ /unit

$$T(2000) = 2000 \times .93 + \frac{2000}{2000} \times 10 + \frac{2000}{2} \times 0.16 = \underline{2030}$$

MODEL #3

Production / Build up Model :



e.g. $p = 100$ unit/day ↑

$d = 20$ unit/day ↓

$p - d = 80$ unit/day ↑

if $t_p = 20$ days

$Q = 20 \times 100 = 2000$ unit

$Q_m = 2000 - 400$ Unit

$Q_m = 1600$ unit

This model is Similar to 1st model EOQ the only difference is the inventory build up is gradual rather than instant. One inventory cycle of 'T' time period can be devided into two parts (1) t_p

~~(i) 2 par~~ (2) $T - t_p$

(1) t_p (production time) - In this period there is production and inventory increase with a constant rate of $(p-d)$ unit/time

(2) $(T-t_p)$ → In this period there is no production and only consumpsion and inventory is consumed with a costant rate d unit/time

Notations

p - production or build up rate (/)

d - demand / consumption rate (/ time)

t_p - production/manufacturing cycle time

$$Q = t_p \cdot p \quad \rightarrow \text{Total produced inventory}$$

$$t_p = \frac{Q}{p}$$

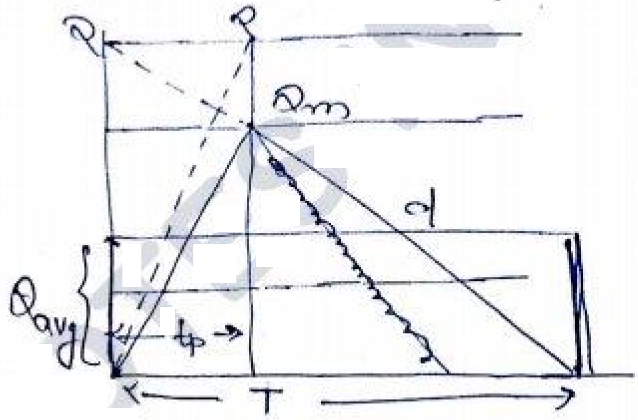
$$Q_m = t_p (p-d) \Rightarrow \boxed{Q_m = Q\left(\frac{p-d}{p}\right)}$$

on hand inventory

Total inventory Cost = setup Cost (S.C.) + Holding Cost (H.C.)

Setup Cost (S.C.) = No. of Setup × Cost/setup

$$\boxed{S.C. = \frac{D}{Q} \cdot C_0}$$



$$H.C. = Q_{avg} \times C_h$$

Area of Rectange = Area of triangle

$$Q_{avg} \times T = \frac{1}{2} T \times Q_m$$

$$Q_{avg} = \frac{Q_m}{2}$$

$$Q_{avg} = \frac{Q}{2}\left(\frac{p-d}{p}\right)$$

$$H.C. = \frac{Q}{2} \cdot C_h \cdot \left(\frac{p-d}{p}\right)$$

Total inventory Cost

$$\boxed{TIC = \frac{D}{Q} \cdot C_0 + \frac{Q}{2} C_h \left(\frac{p-d}{p}\right)}$$

for TIC to be minimum $\frac{d\,TIC}{dQ} = 0$

$$\frac{C_h}{2}\left(\frac{p-d}{p}\right) - \frac{D}{Q^{*2}} C_0 = 0$$

$$\Rightarrow \boxed{Q^* = \sqrt{\frac{2DC_0}{C_h}\left(\frac{p}{p-d}\right)}}$$

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} \sqrt{\frac{p}{p-d}}$$

production factor > 1 (Always)

* $\sqrt{\frac{p}{p-d}}$ is know as production factor

$$\Rightarrow \sqrt{\frac{1}{1-d/p}} \qquad \begin{array}{l} p \to \infty \\ p.f. \to 1 \end{array}$$

At EOQ $\boxed{S.C. = H.C_o}$

$$\boxed{TIC^* = \sqrt{2DC_oC_h} \sqrt{\frac{p-d}{p}}}$$

< 1 So TIC always batter as compare to 1st EOQ Model

Problem 9:- A company require 12000 unit/year of a component X in a year. Component X is made in 30 batches of 400 units each on a machine that produces 8 unit per hour. The company operates for 4000 hrs/yr and it cost Rs 500 to setup the machine. C_h (Holding cost) Rs 10/unit/year find out whether the existing production plant is optimum or not. If not suggest a new

production plan and amount of ~~single~~ saving possible. Also determine production cycle time, maximum inventory level and cycle time corresponding to optimum cycle.

