# Previous Years Paper

## 10th August 2022 (Shift 2)

**Q1.** If A is square matrix of order 3 and A. (Adj(A)) = 10I,
Then the value of $\frac{1}{25}|adj(A)|$ is
(a) 100
(b) 25
(c) 10
(d) 4

**Q2.** Let A and B be two non-singular, square matrices of same order, and
(a) $(AB)^{-1} = B^{-1}.A^{-1}$
(b) $(A+B)^{-1} = B^{-1} + A^{-1}$
(c) adj. A $= |A|.A^{-1}$
(d) $\det(A^{-1}) = [detA]^{-1}$
Choose the correct answer from the options given below
(a) A and B only
(b) B and C only
(c) B and D only
(d) A, C and D only

**Q3.** If $\begin{vmatrix} -1 & a & a^2 \\ -1 & b & b^2 \\ -1 & c & c^2 \end{vmatrix}^2 = \lambda$ and a – b = 1, b – c = 2 and C – a = 3, then the value of $\lambda$ is
(a) 6
(b) 36
(c) 72
(d) 144

**Q4.** If $\begin{bmatrix} 2x-1 & -3 & 6 \\ 3 & 3y-2 & 4 \\ -6 & -4 & 4z-3 \end{bmatrix}$ is skew symmetric matrix, then $xyz$ is equal to
(a) $\frac{1}{2}$
(b) $\frac{2}{3}$
(c) $\frac{1}{4}$
(d) $\frac{3}{4}$

**Q5.** The solution of differential equation
$\sqrt{x+1} - \sqrt{x-1}\frac{dy}{dx} = 0$ is
(a) $y = \sqrt{x^2-1} + \log\left|x + \sqrt{x^2-1}\right| + C$
(b) $y = \sqrt{x^2-1} - \log\left|x + \sqrt{x^2-1}\right| + C$
(c) $y = \sqrt{x^2-1} + \log\left|x + \sqrt{1-x^2}\right| + C$
(d) $y = \sqrt{1-x^2} + \log\left|x + \sqrt{1-x^2}\right| + C$

**Q6.** The point(s) on the curve $\frac{x^2}{9} + \frac{y^2}{64} = 1$, at which the tangents are parallel to x-axis are
(a) $(0, \pm 3)$
(b) $(\pm 8, 0)$
(c) $(0, \pm 8)$
(d) $(\pm 3, 0)$

**Q7.** A die is tossed four times. The probability of getting an odd number at least once, is
(a) $\frac{1}{16}$
(b) $\frac{10}{16}$
(c) $\frac{4}{16}$
(d) $\frac{15}{16}$

**Q8.** A manufacturer of electronic circuit has a stock of 200 resistors, 120 transistors and 150 capacitors and is required to produce two types of circuits A and B. Type A requires 20 resistors, 10 transistors and 10 capacitors. Type B requires 10 resistors, 20 transistors and 30 capacitors. If the profit on type A circuit is ₹50 and that on type B circuit is ₹60, identify the constraints for this LPP, if it was assumed that x circuit B of type A and y circuits of type B was produced by the manufacturer.
(a) $x + 2y \geq 15$
(b) $2x + y \leq 20$
(c) $x + 2y \leq 12$
(d) $x, y \leq 0$
Choose the correct answer from the options given below
(a) A & B only
(b) B & C only
(c) C & D only
(d) A & D only

**Q9.** An energy DRONE is flying along the curve $y = x^2 + 7$. A soldier is placed at (3, 7). The nearest distance of the DRONE from soldier's position is
(a) 2
(b) 3
(c) $\sqrt{5}$
(d) $\sqrt{7}$

**Q10.** The line y = x, partition the area of the circle $(x-1)^2 + y^2 = 1$, into two segments. The area of the major segment is
(a) $\frac{\pi}{2} + \frac{1}{4}$
(b) $\frac{\pi}{4} + \frac{1}{2}$
(c) $\frac{3\pi}{4} + \frac{1}{6}$
(d) $\frac{3\pi}{4} + \frac{1}{2}$

**Q11.** The maximum value of $x^{-x}$ is
(a) $e^{e-1}$
(b) $e^{\frac{1}{e}}$
(c) $e^{\frac{-1}{e}}$
(d) $e^{\frac{-4}{e}}$

**Q12.** If $\int \left(x + \sqrt{x^2-1}\right)^2 dx = \alpha \cdot x + \beta x^3 + \gamma (x^2-1)^{\frac{3}{2}} +$ C,
where C is arbitrary constant, then the value of $3(\alpha + \beta + \gamma)$ is
(a) 44
(b) 23

(c) 11
(d) 1

**Q13.** The probability distribution of a random variable X is

| X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X = x)$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{2}$ |

The variance of x is
(a) $\frac{31}{64}$
(b) $\frac{15}{64}$
(c) $\frac{103}{64}$
(d) 1

**Q14.** $\int_0^1 \frac{dx}{x^2+x+1}$
(a) $\frac{\pi}{3\sqrt{3}}$
(b) $\frac{\pi}{2\sqrt{3}}$
(c) $\frac{\pi}{6\sqrt{3}}$
(d) $\frac{\pi}{\sqrt{3}}$

