

Topics : Application of Derivatives, Solution of Triangle

Type of Questions		M.M., Min.
Comprehension (no negative marking) Q.1 to Q.3	(3 marks, 3 min.)	[9, 9]
Single choice Objective (no negative marking) Q. 4,5,6,7	(3 marks, 3 min.)	[12, 12]
Subjective Questions (no negative marking) Q.8	(4 marks, 5 min.)	[4, 5]

COMPREHENSION (Q. NO. 1 TO 3)

Let $f(x)$ be a function such that it is thrice differentiable in (a, b) . Consider a function

$\phi(x) = f(b) - f(x) - (b-x)f'(x) - \frac{(b-x)^2}{2}f''(x) - (b-x)^3\lambda$. and $\phi(x)$ follows all conditions of Rolle's theorem on $[a, b]$

- If there exist some number $c \in (a, b)$ such that $\phi'(c) = 0$ and $f(b) = f(a) + (b-a)f'(a) + \frac{(b-a)^2}{2}f''(a) + \mu(b-a)^3f'''(c)$, then μ is
 (A) $\frac{1}{2}$ (B) $\frac{1}{6}$ (C) $\frac{1}{8}$ (D) $-\frac{1}{2}$
- Let $f(x) = x^4 - 6x^3 + 12x^2 - 8x + 3$. If Rolle's theorem is applicable to $\phi(x)$ on $[2, 2+h]$ and there exist $c \in (2, 2+h)$ such that $\phi'(c) = 0$ and $\frac{f(2+h)-f(2)}{h^3} = g(c)$, then slope of tangent of curve $y = g(x)$ at $x = 5$ is
 (A) 4 (B) 5 (C) 6 (D) 10
- Let $f(x) = e^{2x}$ and $b = a + h$. If there exists a real number $\theta \in (0, 1)$ such that $\phi(a + \theta h) = 0$ and $\frac{e^{2h} - 1 - 2h - 2h^2}{h^3} = Ae^{B\theta h}$, then the value of $\frac{2B}{A}$ is equal to
 (A) 4 (B) 3 (C) 6 (D) 8
- The curve $y = x^3 + x^2 - x$ has two horizontal tangents. The distance between these two horizontal lines, is
 (A) $\frac{13}{9}$ (B) $\frac{11}{9}$ (C) $\frac{22}{27}$ (D) $\frac{32}{27}$

5. If $a, b > 0$, then minimum value of $y = \frac{b^2}{a-x} + \frac{a^2}{x}$ in $(0, a)$ is
- (A) $\frac{a+b}{a}$ (B) $\frac{ab}{a+b}$ (C) $\frac{1}{a} + \frac{1}{b}$ (D) none of these
6. Find maximum possible area that can be enclosed by a wire of length 20 cm by bending it in form of a circular sector.
- (A) 10 (B) 25 (C) 30 (D) 20
7. If the sides a, b, c of a triangle ABC are the roots of the equation $x^3 - 13x^2 + 54x - 72 = 0$, then the value of $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$ is equal to (with usual notation in $\triangle ABC$)
- (A) $\frac{169}{144}$ (B) $\frac{61}{72}$ (C) $\frac{61}{144}$ (D) $\frac{169}{72}$
8. If $x = e^t \sin t, y = e^t \cos t$, show that $\frac{d^2y}{dx^2} = \frac{-2(x^2 + y^2)}{(x+y)^3}$

Answers Key

1. (B) 2. (A) 3. (B) 4. (D)
5. (D) 6. (B) 7. (C)