Solⁿ:-

$D = 12000$ units/yr

$C_o = 500$ Rs/setup

$C_h = 10$ Rs/unit/year

4000 hrs/yr

So, $d = 3$ unit/hr

$P = 8$ unit/hr.

$$\text{Production facter} = \sqrt{\frac{8}{8-3}} = \sqrt{8/5}$$

1) $$Q^* = \sqrt{\frac{2DC_o}{C_h}\left(\frac{P}{P-d}\right)}$$

$$= \sqrt{\frac{2 \times 12000 \times 500}{10}\left(\frac{8}{8-3}\right)}$$

$Q^* = 1385.64$ unit/setup $\quad TIC^* = Rs\,8660$

$$N^* = \frac{D}{Q^*} = \frac{12000}{1385.64} = 8.66 \text{ Setup/yr}$$

$N = 8, \; Q = 1500$

$N = 9, \; Q = 1333.33$

$N = 10, \; Q = 1200$

Can not take in fraction

$$TIC = \frac{D}{Q} \cdot C_o + \frac{Q}{2} C_h \left(\frac{P-d}{P} \right)$$

a) $N = 8, \quad Q = 1500$

$$TIC = 8 \times 500 + \frac{1500}{2} \times 10 \times \frac{5}{8}$$

$$TIC = Rs\ 8687.5$$

b) $N = 10, \quad Q = 1200$

$$TIC = 10 \times 800 + \frac{1200}{2} \times 10 \times \frac{5}{8}$$

$$= Rs\ 8750$$

So best order size $N = 8$ & $Q = 1500$ Units/setup

2) Amount of saving

$$Q = 400, \quad N = 30$$

$$TIC = 30 \times 800 + \frac{400}{2} \times 10 \times \frac{5}{8}$$

$$= Rs\ 16250$$

$$\text{Saving} = 16250 - 8687.5$$

$$= Rs\ 7562.5$$

3)

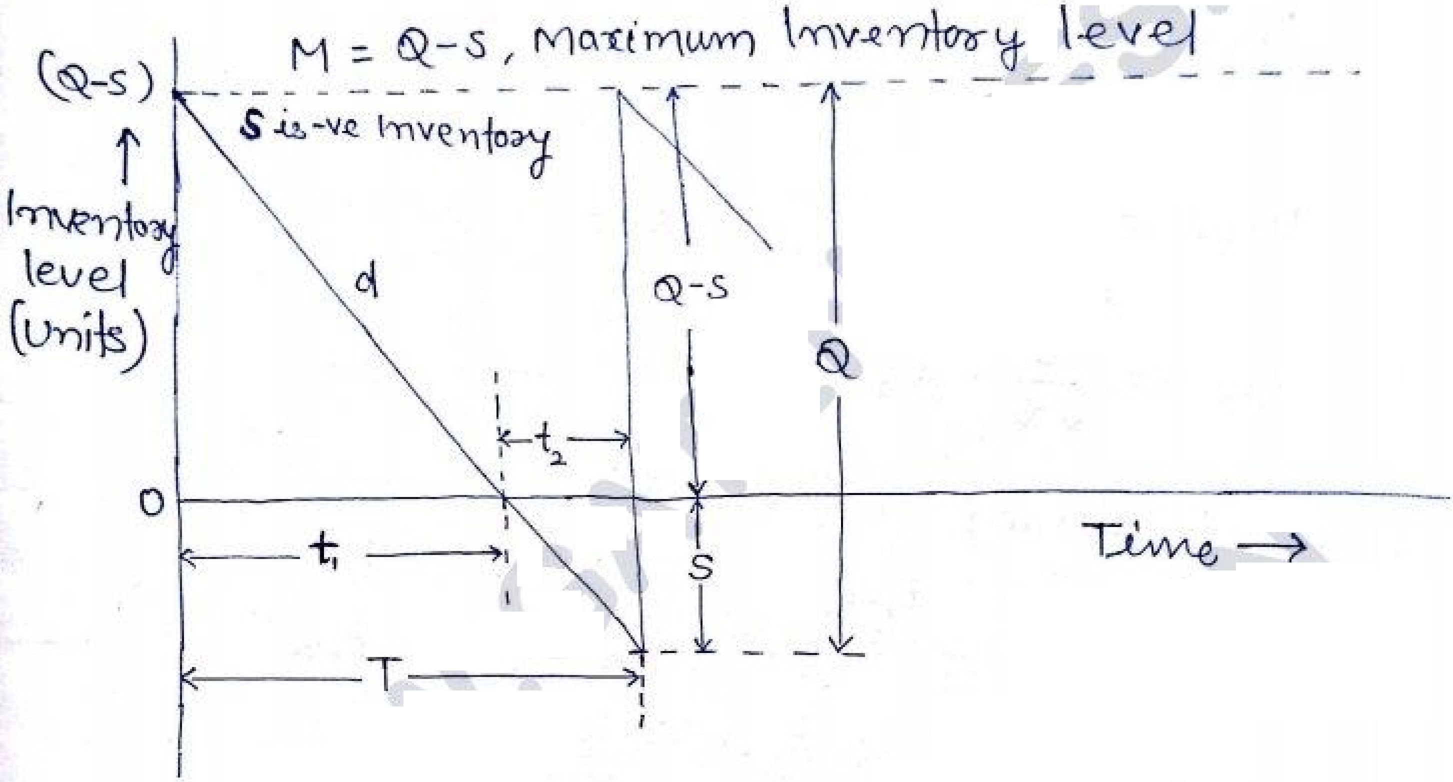
$$t_p = \frac{Q}{P} = \frac{1500}{8} = 187.5 \text{ hr}$$