**Q15.** If the order and degree of the differential equation $\sqrt{\frac{d^2y}{dx^2}} = \left(1+\frac{dy}{dx}\right)^{\frac{1}{3}}$ are a and b respectively, then the value of $a^2+b^2$ is
(a) 2
(b) 3
(c) 5
(d) 13

**Q16.** The integrating factor of differential equation $x\frac{dy}{dx} + 2y = x^2 logx$ is
(a) $\frac{1}{x}$
(b) x
(c) $\frac{1}{x^2}$
(d) $x^2$

**Q17.** General solution of the differential equation $\frac{dy}{dx} + \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} = 0$ is
(a) $\tan^{-1} x + \tan^{-1} y = C$
(b) $\sin^{-1} x - \tan^{-1} y = C$
(c) $x\sqrt{1-y^2} - y\sqrt{1-x^2} = C$
(d) $\sin^{-1} x - \tan^{-1} y = C$
(e) $\cos^{-1} x + \cos^{-1} y = C$
(where C is arbitrary constant)
Choose the correct answer from the options given below
(a) B and C only
(b) B, D and E only
(c) A and D only
(d) A and B only

**Q18.** The absolute maximum value of y = $x^3$ - $3x + 2, 0 \leq x \leq 2$, is
(a) 4
(b) 6
(c) 2
(d) 0

**Q19.** Match List I with List II

| List-I | List-II |
|---|---|
| A. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx$ | I. $\frac{\pi}{2}$ |
| B. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx$ | II. $\frac{\pi}{4}$ |
| C. $\int_0^{\frac{\pi}{2}} \frac{1}{1+\tan x} dx$ | III. 0 |
| D. $\int_0^{\pi} \lvert\cos x\rvert dx$ | IV. 2 |

(a) A-I, B-III, C-II, D-IV
(b) A-III, B-I, C-II, D-IV
(c) A-II, B-III, C-IV, D-I
(d) A-IV, B-III, C-I, D-II

**Q20.** The portion of the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ that lies in the first quadrant is
(a) $\pi ab$
(b) $\frac{\pi ab}{2}$
(c) $\frac{\pi ab}{4}$
(d) $2\pi ab$

**Q21.** $\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx =$
(a) $\frac{e^x}{1+x^2}$
(b) $\frac{-e^x}{1+x^2} + C$
(c) $\frac{e^x}{(1+x^2)^2} + C$
(d) $\frac{-e^x}{(1+x^2)^2} + C$

**Q22.** $\int \frac{\sin(\tan^{-1} x)}{1+x^2} dx =$
(a) $-\cos(\tan^{-1} x) + C$
(b) $\cos(\tan^{-1} x) + C$
(c) $\tan(\cos^{-1} x) + C$
(d) $-\tan(\cos^{-1} x) + C$

**Q23.** The distance of the plane $\vec{r}.\left(\frac{2}{7}\hat{\imath} + \frac{3}{7}\hat{\jmath} - \frac{6}{7}\hat{k}\right) = 2$ from the origin is
(a) 1 unit
(b) 2 units
(c) 7 units
(d) 8 units

**Q24.** If $|\vec{a}| = 8, |\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, then the value $\vec{a} \cdot \vec{b}$ is
(a) $2\sqrt{3}$
(b) $12\sqrt{3}$
(c) $8\sqrt{3}$
(d) $6\sqrt{3}$

**Q25.** The direction ratios of the line $\frac{1-x}{3} = \frac{7y-14}{2} = \frac{z-3}{2}$ are
(a) $-3, \frac{1}{7}, 2$
(b) $-3, 2, 2$
(c) $-3, \frac{2}{7}, 1$
(d) $-21, 2, 14$

**Q26.** The vertices of a closed convex polygon representing the feasible region of the LPP with objective function $z = 5x + 3y$ are (0, 0,) (3, 1), (1, 3) and (0, 2). The maximum value of $z$ is
(a) 6
(b) 18
(c) 14
(d) 15

**Q27.** $\int \frac{x^2+1}{x^4+1} dx =$
(a) $\log(x^4+1)+C$
(b) $\log\left(\frac{x^2-1}{x^2+1}\right)+C$
(c) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+c$
(d) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}x}\right)+c$

**Q28.** $\int_0^{4\pi} \frac{x}{1+|\cos x|} dx =$
(a) $4\pi$
(b) $16\pi$
(c) $32\pi$
(d) $64\ \pi$

**Q29.** A man is known to speak truth 4 out of 5 times. He throws a die and reports that five appears. Then the probability that actual five appears on the dice is
(a) $\frac{3}{8}$
(b) $\frac{4}{9}$
(c) $\frac{5}{9}$
(d) $\frac{5}{8}$

**Q30.** Five numbers taken out from numbers 1-30 and arrange them in ascending order. The probability that the third number will be 20 is
(a) $\frac{{}^{20}C_2 \times {}^{10}C_2}{{}^{30}C_5}$
(b) $\frac{{}^{19}C_2 \times {}^{10}C_2}{{}^{30}C_5}$
(c) $\frac{{}^{19}C_2 \times {}^{11}C_3}{{}^{30}C_5}$
(d) $\frac{{}^{19}C_2 \times {}^{11}C_2}{{}^{30}C_5}$

**Q31.** Bag I contain 4 red and 5 black balls, while another Bag II contain 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be black. Then the probability that it was drawn from Bag II, is
(a) $\frac{54}{109}$
(b) $\frac{51}{109}$
(c) $\frac{64}{109}$
(d) $\frac{54}{119}$

**Q32.** A card is picked at random from a pack of 52 playing cards. If the picked card is a queen, then probability of card to be of spade type also, is
(a) $\frac{1}{3}$
(b) $\frac{4}{13}$
(c) $\frac{1}{4}$
(d) $\frac{1}{2}$

**Q33.** The differential equal representing family of curves $y = ae^{mx} + be^{nx}$, where $a$ and $b$ are arbitrary constants, is
(a) $\frac{d^2y}{dx^2}+(m+n)\frac{dy}{dx}+y=0$
(b) $\frac{d^2y}{dx^2}+mn\frac{dy}{dx}+(m+n)y=0$
(c) $\frac{d^2y}{dx^2}-(m+n)\frac{dy}{dx}+mny=0$
(d) $\frac{d^2y}{dx^2}+(m+n)\frac{dy}{dx}+mny=0$

**Q34.** The interval in which the function, $f(x) = 7 - 4x - x^2$ is strictly increasing is
(a) $(-\infty, \infty)$
(b) $(-2, \infty)$
(c) $(-\infty, -2)$
(d) $(-\infty, -2]$

**Q35.** The area (in square units) of minor segment of the circle $x^2 + y^2 = 25$ cut off by the line $x = \frac{5}{2}$ is
(a) $25\left(\frac{\pi}{4}-\frac{\sqrt{3}}{2}\right)$
(b) $\frac{25}{12}(4\pi - 3\sqrt{3})$
(c) $\frac{25}{12}(3\pi - 4\sqrt{3})$
(d) $25\left(\frac{\sqrt{3}}{2}+\frac{\pi}{4}\right)$

**Q36.** If $\int_0^{\frac{\pi}{2}} \sqrt{\tan x}\ dx = \frac{\lambda}{2}$, then the value of $\lambda$ is
(a) $\pi$
(b) $-1$
(c) $\frac{\pi}{2}$
(d) $\frac{3}{\sqrt{2}}$

**Q37.** Which of the following is/are true?
A. A relation $R$ on a set $A$ is called an equivalence relation, if it is reflexive, symmetric and transitive.
B. The function $f: R \to R$ defined by $f(x) = e^x$ is not one-one.
C. The one-one function is also known as injective function.
D. The onto function is also known as surjective function.
E. A function $f: X \to Y$ is said to be many-one, if two or more than two elements in set X have the different image in set Y.

Choose the correct answer from the option given below:
(a) A, C, D only
(b) B, C, D only
(c) C, D, E only
(d) B, D, E only

**Q38.** If $|\vec{a}| = 3|\vec{b}|, |\vec{b}| = 2$ and angle between $\vec{a}$ and $\vec{b}$ is $60°$, then $|\vec{a} - \vec{b}|$ is equal to:
(a) 14
(b) $2\sqrt{7}$
(c) 28
(d) 25

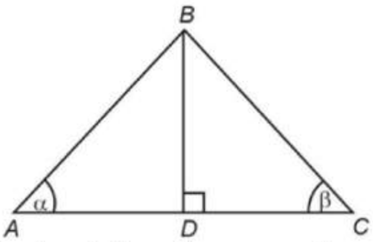
**Q39.** If $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and square matrix satisfy AB = $8I$, then the value of |B| is:
(a) 512
(b) 64
(c) 32
(d) 8

**Q40.** The angle between the vectors $\hat{\imath} - \hat{\jmath}$ and $\hat{\jmath} - \hat{k}$ is:
(a) $\frac{5\pi}{6}$

(b) $\frac{-\pi}{3}$
(c) $\frac{2\pi}{3}$
(d) $\frac{\pi}{3}$

**Directions 41-45**

Passage: Two men on either side of a pole of 30 m high observe its top at angles of elevation α and β respectively. The distance between the two men is $40\sqrt{3}$ m and the distance between the first man at A and the pole is $30\sqrt{3}$ m.




Based on the above information, answer the question:

**Q41.** The value of α is
(a) $\frac{\pi}{3}$
(b) $\frac{\pi}{4}$
(c) $\frac{\pi}{6}$
(d) $\frac{\pi}{8}$

**Q42.** $\tan(\alpha + \beta) =$
(a) $\sqrt{3} + \frac{1}{\sqrt{3}}$
(b) $\sqrt{3} - \frac{1}{\sqrt{3}}$
(c) $2\sqrt{3} - 1$
(d) Infinite (does not exist)

**Q43.** Area of $\Delta ABD$ is
(a) $150\sqrt{3}\, m^2$
(b) $900\sqrt{3}\, m^2$
(c) $450\sqrt{3}\, m^2$
(d) $100\sqrt{3}\, m^2$

**Q44.** $\angle ABC$ is equal to:
(a) $\frac{\pi}{4}$
(b) $\frac{\pi}{6}$
(c) $\frac{\pi}{2}$
(d) $\frac{\pi}{3}$

**Q45.** The value of $\frac{1}{AB^2} + \frac{1}{BC^2}$ is
(a) $\frac{1}{BD^2}$
(b) $\frac{1}{AD^2}$
(c) $\frac{1}{CD^2}$
(d) $\frac{1}{AC^2}$

**Directions 46-50**

Passage:
Manjeet wants to donate a rectangular plot of land for a school in his village. When he has asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if its length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 $m^2$. If length and breadth of the plot are x and y respectively, then based on above information answer the question.