$$Q_m = Q\left(\frac{P-d}{P}\right) = 1500 \times \frac{5}{8} = 937.5 \text{ Unit/setup}$$

$$T = \frac{Q}{d} = \frac{1500}{3} = 500 \text{ hrs/setup.}$$

MODEL #4

Shortage/stockout/Back order :



This model is similar to 1st model EOQ the only difference that shortages are allowed. Planned shortage or back order is a condition where customer place an order and finds that inventory is out of stock then he wait for next shipment to get this order fullfilled

Notations

S - No. of Unit short or back ordered

C_b - back order or shortage cost/back order/year (Rs/Unit/yr)

or C_s -

Total inventory Cost

$$TIC = O.C. + H.C. + \text{shortage Cost } (S.C)$$

$$O.C. = \frac{D}{Q} \cdot C_o$$

$$H.C. = \frac{(Q-S)^2}{2Q} \cdot C_h$$

$$S.C. = \frac{S^2}{2Q} \cdot C_b$$

H.C. for period T

$$= \left(\frac{Q-S}{2}\right) \cdot t_1 \cdot C_h$$

$$\therefore (Q-S) = t_1 \cdot d$$

$$Q = T \cdot d$$

$$\frac{t_1}{T} = \frac{Q-S}{Q}$$

$$t_1 = \left(\frac{Q-S}{Q}\right) \cdot T$$

H.C. for period T

$$= \left(\frac{Q-S}{2}\right) \cdot \left(\frac{Q-S}{Q}\right) T \cdot C_h$$

$$= \frac{(Q-S)^2}{2Q} \cdot T \cdot C_h$$

$$\text{Annual Holding Cost} = \frac{(Q-S)^2}{2Q} \cdot T \cdot N \cdot C_h$$

$$\text{Annual H.C.} = \frac{(Q-S)^2}{2Q} \cdot C_h$$

Shortage cost for period T

$$S.C. = \frac{S}{2} \cdot t_2 \cdot C_B$$

$$\because S = t_2 \cdot d$$

$$Q = T \cdot d$$

$$\Rightarrow \quad t_2 = \frac{S}{Q} T$$

S.C. for period T

$$= \frac{S}{2} \cdot \frac{S}{Q} \cdot C_b T = \frac{S^2}{2Q} \cdot C_b \cdot T$$

$$\text{Annual shortage Cost} = \frac{S^2}{2Q} \cdot C_b \cdot T \cdot N$$

$$\text{Annual S.C.} = \frac{S^2}{2Q} \cdot C_b$$

So the total inventory Cost

$$\boxed{TIC = \frac{D}{Q} C_0 + \frac{(Q-S)^2}{2Q} \cdot C_h + \frac{S^2}{2Q} \cdot C_b}$$

Q, S Are variable

for TIC to be minimum

$$\boxed{Q^* = \sqrt{\frac{2DC_0}{C_h}} \underbrace{\sqrt{\frac{C_b + C_h}{C_b}}}}$$ ← Cost Factor > 1 (Alway)

$$\left(\sqrt{1 + \frac{C_h}{C_b}}\right) \quad C_b \to \infty$$

if $C_b \to \infty$ it become 1st EOQ

At EOQ $\boxed{O.C. = H.C + S.C}$

Total inventory cost

$$\boxed{TIC^* = \sqrt{2DC_oC_h}\;\underbrace{\sqrt{\frac{C_b}{C_b+C_h}}}_{<1\,(\text{Always})}}$$

Optimum number of units back ordered/short

$$\boxed{(Q-S)\times C_h = S\times C_b}$$

$$(Q^*-S^*)\times C_h = S^*\times C_b$$

$$\frac{Q^*-S^*}{S^*} = \frac{C_b}{C_h}$$

adding +1 both side

$$\frac{Q^*}{S^*} = \frac{C_b+C_h}{C_h}$$

$$\boxed{S^* = Q^*\cdot\left(\frac{C_h}{C_b+C_h}\right)}$$

maximum Inventory

$$M^* = Q^* - S^*$$

$$\boxed{M^* = Q^*\,\frac{C_b}{C_b+C_h}}$$

Problem 10: A dealer supply following information

$D = 10,000$ Units , $C = Rs.20/unit$, $C_o = Rs\ 10/order$

$C_h = 20\%$ of C ,

Dealer is considering the possibility of back ordering and dealer estimated that annual cost of back ordering is 25% of C