**Q46.** The length (x) and breadth (y) of plot satisfy equations:
(a) x – y = 50, 2x – y = 550
(b) x – y = 50, 2x + y = 550
(c) x + y = 50, 2x + y = 550
(d) x + y = 50, 2x + y = 550

**Q47.** The linear equation involving x and y are written in matrix form as:
(a) $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$
(b) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$
(d) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -50 \\ -550 \end{bmatrix}$

**Q48.** The length of the plot is:
(a) 150 m
(b) 400 m
(c) 200 m
(d) 320 m

**Q49.** The breadth of plot is:
(a) 150 m
(b) 200 m
(c) 430 m
(d) 350 m

**Q50.** Area of the rectangular plot is:
(a) 60,000 sq. m
(b) 3,000 sq. m
(c) 25,000 sq. m
(d) 30,000 sq. m

# SOLUTIONS

**S1.** **Ans. (d)**

**Sol.** $A(\text{Adj}(A)) = 10I$

$|A|I = 10I$

$\Rightarrow \ |A| = 10$

$\frac{1}{25}|Adj(A)| = \frac{|A|^2}{25} = \frac{100}{25}$

$=4$

**S2.** **Ans. (d)**

**Sol.** (a) $(AB)^{-1} = B^{-1}A^{-1}$

(b) $(A+B)^{-1} \neq B^{-1} + A^{-1}$

(c) $A\ adjA = |A|I$

$adj(A) = |A|\frac{I}{A}$

$= |A|A^{-1}$

(d) $\det(A^{-1}) = (detA)^{-1}$

**S3.** **Ans. (b)**

**Sol.** $\begin{vmatrix} -1 & a & a^2 \\ -1 & b & b^2 \\ -1 & c & c^2 \end{vmatrix}^2 = (-1)^2 \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}^2 = \lambda$

$= (a-b)^2(b-c)^2(c-a)^2 = \lambda$

$= (1)^2(2)^2(3)^2 = \lambda$

$= 36$

**S4.** **Ans. (c)**

**Sol.** $\begin{bmatrix} 2x-1 & -3 & 6 \\ 3 & 3y-2 & 4 \\ -6 & -4 & 4z-3 \end{bmatrix} = A$

If A is skew symmetric

$\Rightarrow \ aii = 0$

$2x - 1 = 0 \Rightarrow x = \frac{1}{2}$

$3y - 2 = 0 \Rightarrow y = \frac{2}{3}$

$4z - 3 = 0 \Rightarrow z = \frac{3}{4}$

Now, $xyz = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$

**S5.** **Ans. (a)**

**Sol.** $\sqrt{x+1} - \sqrt{x-1}\frac{dy}{dx} = 0$

$\int \frac{\sqrt{x+1}}{\sqrt{x-1}} dx = \int dy$

$\int \frac{x+1}{\sqrt{x^2-1}} dx = y$

$\int \frac{x}{\sqrt{x^2-1}} dx + \int \frac{dx}{\sqrt{x^2-1}} = y$

$y = \sqrt{x^2-1} + \log\left|x + \sqrt{x^2-1}\right| + C$

**S6.** **Ans. (c)**

**Sol.** $\frac{x^2}{9} + \frac{y^2}{64} = 1$

$\frac{2x}{9} + \frac{2yy'}{64} = 0$

$y' = -\frac{64}{9}\frac{x}{y} = 0$

$\Rightarrow \ x = 0$

Now, $\frac{0}{9} + \frac{y^2}{64} = 1$

$y = \pm 8$

**S7.** **Ans. (d)**

**Sol.** P(getting odd at least one) = 1 – P(even)

$= 1 - \frac{3^4}{6^4}$

$= 1 - \frac{1}{2^4}$

$= \frac{15}{16}$

**S8.** **Ans. (b)**

**Sol.** x circuits of type-A

y circuits of type-B

z = 50x + 60y

Now A need 20 R, 10 T, 10 C

B need 10 R, 20 T, 30 C

Stock 200 R, 120 T, 150 C

$20x + 10y \leq 200 \Rightarrow 2x + y \leq 20$

$10x + 20y \leq 120$

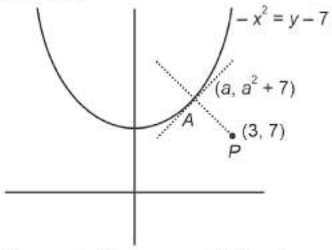
$\Rightarrow x + 2y \leq 12$

$10x + 30y \leq 150 \Rightarrow x + 3y \leq 15$

& $x \geq 0, y \geq 0$

**S9.** **Ans. (c)**

**Sol.**




Nearest distance will lie along comes nearest

Slope at A

2x = y'

2a = y'

Slope of normal $= \frac{-1}{2a}$

$m_{pA} = \frac{a^2+7-7}{a-3}$

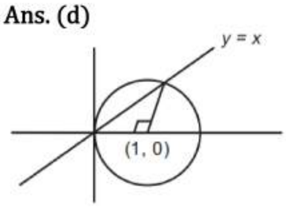
$\therefore \ \frac{a^2}{a-3} = \frac{-1}{2a}$

$2a^3 + a - 3 = 0$

$\Rightarrow a = 1$

**S10.** **Ans. (d)**

**Sol.**




Area $= \pi(1)^2 - \left[\frac{90}{360}\pi(1)^2 - \frac{1}{2} \times 1 \times 1\right]$

$= \frac{3\pi}{4} + \frac{1}{2}$

**S11.** **Ans. (b)**

**Sol.** $f(x) = x^{-x}$

$\log f(x) = -xlogx$

$\frac{f'(x)}{f(x)} = -[1 + logx]$

$f'(x) = -x^{-x}[1 + \log x] = 0$

$x = \frac{1}{e}$

$\therefore\ f(x) = \left(\frac{1}{e}\right)^{-\frac{1}{e}}$

$f(x)_{max} = e^{\frac{1}{e}}$

**S12. Ans. (d)**

**Sol.** $\int \left(x + \sqrt{x^2-1}\right)^2 dx = \alpha \cdot x + \beta x^3 + \gamma (x^2-1)^{\frac{3}{2}} + C$

$= \int \left(x^2 + x^2 - 1 + 2x\sqrt{x^2-1}\right) dx$

$= \int 2x^2 dx - \int dx + 2\int x\sqrt{x^2-1}\, dx$

$= \frac{2x^3}{3} - x + 2I$

Now,

$I = \int x\sqrt{x^2-1} dx$

Let $x^2 - 1 = t^2$

2x dx = 2t dt

$= \int t \cdot t\, dt = \int t^2 dt = \frac{t^3}{3} + C'$

$= \frac{\left(\sqrt{(x^2-1)}\right)^3}{3}$

Here $\alpha = 0, \beta = 0, \gamma = \frac{1}{3}$

$\Rightarrow 3(\alpha + \beta + \gamma) = 1$

**S13. Ans. (c)**

**Sol.** $\sum P_i x_i^2 - (\text{ Mean })^2$

$\left(0 \times \left(\frac{1}{4}\right) + 1^2 \times \left(\frac{1}{8}\right) + 2^2 \times \left(\frac{1}{8}\right) + 3^2 \times \left(\frac{1}{2}\right)\right)$

$- \left(0 \times \frac{1}{4} + 1 \times \frac{1}{8} + 2 \times \frac{1}{8} + 3 \times \frac{1}{2}\right)^2$

$= \frac{41}{8} - \left(\frac{15}{8}\right)^2$

$= \frac{41}{8} - \frac{225}{64} = \frac{103}{64}$

**S14. Ans. (a)**

**Sol.** $\int_0^1 \frac{dx}{x^2+x+1} =$

$= \int_0^1 \frac{dx}{\left(x+\frac{1}{2}\right)^2 + 1 - \frac{1}{4}}$

$= \int_0^1 \frac{dx}{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}}$

$= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)\Bigg]_0^1$

$= \frac{2}{\sqrt{3}}\left[\tan^{-1}\sqrt{3} - \tan^{-1}\frac{1}{\sqrt{3}}\right]$

$= \frac{2}{\sqrt{3}}\left[\frac{\pi}{3} - \frac{\pi}{6}\right] = \frac{2}{\sqrt{3}} \times \frac{\pi}{6} = \frac{\pi}{3\sqrt{3}}$

**S15. Ans. (d)**

**Sol.** $\sqrt{\frac{d^2y}{dx^2}} = \left(1 + \frac{dy}{dx}\right)^{\frac{1}{3}}$

Taking power 6 on both sides

$\left(\frac{d^2y}{dx^2}\right)^3 = \left(1 + \frac{dy}{dx}\right)^2$

$\therefore\ Order = 2 = a$

Degree = 3 = b

$\therefore\ a^2 + b^2 = 4 + 9$

$= 13$

**S16. Ans. (d)**

**Sol.** $x\frac{dy}{dx} + 2y = x^2 \log x$

$\frac{dy}{dx} + \left(\frac{2}{x}\right) y = x\log x$

$\text{IF} = e^{\int \frac{2}{x} dx} = e^{2\ln x} = x^2$

**S17. Ans. (b)**

**Sol.** $\frac{dy}{\sqrt{1-y^2}} = -\frac{dx}{\sqrt{1-x^2}}$

Integrating both sides

$\sin^{-1} y = -\sin^{-1} x + C$

$\sin^{-1} x + \sin^{-1} y = C$

OR $\sin^{-1} x - \cos^{-1} y = C$

OR $\cos^{-1} x + \cos^{-1} y = C$

**S18. Ans. (a)**

**Sol.** $\frac{dy}{dx} = 3x^2 - 3 = 0 \Rightarrow x = 1$

$y = f(x) = x^3 - 3x + 2$

$f(0) = 2$

$f(2) = 4$

$f(1) = 0$

**S19. Ans. (b)**

**Sol.** $I_1 = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (\sin x)^7\, dx = 0$ as $(\sin x)^7$ is odd function

$I_2 = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\, dx = 2\int_{2}^{\frac{\pi}{2}} \sin^2 x\, dx = x - \frac{\sin 2x}{2}\Big|_0^{\frac{\pi}{2}} = \frac{\pi}{2}$