Then determine

i) Optimum No. of unit to be ordered.

ii) Quantity to be back ordered.

iii) Would you recommend to allow back ordering if so annual cost saving by adopting the policy of back ordering

Soln:-

$C_h = Rs\ 4$

$C_b = Rs\ 5$

(i) $$Q^* = \sqrt{\frac{2DC_o}{C_h}}\sqrt{\frac{C_h+C_b}{C_b}}$$

$$Q^* = \sqrt{\frac{2 \times 10000 \times 10}{20 \times 20/100}}\sqrt{\frac{4+5}{5}}$$

$$Q^* = 300 \text{ Units}$$

(ii) $$S^* = \frac{4}{4+5} \times 300 = \frac{1200}{9} = 133.33 \text{ Unit}$$

(iii) Yes

a) without back order cost

$$TIC^* = \sqrt{2DC_oC_h}$$

$$= \sqrt{2 \times 10000 \times 10 \times 4}$$

$$TIC^* = Rs\ 894.4$$

b) with back order

$$TIC^* = \sqrt{2DC_oC_h}\sqrt{\frac{C_b}{C_b+C_h}}$$

$$TIC^* = 894.4 \times \frac{5}{9}$$

$$TIC^* = Rs\,666.6$$

$$\text{saving} = 894.4 - 666.6$$

$$= Rs.\,227.76$$

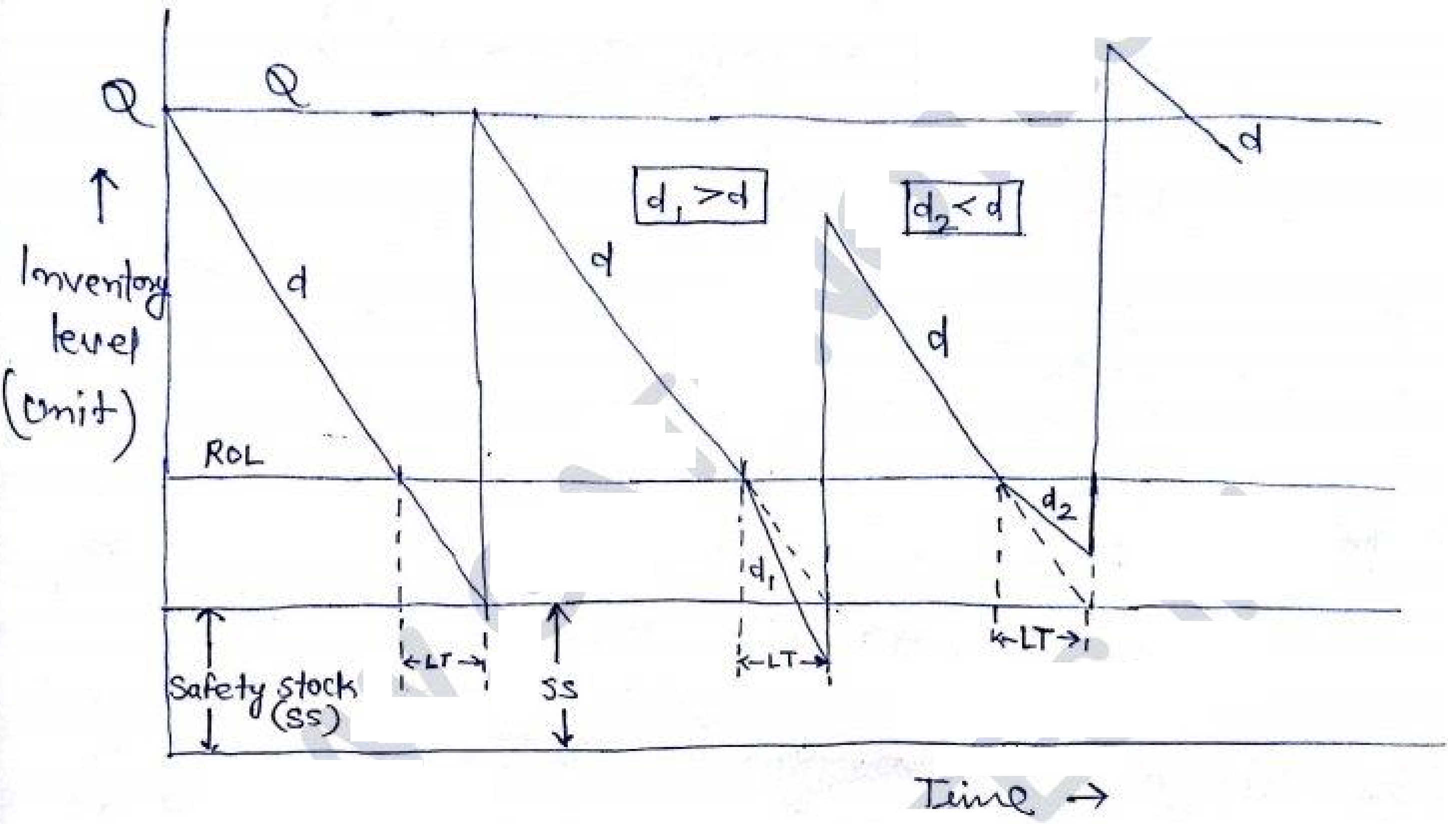
*Combined Production & shortage

$$Q^* = \sqrt{\frac{2DC_o}{C_h}}\sqrt{\left(\frac{P}{P-d}\right)}\sqrt{\left(\frac{C_b+C_h}{C_b}\right)}$$

$$TIC^* = \sqrt{2DC_oC_h}\sqrt{\left(\frac{P-d}{P}\right)}\sqrt{\left(\frac{C_b}{C_b+C_h}\right)}$$

Probabilistic Model:-

Demand Rate and lead time are not Costant



factor Incouraging higher safety Stock:-

1) When the demand rate and lead time variation are more and fluctuating
2) When the inventory holding Cost is less and is not of much concern
3) When the loose due to absence due to inventory that is shortage cost is very high.
4) When the number of orders in a year are more
5) To have batter costumer satisfaction

Reoder level Given by -

ROL = Average Demand during LT (ADDLT) + Safety stock (SS)

$$\boxed{ROL = LT \times d + SS}$$

$$\boxed{Q_{avg} = \frac{Q}{2} + SS}$$

MODEL #1

Demand-Profit Model/static Inventory Model.