$I_3 = \int_0^{\frac{\pi}{2}} \frac{1}{1+\tan x} dx = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx$ ... (1)

$I_3 = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x} dx$ ...(2)

Adding (1) and (2)

$2I_3 = \int_0^{\frac{\pi}{2}} 1 dx \Rightarrow I_3 = \frac{\pi}{4}$

$I_4 = \int_0^{\pi} |\cos x|\, dx = 2\int_0^{\frac{\pi}{2}} \cos x\, dx = 2\sin x\Big|_0^{\frac{\pi}{2}} = 2$

**S20. Ans. (c)**

**Sol.** Area of ellipse $= \pi ab$

In 1st quadrant only area $= \frac{\pi ab}{4}$

**S21. Ans. (a)**

**Sol.** $\int e^x \left(\frac{1+x^2-2x}{(1+x^2)^2}\right) dx = \int e^x \left(\frac{1}{1+x^2} - \frac{2x}{(1+x^2)^2}\right) dx$

If $f(x) = \frac{1}{1+x^2}$ then $f'(x) = \frac{-2x}{(1+x^2)^2} dx$

$\int e^x(f(x) + f'(x))dx = e^x f(x) + c$

$= \frac{e^x}{1+x^2} + C$

**S22. Ans. (a)**

**Sol.** Let $\tan^{-1} x = t$

$\Rightarrow \frac{1}{1+x^2} dx = dt$

$I = \int \sin t\, dt$

$= -\cos(\tan^{-1} x) + C$

**S23. Ans. (b)**

**Sol.** Equation of plane can be given as

2x + 3y – 6z = 14

Distance from (0, 0, 0)

$$= \left|\frac{14}{\sqrt{2^2+(3)^2+(-6)^2}}\right| = 2 \text{ units}$$

**S24. Ans. (b)**

**Sol.** $|\vec{a}|^2|\vec{b}|^2 = |\vec{a}\times\vec{b}|^2 + (\vec{a}\cdot\vec{b})^2$

$576 = 144 + (\vec{a}\cdot\vec{b})^2$

$\Rightarrow \vec{a}\cdot\vec{b} = \sqrt{432} = 12\sqrt{3}$

**S25. Ans. (d)**

**Sol.** $\frac{x-1}{-3} = \frac{y-2}{2/7} = \frac{z-3}{2}$

So, the direction ratios are $\left(-3, \frac{2}{7}, 2\right)$

OR $(-21, 2, 14)$

**S26.** And. (b)

**Sol.** $z(0,0) = 0$

$z(3,1) = 5.3 + 3.1 = 18$

$z(1,3) = 5.1 + 3.3 = 14$

$z(0,2) = 5.0 + 3.2 = 6$

So $z_{max}$ is 18 at (3, 1)

**S27. Ans. (c)**

**Sol.** $I = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx = \int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+2}$

Let $x - \frac{1}{x} = t$ then $\left(1+\frac{1}{x^2}\right)dx = dt$

$I = \int \frac{dt}{t^2+2} = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right) + C$

$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right) + C = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + C$

**S28. Ans. (b)**

**Sol.** $I = \int_0^{4\pi} \frac{x}{1+|\cos x|}dx$ ... (i)

Also $I = \int_0^{4\pi} \frac{4\pi - x}{1+|\cos(4\pi - x)|}dx$ .... (ii)

(i) + (ii), $2I = \int_0^{4\pi} \frac{4\pi}{1+|\cos x|}dx$

$\Rightarrow I = \left(\int_0^{4\pi} \frac{1}{1+|\cos x|}dx\right)2\pi$

$= 8\pi \int_0^{\pi} \frac{1}{1+|\cos x|}dx$

$= 8\pi\left(\int_0^{\frac{\pi}{2}} \frac{dx}{1+\cos x} + \int_{\frac{\pi}{2}}^{\pi} \frac{dx}{1-\cos x}\right)$

$= 8\pi\left(\tan\left(\frac{x}{2}\right)\Big|_0^{\frac{\pi}{2}} + \left(-\cot\frac{x}{2}\right)\Big|_{\frac{\pi}{2}}^{\pi}\right)$

$= 8\pi \times 2 = 16\pi$

**S29. Ans. (b)**

**Sol.** Given $P(T) = \frac{4}{5}$

So, required probability

$= \frac{P(T).\frac{1}{6}}{P(T).\frac{1}{6}+P(F).\frac{5}{6}}$

$= \frac{\frac{4}{5}\frac{1}{6}}{\frac{4}{5}\frac{1}{6}+\frac{1}{5}\frac{5}{6}} = \frac{4}{9}$

**S30. Ans. (b)**

**Sol.** $S = \{1, 2, 3 \ldots\ldots, 30\}$

Required probability $= \frac{{}^{19}C_2 \times {}^{10}C_2}{{}^{30}C_5 \times 1}$

**S31. Ans. (a)**

**Sol.** $P\left(\frac{R}{B_1}\right) = \frac{4}{9}, P\left(\frac{B}{B_1}\right) = \frac{5}{9}$

$P\left(\frac{R}{B_2}\right) = \frac{5}{11}, P\left(\frac{B}{B_2}\right) = \frac{6}{11}$

$P\left(\frac{B_2}{B}\right) = \frac{P(B_2).P\left(\frac{B}{B_2}\right)}{P(B_1).P\left(\frac{B}{B_1}\right) + P(B_2).P\left(\frac{B}{B_2}\right)}$

$= \frac{\frac{1}{2}\cdot\frac{6}{11}}{\frac{1}{2}\cdot\frac{5}{9}+\frac{1}{2}\cdot\frac{6}{11}}$

$= \frac{\frac{6}{11}}{\frac{109}{99}} = \frac{54}{109}$

**S32. Ans. (c)**

**Sol.** Let $Q$: card is Queen

$S$: card is Spade

$then\ P\left(\frac{\overline{S}}{Q}\right) = \frac{P(S\cap Q)}{P(Q)} = \frac{\frac{1}{52}}{\frac{4}{52}} = \frac{1}{4}$

**S33. Ans. (c)**

**Sol.** $y = ae^{mx} + be^{nx}$

$\Rightarrow \frac{dy}{dx} = ame^{mx} + nbe^{nx}$

$\Rightarrow \frac{d^2y}{dx^2} = am^2e^{mx} + n^2be^{nx}$

$\therefore \frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = am^2e^{mx} + bn^2e^{nx} - (m+n)[ame^{mx} + nbe^{nx}] + mn[ae^{mx} + be^{nx}]$

$= 0$

**S34. Ans. (c)**

**Sol.** $f(x) = 7 - 4x - x^2$

then $f'(x) = -4 - 2x$

for strictly increasing

$f'(x) > 0$

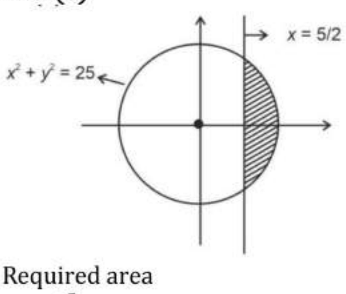
$\Rightarrow -(4+2x) > 0$

$\Rightarrow x < -2$

$\Rightarrow x \in (-\infty, -2)$

**S35. Ans. (b)**

**Sol.**




Required area

$= 2\int_{\frac{5}{2}}^{5} \sqrt{25-x^2}dx$

$= 2\left[\frac{x}{2}\sqrt{25-x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right]_{\frac{5}{2}}^{5}$

$= \left\{5\cdot 0 + 25\cdot\frac{\pi}{2}\right\} - \left\{\frac{5}{2}\sqrt{\frac{75}{4}} + 25\cdot\frac{\pi}{6}\right\}$

$= \frac{25\pi}{2} - \frac{25}{4}\sqrt{3} - \frac{25\pi}{6} = \frac{25\pi}{3} - \frac{25}{4}\sqrt{3}$

**S36. Ans. (a)**

**Sol.** Let $I = \int_0^{\frac{\pi}{2}} \sqrt{\tan x}\, dx$

$$I = \int_0^{\frac{\pi}{2}} \sqrt{\cot x}\, dx$$

$$2I = \int_0^{\frac{\pi}{2}} (\sqrt{\tan x} + \sqrt{\cot x}) dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sqrt{\sin\ x\ \cos x}} dx$$

Let $\sin x - \cot x = t$

$\Rightarrow\ (\cos x + \sin x)dx = dt$

$$\therefore\ 2I = \int_{-1}^{1} \frac{\sqrt{2}dt}{\sqrt{1-t^2}}$$

$$\Rightarrow\ I = \frac{1}{\sqrt{2}}.2\int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$\Rightarrow\ = \sqrt{2}\,(\sin^{-1} t)_0^1 = \frac{\pi}{\sqrt{2}}$$

**S37. Ans. (a)**

**Sol.** (A) Definition

(B) $f(x) = e^x$ is a one-one function

($\because f'(x) = e^x > 0$ for all $x \in R$ )

(C) and (D) are also correct

**S38. Ans. (b)**

**Sol.** Given

$|\vec{a}| = 3|\vec{b}|, |\vec{b}| = 2$

and let $\theta$ be the angle between $\vec{a}$ and $\vec{b}$, then $\theta = 60°$

Now, $|\vec{a} - \vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}\,|^2 - 2\vec{a}.\vec{b}}$