In this model Demand is uncertain and decision is based upon single order that is reordering is not permitted. This model is used for perishable items like vegetables, fruits, flowers etc. or for those items which become outdated very fast.

D - Demand

S - supply

p - profit/unit

l - loss/unit

1) if $D > S$ $\longrightarrow$ $(D-S) \cdot p$ (Potential profit loss

2) if $S > D$ $\longrightarrow$ $(S-D) \cdot l$ (loss due to unsold items)

$$\boxed{p = S_p - C + C_b}$$

$$\boxed{l = C - C_s + C_h}$$

C_s - scrap price

C_h - holding price.

p - potential profit loss/unit because of not meeting the demand

S_p - selling price / unit

C - purchasing Cost per Unit

C_b - back order or shortage Cost or good will loss per unit

l - Unsold item loss per unit

C_s - Salvage or Scrap value

C_h - Holding Cost per Unit

In this model in order to maximize our profit we select the ordering quantity 'S' in such a manner

$$\boxed{P(S-1) < \frac{p}{p+l} \leq P(S)}$$

where

$P(S-1)$ - Cummulative or additive probability of the demand for $(S-1)$ unit.

$P(S)$ - Cummulative probability of the demand for (S) unit.

eg

(Give) Demand	(Give) probability	(find) cummulative probability
1	0.06	0.06
2	0.11	0.17
3	0.09	0.26
4	0.08	0.34
5	0.14	0.48
6	0.13	0.61
⋮	⋮	⋮
		1

Suppose $\frac{p}{p+l} = 0.45$

$P(S-1)$ 0.45 $P(S)$

So ordering Quantity '5' to maximize profit

if

$c = 0, C_s = 0, S_p = 0$

$p = C_b$
$l = C_h$

$$\boxed{P(S-1) < \frac{C_b}{C_b + C_h} \leq P(S)}$$

Problem 11: A shop keeper purchase a seasonal product at the begining of season and can not reorder. The item cost Rs.30 and sold at Rs.75 each. For any item that cannot meet for demand, he had estimated a good will loss of Rs. 20. Any item unsold will have a salvage value of Rs.15 and holding cost during the period 20% of unit price find the optimum number of units to be stock to optimise profit.