$= \sqrt{36 + 4 - 2\cdot 6\cdot 2\cdot \frac{1}{2}}$

$= \sqrt{28} = 2\sqrt{7}$

**S39. Ans. (b)**

**Sol.** $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

$|A| = 1(-4) + 1(7) + 1(5)$

$= 8$

and AB = 8I

$\Rightarrow\ |AB| = |8I|$

$\Rightarrow\ |B| = \frac{8^3}{8} = 64$

**S40. Ans. (c)**

**Sol.** Let $\vec{a} = \hat{\imath} - \hat{\jmath}, \vec{b} = \hat{\jmath} - \hat{k}$

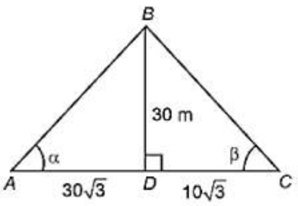
and let $\theta$ be the angle between $\vec{a}$ and $\vec{b}$, then

$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{-1}{\sqrt{2}\sqrt{2}} = \frac{-1}{2}$

$\therefore \theta = \frac{2\pi}{3}$

**S41. Ans. (c)**

**Sol.**

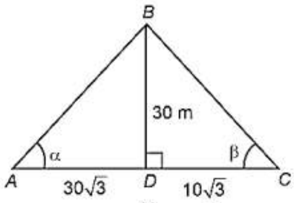



So, $\tan\alpha = \frac{30}{30\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\therefore\ \alpha = 30°$ or $\frac{\pi}{6}$

**S42. Ans. (d)**

**Sol.**




So, $\tan\alpha = \frac{30}{30\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\therefore\ \alpha = 30°$ or $\frac{\pi}{6}$

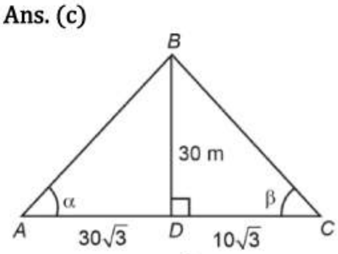
$\tan\ \beta = \frac{30}{10\sqrt{3}} = \sqrt{3}$

$\therefore\ \beta = \frac{\pi}{3}$

So, $\tan(\alpha + \beta\,)\ \tan\left(\frac{\pi}{2}\right) \to \infty$ (does not exist)

**S43. Ans. (c)**

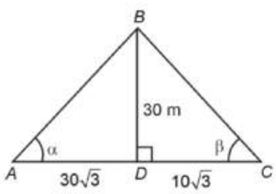
**Sol.**




Area of $\Delta ABD = \frac{1}{2} \times 30\sqrt{3} \times 30 = 450\sqrt{3}m^2$

**S44. Ans. (C)**

**Sol.**




So, $\tan\alpha = \frac{30}{30\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\therefore\ \alpha = 30°\ or\ \frac{\pi}{6}$

$\tan\beta = \frac{30}{10\sqrt{3}} = \sqrt{3}$

$\therefore \beta = \frac{\pi}{3}$

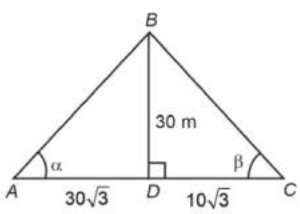
So, $\tan(\alpha + \beta) = \tan\left(\frac{\pi}{2}\right) \to \infty$ (does not exist)

Area of $\Delta ABD = \frac{1}{2} \times 30\sqrt{3} \times 30 = 450\sqrt{3}m^2$

$\angle ABC = \frac{\pi}{2}$

**S45. Ans. (A)**

**Sol.**




So, $\tan\alpha = \frac{30}{30\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\therefore\ \alpha = 30° \ or\ \dfrac{\pi}{6}$

$\tan\beta = \dfrac{30}{10\sqrt{3}} = \sqrt{3}$

$\therefore \beta = \dfrac{\pi}{3}$

So, $\tan(\alpha + \beta) = \tan\left(\frac{\pi}{2}\right) \rightarrow \infty$ (does not exist)

Area of $\Delta ABD = \frac{1}{2} \times 30\sqrt{3} \times 30 = 450\sqrt{3} m^2$

$\angle ABC = \dfrac{\pi}{2} - \alpha$

$= \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

$\frac{1}{AB^2} + \frac{1}{BC^2} = \frac{1}{30^2(4)} + \frac{1}{1200} = \frac{1}{3600} + \frac{1}{1200} = \frac{4}{3600} = \frac{1}{900}$

$= \dfrac{1}{BD^2}$

**S46. Ans. (b)**

**Sol.** (x – 50) (y + 50) = xy

$\Rightarrow x - y = 50$ ... $(i)$

(x – 10) (y – 20) = xy – 5300

⇒ 2x + y = 550 ...(ii)

From these equations

y = 200 m

y = 150 m

**S47.** Ans. (a)

**Sol.** $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ -550 \end{bmatrix}$

**S48.** Ans. (c)

**Sol.** $l$ = length of plot = 200 m

**S49.** Ans. (a)

**Sol.** Breadth of the plot $= b = 150\,m$

**S50. Ans. (d)**

**Sol.** Area of plot = 200 × 150 = 30,000 $m^2$