Demand	1	2	3	4	5	6	7	8	9	10
probability	0.09	0.10	0.13	0.11	0.07	0.08	0.10	0.12	0.14	0.06

Soln

Cummulative probability	0.09	0.19	0.32	0.43	0.50	0.58	0.68	0.80	0.94	1

$C = Rs30$

$S_p = Rs. 75$

$C_s = Rs. 15$

$C_h = \frac{20 \times 30}{100} = 6 Rs$

$C_b = 20$

$p = 75 - 30 + 20 = 65\ Rs$

$l = 30 - 15 + 6 = 21\ Rs$

$$\frac{p}{l + p} = 0.758$$

$$\boxed{S = 8}$$

Problem 12:- find the shortage cost raise when holding cost is Rs. 3 and demand in probability is as given below with optimum stock level of 9 unit

Demand	1	2	3	4	5	6	7	8	9	10	11	12
Prob.	0.05	0.06	0.09	0.12	0.08	0.07	0.13	0.15	0.03	0.07	0.09	0.06
Cu prob	0.05	0.11	0.20	0.32	0.40	0.47	0.60	0.75	0.78	0.85	0.94	1

Sol

$P(S-1)$ (0.75) $\quad$ $P(S)$ (0.78)

$C_h = \text{Rs. } 3 \qquad S = 9$

$P(S) = 0.78$

$P(S-1) = 0.75$

$$P(S-1) < \frac{C_b}{C_b + C_h} \le P(S)$$

$$0.75 < \frac{C_b}{C_b + 3} \le 0.78$$

$$0.75 = \frac{C_b}{C_b + 3} \Rightarrow C_b = \text{Rs } 9$$

$$0.78 = \frac{C_b}{C_b + 3} \Rightarrow C_b = \text{Rs } 10.63$$

$$\text{Rs } 9.00 \le C_b \le \text{Rs } 10.63$$

MODEL # 2

Service level Model :-

This model is prefferred where the different Cost factor involve in inventory are not known exactly it is based upon probability theory and the amount of safety stock is kept according to level of service managment wants to achive

$$\text{Service level} = \left(\frac{\text{Number of units supplied without delay}}{\text{Total Number of unit demanded}}\right)_{LT}$$

$S \rightarrow 0 \text{ to } 1$

$S\% \rightarrow 0 \text{ to } 100\%$

95% service level is the standerd value and it means that 95% of the customers order on an average are fullfilled during lead time and only 5% of Customers order on an average are rejected during lead time.

when the demand during lead time may be approxemated by a normal distrubution with certain average ($\bar{x}$ or μ) and standard deviation (σ)

then reorder level is given by

$$\boxed{ROL = \bar{X} + Z\sigma}$$

$$\boxed{Z\sigma = SS}$$ → safety stock

where $\bar{X}$ = Average demand during lead time

$$\boxed{\bar{X} = LT \times d}$$

σ = Standard deviation for the demand variation during lead time and

Z = Standard normal variant whose value depends upon the service level required

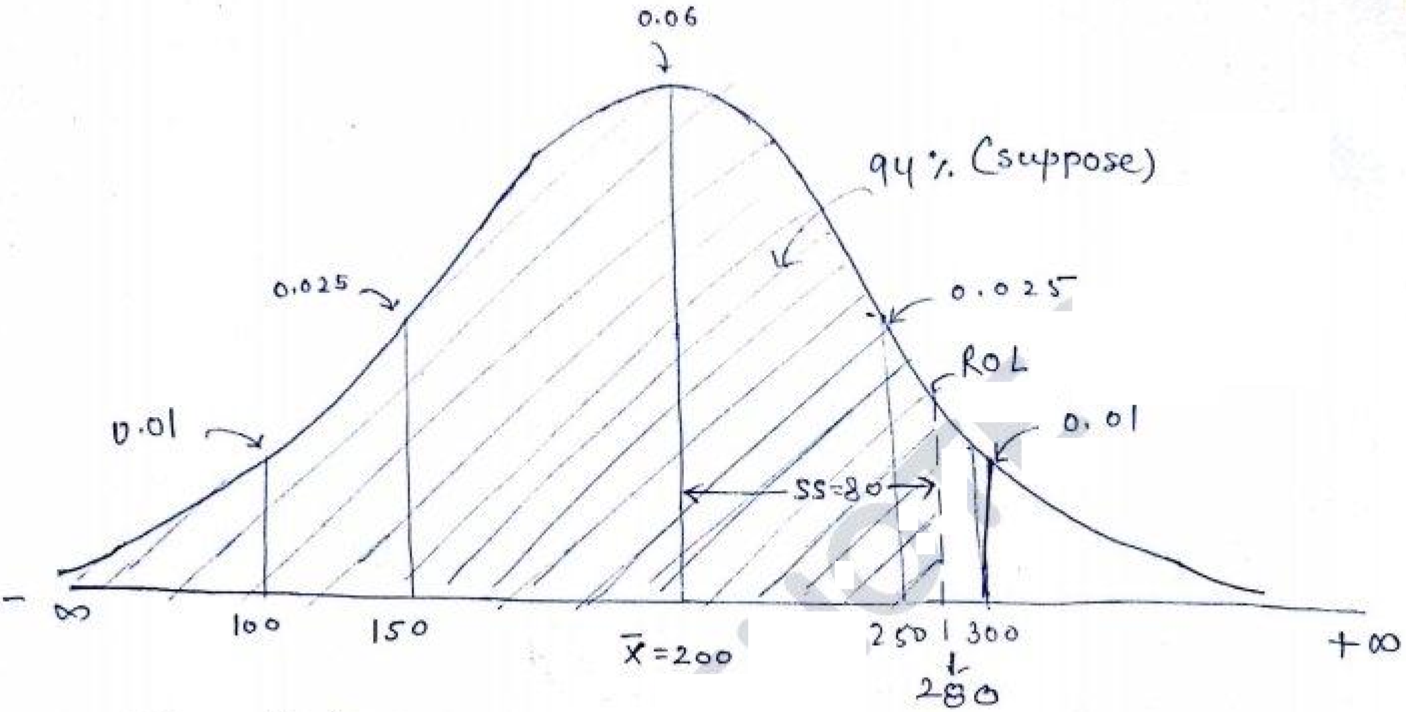
standard values for Z

Z	service level (%)
0.84	80%
1.28	90%
1.645	95%
2.33	99%

eg. x_1, x_2, x_3

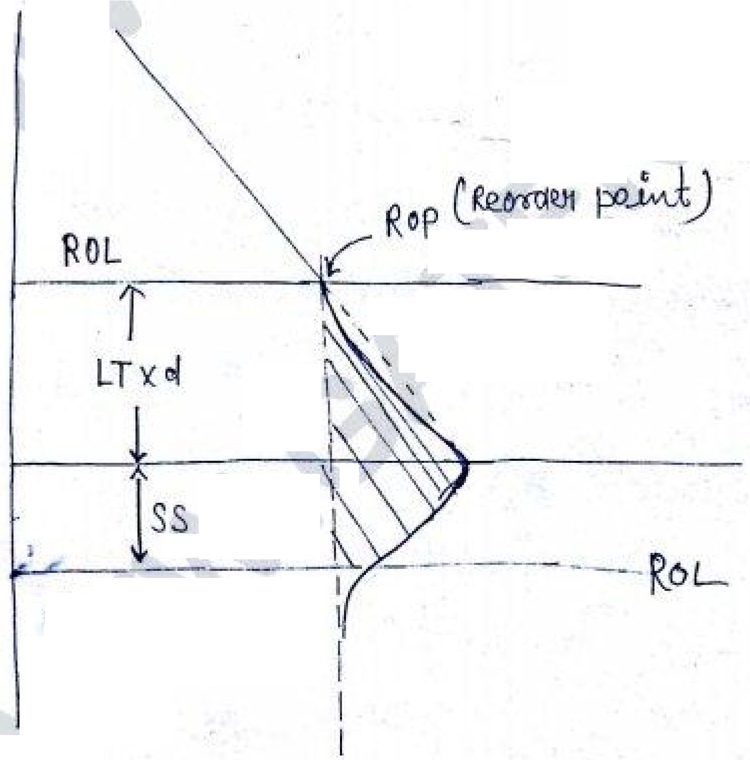
$$\bar{x} = \frac{x_1 + x_2 + x_3}{3} \qquad \sigma = \sqrt{\frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+(x_3-\bar{x})^2}{3}}$$

$SS = Z.\sigma$
↓
constant



LT = 10days
d = 20 unit/days
$\bar{X}$ = 200 units
SS = 80 unit
service level = 94%

Shaded Area Represent the service level
or service level = Area up to ROL



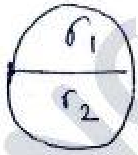
Shaded Area = Service level

Note: lead time is one complete cycle and σ should be always corrosponding to lead time while computing safety stock.

* one cycle consist of two parts

1^{st} half - σ_1

2^{nd} half - σ_2



then σ for complete cycle

$$\sigma^2 = \sigma_1^2 + \sigma_2^2$$

$$\boxed{\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}}$$

Problem 13: Average weekly demand is of 800 unit and weekly standard deviation is of 100 units. Unit price is Rs 75 and holding cost Rs 0.2/unit/week. if the lead time is of 4 week then for 95% service level determine.

i) Safety stock ii) Reorder level (iii)

(iii) Annual cost of maintaining safety stock.

Soln. $d = 800$ unit $\quad LT = 4$ week

$\sigma = 100 \quad$ 95% service level

$z = 1.645$

$C = Rs\ 75$

$C_h = 0.2$/unit/week.

Average demand during lead time $\bar{X} = 800$ unit/week

(i) Safety stock (SS) $= Z \cdot \sigma$

$= 1.645 \times 200$

SS $= 329$ unit

* σ for 4 week

$\sigma' = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2}$

$= \sqrt{4\sigma^2}$

$\sigma' = 2\sigma$

(ii) ~~ROL = ...~~

= ~~...~~

* As lead time is of 4 week and σ is given weekly so, converting σ corresponding to LT

$\sigma'^2 = \sigma^2 + \sigma^2 + \sigma^2 + \sigma^2$

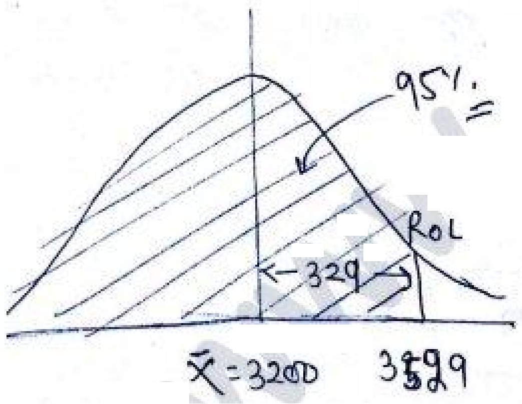
$\sigma' = 2\sigma = 200$ unit

ROL $= \bar{x} + z\sigma$ $\quad \boxed{\bar{X} = LT \times d}$

$= 4 \times 800 + 329$

$= 3529$ Unit

(iii) Cost of maintaining SS $=$ SS $\times C_h$



$C_h = 0.2$ /unit/week

52 week/yr

$C_h =$ Rs 10.4 /unit/yr

Cost $= 329 \times 10.4$

$=$ Rs 3421.6

Actual demand during lead time

210, 180, 240, 200, 260, 190, 230

So Safety Stock

$$\boxed{SS = \begin{pmatrix}\text{Max. demand during} \\ \text{lead time}\end{pmatrix} - \begin{pmatrix}\text{Average demand} \\ \text{during lead time}\end{pmatrix}}$$

So

$SS = 260 - 215.$

$SS = 45$ Unit

Problem 14: For a production system annual consumption is 20,000 units and cost of per unit is Rs. 4. ordering cost Rs 60/order and inventory carrying cost Rs. 0.6/unit/year. Past lead time are 15, 20, 18, 12, 22, 27 day. If there are 250 working day in a year then calculate

(i) EOQ (ii) Safety stock (SS) (iii) Reorder level (ROL)

(iv) Average stock in inventory.

Sol^n (i) $Q^* = \sqrt{\frac{2 D C_o}{C_h}} = \sqrt{\frac{2 \times 20000 \times 60}{0.6}} = 2000$ Unit

(ii) $\boxed{SS = \text{Max. DDLT} - \text{Avg DDLT}}$

$\boxed{SS = (\text{Max. LT} - \text{Avg. LT}) \times d}$

$= (27 - 19) \times \frac{20,000}{250}$

$SS = 640$ Unit

(iii) $ROL = Avg.\,LT \times d + SS$

$$= 19 \times \frac{20000}{250} + 640$$

$ROL = 2160$ Unit

(iv) Average Stock

$$Q_{avg} = \frac{Q^*}{2} + SS$$

$$= \frac{2000}{2} + 640$$

$$= 1640 \text{ Units}$$

Inventory Classification and Control :-

(1) ABC Control (Based on Pareto law / 80-20 law)

	Usage %	item %
A	50-60%	10-20%
B	30-40%	30-40%
C	10-20%	50-60%

e.g.

Item	items % (100/No. of item)	Demand (D)	Unit price (C)	Usage (D.C) Value	Usage (%)	Usages % Decreasing order	
1	10%	400	70	28000	$\left(\frac{28000}{\Sigma x}\right)\times 100$	32%	53% Ⓐ
2	10%	600	80	4800	$\left(\frac{4800}{\Sigma x}\right)\times 100$	21%	
3	10%	500	200	1 lakh	$\left(\frac{1\ lakh}{\Sigma x}\right)\times 100$	16%	33% Ⓑ
⋮	⋮	⋮	⋮	⋮	⋮	9%	
						8%	
						5%	14% Ⓒ
						3%	
10	10%						
	100%			ΣX	100%		

In ABC Control inventory items are classified into A, B, and C category depending upon their usages value. For A category item inventory is kept almost nill and frequent review is done, on the other hand C category items large amount of inventory is kept and it is reviewed after a long period.

2) VED (Vital, Essential and Desirable)

Inventories are classified on the basis of importance of inventory items for the production system

V - very imp. (production can't run)

E - skip for short type

D - No. effect on production

3) HML (High Medium low) :- ⇒ c → (R/unit)

Inventories are classified on the basis of unit price of inventory

4) SDE (scarce Difficult easy) :-

Inventories are classified on the basis of availability of inventory item.

3